The equations of the Unified Physics

Stergios Pellis
sterpellis@gmail.com
ORCID iD: 0000-0002-7363-8254
Greece
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Abstract

In this paper will be presented the equations of Unified Physics. The beautiful equations of the unification of the fundamental interactions. We calculate the unity formulas that connect the coupling constants of the fundamental forces. These equations are applicable for all energy scales. Also we present the unification of atomic physics and cosmology and the formulas for the cosmological constant. It will discover a new simple Large Number Hypothesis which calculates the Mass,the Age and the Radius of the universe. The diameter of the observable universe will be calculated to be equal to the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron. We will prove the shape of the Universe is Poincaré dodecahedral space. We propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy.

Keywords

Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant , Unification of the microcosm and the macrocosm , Poincaré dodecahedral space

1. Introduction

In [1] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. We propose in [2] , [3] and [4] the exact formula for the fine-structure constant α with the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}=137.035999164... \qquad (1)$$

Also we propose in [4] , [5] and [6] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2=137.035999078... \qquad (2)$$

We propose in [7] the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \Rightarrow \mu=1836.15267343... \qquad (3)$$

$$7\cdot\mu^3=165^3\cdot\ln^{11}10 \Rightarrow \mu=1836.15267392... \qquad (4)$$

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}=1836.15267343... \qquad (5)$$

Also was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42) \tag{6}$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12 \tag{7}$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231 \tag{8}$$

$$\mu-807\cdot\alpha=1205\cdot\pi-518\cdot\varphi-411\cdot e \tag{9}$$

In [8] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot10^2\cdot\cos(\mu\cdot\alpha^{-1})+13^2=0 \tag{10}$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \tag{11}$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \tag{12}$$

It was presented in [9] the mathematical formulas that connects the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1 \tag{13}$$

$$\mu^2=4\cdot e^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{14}$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A^2 \tag{15}$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=1 \tag{16}$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{17}$$

$$\mu^2=4\cdot e^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \tag{18}$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{19}$$

In [10] we presented the recommended value for the strong coupling constant:

$$\alpha_s=\frac{Euler'\,number}{Gerford's\ constant}=\frac{e}{e^{\pi}}=e^{1-\pi}=0,11748.. \tag{20}$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. Also for the value of the strong coupling constant we have the equivalent expressions $\alpha_s=e\cdot e^{-\pi}=e\cdot i^{2i}=i^{-2i/\pi}\cdot i^{2i}=i^{2i-(2i/\pi)}=i^{2i}$. From Euler's identity resulting the beautiful formulas:

$$e^i=i^2\cdot\alpha_s^i \tag{21}$$

$$e^i+\alpha_s^i=0 \tag{22}$$

2. Unification of the fundamental interactions

In the papers [11] , [12] , [13] and [14] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant α_s and the weak coupling constant α_w. We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s=10^7\cdot\alpha_w \tag{23}$$

$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w \quad (24)$$

$$e^\pi \cdot \alpha_s^2 = 10^7 \cdot \alpha_w \quad (25)$$

$$\alpha_s^{2i} = i^2 \cdot 10^{7i} \cdot \alpha_w^i \quad (26)$$

Resulting the unity formulas that connects the strong coupling constant α_s and the fine-structure constant α:

$$\alpha_s \cdot \cos\alpha^{-1} = i^{2i} \quad (27)$$

$$\cos\alpha^{-1} = \frac{\alpha_s^{-1}}{e^\pi} \quad (28)$$

The figure 1 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^π.

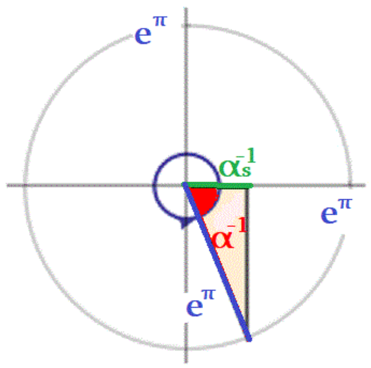

Figure 1. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius e^π.*

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$\alpha_s \cdot \cos\alpha^{-1} = i^{2i} \quad (29)$$

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i} \quad (30)$$

$$e^\pi \cdot \alpha_s \cdot \cos\alpha^{-1} = 1 \quad (31)$$

$$e^\pi \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \quad (32)$$

$$\alpha_s^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^i = 2^i \cdot i^2 \quad (33)$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

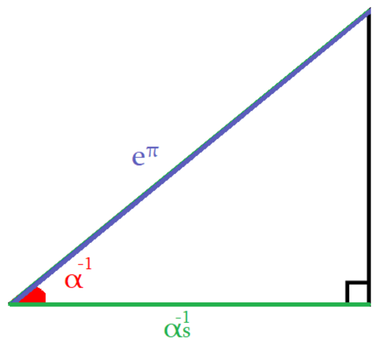

Figure 2. *Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.*

The electroweak theory,in physics,is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^{7}\cdot \alpha_{w}\cdot \cos\alpha^{-1}=e\cdot i^{2i} \tag{34}$$

$$10^{7}\cdot \alpha_{w}\cdot (e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i} \tag{35}$$

$$10^{7}\cdot e^{\pi}\cdot \alpha_{w}\cdot \cos\alpha^{-1}=e \tag{36}$$

$$10^{7}\cdot e^{\pi}\cdot \alpha_{w}\cdot (e^{i/\alpha}+e^{-i/\alpha})=2\cdot e \tag{37}$$

The figure 3 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^{7}\cdot e^{\pi-1}$.

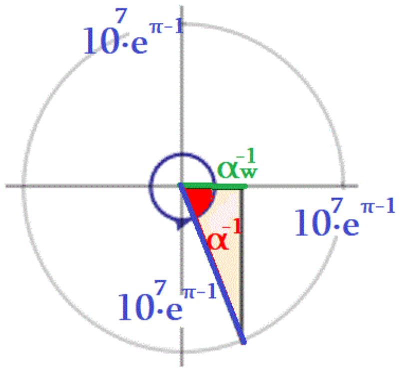

Figure 3. *The angle in α^{-1} radians.*

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.

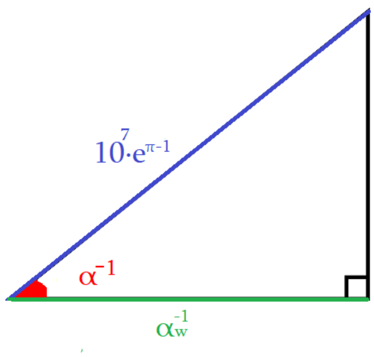

Figure 4. *Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions*

Resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$10^{7}\cdot \alpha_{w}\cdot \cos\alpha^{-1}=\alpha_{s} \tag{38}$$

$$\cos\alpha^{-1}=\frac{\alpha_s\alpha_w^{-1}}{10^7} \tag{39}$$

The figure 5 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^{7}.

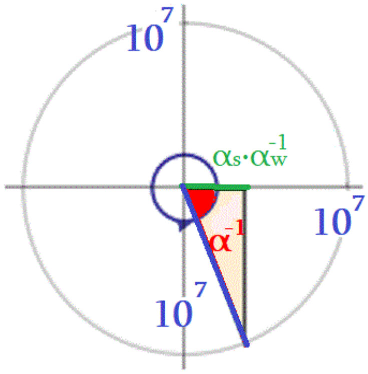

Figure 5. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7.*

The figure 6 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.

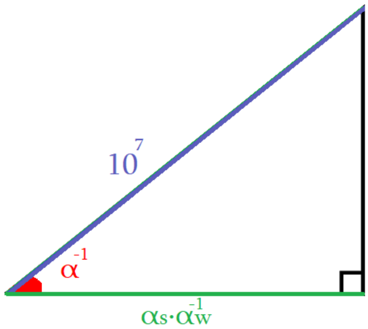

Figure 6. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.*

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = \alpha_s \tag{40}$$

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s \tag{41}$$

Resulting the unity formula that connects the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \tag{42}$$

$$\alpha^{-2} \cdot \cos^2\alpha^{-1} = 4 \cdot \alpha_G \cdot N_A^2 \tag{43}$$

The figure 7 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

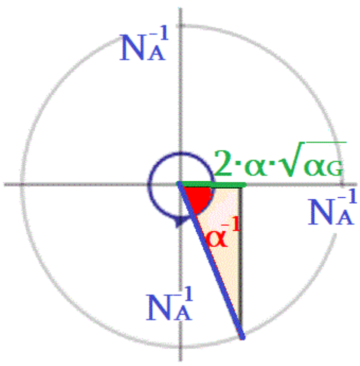

Figure 7. *The angle in α^{-1} radians.*

The figures 8 and 9 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

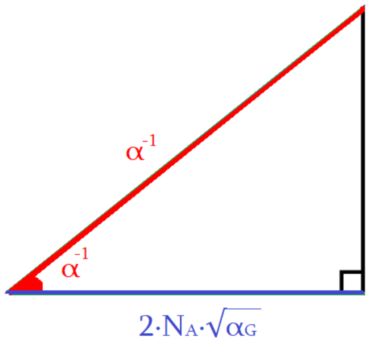

Figure 8. *First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions*

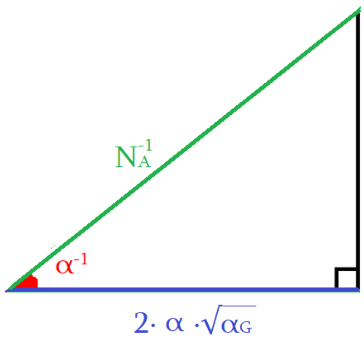

Figure 9. *Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \tag{44}$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot N_A^2 \tag{45}$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2 \tag{46}$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \tag{47}$$

$$2 \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i} \tag{48}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i} \tag{49}$$

$$2 \cdot e^{\pi} \cdot a_s \cdot a \cdot a_G^{1/2} \cdot N_A = 1 \tag{50}$$

$$2 \cdot e^{4\pi} \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1 \tag{51}$$

$$e^{4\pi} \cdot a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = 1 \tag{52}$$

The figure 10 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.

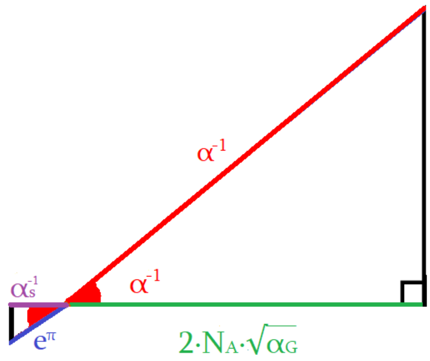

Figure 10. *Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions*

We reached the conclusion of the dimensionless unification of the weak nuclear,the gravitational and electromagnetic forces:

$$4\cdot10^{14}\cdot a_w^2\cdot a^2\cdot a_G\cdot N_A^2=i^{4i}\cdot e^2 \quad (53)$$

$$4\cdot10^{14}\cdot a^2\cdot \cos^2 a^{-1}\cdot a_w^2\cdot a_G\cdot N_A^2=i^{8i} \quad (54)$$

$$10^{14}\cdot a^2\cdot(e^{i/a}+e^{-i/a})^2\cdot a_w^2\cdot a_G\cdot N_A^2=i^{8i} \quad (55)$$

$$4\cdot10^{14}\cdot e^{2\pi}\cdot a_w^2\cdot a^2\cdot a_G\cdot N_A^2=e^2 \quad (56)$$

$$4\cdot10^{14}\cdot e^{2\pi}\cdot a^2\cdot \cos^2 a^{-1}\cdot a_w^2\cdot a_G\cdot N_A^2=1 \quad (57)$$

$$10^{14}\cdot e^{4\pi}\cdot a^2\cdot(e^{i/a}+e^{-i/a})^2\cdot a_w^2\cdot a_G\cdot N_A^2=i^{8i} \quad (58)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions.

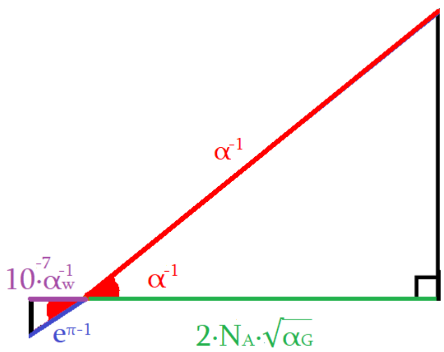

Figure 11. *Geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions*

Resulting the unity formula that connect the strong coupling constant a_s,the weak coupling constant a_w,the fine-structure constant a and the gravitational coupling constant $a_{G(p)}$ for the proton:

$$4\cdot10^{14}\cdot N_A^2\cdot a_w^2\cdot a^2\cdot a_{G(p)}=\mu^2\cdot a_s^2 \quad (59)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$a_s^2=4\cdot10^{14}\cdot a_w^2\cdot a^2\cdot a_G\cdot N_A^2 \quad (60)$$

$$\alpha_s \cdot \cos\alpha^{-1} = 4 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G \quad (61)$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) \quad (62)$$

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational

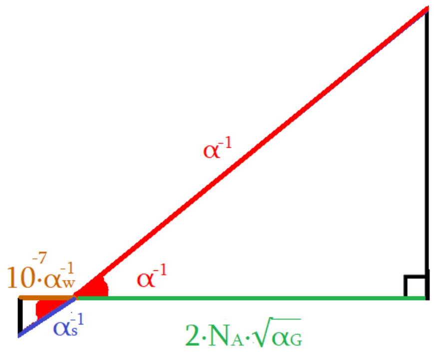

Figure 12. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions*

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron,the gravitational coupling constant of the proton $\alpha_{G(p)}$,the strong coupling constant α_s and the weak coupling constant α_w:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (63)$$

$$\mu^2 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2 \quad (64)$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (65)$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \quad (66)$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (67)$$

$$\mu \cdot \alpha_s = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (68)$$

$$\mu \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \quad (69)$$

The figure 13 below shows the geometric representation of the dimensionless unification of the fundamental interactions.

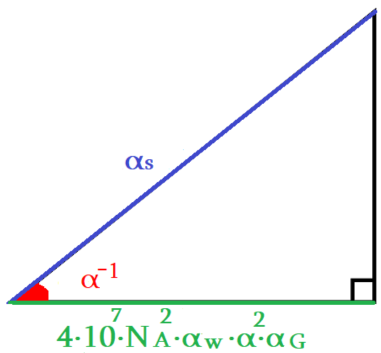

Figure 13. *Geometric representation of the Dimensionless unification of the fundamental interactions*

These equations are applicable for all energy scales. In [15] and [16] we found the expressions for the gravitational constant:

$$G = (2e\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \quad (70)$$

$$G = i^{4i}(2\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \quad (71)$$

$$G = i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \quad (72)$$

$$G = \alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \quad (73)$$

It presented the theoretical value of the Gravitational constant $G=6.67448\times10^{-11}$ $m^3/kg\cdot s^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring 6.674184×10^{-11} $m^3/kg\cdot s^2$ and 6.674484×10^{-11} $m^3/kg\cdot s^2$-of-swinging and angular acceleration methods, respectively.

3. Unification of atomic physics and cosmology

In [17] and [18] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant α,which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant α_g. It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant α_g is an equivalent way to express the biggest issue in theoretical physics. The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3} = \sqrt{\frac{\alpha_G^3}{\alpha^6}} = 1.886837\times10^{-61} \quad (74)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5}\times10^{-60} = 1,886837\times10^{-61} \quad (75)$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3} \quad (76)$$

$$\alpha_g=i^{6i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-3} \quad (77)$$

$$\alpha_g=i^{6i}\cdot e^3\cdot(2\cdot10^7\cdot\alpha_w\cdot\alpha^3\cdot N_A)^{-3} \quad (78)$$

$$\alpha_g=(10^7\cdot\alpha_w\cdot\alpha_G^{1/2}\cdot e^{-1}\cdot\alpha_s^{-1}\cdot\alpha^{-1})^3 \quad (79)$$

$$\alpha_g^2=(10^{14}\cdot\alpha_w^2\cdot\alpha_G\cdot e^{-2}\cdot\alpha_s^{-2}\cdot\alpha^{-2})^3 \quad (80)$$

$$\alpha_g=10^{21}\cdot i^{6i}\cdot\alpha_w^3\cdot\alpha_G^{3/2}\cdot\alpha_s^{-6}\cdot\alpha^{-3} \quad (81)$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g^2=10^{42}\cdot i^{12i}\cdot\alpha_w^6\cdot\alpha_G^3\cdot\alpha_s^{-12}\cdot\alpha^{-6} \quad (82)$$

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$l_{pl}^2 \cdot \Lambda=(2\cdot e\cdot a^2\cdot N_A)^{-6} \tag{83}$$

$$l_{pl}^2 \cdot \Lambda=i^{12i}\cdot(2\cdot a_s\cdot a^2\cdot N_A)^{-6} \tag{84}$$

$$l_{pl}^2 \cdot \Lambda=i^{12i}\cdot e^6\cdot(2\cdot 10^7\cdot a_w\cdot a^3\cdot N_A)^{-6} \tag{85}$$

$$e^6\cdot a_s^6\cdot a^6\cdot l_{pl}^2\cdot\Lambda=10^{42}\cdot a_G^3\cdot a_w^6 \tag{86}$$

$$a_s^{12}\cdot a^6\cdot l_{pl}^2\cdot\Lambda=10^{42}\cdot i^{12i}\cdot a_G^3\cdot a_w^6 \tag{87}$$

For the cosmological constant equals:

$$\Lambda=\left(2e\alpha^2\mathrm{N_A}\right)^{-6}\frac{c^3}{G\hbar} \tag{88}$$

$$\Lambda=i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{89}$$

$$\Lambda=i^{12i}e^6\left(2\,10^7\alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{90}$$

$$\Lambda=10^{42}\left(\frac{\alpha_G\alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{91}$$

$$\Lambda=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{92}$$

In [19] we found the Equations of the Universe:

$$\frac{\Lambda G\hbar}{c^3}=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{93}$$

$$e^{6\pi}\frac{\Lambda G\hbar}{c^3}=10^{42}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{94}$$

We proposed a possible solution for the cosmological parameters. The density parameter for normal baryonic matter is:

$$\Omega_B=e^{-\pi}=i^{2i}=0{,}04321=4{,}32\% \tag{95}$$

The density parameter for dark matter is:

$$\Omega_D=6\cdot e^{-\pi}=6\cdot i^{2i}=0.25928=25.92\% \tag{96}$$

The density parameter for the dark energy is:

$$\Omega\Lambda=17\cdot e^{-\pi}=17\cdot i^{2i}=0.73463=73.46\% \quad (97)$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0=24\cdot e^{-\pi}=24\cdot i^{2i}=1.03713 \quad (98)$$

A positively curved universe is described by elliptic geometry,and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold,such as the Poincaré dodecahedral space,all of which are quotients of the 3-sphere. In [20] we proposed a possible solution for the Equation of state in cosmology. From the dimensionless unification of the fundamental interactions the state equation w has value:

$$w=-24\cdot e^{-\pi}=-24\cdot i^{2i}=-1.03713 \quad (99)$$

4. Length,Mass,Energy and Time scales

In [21] , [22] and [23] we presented the law of the gravitational fine-structure constant αg followed by ratios of maximum and minimum theoretical values for natural quantities.This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length l,time t,speed υ and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{\max}} = \frac{t_{min}}{t_{\max}} = \frac{\upsilon_{min}}{\upsilon_{\max}} = \frac{T_{min}}{T_{max}} \quad (100)$$

Energy E, mass M, action A,momentum P and entropy S have another min/max ratio,which is the square of αg:

$$\alpha_g^2 = \frac{E_{min}}{E_{\max}} = \frac{M_{min}}{M_{\max}} = \frac{A_{min}}{A_{\max}} = \frac{P_{min}}{P_{\max}} = \frac{S_{min}}{S_{max}} \quad (101)$$

Force F has min/max ratio which is αg^4:

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (102)$$

Mass density has min/max ratio which is αg^5:

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (103)$$

A Planck length l_{pl} is about 10^{-20} times the diameter of a proton,meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck l_{pl} defined as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The classical electron radius is a combination of fundamental physical quantities that define a length scale for problems involving an electron interacting with electromagnetic radiation. The classical electron radius is given as:

$$r_e = \alpha^2 \alpha_0 = \frac{\hbar\alpha}{m_e c} = \frac{\lambda_c \alpha}{m_e c^2} = \frac{\mu_0}{4\pi}\frac{q_e^2}{m_e} = \frac{k_e q_e^2}{m_e c^2} = \frac{\alpha^3}{4\pi R_\infty}$$

The Bohr radius a_0 is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius a_0 is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

The proton radius r_p is the distance from the center of the proton to the tip of the proton. The proton radius r_p is an unanswered physics problem related to the size of the proton. In atomic physics, there are two common and "natural" scales of length. The first scale of length is given by Compton's wavelength of electrons. Using the de Broglie equation, we get that Compton's wavelength is the wavelength of a photon whose energy is the same as the rest mass of the particle,or mathematically speaking: The Compton wavelength of a particle is equal to the wavelength of a photon whose energy is the same as the mass of that particle. It was introduced by Arthur Compton in his explanation of the scattering of photons by electrons. The standard Compton wavelength λ,of a particle is given by $\lambda_c=h/m\cdot c$. Thus respectively the Compton wavelength λ_c of the electron with mass m_e is given by the formula:

$$\lambda_c = \frac{2\pi r_e}{\alpha} = \frac{h}{m_e c}$$

Sometimes the Compton wavelength is expressed by the reduced Compton $\bar{\lambda}_c$ wavelength. When the Compton λ_c wavelength is divided by $2\cdot\pi$,we obtain the reduced Compton $\bar{\lambda}_c$ wavelength,ie the Compton wavelength for 1 radius instead of $2\cdot\pi$ rad $\bar{\lambda}_c=\lambda_c/2\cdot\pi$. The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}}\frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2\alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha\frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha\alpha_0}\right)^2$$

A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length l_{pl}. The smallest components of spacetime will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length $2\cdot e\cdot l_{pl}$. Perhaps for the minimum distance l_{min} apply:

$$l_{min}=2\cdot e\cdot l_{pl} \tag{104}$$

From expressions apply:

$$\cos\alpha^{-1}=e^{-1}$$

$$\cos\alpha^{-1}\cdot l_{min}=2\cdot l_{pl}$$

$$\cos\alpha^{-1} = \frac{2l_{pl}}{l_{min}} \tag{105}$$

The figures 14 below show the geometric representation of the fundamental unit of length.

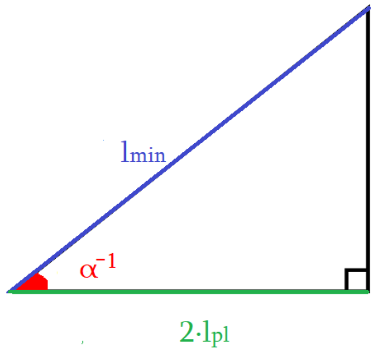

Figure 14. *Geometric representation of the fundamental unit of length.*

For the Bohr radius a_0 apply:

$$a_0 = N_A \cdot l_{min}$$

$$a_0 = 2 \cdot e \cdot N_A \cdot l_{pl} \tag{106}$$

The figures 15 below show the geometric representation of the relationship between the Bohr radius and the Planck length.

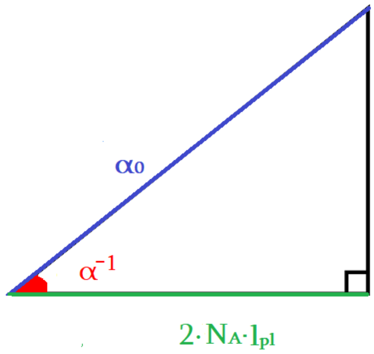

Figure 15. *Geometric representation of the relationship between the Bohr radius and the Planck length.*

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as $L_H = c \cdot H_0^{-1}$. It is equivalent to 4.420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting $D = c \cdot H_0^{-1}$ into the equation for Hubble's law, $u = H_0^{-1} \cdot D$. For the density parameter for dark energy apply:

$$\Omega_\Lambda = \left(\frac{L_H}{R_d}\right)^2 = \frac{L_H^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\%$$

So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (107)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (108)$$

The figure 16 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

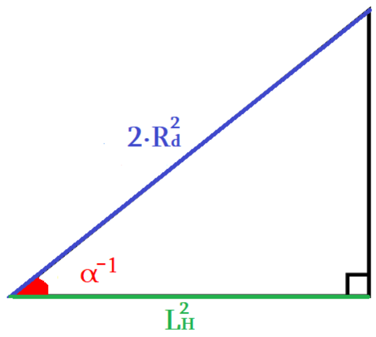

Figure 16. *Geometric representation of the relationship between the de Sitter radius and the Hubble length.*

The maximum distance l_{max} corresponds to the distance of the universe $l_U = c \cdot H_0^{-1}$. Length l has the min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (109)$$

The maximum distance l_{max} corresponds to the distance of the universe:

$$l_{max} = L_H = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{min} \quad (110)$$

The figure 17 shows the geometric representation of the relationship between the maximum distance and the Planck length.

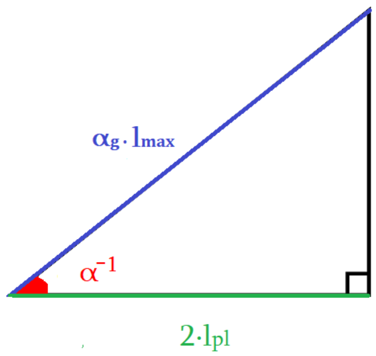

Figure 17. *Geometric representation of the relationship between the maximum distance and the Planck length.*

The value of the maximum distance l_{max} is $l_{max} = 4.656933 \times 10^{26}$ m. In [24] , [25] , [26] and [27] we presented the Dimensionless theory of everything. In [28] we presented the New Large Number Hypothesis of the universe. The figure 18 shows the geometric representation of the relationship between the radius of the universe with the Planck length.

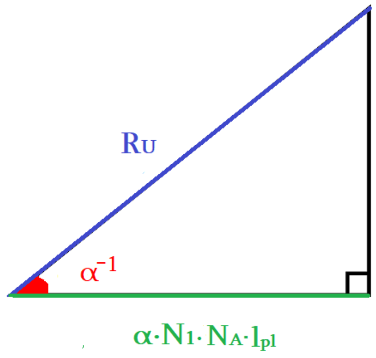

Figure 18. *Geometric representation of the relationship between the radius of the universe with the Planck length*

The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \bar{\lambda}_c \tag{111}$$

Diameter of the universe=ratio of electric force to gravitational force×reduced Compton wavelength of the electron

So the expressions for the radius of the observable universe are:

$$\frac{2R_U}{r_e} = \frac{N_1}{\alpha} \tag{112}$$

$$\frac{2R_U}{r_e} = \frac{1}{\mu \alpha_G} \tag{113}$$

$$\frac{2R_U}{\alpha_0} = \alpha N_1 \tag{114}$$

$$\frac{2R_U}{l_{min}} = \alpha N_1 N_A \tag{115}$$

The expression between the radius of the observable universe R_U with the Planck length l_{pl} is:

$$R_U = e \cdot \alpha \cdot N_1 \cdot N_A \cdot l_{pl} \tag{116}$$

The expression between the radius of the observable universe R_U with the minimum distance l_{min} is:

$$2 \cdot R_U = \alpha \cdot N_1 \cdot N_A \cdot l_{min} \tag{117}$$

So apply the expressions for the radius of the observable universe:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 \tag{118}$$

$$R_U = \frac{N_1}{2\alpha} r_e \tag{119}$$

$$R_U = \frac{1}{2\mu\alpha_G} r_e \tag{120}$$

$$R_U = \frac{m_{pl}^2 r_e}{2m_e m_p} \tag{121}$$

$$R_U = \frac{\hbar c r_e}{2Gm_e m_p} \tag{122}$$

$$R_U = \frac{\alpha\hbar}{2Gm_e^2 m_p} \tag{123}$$

For the value of the radius of the universe apply $R_U=4.38\times10^{26}$ m. The expressions for the gravitational constant are:

$$G = \frac{\hbar c r_e}{2m_e m_p} \frac{1}{R_U} \tag{124}$$

$$G = \frac{\alpha\hbar}{2m_e^2 m_p} \frac{1}{R_U} \tag{125}$$

The Planck mass m_{pl} appears everywhere in astrophysics,cosmology,quantum gravity,string theory,etc. Its mass is enormous compared to any subatomic particle and even the mass of heavier atoms. The mass Planck m_{pl} can be defined by three fundamental natural constants,the speed of light in vacuum c,the reduced Planck constant $\hbar$ and the gravity constant G as:

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_{pl} c} = \frac{\mu_0 q_{pl}^2}{4\pi l_{pl}}$$

In [29] J.Forsythe and T. Valev found an extended mass relation for seven fundamental masses. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum "Gravity Atom" whose gravitational potential is equal to electrostatic potential , Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. In [30] we presented the mass scale law of the Universe. We found a mass relation for fundamental masses:

$$M_n=\alpha^{-1}\cdot\alpha_g^{(2-n)/3}\cdot m_e \tag{126}$$

$$n=0,1,2,3,4,5,6$$

For n=0 M_0 is the minimum mass M_{min}:

$$M_0=M_{min}=\alpha^{-1}\cdot\alpha_g^{(2-0)/3}\cdot m_e$$

$$M_0=M_{min}=\alpha^{-1}\cdot\alpha_g^{2/3}\cdot m_e \tag{127}$$

For n=1 M_1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass M_{Un},most likely a yet unobserved light particle:

$$M_1=M_{Un}=\alpha^{-1}\cdot\alpha_g^{(2-1)/3}\cdot m_e$$

$$M_1=M_{Un}=\alpha^{-1}\cdot\alpha_g^{1/3}\cdot m_e \qquad (128)$$

For n=2 M_2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass M_{pi}:

$$M_2=M_{pi}=\alpha^{-1}\cdot\alpha_g^{(2-2)/3}\cdot m_e$$

$$M_2=M_{pi}=\alpha^{-1}\cdot m_e \qquad (129)$$

For n=3 M_3 is the Planck mass m_{pl}:

$$M_3=m_{pl}=\alpha^{-1}\cdot\alpha_g^{(2-3)/3}\cdot m_e$$

$$M_3=m_{pl}=\alpha^{-1}\cdot\alpha_g^{-1/3}\cdot m_e \qquad (130)$$

For n=4 is the central mass of a hypothetical quantum "Gravity Atom" M_{GA}.

$$M_4=M_{GA}=\alpha^{-1}\cdot\alpha_g^{(2-4)/3}\cdot m_e$$

$$M_4=M_{GA}=\alpha^{-1}\cdot\alpha_g^{-2/3}\cdot m_e \qquad (131)$$

For n=5 is of the order of the Eddington mass M_{Edd} limit of the most massive stars:

$$M_5=M_{Edd}=\alpha^{-1}\cdot\alpha_g^{(2-5)/3}\cdot m_e$$

$$M_5=M_{Edd}=\alpha^{-1}\cdot\alpha_g^{-1}\cdot m_e \qquad (132)$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe M_U:

$$M_6=M_U=\alpha^{-1}\cdot\alpha_g^{(2-5)/3}\cdot m_e$$

$$M_6=M_U=\alpha^{-1}\cdot\alpha_g^{-4/3}\cdot m_e \qquad (133)$$

The similar mass relation for seven fundamental masses is:

$$M_n=\alpha_g^{-n/3}\cdot M_{min} \qquad (134)$$

$$n=0,1,2,3,4,5,6$$

For n=0 M_0 is the minimum mass M_{min}:

$$M_0=M_{min} \qquad (135)$$

For n=1 M_1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass M_{Un},most likely a yet unobserved light particle:

$$M_1=M_{Un}=\alpha_g^{-1/3}\cdot M_{min} \qquad (136)$$

For n=2 M_2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass M_{pi}:

$$M_2=M_{pi}=\alpha_g^{-2/3}\cdot M_{min} \qquad (137)$$

For n=3 M_3 is the Planck mass m_{pl}:

$$M_3=m_{pl}=\alpha_g^{-1}\cdot M_{min} \qquad (138)$$

For n=4 is the central mass of a hypothetical quantum "Gravity Atom" M_{GA}:

$$M_4=M_{GA}=\alpha_g^{-4/3}\cdot M_{min} \qquad (139)$$

For n=5 is of the order of the Eddington mass M_{Edd} limit of the most massive stars:

$$M_5=M_{Edd}=\alpha_g^{-5/3}\cdot M_{min} \tag{140}$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe M_U:

$$M_6=M_U=\alpha_g^{-2}\cdot M_{min} \tag{141}$$

The following applies to the minimum mass M_{min}:

$$\mathrm{M_{min}}c^2 = \frac{\hbar}{t_{max}}$$

$$\mathrm{M_{min}}c^2 = \hbar \mathrm{H_0}$$

$$\mathrm{M_{min}} = \frac{\hbar \mathrm{H_0}}{c^2} \tag{142}$$

$$\mathrm{M_{min}} = \frac{\hbar}{c l_{max}} \tag{143}$$

So apply the expressions:

$$\mathrm{M_{min}} = \frac{\hbar}{c}\sqrt{\Lambda} \tag{144}$$

$$\mathrm{M_{min}} = \frac{m_{pl}^2}{\mathrm{M}_{max}} \tag{144}$$

$$\mathrm{M_{min}} = \frac{m_{pl}^2}{\mathrm{M}_{max}} \tag{145}$$

Therefore for the minimum mass M_{min} apply:

$$\mathrm{M_{min}} = \alpha_g m_{pl} \tag{146}$$

$$\mathrm{M_{min}} = \frac{\alpha_G}{\alpha^3} m_e \tag{147}$$

$$\mathrm{M_{min}} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \tag{148}$$

$$M_{min}=(2\cdot e\cdot N_A)^{-2}\cdot \alpha^{-1}\cdot m_e \tag{149}$$

Mass M have max/min ratio,which is the square of α_g:

$$\alpha_g^2 = \frac{\mathrm{M}_{min}}{\mathrm{M}_{max}} \tag{150}$$

For the maximum mass M_{max} applies:

$$M_{\max} = \frac{F_{\max} l_{max}}{c^2} \tag{151}$$

$$M_{\max} = \frac{m_{pl}^2}{M_{min}} \tag{152}$$

The expressions for the mass of the observable universe are:

$$M_U = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e \tag{153}$$

$$M_U = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e \tag{154}$$

$$M_U = (2 \cdot e \cdot \alpha^2 \cdot N_A)^2 \cdot N_1 \cdot m_p \tag{155}$$

$$M_U = \mu \cdot \alpha \cdot N_1^2 \cdot m_p \tag{156}$$

For the value of the mass of the observable universe M_U apply $M_U = 1.153482 \times 10^{53}$ kg. The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu\alpha N_1^2 = 6.9 \times 10^{79} \tag{157}$$

$$\frac{M_U}{m_p} = \left(2e\alpha^2 N_A\right)^2 N_1 \tag{158}$$

$$\frac{M_U}{m_p} = \frac{N_1}{\alpha_g^{\frac{2}{3}}} \tag{159}$$

$$\frac{M_U}{m_p} = \left(\frac{r_e}{l_{pl}}\right)^2 N_1 \tag{160}$$

Also apply the expressions:

$$m_{pl} \cdot l_{max} = M_U \cdot l_{pl} \tag{161}$$

$$l^2_{max} \cdot M_{min} = l^2_{min} \cdot M_U \tag{162}$$

The expressions for the relationship between the mass of the observable universe M_U with the radius of the universe R_U are:

$$\frac{M_U}{R_U^2} = 4\alpha\mu^2 \frac{m_e}{r_e^2} \tag{163}$$

$$\frac{M_U}{R_U^2} = 16 \frac{m_p}{r_p r_e} \tag{164}$$

$$\frac{M_U}{R_U^2} = \frac{64}{\alpha} \frac{m_e}{r_p^2} \tag{165}$$

$$\frac{M_U}{m_p} = \alpha\mu\left(\frac{2R_U}{r_e}\right)^2 \tag{166}$$

$$\frac{\frac{M_U}{m_p}}{\left(\frac{2R_U}{r_e}\right)^2} = \alpha\mu \tag{167}$$

R. Adler in [31] calculated the energy ratio in cosmology,the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies,the rest energy of the electron E_e,and the binding energy of the hydrogen atom E_H. The rest energy of the electron E_e is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom E_H is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales,the Planck energy density ρ_{pl},and the density of the dark energy ρ_Λ. The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density ρ_Λ is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So with expression (58) for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \tag{168}$$

The Planck time t_{pl} is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl}c^2}$$

For the minimum distance l_{min} apply:

$$l_{min}=2 \cdot e \cdot l_{pl}$$

So for the minimum time t_{min} apply:

$$t_{\min} = \frac{l_{min}}{c} \tag{169}$$

$$t_{\min} = \frac{2el_{pl}}{c} \tag{170}$$

$$t_{min}=2 \cdot e \cdot t_{pl} \tag{171}$$

From expressions apply:

$$\cos a^{-1}=e^{-1}$$

$$\cos a^{-1} \cdot t_{min}=2 \cdot t_{pl} \tag{172}$$

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \tag{173}$$

The figures 19 below show the geometric representation of the fundamental unit of time.

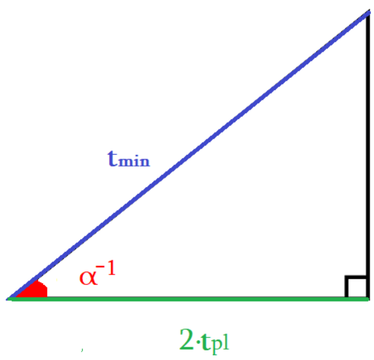

Figure 19. *Geometric representation of the fundamental unit of time.*

The maximum time period t_{max} is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe $t_U=H_0^{-1}$. Therefore $t_{max}=t_U=H_0^{-1}$. Time t has the min/max ratio which is:

$$\alpha_g = \frac{t_{min}}{t_{max}}$$

$$a_g=t_{min} \cdot H_0 \tag{175}$$

$$a_g=2 \cdot e \cdot t_{pl} \cdot H_0 \tag{176}$$

The gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. Changes in electrical membrane potential generate neuronal action potentials. Oscillatory activity of neurons is connected to these spikes. The oscillation of the single neuron can be observed in fluctuations at the threshold of the membrane potential. The time quantum in the brain t_B,the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \tag{177}$$

For the age of the universe apply:

$$\mathrm{T}_U = \frac{R_U}{c} \tag{178}$$

$$\mathrm{T}_U = \frac{\mathrm{N}_1 r_e}{2ac} \tag{179}$$

$$\mathrm{T}_U = \frac{r_e}{2\mu\alpha_G c} \tag{180}$$

$$\mathrm{T}_U = \frac{\alpha \mathrm{N}_1 \alpha_0}{2c} \tag{181}$$

$$\mathrm{T}_U = \frac{\alpha\hbar}{2cGm_e^2 m_p} \tag{182}$$

$$\mathrm{T}_U = \frac{\hbar r_e}{2Gm_e m_p} \tag{183}$$

For the value of the age of the universe apply $T_U=1.46\times10^{18}$ s. The expressions for the gravitational constant are:

$$G = \frac{\alpha\hbar}{2cm_e^2 m_p}\frac{1}{\mathrm{T}_U} \tag{184}$$

$$G = \frac{\hbar r_e}{2m_e m_p}\frac{1}{\mathrm{T}_U} \tag{185}$$

In July 2019, astronomers reported that a new method to determine the Hubble constant, and resolve the discrepancy of earlier methods, has been proposed based on the mergers of pairs of neutron stars, following the detection of the neutron star merger of GW170817, an event known as a dark siren. Their measurement of the Hubble constant is 73.3±5.3 (km/s)/Mpc. Laurent Nottale in [32] assumed a large-number relation:

$$\alpha\frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}}\right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g:

$$\alpha\frac{m_{pl}}{m_e}=\left(l_{pl}\sqrt{\Lambda}\right)^{-\frac{1}{3}}$$

$$\alpha_g=l_{pl}\sqrt{\Lambda}$$

$$\alpha_g=\sqrt{\frac{G\hbar\Lambda}{c^3}}$$

In the papers [33] was presented the theoretical value for the Hubble Constant. The density parameter for dark energy is defined as:

$$\Omega_\Lambda=\frac{\Lambda c^2}{3\mathrm{H}_0^2}$$

Also for the density parameter for dark energy apply:

$$\Omega_\Lambda=\frac{c^2}{R_d^2\mathrm{H}_0^2}$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda=\left(\frac{L_\mathrm{H}}{R_d}\right)^2=\frac{L_\mathrm{H}^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is $\Omega_\Lambda=2\cdot e^{-1}=0.73576=73.57\%$. So from this expression apply:

$$2\cdot Rd^2=e\cdot L_H^2 \qquad (186)$$

So apply the expression:

$$\cos\alpha^{-1}=\frac{L_\mathrm{H}^2}{2R_d^2} \qquad (187)$$

The figure 20 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

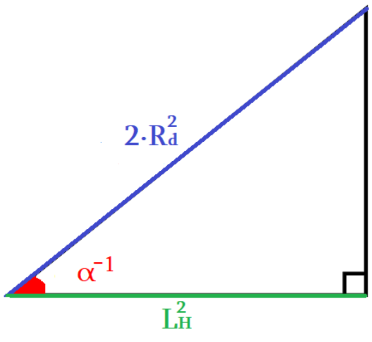

Figure 20. *Geometric representation of the relationship between the de Sitter radius and the Hubble length.*

For the cosmological constant Λ equals:

$$\Lambda = \frac{6H_0^2}{ec^2} \tag{188}$$

So apply the expressions:

$$\frac{6H_0^2}{\Lambda c^2} = e \tag{189}$$

$$\frac{H_0^2}{\Lambda c^2} = \frac{e}{6} \tag{190}$$

$$\cos \alpha^{-1} = \frac{\Lambda c^2}{6H_0^2} \tag{191}$$

The figure 21 shows the geometric representation of the relationship between the Hubble constant and cosmological constant.

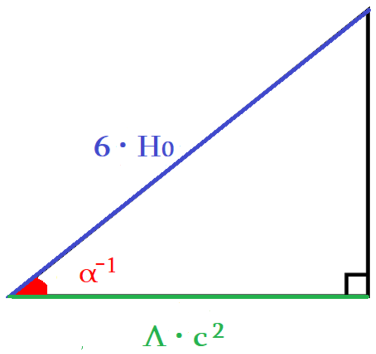

Figure 21. *Geometric representation of the relationship between the Hubble constant and cosmological constant.*

So the formula for the Hubble Constant is:

$$H_0 = c\sqrt{\frac{e}{6}\Lambda} \tag{192}$$

For the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

So the formulas for the Hubble Constant are:

$$H_0 = \frac{\alpha_g}{t_{pl}}\sqrt{\frac{e}{6}} \tag{193}$$

$$H_0 = \frac{\alpha_g c}{l_{pl}}\sqrt{\frac{e}{6}} \tag{194}$$

Also apply the expression:

$$(H_0 t_{pl})^2 = \frac{e}{6}\alpha_g^2 \tag{195}$$

These equations calculate the theoretical value of the Hubble Constant $H_0 = 2.355683 \times 10^{-18}\ s^{-1} = 72.69$ (km/s)/Mpc.

The cosmological constant Λ equals:

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{c l_{pl}^2}{r_e^2}\sqrt{\frac{e}{6}} \tag{196}$$

Also the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \alpha_g \sqrt{\frac{e c^5}{6 G\hbar}} \tag{197}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{e}{6}\alpha_g^2 \tag{198}$$

The cosmological constant Λ equals:

$$\Lambda = \frac{G}{\hbar^4}\left(\frac{m_e}{a}\right)^6$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{c m_e^3}{\alpha^3 \hbar^2}\sqrt{\frac{eG}{6}} \tag{199}$$

From the dimensionless unification of the atomic physics and the cosmology apply:

$$\alpha_g = (2\cdot e\cdot \alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2\cdot e\cdot \alpha^2\cdot N_A)^{-6} \tag{200}$$

$$(2\cdot e\cdot \alpha^2\cdot N_A)^6\cdot l_{pl}^2 \cdot \Lambda = 1 \tag{201}$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 \mathrm{N_A}\right)^{-6}\frac{c^3}{G\hbar} \tag{202}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{1}{\left(2e\alpha^2 \mathrm{N_A}\right)^3}\sqrt{\frac{e c^5}{6 G\hbar}} \tag{203}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{1}{6e^5\left(2\alpha^2\mathrm{N_A}\right)^6} \tag{204}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g = i^{6i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-6} \tag{205}$$

$$(2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \tag{206}$$

For the cosmological constant equals:

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{207}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \frac{i^{6i}}{\left(2\alpha_s\alpha^2\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{208}$$

$$\mathrm{H}_0 = \frac{1}{\left(2e^{\pi}\alpha_s\alpha^2\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{209}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{e}{48\left(e^{\pi}\alpha_s\alpha^2\mathrm{N_A}\right)^3} \tag{210}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-6} \tag{211}$$

$$(2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \tag{212}$$

For the cosmological constant equals:

$$\Lambda = i^{12i}e^6\left(2\,10^7\alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{213}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \frac{i^{6i}e^3}{\left(2\,10^7\alpha_w\alpha^3\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{214}$$

$$H_0=\frac{1}{\left(2\,10^7 e^{\pi-1}\alpha_w\alpha^3 N_A\right)^3}\sqrt{\frac{ec^3}{6G\hbar}} \tag{215}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2=10^{42}\left(\frac{\alpha_G a_w^2}{e^2\alpha_s^2\alpha^2}\right)^3 \tag{216}$$

$$l_{pl}^2\Lambda=10^{42}\left(\frac{\alpha_G\alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3 \tag{217}$$

$$e^6\cdot \alpha s^6\cdot \alpha^6\cdot lpl^2\cdot \Lambda=10^{42}\cdot \alpha G^3\cdot \alpha w^6 \tag{218}$$

For the cosmological constant equals:

$$\Lambda=10^{42}\left(\frac{\alpha_G\alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{219}$$

So the formula for the Hubble Constant is:

$$H_0=10^{21}\left(\frac{\alpha_w\sqrt{\alpha_G}}{e\alpha_s\alpha}\right)^3\sqrt{\frac{ec^5}{6G\hbar}} \tag{220}$$

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5}=\frac{10^{42}}{6e^5}\left(\frac{\alpha_w^2\alpha_G}{\alpha_s^2\alpha^2}\right)^3 \tag{221}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{222}$$

$$l_{pl}^2\Lambda=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{223}$$

$$\alpha s^{12}\cdot \alpha^6\cdot lpl^2\cdot \Lambda=10^{42}\cdot i^{12i}\cdot \alpha G^3\cdot \alpha w^6 \tag{224}$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{225}$$

So the formulas for the Hubble Constant are:

$$H_0 = \left(\frac{10^7 i^{2i} \alpha_w \sqrt{\alpha_G}}{\alpha\alpha_s^2}\right)^3 \sqrt{\frac{ec^5}{6G\hbar}} \tag{226}$$

$$H_0 = \left(\frac{10^7 \alpha_w \sqrt{\alpha_G}}{e^{\pi}\alpha\alpha_s^2}\right)^3 \sqrt{\frac{ec^5}{6G\hbar}} \tag{227}$$

Also apply the expression:

$$\frac{6G\hbar H_0^2}{ec^5} = \left(\frac{10^{14}\alpha_w^2\alpha_G}{e^{2\pi}\alpha^2\alpha_s^4}\right)^3 \tag{228}$$

$$6e^{6\pi}\frac{G\hbar H_0^2}{c^5} = e\left(\frac{10^{14}\alpha_w^2\alpha_G}{\alpha^2\alpha_s^4}\right)^3 \tag{229}$$

$$6e^{5\pi}\frac{G\hbar H_0^2}{c^5} = \frac{1}{\alpha_s^{11}}\left(\frac{10^{14}\alpha_w^2\alpha_G}{\alpha^2}\right)^3 \tag{230}$$

So the Equations of the Universe are:

$$6e^{5\pi}\frac{G\hbar H_0^2}{c^5} = 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{11}} \tag{231}$$

$$e^{7\pi}\frac{G\hbar\Lambda^2}{cH_0^2} = 6\cdot 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{13}} \tag{232}$$

5. Poincaré dodecahedral space

In 2003 J.-P. Luminet in [34] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called "well-proportioned spaces" because they have a similar extent in all three dimensions. More specifically,we discovered that the best candidate to fit the observed power spectrum is a well-proportioned space called the Poincaré dodecahedral space. This space may be represented by a polyhedron with 12 pentagonal faces, with opposite faces being "glued" together after a twist of 36°. This is the only consistent way to obtain a spherical (i.e. positively curved) space from a dodecahedron: if the twist was 108°, for example, we would end up with a radically different hyperbolic space. The Poincaré dodecahedral space is essentially a multiply connected variant of a simply connected hypersphere, although its volume is 120 times smaller. A rocket leaving the dodecahedron through a given face immediately re-enters through the opposite face,and light propagates such that any observer whose line-of-sight intercepts one face has the illusion of seeing a slightly rotated copy of their own dodecahedron. This means that some photons from the cosmic microwave background, for example, would appear twice in the sky. The power spectrum associated with the Poincaré dodecahedral space is different from that of a flat space because the fluctuations in the cosmic microwave background will change as a function of their wavelengths. In other words, due to a cut-off in space corresponding to the size of the dodecahedron, one expects fewer fluctuations at large angular scales than in an infinite flat space, but at small angular scales one must recover the same pattern as in the flat infinite space. In order to calculate the power spectrum we varied the mass-energy density of the dodecahedral universe and computed the quadrupole and the octopole modes relative to the WMAP data. To our delight,we found a small interval of values over which both these modes matched the observations perfectly. Moreover,the best fit occurred in the range $1.01<\Omega<1.02$,which sits comfortably with the observed value.

In [35] , [36] and [37] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy. The Poincaré dodecahedral space therefore accounts for the lack of large-scale fluctuations in the microwave background and also for the slight positive curvature of space inferred from WMAP and other observations. Moreover, given the observed values of the mass-energy densities and of the expansion rate of the universe, the size of the dodecahedral universe can be calculated. We found that the smallest dimension of the Poincaré dodecahedron space is 43 billion light-years, compared with 53 billion light-years for the "horizon radius" of the observable universe. Moreover,the volume of this universe is about 20% smaller than the volume of the observable universe. (There is a common misconception that the horizon radius of a flat universe is 13.7 billion light-years, since that is the age of the universe multiplied by the speed of light. However, the horizon radius is actually much larger because photons from the horizon that are reaching us now have had to cross a much larger distance due to the expansion of the universe.) If physical space is indeed smaller than the observable universe, some points on the map of the cosmic microwave background will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998,these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same. This "lensing" effect,which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the Poincaré dodecahedral model actually predicts six pairs of diametrically opposite matched circles with an angular radius of 10-50°,depending on the precise values of cosmological parameters such as the mass-energy density. An expression for the critical density is found by assuming Λ to be zero and setting the normalized spatial curvature,k,equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c = \frac{3H_0^3}{8\pi G}$$

The sum of the contributions to the total density parameter Ω_0 at the current time is $\Omega_0=1.02\pm0,02$. Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda=0.73$,$\Omega_D=0.23$ and $\Omega_B=0.04$. The figure 17 shows the Geometric representation of the density parameter for the baryonic matter.

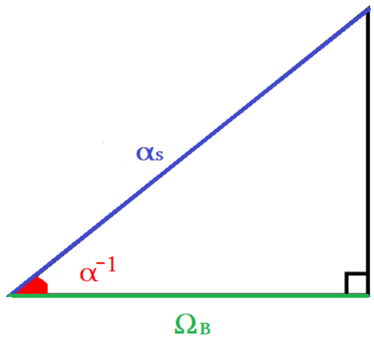

Figure 17. *Geometric representation of the the density parameter for the baryonic matter*

The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B=0.044\pm0.004$. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\% \tag{233}$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^{i}+1=0 \tag{234}$$

$$\Omega_B^{i}=i^2 \tag{235}$$

$$\Omega_B^{2i}=1 \tag{236}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B=e^{-1}\cdot a_s \quad (237)$$

$$\Omega_B=a_w^{-1}\cdot a_s^2\cdot 10^{-7} \quad (238)$$

$$\Omega_B=2^{-1}\cdot a_s\cdot(e^{i/a}+e^{-i/a}) \quad (239)$$

$$\Omega_B=2\cdot N_A\cdot a_s\cdot a\cdot a_G^{1/2} \quad (240)$$

$$\Omega_B=2^{-1}\cdot e^{-1}\cdot 10^7\cdot a_w\cdot(e^{i/a}+e^{-i/a}) \quad (241)$$

$$\Omega_B=2\cdot 10^7\cdot N_A\cdot e^{-1}\cdot a_w\cdot a\cdot a_G^{1/2} \quad (242)$$

$$\Omega_B=10^{-7}\cdot a_g^{1/3}\cdot a_s^2\cdot a\cdot a_w^{-1}\cdot a_G^{-1/2} \quad (243)$$

In [38] we presented the solution for the Density Parameter of Dark Energy. The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is $\Omega_\Lambda=0.73\pm0.04$. With 73% of the influence on the expansion of the universe in this era,dark energy is viewed as the dominant influence on that expansion. The density parameter for dark energy is defined as:

$$\Omega_\Lambda=\frac{\Lambda c^2}{3\mathrm{H}_0^2}$$

The cosmological constant is the inverse of the square of a length L:

$$L=\sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d=\sqrt{3}L$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda=\frac{c^2}{R_d^2\mathrm{H}_0^2}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H=c\cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. So for the density parameter for dark energy apply:

$$\Omega_\Lambda=\left(\frac{L_\mathrm{H}}{R_d}\right)^2=\frac{L_\mathrm{H}^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot e^{-1}=0.73576=73.57\% \quad (244)$$

So from expression apply:

$$2\cdot R_d^2=e\cdot L_H^2 \quad (245)$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot \cos a^{-1} \quad (246)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \quad (247)$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/\alpha} + e^{-i/\alpha} \quad (248)$$

The figure 18 shows the geometric representation of the density parameter for dark energy.

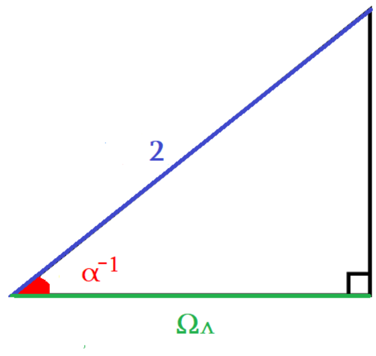

Figure 18. *Geometric representation of the the density parameter for the dark energy*

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (249)$$

The figure 19 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

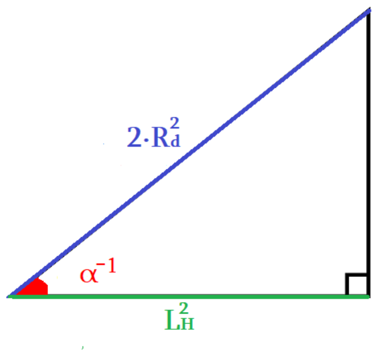

Figure 19. *Geometric representation of the relationship between the de Sitter radius and the Hubble length*

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot 10^{-7} \alpha_s \cdot \alpha_w^{-1} \quad (250)$$

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot \alpha_s^{-1} \quad (251)$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot \alpha_w^{-1} \quad (252)$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1} \quad (253)$$

$$\Omega_\Lambda = 4 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \quad (254)$$

$$\Omega_\Lambda = i^{8i} \cdot \alpha^{-2} \cdot \alpha_s^{-4} \cdot \alpha_G^{-1} \cdot N_A^{-2} \quad (255)$$

$$\Omega_\Lambda = 10^{7} \cdot i^{4i} \cdot \alpha^{-1} \cdot \alpha_w^{-1} \cdot \alpha_G^{-1/2} \cdot N_A^{-1} \quad (256)$$

$$\Omega_\Lambda=8\cdot 10^7\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G\cdot \alpha_s^{-1} \tag{257}$$

The figure 20 shows the geometric representation of the density parameter of dark matter.

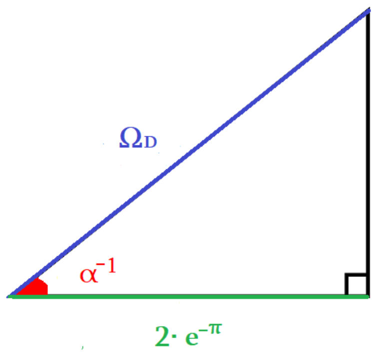

Figure 20. *Geometric representation of the density parameter of dark matter.*

The figure 21 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.

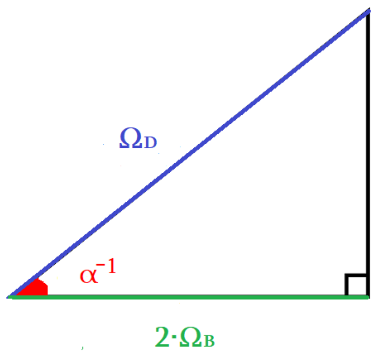

Figure 21. *Geometric representation of the relationship between the density parameter of dark and baryonic matter.*

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D=0.23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2\cdot e^{1-\pi}=2\cdot e\cdot i^{2i}=0.2349=23.49\% \tag{258}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2\cdot \alpha_s \tag{259}$$

$$\Omega_D=2\cdot 10^7\cdot e^{-1}\cdot \alpha_w \tag{260}$$

$$\Omega_D=2\cdot (i^{2i}\cdot 10^7\cdot \alpha_w)^{1/2} \tag{261}$$

$$\Omega_D=4\cdot i^{2i}\cdot (e^{i/\alpha}+e^{-i/\alpha})^{-1} \tag{262}$$

$$\Omega_D=10^7\cdot \alpha_w\cdot (e^{i/\alpha}+e^{-i/\alpha}) \tag{263}$$

$$\Omega_D=4\cdot 10^7\cdot \alpha_w\cdot \alpha\cdot \alpha_G^{1/2}\cdot N_A \tag{264}$$

$$\Omega_D=16\cdot 10^7\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G\cdot (e^{i/\alpha}+e^{-i/\alpha})^{-1} \tag{265}$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D=2\cdot e\cdot \Omega_B \tag{266}$$

The relationship between the density parameter of dark energy,dark matter and baryonic matter is:

$$\Omega_D \cdot \Omega_\Lambda = 4 \cdot \Omega_B \qquad (267)$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda = e^{-\pi} + 2 \cdot e^{1-\pi} + 2 \cdot e^{-1} = 1.0139 \qquad (268)$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold (such as the Poincaré dodecahedral space),all of which are quotients of the 3-sphere. Poincaré dodecahedral space is a positively curved space,colloquially described as "soccer ball-shaped",as it is the quotient of the 3-sphere by the binary icosahedral group,which is very close to icosahedral symmetry,the symmetry of a soccer ball. This was proposed by Jean-Pierre Luminet and colleagues in 2003 and an optimal orientation on the sky for the model was estimated in 2008. When the universe expands sufficiently,the cosmological constant L becomes more important than the energy density of matter in determining the fate of the universe. If $\Lambda>0$ there will be an approximately exponential expansion. This seems to be happening now in our universe. In [39] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP’s observations with no fine-tuning required. Circle searching (Cornish,Spergel and Starkman,1998) may confirm the model’s topological predictions,while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0 \approx 1.013 > 1$$

If confirmed,the model will answer the ancient question of whether space is finite or infinite,while retaining the standard Friedmann-Lemaˆıtre foundation for local physics. The Poincaré dodecahedral space is a dodecahedral block of space with opposite faces abstractly glued together, so objects passing out of the dodecahedron across any face return from the opposite face. Light travels across the faces in the same way, so if we sit inside the dodecahedron and look outward across a face, our line of sight re-enters the dodecahedron from the opposite face. We have the illusion of looking into an adjacent copy of the dodecahedron. If we take the original dodecahedral block of space not as a Euclidean dodecahedron (with edge angles ≈ 117°) but as a spherical dodecahedron (with edge angles exactly 120°),then adjacent images of the dodecahedron fit together snugly to tile the hypersphere, analogously to the way adjacent images of spherical pentagons (with perfect 120° angles) fit snugly to tile an ordinary sphere. Thus the Poincar´e space is a positively curved space, with a multiply connected topology whose volume is 120 times smaller than that of the simply connected hypersphere (for a given curvature radius).

The Poincar´e dodecahedral space’s power spectrum depends strongly on the assumed mass-energy density parameter Ω_0. The octopole term ($\ell=3$) matches WMAP’s octupole best when $1.010<\Omega_0<1.014$. Encouragingly, in the subinterval $1.012<\Omega_0<1.014$ the quadrupole ($\ell=2$) also matches the WMAP value. More encouragingly still, this subinterval agrees well with observations, falling comfortably within WMAP’s best fit range of $\Omega_0=1.02\pm0,02$. The excellent agreement with WMAP’s results is all the more striking because the Poincaré dodecahedral space offers no free parameters in its construction. The Poincar´e space is rigid, meaning that geometrical considerations require a completely regular dodecahedron. By contrast, a 3-torus, which is nominally made by gluing opposite faces of a cube but may be freely deformed to any parallelepiped, has six degrees of freedom in its geometrical construction. Furthermore, the Poincar´e space is globally homogeneous, meaning that its geometry-and therefore its power spectrum - looks statistically the same to all observers within it. By contrast a typical finite space looks different to observers sitting at different locations. Confirmation of a positively curved universe ($\Omega_0>1$) would require revisions to current theories of inflation, but the jury is still out on how severe those changes would be. Some researchers argue that positive curvature would not disrupt the overall mechanism and effects of inflation, but only limit the factor by which space expands during the inflationary epoch to about a factor of ten. Others claim that such models require fine-tuning and are less natural than the infinite flat space model. Having accounted for the weak observed quadrupole, the Poincaré dodecahedral space will face two more experimental tests in the next few years:

The Cornish-Spergel-Starkman circles–in–the–sky method predicts temperature correlations along matching circles in small multi connected spaces such as this one. When $\Omega_0\approx1.013$ the horizon radius is about 0,38 in units of the curvature radius, while the dodecahedron’s inradius and outradius are 0.31 and 0.39, respectively, in the same units; as a result, the volume of the physical space is only 83% the volume of the horizon sphere. In this case the horizon sphere self intersects in six pairs of circles of angular radius about 35°,making the dodecahedral space a good candidate for circle detection if technical problems (galactic foreground removal, integrated Sachs-Wolfe effect, Doppler effect of plasma motion) can be overcome. Indeed the Poincaré dodecahedral space makes circle searching easier than in the general case, because the six pairs of matching circles must a priori lie in a symmetrical pattern like the faces of a dodecahedron, thus allowing the searcher to slightly relax the noise tolerances without increasing the danger of a false positive. The Poincaré dodecahedral space predicts $\Omega_0 \approx 1.013>1$. The upcoming Planck surveyor data (or possibly even the existing WMAP data in conjunction with other data sets) should determine Ω_0 to within 1%. Finding $\Omega_0<1.01$ would refute the Poincaré space as a cosmological model, while $\Omega_0>1.01$ would provide strong evidence in its favor.

6. Conclusions

We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i$$

It presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$\alpha_s^2=i^{2i}\cdot 10^7\cdot\alpha_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i}$$

The dimensionless unification of the strong nuclear,the weak nuclear and electromagnetic interactions:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot\alpha_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1$$

$$16\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=(e^{i/\alpha}+e^{-i/\alpha})^2$$

The dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4\cdot\alpha_s^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=i^{4i}$$

$$\alpha^2\cdot(e^{i/\alpha}+e^{-i/\alpha})\cdot\alpha_s^4\cdot\alpha_G\cdot N_A^2=i^{8i}$$

The dimensionless unification of of the weak nuclear,the gravitational and the electromagnetic interactions:

$$4\cdot 10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=i^{4i}\cdot e^2$$

$$10^{14}\cdot\alpha^2\cdot(e^{i/\alpha}+e^{-i/\alpha})^2\cdot\alpha_w^2\cdot\alpha_G\cdot N_A^2=i^{8i}$$

The dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot 10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2$$

$$8\cdot 10^7\cdot N_A^2\cdot\alpha_w\cdot\alpha^2\cdot\alpha_G=\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})$$

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron,the gravitational coupling constant of the proton $\alpha_{G(p)}$,the strong coupling constant α_s and the weak coupling constant α_w:

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2$$

$$\mu^2\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_{G(p)}\cdot N_A^2$$

$$\mu\cdot N_1\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^3\cdot N_A^2$$

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1$$

$$\mu^3\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_{G(p)}^2\cdot N_A^2\cdot N_1$$

$$\mu\cdot \alpha_s=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2$$

$$\mu\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_G\cdot \alpha_{G(p)}\cdot N_A^2\cdot N_1$$

We found the formula for the Gravitational constant:

$$G=(2e\alpha N_A)^{-2}\frac{\hbar c}{m_e^2}$$

$$G=i^{4i}(2\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2}$$

$$G=i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

$$G=\alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$l_{pl}^2\cdot \Lambda=(2\cdot e\cdot \alpha^2\cdot N_A)^{-6}$$

$$l_{pl}^2\cdot \Lambda=i^{12i}\cdot (2\cdot \alpha_s\cdot \alpha^2\cdot N_A)^{-6}$$

$$l_{pl}^2\cdot \Lambda=i^{12i}\cdot e^6\cdot (2\cdot 10^7\cdot \alpha_w\cdot \alpha^3\cdot N_A)^{-6}$$

$$e^6\cdot \alpha_s^6\cdot \alpha^6\cdot l_{pl}^2\cdot \Lambda=10^{42}\cdot \alpha_G^3\cdot \alpha_w^6$$

$$\alpha_s^{12}\cdot \alpha^6\cdot l_{pl}^2\cdot \Lambda=10^{42}\cdot i^{12i}\cdot \alpha_G^3\cdot \alpha_w^6$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} e^6 \left(2\,10^7 \alpha_w a^3 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \frac{c^3}{G\hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3$$

We presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities. Length l,time t,speed υ and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{\max}} = \frac{t_{min}}{t_{\max}} = \frac{\upsilon_{min}}{\upsilon_{\max}} = \frac{T_{min}}{T_{max}}$$

Energy E, mass M, action A,momentum P and entropy S have another min/max ratio,which is the square of α_g:

$$\alpha_g^2 = \frac{E_{min}}{E_{\max}} = \frac{M_{min}}{M_{\max}} = \frac{A_{min}}{A_{\max}} = \frac{P_{min}}{P_{\max}} = \frac{S_{min}}{S_{max}}$$

Force F has min/max ratio which is α_g^4:

$$\alpha_g^4 = \frac{F_{min}}{F_{max}}$$

Mass density has min/max ratio which is α_g^5:

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}}$$

Perhaps for the minimum distance l_{min} apply:

$$l_{min}=2\cdot e\cdot l_{pl}$$

The maximum distance l_{max} is:

$$l_{max}=L_H=c\cdot H_0^{-1}=\alpha_g^{-1}\cdot l_{min}$$

For the minimum mass M_{min} apply:

$$M_{min}=\frac{m_{pl}^2}{M_{max}}=\alpha_g m_{pl}=\frac{\alpha_G}{\alpha^3}m_e=\frac{\sqrt[3]{\alpha_g^2}}{\alpha}m_e$$

From the dimensionless unification of the fundamental interactions we discover a new simple Large Number Hypothesis which calculates the Mass,the Age and the Radius of the universe. The expressions for the mass of the observable universe are:

$$M_U=\alpha^{-1}\cdot\alpha_g^{-4/3}\cdot m_e=\alpha^3\cdot\alpha_G^{-2}\cdot m_e=(2\cdot e\cdot\alpha^2\cdot N_A)^2\cdot N_1\cdot m_p=\mu\cdot\alpha\cdot N_1^2\cdot m_p=1.153482\times10^{53}\text{ kg}$$

The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd}=\frac{M_U}{m_p}=\mu\alpha N_1^2=\frac{N_1}{\alpha_g^{\frac{2}{3}}}=\left(2e\alpha^2N_A\right)^2N_1=\left(\frac{r_e}{l_{pl}}\right)^2N_1=6.9\times10^{79}$$

The diameter of the observable universe will be calculated to be equal to the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2\cdot R_U=N_1\cdot\bar{\lambda}_c$$

The expressions for the radius of the observable universe are:

$$R_U=\frac{\alpha N_1}{2}\alpha_0=\frac{N_1}{2\alpha}r_e=\frac{1}{2\mu\alpha_G}r_e=\frac{m_{pl}^2r_e}{2m_em_p}=\frac{\hbar cr_e}{2Gm_em_p}=\frac{\alpha\hbar}{2Gm_e^2m_p}$$

We Found the value of the radius of the universe $R_U=4.38\times10^{26}$ m. The expressions for the radius of the observable universe are:

$$T_U=\frac{R_U}{c}=\frac{N_1r_e}{2\alpha c}=\frac{r_e}{2\mu\alpha_Gc}=\frac{\alpha N_1\alpha_0}{2c}=\frac{\alpha\hbar}{2cGm_e^2m_p}=\frac{\hbar r_e}{2Gm_em_p}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}}=\frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5}\times10^{-120}$$

Perhaps for the minimum time t_{min} apply:

$$t_{min}=2\cdot e\cdot t_{pl}$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter,dark matter and dark energy:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\%$$

$$\Omega_\Lambda=2\cdot e^{-1}=0.7357=73.57\%$$

$$\Omega_D=2\cdot e^{1-\pi}=2\cdot e\cdot i^{2i}=0.2349=23.49\%$$

The sum of the contributions to the total density parameter at the current time is $\Omega_0=1.0139$. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry,and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold,such as the Poincaré dodecahedral space,all of which are quotients of the 3-sphere. These results prove that the weather space is finite. The state equation w has value:

$$w=-24\cdot e^{-\pi}=-24\cdot i^{2i}=-1.037134$$

For as much as $w<-1$,the density actually increases with time. The Equations of the Universe are:

$$6e^{5\pi}\frac{G\hbar H_0^2}{c^5}=10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{11}}$$

$$e^{7\pi}\frac{G\hbar\Lambda^2}{cH_0^2}=6\cdot 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{13}}$$

References

[1] Pellis, Stergios, Unification Archimedes constant π, golden ratio φ, Euler's number e and imaginary number i (October 10, 2021)
http://dx.doi.org/10.2139/ssrn.3975869
[2] Pellis, Stergios, Exact formula for the Fine-Structure Constant α in Terms of the Golden Ratio φ (October 13, 2021)
http://dx.doi.org/10.2139/ssrn.4160769
[3] Pellis, Stergios, Fine-Structure Constant from the Golden Angle, the Relativity Factor and the Fifth Power of the Golden Mean (September 5, 2022)
http://dx.doi.org/10.2139/ssrn.4247343
[4] Pellis, Stergios, Exact expressions of the fine-structure constant (October 20, 2021)
http://dx.doi.org/10.2139/ssrn.3975861
[5] Pellis, Stergios, Fine-structure constant from the Archimedes constant (October 11, 2022)
http://dx.doi.org/10.2139/ssrn.4245208
[6] Pellis, Stergios, Fine-Structure Constant from the Madelung Constant (July 27, 2022)
http://dx.doi.org/10.2139/ssrn.4174644
[7] Pellis, Stergios, Exact mathematical expressions of the proton to electron mass ratio (October 10, 2021)
http://dx.doi.org/10.2139/ssrn.3967998
[8] Pellis, Stergios, Unity formula that connect the fine-structure constant and the proton to electron mass ratio (November 8, 2021)
http://dx.doi.org/10.2139/ssrn.3963425
[9] Pellis, Stergios, Exact mathematical formula that connect 6 dimensionless physical constants (October 17, 2021)
http://dx.doi.org/10.2139/ssrn.3963427
[10] Pellis, Stergios, Theoretical value for the strong coupling constant (January 1, 2022)
http://dx.doi.org/10.2139/ssrn.3998175
[11] Pellis, S. (2023) Unity Formulas for the Coupling Constants and the Dimensionless Physical Constants. Journal of High Energy Physics, Gravitation and Cosmology, 9, 245-294.
https://doi.org/10.4236/jhepgc.2023.91021
[12] Pellis, Stergios, Dimensionless Unification of the Fundamental Interactions (August 27, 2022)
http://dx.doi.org/10.2139/ssrn.4201780
[13] Pellis, Stergios, Unification of the fundamental interactions (2022)
DOI: 10.13140/RG.2.2.12296.70405
[14] Pellis, Stergios, Unification of the Fundamental Forces (2022)
DOI: 10.13140/RG.2.2.33651.60967
[15] Pellis, Stergios, Theoretical Value of the Gravitational Constant (May 7, 2023)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4442074
[16] Pellis, Stergios, Formula for the Gravitational constant (January 2023)

DOI: 10.13140/RG.2.2.19656.60166
[17] Pellis, Stergios, Dimensionless Solution for the Cosmological Constant (September 14, 2022)
http://dx.doi.org/10.2139/ssrn.4219292
[18] Pellis, Stergios, Unification of atomic physics and cosmology (2022)
DOI: 10.13140/RG.2.2.11493.88804
[19] Pellis, Stergios, The Equation of the Universe (2023)
DOI: 10.13140/RG.2.2.24768.40960
[20] Pellis, Stergios, Equation of state in cosmology (December 2022)
DOI: 10.13140/RG.2.2.17952.25609/1
[21] Pellis, Stergios, Maximum and Minimum Values for Natural Quantities (December 10, 2022)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4306280
[22] Pellis, Stergios, The coupling constant for the fifth force (April 2023)
DOI: 10.13140/RG.2.2.32481.99686
[23] Pellis, Stergios, Gravitational fine-structure constant (December 2022)
DOI: 10.13140/RG.2.2.34770.43206/2
[24] Pellis, Stergios, Dimensionless theory of everything (June 5, 2023)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4469668
[25] Pellis, Stergios, The Dimensionless Equations of the Universe (June 23, 2023)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4488688
[26] Pellis, Stergios, Unification of the Microcosm and the Macrocosm (April 24, 2023)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4429653
[27] The Dimensionless Fractal Universe (August 2023)
DOI: 10.13140/RG.2.2.25324.54407/1
[28] Pellis, Stergios, New Large Number Hypothesis of the universe (August 1, 2023)
Available at SSRN: http://dx.doi.org/10.2139/ssrn.4529088
[29] Forsythe, C. J. & Valev, D. T. (2014). Extended mass relation for seven fundamental masses and new evidence of large number hypothesis. Physics International, 5(2), 152-158.
https://doi.org/10.3844/pisp.2014.152.158
[30] Pellis, Stergios, The mass scale law of the Universe (August 2023)
DOI: 10.13140/RG.2.2.20474.88003/1
[31] R. Adler Comment on the cosmological constant and a gravitational alpha (2011)
https://arxiv.org/pdf/1110.3358.pdf
[32] L. Nottale, Mach's principle,Dirac's large numbers and the cosmological constant problem (1993)
http://www.luth.obspm.fr/%7Eluthier/nottale/arlambda.pdf
[33] Pellis, Stergios, Theoretical value for the Hubble Constant (August 2023)
DOI: 10.13140/RG.2.2.24461.10729/3
[34] J.-P. Luminet A cosmic hall of mirrors (2005)
https://arxiv.org/abs/physics/0509171
[35] Pellis, Stergios, Poincaré Dodecahedral Space Solution of The Shape of The Universe (December 31, 2022).
http://dx.doi.org/10.2139/ssrn.4316617
[36] Pellis, Stergios, The Shape of The Universe (March 2023).
doi: 10.13140/RG.2.2.16034.09922
[37] Pellis, Stergios,Solution for the cosmological parameters (December 2022).
DOI: 10.13140/RG.2.2.23917.67047/2
[38] Pellis, Stergios, Solution for the Density Parameter of Dark Energy (December 25, 2022).
http://dx.doi.org/10.2139/ssrn.4406279
[39] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq, J.-P. Uzan Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (2003)
https://arxiv.org/abs/astro-ph/0310253