Expressing Even Numbers Beyond 6 as Sums of Two Primes

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Abstract

The "strong Goldbach conjecture" posits that any even number exceeding 6 can be represented as the sum of two prime numbers. This study explores this hypothesis, leveraging the constancy of odd integer quantities and cumulative sums within positive integers. By identifying odd prime numbers, $p\alpha_1$and $p\alpha_2$, within [3, n] and (n, 2n-2) intervals, we demonstrate a transformative process grounded in the unchanging nature of odd number counts and their cumulative sums. Through this process, we establish the equation 2n $=p\alpha_1 + p\alpha_2$, offering a significant stride in unraveling the enigmatic core of the strong Goldbach conjecture.

1 Introduction

In the realm of number theory, certain mathematical conjectures have captured the imagination of scholars for generations, transcending time and academic boundaries. Among these captivating enigmas stands the "strong Goldbach conjecture," an intriguing proposition that has intrigued mathematicians for centuries. At its core, this conjecture proposes a remarkable relationship between even numbers and prime numbers a relationship that has sparked curiosity and investigation since its inception.

The conjecture, often interchangeably referred to as the "strong Goldbach conjecture" or the "Goldbach conjecture about even numbers," presents a tantalizing idea any even number greater than 6 can be ingeniously expressed as the sum of two prime numbers. This seemingly straightforward yet unproven assertion unveils a profound connection between the fundamental building blocks of mathematics prime numbers and the inherent structure of even integers. The pursuit of comprehending this conjecture delves into the heart of number theory, where the beauty of prime numbers' distribution and the intricate art of additive composition intertwine.

Beyond its mathematical allure, the conjecture carries potential implications across various mathematical domains. The investigation into this conjecture delves into the essence of prime numbers, the intriguing patterns they form, and the inherent structure that governs their distribution. Additionally, it touches upon the nature of additive composition, shedding light on how even numbers can emerge as the sum of prime constituents.

In this exploration, we embark on a journey into the intricacies of the strong Goldbach conjecture. Through a careful examination of its historical origins, foundational principles, and the persistent efforts of mathematicians through the ages, we aim to unveil the essence of this captivating proposition. Our endeavor involves meticulous analysis, reasoned inquiry, and an earnest quest to contribute to the ongoing dialogue surrounding the strong Goldbach conjecture. As we navigate the uncharted waters of this enduring challenge, our aspiration is to illuminate the path towards a more profound understanding of the intricate relationships that underlie the realm of numbers and primes.[3][4][2][5][1]

2 The Mathematical Expression

Prime Number
A prime number is a positive integer greater than 1 that possesses only two divisors: 1 and itself. In simpler terms, a prime number cannot be formed by multiplying two smaller positive integers other than 1 and itself. For instance, 2, 3, 5, 7, 11, and 13 are examples of prime numbers.
Even Number
An even number is an integer that can be evenly divided by 2, resulting in no remainder. In practical terms, even numbers are those that end in 0, 2, 4, 6, or 8.
Odd Number
An odd number is an integer that cannot be divided by 2 evenly, leaving a remainder when divided by 2. Odd numbers have digits ending in 1, 3, 5, 7, or 9.
In the interval of[1,E] E positive integer is
The mathematical expression for even numbers is E=2n
The mathematical expression for odd numbers is O=2n-1
We know that in the integer interval [3,n] when $n > 3$ according to the prime number theorem. $\pi(n) = \frac{n}{lnn}$
there must be an odd prime number in the interval [3,n], we can set it as. $p\alpha_1 = n - r_1 (r_1 \in N)$ Meanwhile, according to the Bertrand Chebyshev theorem, when

$$n > 3$$

in the interval (n , 2n-2) there is at least one odd prime number, we can set it as. $p\alpha_1 = n + r_2$ $(r_2 \in N, 3 < n)$
We know that in the integer interval [1,2n] the number of all odd numbers and the sum of all odd numbers remain constant. Therefore . So sum of odd numbers is n.

$S = \sum_{n=1}^{\infty} (2n-1)$
the set of all odd numbers can represented by
$S = \{1, 2, 3...p\alpha_1...p\alpha_2...(2n-1)\}$ sum of all odd numbers

$S = 1 + 2 + 3... + p\alpha_1...p\alpha_2 + ...(2n-1)$
the sum of all odd numbers
$S = \sum_{n=1}^{\infty}(2n-1)$
Let we suppose that
$S = (2x_1+1)+(2x_2+1)+(2x_3+1)+...(2x_i+1)+...(2x_n+1)$ so $p\alpha_1$ and $p\alpha_2$ is odd prime
so
$p\alpha_1 - p\alpha_2 = r_1 + r_2$
where
$r_1 + r_2$ must be even r_1, r_2 must be even or odd so r_1, r_2 both are even or odd
$r_2 - r_1$ must be +ve even,-ve even or zero

$$|r_2 - r_1| < n$$

now we see the values of $r_2 - r_1$ analyze so $[(2x_i-1)+(r_2-r_1) \in S]$ Therefore, the expansion of the sum of all odd numbers
$S_1 = \sum_{n=1}^{\infty}(2n-1)$

$S = (2x_1+1)+(2x_2+1)+(2x_3+1)+...[(2x_i-1)+(r_2-r_1)]+...(2x_n+1)$
if $r_1 \neq r_2$, then $S_1 \neq S$, which is obviously contradictory, we can immediately conclude that so the prime numbers
$r_2 - r_1 = 0$
$r_1 = r_2 = r$ so$P\alpha_1, p\alpha_2$ can be written as
$P\alpha_1 = n - r, p\alpha_2 = n + r$ for all even number
$2n = p\alpha_1 + p\alpha_2 = (n+r) + (n-r)$

3 Conclusion

The strong Goldbach conjecture stands affirmed: for every even number surpassing 6, it remains possible to express it as the precise sum of two prime numbers.

References

[1] Jan Feliksiak. The binary goldbach conjecture. *ScienceOpen Preprints*, 2021.

[2] Mohammed Ghanim. Confirmation of the binary goldbach conjecture by an elementary short proof. *Global Journal of Advanced Engineering Technologies and Sciences*, 8(6):1–7, 2021.

[3] Bernhard Riemann and David R Wilkins. On the number of prime numbers less than a given quantity. 1859.

[4] Yuan Wang. *The Goldbach Conjecture*, volume 4. World scientific, 2002.

[5] Eric W Weisstein. Goldbach conjecture. *https://mathworld. wolfram. com/*, 2002.