New Approaches to Riemann Hypothesis Solution

Miroslav Súkeník and Magdaléna Súkeníková
Slovak University of Technology, Radlinského 9, 812 37 Bratislava. Slovakia
(sukenik4@gmail.com)

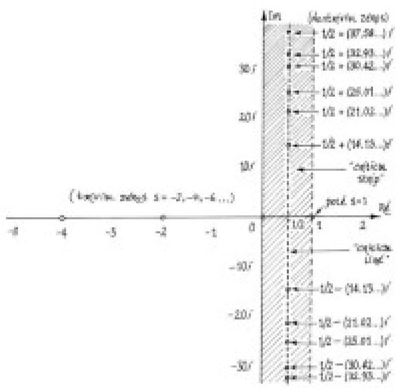
Abstract: *In the article, we assume that the Golden Ratio plays a fundamental role in a localosation of non-trivial zero points of the Riemann Zeta function on the critical line s = 0.5.*

1: Golden Ratio and Zeta Zeros

Using digital quantum tomography, in August 2023 it was possible to visualize a pair of entangled photons (they have opposite momentum and spin). [1] It is actually a display of the interference of the wave functions of both photons, for which a high-speed camera was used. Picture 1. shows a visualization that is identical to the well-known Tao symbol.

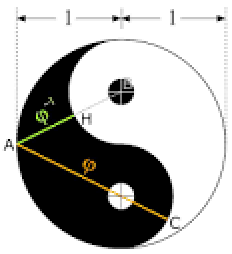
Picture 1: *Diphotons quantum entanglement visualisation*

For each pair of non-trivial zero points of the Zeta function, the same visualization should apply, because both complexly combined zero points are in mutual opposition, see fig. 2.



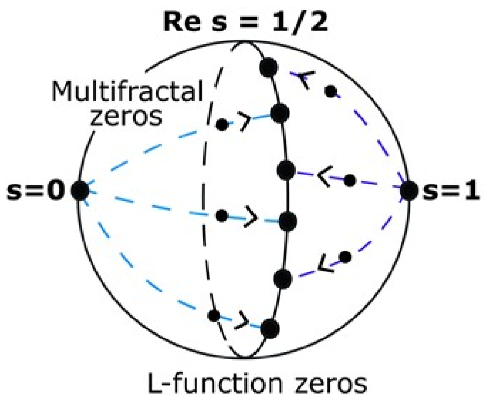
Picture 2: *Zero Points Zeta Functions*

Now we have to ask ourselves what the Tao symbol has to do with Riemann's Zeta function.
An interesting feature of the Tao symbol is the presence of the Golden Ratio, see fig. 3.



Picture 3: *Golden Ratio in Tao Symbol*

It is very interesting, but the Golden Ratio also appears in connection with the Zeta function [2], [3], [4]. If we create a Riemann Sphere in the critical belt with s = 0 and s = 1, the critical line s = 0.5 transforms into a circle, see Fig. 4



Picture 4: *Transformation of the critical line on the Riemann Sphere.*

There are strong indications that for complexly joined pairs of nontrivial zero points located on the Riemann Sphere with real value s = 0.5, the Golden Ratio always holds.

The Golden Ratio can be expressed in the form:

$$\tau - \frac{1}{\tau} = 1 \qquad (1)$$
$$\text{where } \tau = 1.61803\ldots$$

There is an inverse relationship between the Zeta function and the Moebius function

$$\frac{1}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s} \qquad (2)$$

Furthermore, the relationship between the Euler function phi and the Moebius function it holds:

$$\frac{\varphi(n)}{n} = \sum_{d/n} \frac{\mu(d)}{d} \tag{3}$$

It can be proved that the Golden Ratio can be expressed using reciprocal relations as follows:

$$\tau = 2\cos\left(\frac{\pi}{5}\right) = -\sum_{n=1}^{\infty} \frac{\varphi(n)}{n} \log\left(1 - \frac{1}{\tau^n}\right)$$

$$\frac{1}{\tau} = 2\sin\left(\frac{\pi}{10}\right) = -\sum_{n=1}^{\infty} \frac{\mu(n)}{n} \log\left(1 - \frac{1}{\tau^n}\right) \qquad (4), (5)$$

Relations (1) to (5) can then be used in the proof of the Riemann Hypothesis.

2: CONCLUSIONS

We assume that in order to place all non-trivial zeta zeros on the critical line, the Golden Ratio must be apply. There are cannot be any non-trivial zero point that lies outside the critical line and does not meet the Golden Ratio condition.

References:

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[2] Shaimaa Said Soltan „ Relation Between the Golden Ratio Phi and Zeta Function SUM"
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[3] Frantz Olivier „ The Riemann Hypothesis : The Vision and How We Proceed"
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