Common Points of Parallel Lines and Division by Zero Calculus

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Abstract: In this note, we will consider some common points of two parallel lines on the plane from the viewpoint of the division by zero calculus. Usually, we will consider that there are no common points or the common point is the point at infinity for two parallel lines. We will, surprisingly, introduce a new common point for two parallel lines from the viewpoint of the division by zero calculus.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, point at infinity, parallel lines.

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1 Introduction

In this note, we will consider some common points of two parallel lines on the plane from the viewpoint of the division by zero calculus. Usually, we will consider that there are no common points or the common point is the point at infinity for two parallel lines. We will introduce a new common point for two parallel lines from the viewpoint of the division by zero calculus.

2 Result

Theorem: *For two disjoint circles C_1 and C_2 with same radii on the plane, we consider the parallel lines that are common tangential lines for two circles. When we consider that the parallel lines were obtained by changing of the radius of the circle C_1 (C_2 is fixed), the common point of the parallel lines exists and it is given by the center point of the circle C_2 in the sense of the division by zero calculus.*

3 Proof of the Theorem

Firstly, we just recall the definition of the division by zero calculus. For a function $y = f(x)$ which is n order differentiable at $x = a$, we will **define** the value of the function, for $n > 0$

$$\frac{f(x)}{(x-a)^n}$$

at the point $x = a$ by the value

$$\frac{f^{(n)}(a)}{n!}.$$

For the important case of $n = 1$,

$$\frac{f(x)}{x-a}|_{x=a} = f'(a). \tag{3.1}$$

In particular, the values of the functions $y = 1/x$ and $y = 0/x$ at the origin $x = 0$ are zero. **We write them as $1/0 = 0$ and $0/0 = 0$, respectively.** Of course, the definitions of $1/0 = 0$ and $0/0 = 0$ are not usual ones in the sense: $0 \cdot x = b$ and $x = b/0$ in the usual sense (however, in the sense of the Moore-Penrose generalized solution, our definition is the same). Our division by zero is given in this sense and is not given by the usual sense as in stated in [1, 2, 3, 4].

In particular, note that for $a > 0$

$$\left[\frac{a^n}{n}\right]_{n=0} = \log a.$$

This will mean that the concept of division by zero calculus is important.

Without loss of generality, we consider two circles C_R and C_r on the x, y plane such that for fixed a with $0 < R + r < a$

$$C_R = \{(x, y); x^2 + y^2 = R^2\}$$

and

$$C_r = \{(x, y); (x - a)^2 + y^2 = r^2\}.$$

Then, for $R > r$, we have the common point $(X, 0)$ of the two tangential lines of two circles as

$$X = \frac{aR}{R - r} = a + \frac{ar}{R - r}.$$

Therefore, as a function in R for fixed r, by the division by zero calculus, we have the desired result $X = a$ for $R = r$.

Meanwhile, from the identity

$$X = -\frac{aR}{r - R},$$

as a function in r (R is fixed), by the division by zero calculus, we have the desired result $X = 0$ for $r = R$.

Remarks

For any two parallel lines, we can consider that its common point is the point at infinity by the steleographic projection to the Riemann sphere and the point at infinity is represented by 0 in the division by zero calculus mathematics and so we can consider that any parallel lines have the common point zero (the origin).

This fact was shown with many examples. See [1, 2, 3, 4].

In this sense, the common point which is introduced here is a special type common point for parallel lines. Its meanings are open problems.

4 Conclusion

A new concept in mathematics:

There are geometric shapes here. There are mathematical propositions that change depending on the process of creating the figure. This seems to have some philosophical meaning. (2023.9.10.6:38)

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