WHY EINSTEIN WAS RIGHT ABOUT UNIFIED FIELD THEORY?

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Abstract. Goal of physics is to arrive at new theory still rooted in old theory. From Newtonian physics to Einstein and most probably to quantum gravity and then to unified field theory that is most accepted view on how physics theories should progress. But what if this assumption is not true? What if from Newton to Einstein and then to Extended Einstein? What if final step does not take into account quantum physics? This seems absurd at first but I will explore this possibility. First question that arises is that basics or fundamental physics should stay at basic level same, basic physics is classical physics and then from it Relativity emerges as result of Maxwell equations. Then a new physic is born, quantum physics that changes the view of classical physics into probability theory with no way to say what will happen exactly. This approach was most successful theory of twenty century and to this day proves itself with amazing accuracy. On the other side of physics there is another successful theory that is general relativity, that proves itself with any experiment. But both theories are incomplete, quantum physics lacks gravity, relativity breaks down at singularities that emerge from it in crucial of universe evolution moment that is the moment of big bang itself. Most physics community effort goes into trying to quantize gravity one way or another, or in general unify quantum physics with gravity and extensions of standard model. But there should be equal amount of tries to do the opposite, try to make quantum physics consistent with gravity first. By that I mean that quantum should be relativized same way theory of relativity needs to be quantized. My approach will be even another one, I will assume that only step forward is to extend relativity into more general theory without need to quantize it at all. And that its generalization will be good enough to explain quantum effects. This may seem absurd at first but as I will show it gives pretty good results and in general is a good but very complicated model of spacetime curvature. Field a physical field should be threat as field a continuous one with no gaps with empty parts of that field. Same should apply to matter field, matter should be threat as one field that is continuous in all space and that is goal of this work.

Date: September 11, 2023.

Contents

1. Expanding Einstein

1.1. **EFE- Einstein field equations.**

Einstein field equations are basics theory of gravitation. They relate spacetime curvature with matter-energy content of universe. But problem with field equations is that they break at edge of a black hole, where all geodesic end. This geodesic incompleteness is a crucial problem that breaks down whole model in ultimate sense. Model works perfect till there is a singularity of a black hole or begin of universe. Another problem with that theory that is not a real problem but seems there could be something wrong is dark matter, as long as dark energy can be thought just as cosmological constant, problem with dark matter is that i could be new type of particle or something is missing in gravity theory.

EFE can be written as ten equations where key is Einstein tensor [1][2][3][4] that is derived from Banachi identity. This makes conservation of energy possible and energy momentum tensor zero covariant derivative, equation can be written:

$$G_{ab} = \kappa T_{ab} \tag{1.1}$$

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa T_{ab} \tag{1.2}$$

Where both parts of equation give zero covariant derivative that can be written, whee I will use covariant derivative with index up so it's multiply by metric tensor:

$$g^{bk} \nabla_k = \nabla^b \tag{1.3}$$

$$\nabla^b G_{ab} = \nabla^b \kappa T_{ab} = 0 \tag{1.4}$$

$$\nabla^b \left(R_{ab} - \frac{1}{2} R g_{ab} \right) = \kappa \nabla^b T_{ab} = 0 \tag{1.5}$$

Vacuum solutions are arrived at when energy momentum tensor [5] is equal to zero so when there is no matter present that is equal for vanishing of Einstein tensor to be equal to zero $G_{ab} = 0$. But it does not mean that spacetime is flat when Einstein tensor vanishes. So it means that matter field is not a continuous field and this will be key hint for extending that equation.

Where i did use normal letters for spacetime indexes, not Greek ones as it is written most of the time. But still convention I will be using here is that Latin letters have indexes ranging from zero to three. So they both represent space and time coordinates.

1.2. **EEFE- Extended Einstein field equations.** Now why does matter field should not vanish in any point? It seems absurd but its only way to arrive at extension of Einstein field equations. And it has one big advantage it can explain in principle dark matter. Matter will extend as a field in whole space, that could explain why there is missing matter in the universe, it's stored in field itself.

But where to start? That is key question, let first write EFE but expand all elements to curvature tensor:

$$R^c_{bcd} - \frac{1}{2} R^c_{ncm} g^{nm} g_{bd} = \kappa T_{bd} \tag{1.6}$$

Where I did use bd index as base one. Now we can see a reaping pattern here, there is curvature tensor then it's contractions. Only thing that does not follow this rule is energy momentum tensor that does not have any contractions. What if this symmetry of contraction is preserve so let me rewrite this equation:

$$R^c_{bcd} - \frac{1}{2} R^c_{ncm} g^{nm} g_{bd} = \kappa T^c_{bcd} \tag{1.7}$$

Now there is a clear pattern, in truth we are acting on four rank tensors in whole equation. On one side of equation there is curvature tensor and it's contractions that are a bit complex on left side there is some unknown tensor that reduces to energy momentum tensor by contraction. So EFE are not a final word as it can be seen we can calculate curvature tensor directly by using one metric tensor that will contraction cancel one contraction, and this will lead to:

$$g^{kc} R^c_{bcd} - \frac{1}{2} g^{kc} R^c_{ncm} g^{nm} g_{bd} = \kappa T^c_{bcd} \tag{1.8}$$

$$R^c_{bkd} - \frac{1}{2} R^c_{nkm} g^{nm} g_{bd} = \kappa T^c_{bkd} \tag{1.9}$$

$$R^c_{bkd} - \frac{1}{2} R^c_{k} g_{bd} = \kappa T^c_{bkd} \tag{1.10}$$

$$g_{pc} R^c_{bkd} - \frac{1}{2} g_{pc} R^c_{k} g_{bd} = g_{pc} \kappa T^c_{bkd} \tag{1.11}$$

$$R_{pbkd} - \frac{1}{2} R_{pk} g_{bd} = \kappa T_{pbkd} \tag{1.12}$$

EEFE can be written in two forms as seen here mixed and fully covariant, I will use fully covariant one for all calculations presented here but they are both equal from fact it's a tensor equation.

2. Properties of EEFE

2.1. **Conservation.** I can easy prove that indeed this new tensor has covariant derivative equal to zero on both side of equation [6], it's pretty simple task from fact how this tensor is derived. I will write EFE and i know that it's covariant derivative is zero so I can just multiply both side of equation by same metric tensor that will give me, where I use fact that covariant derivative of metric tensor is equal to zero so I threat it as a constant:

$$\nabla^d \left(R^c_{bcd} - \frac{1}{2} R^c_{ncm} g^{nm} g_{bd} \right) = \kappa \nabla^d T^c_{bcd} = 0 \tag{2.1}$$

$$\nabla^d g^{kc} \left(R^c_{bcd} - \frac{1}{2} R^c_{ncm} g^{nm} g_{bd} \right) = \kappa \nabla^d g^{kc} T^c_{bcd} = 0 \tag{2.2}$$

$$\nabla^d \left(R^c_{bkd} - \frac{1}{2} R^c_{nkm} g^{nm} g_{bd} \right) = \kappa \nabla^d T^c_{bkd} = 0 \tag{2.3}$$

Now contraction of those equation will naturally lead to EFE that is obvious fact from previous equations, but will write it:

$$g^{kp} R_{pbkd} - \frac{1}{2} g^{kp} R_{pk} g_{bd} = \kappa g^{kp} T_{pbkd} \tag{2.4}$$

$$R_{bd} - \frac{1}{2} R g_{bd} = \kappa T_{bd} \tag{2.5}$$

2.2. **Non vanishing energy momentum tensor and dimensions.** This new extended Einstein tensor has one key property, when matter field vanishes it gives a flat spacetime. I can prove it by writing that tensor and setting it zero:

$$R_{pbkd} - \frac{1}{2} R_{pk} g_{bd} = 0 \tag{2.6}$$

Now set indexes so Riemann tensor vanishes:

$$R_{pkkk} - \frac{1}{2} R_{pk} g_{kk} = 0 \tag{2.7}$$

$$-\frac{1}{2} R_{pk} g_{kk} = 0 \tag{2.8}$$

$$R_{pk} = 0 \tag{2.9}$$

Now plug it into equation gives that Riemann tensor is always zero:

$$R_{pbkd} = 0 \tag{2.10}$$

It means that energy momentum extended tensor has to not vanish at every point of space to not give flat spacetime. Finally last property is that there is only equal amount of unknowns in four dimensional spacetime. This can be easy seen from fact that Ricci tensor with metric tensor has total twenty components that are independent. And Riemann tensor gives twenty independent components in four dimensional spacetime. And to prove it's only case I can take number of independent Riemann components and set it equal to two times second order symmetric tensor components that gives:

$$\frac{n^2 \left(n^2 - 1\right)}{12} = n \left(n + 1\right) \tag{2.11}$$

$$\frac{n^4 - n^2}{12} = n^2 + n \tag{2.12}$$

$$\frac{n^3 - n}{12} = 1 + n \tag{2.13}$$

$$\frac{n^3 - n}{12} - n - 1 = 0 \tag{2.14}$$

Solution to this equation [7] is $4, -1, -3$ so from fact that number of dimensions is always a positive number there is only four left.

3. Predictions

3.1. **Dark Matter.** Dark matter is in this EEFE not a new type of particle but field of matter itself that extends in space. I will do simplest possible approximation of value of this field by using simple integral. First i start off by writing density with function of mass $M(R)$. Then will assume that in units of radius that gives surface of matter r_0 i will get rest mass contained in that radius. And finally write density dived by rest mass that will be close to one over radius to third. From it its straight forward to calculate integral that gives total mass change from surface to infinity [8]:

$$\rho = \frac{M(R)}{\frac{4}{3}\pi R^3} \tag{3.1}$$

$$\int_0^{r_0} \frac{M(R)}{\frac{4}{3}\pi} dr = M_0 \tag{3.2}$$

$$\frac{\rho}{\int_0^{r_0} M(R)\, dr} \approx \frac{1}{R^3} \tag{3.3}$$

$$\int_1^{\infty} \int_0^{\pi} \int_0^{2\pi} \frac{1}{R^3} dr d\theta d\varphi = \pi^2 \tag{3.4}$$

This is simplest calculation for dark matter that is just extend matter field. I assumed spherical coordinate system. In more general case, mass density function decreases proportional to radius cubed. From it I can calculate how much there is dark matter compared to normal matter:

$$\frac{\pi^2 - 1}{\pi^2} \approx 90\% \tag{3.5}$$

This whole dark matter calculation is a approximation but a good enough one.

3.2. **Universe expansion.** Good test of this theory is predicting the cosmological constant. From previous subsection approximation of amount of dark matter is equal to π^2 now I will use perfect fluid model but with pressure calculation that will be equal to one third escape velocity of universe squared dived by speed of light squared. I can write energy momentum tensor for perfect fluid as:

$$T_{00} = \rho_0 \pi^2 \tag{3.6}$$

$$T_{aa} = \frac{1}{3}\frac{2GM_0}{c^2 R}\rho_0 \pi^2 \tag{3.7}$$

Now lets go back to field equation, I will assume only diagonal elements of both metric tensor and Ricci tensor and energy momentum extended tensor, so field equation in this case turns into:

$$R_{pbpb} - \frac{1}{2}R_{pp}g_{bb} = \kappa T_{pbpb} \tag{3.8}$$

Now I want to calculate zero-zero component of Ricci tensor that will be equal to:

$$R_{0b0b} - \frac{1}{2}R_{00}g_{bb} = \kappa T_{pbpb} \tag{3.9}$$

$$g^{bb}R_{0b0b} - 2R_{00} = \kappa g^{bb}T_{0b0b} \tag{3.10}$$

$$R_{00} = -\kappa T_{00} \tag{3.11}$$

Rest of components I can calculate in same way so I will finally get [9]:

$$g^{bb}R_{pbpb} - \frac{1}{2}g^{bb}R_{pp}g_{bb} = \kappa g^{bb}T_{pbpb} \tag{3.12}$$

$$R_{pp} = -\kappa T_{pp} \tag{3.13}$$

Now I can divide Ricci tensor by Einstein constant to arrive at energy momentum part and i will sum elements of space part of energy momentum tensor [9]:

$$\frac{1}{\kappa}g^{ii}R_{ii} = -g^{ii}T_{ii} \tag{3.14}$$

$$\frac{1}{\kappa}R = -\left(\frac{2GM_0}{c^2 R}\right)\rho_0 \pi^2 \tag{3.15}$$

$$\frac{1}{\kappa}R = -2.1008 \cdot 10^{-27}[kg/m^3] \tag{3.16}$$

Now from previous equations I can calculate Ricci scalar as equal to:

$$R = -\kappa T \tag{3.17}$$

Now plug in all the numbers and i finally get [10] :

$$R = -\kappa \rho_0 \pi^2 \left(1 + \frac{2GM_0}{c^2 R}\right) \tag{3.18}$$

$$R = -1.166 \cdot 10^{-52}\ [m^{-2}] \tag{3.19}$$

References

[1] https://mathworld.wolfram.com/RicciCurvatureTensor.html

[2] https://mathworld.wolfram.com/RiemannTensor.html

[3] https://mathworld.wolfram.com/EinsteinFieldEquations.html

[4] https://mathworld.wolfram.com/EinsteinTensor.html

[5] https://www.astro.gla.ac.uk/users/martin/teaching/gr1/gr1_sec08.pdf

[6] https://mathworld.wolfram.com/ContractedBianchiIdentities.html

[7] https://www.wolframalpha.com/input?i=%28n%5E3-n%29%2F12-n-1%3D0

[8] https://www.wolframalpha.com/input?i=int+%5B%2F%2Fmath%3A1%2Frho%5E3%2F%2F%5D+%5B%2F%2Fmath%3Adrho+dphi+dtheta%2F%2F%5D+%2C+rho%3D%5B%2F%2Fmath%3A1%2F%2F%5D..%5B%2F%2Fmath%3Ainfinity%2F%2F%5D%2C+phi%3D%5B%2F%2Fmath%3A0%2F%2F%5D..%5B%2F%2Fmath%3Api%2F%2F%5D%2C+theta%3D%5B%2F%2Fmath%3A0%2F%2F%5D..%5B%2F%2Fmath%3A2pi%2F%2F%5D+

[9] https://www.wolframalpha.com/input?i=1.5*10%5E53+kg%2F%28%284.4*10%5E26+m%29%5E3*%284%2F3+pi+%29%29%28%282*1.5*10%5E53+kg%2Fplanck+mass%29%28planck+lenght%2F4.4*10%5E26m%29pi%5E2%29

[10] https://www.wolframalpha.com/input?i=1.5*10%5E53+kg%2F%28%284.4*10%5E26+m%29%5E3*%284%2F3+pi+%29%29%28%28%282*1.5*10%5E53+kg%2Fplanck+mass%29%28planck+lenght%2F4.4*10%5E26m%29pi%5E2%29%2Bpi%5E2%29%288+pi+G%2Fc%5E2%29

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