Proof of the Odd Golbach's Conjecture

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Abstract
In this work a proof is presented provided that the original Goldbach's conjecture(Goldbach's) has been verified.

Introduction

The "Odd Goldbach Conjecture," also known as the "Weak Goldbach Conjecture," states that every odd integer greater than 5 can be expressed as the sum of three prime numbers.

This conjecture is a variation of the original Goldbach Conjecture, which deals with the representation of even integers as the sum of two prime numbers. In the case of the Odd Goldbach Conjecture, you're looking for representations of odd numbers as the sum of three prime numbers.

For instance:

7 = 2 + 2 + 3
9 = 2 + 2 + 5
11 = 2 + 2 + 7 = 3 + 3 + 5
13 = 2 + 2 + 9 = 3 + 3 + 7 = 5 + 5 + 3
and so on...
Similar to the original Goldbach Conjecture, the Odd Goldbach Conjecture has been verified for many cases through computational searches and numerical evidence, but a rigorous proof or counterexample remains unknown. It's considered a challenging problem in number theory, and mathematicians continue to explore its intricacies in their pursuit of a solution.

In this work a proof is presented provided that the original Goldbach's conjecture(Goldbach's) has been verified.

Proof

Since Goldbach's conjecture is assumed to be true, it follows that for every natural number $n>1$, there exist two prime numbers a and b such that $a+b=2n$. This is also valid for $2(n+1)$.

Thus, we have $2(n+1)+1=2n+2+1=a+b+3$, representing an odd number greater than 5 as the sum of three primes. This result substantiates the expression of odd numbers as such triple prime sums. Given that the selection of n is arbitrary, the conjecture is thereby demonstrated.

Conclusion

A proof of Goldbach's conjecture holds the potential to yield numerous corollaries, similar to the one presented here, as well as implications for other conjectures like the Twin Primes Conjecture (Tao, Terry).

Reference

- Tao, Terry, Ph.D. (presenter) (7 October 2014). *Small and large gaps between the primes* (video lecture). UCLA Department of Mathematics – via YouTube.