

# Proof that the Center of Buoyancy is Equal to the Center of Hydrostatic Pressure (Part 2:) Semi-Submerged Circular Cylinder and Triangular Prism

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## Summary

We recently proved that “the center of buoyancy of floating bodies is equal to the center of hydrostatic pressure”. This subject was an unsolved problem in physics and naval architecture, even though the buoyancy taught by Archimedes' principle can be obtained clearly by the surface integral of hydrostatic pressure. Then we thought that the reason why the vertical position of the center of pressure could not be determined was that the horizontal force would be zero due to equilibrium in the upright state.

As a breakthrough, we dared to assume the left-right asymmetric pressure field by inclining the floating body with heel angle  $\theta$ . In that state, the force and moment due to hydrostatic pressure were calculated correctly with respect to the tilted coordinate system fixed to the body. By doing so, we succeeded in determining the center of pressure. Then, by setting the heel angle  $\theta$  to zero in order to make it upright state, it could be proved that the center of hydrostatic pressure is equal to the well-known center of buoyancy, *i.e.*, the centroid of the cross-sectional area under the water surface.

As noted above, we have already proved this problem for rectangular and arbitrarily shaped cross-sections, and published them here on *viXra.org* in English. Although the case of a semi-submerged circular cylinder and a triangular prism are also included in the proof of arbitrary shapes, we prove for each shape separately in this 2<sup>nd</sup> report, since they are two typical cross-sectional shapes along with rectangles. However, there is an essential difference in the proof between the two shapes. The reason is why the former does not change its underwater shape when inclined laterally, while the latter, like the rectangle, changes its cross-sectional shape when inclined. The present paper provides clear proofs for both shapes.

*Keywords* : **Center of Buoyancy, Hydrostatic Pressure, Archimedes' Principle,  
 Surface Integral, Semi-Submerged Circular Cylinder, Triangular Prism**

## 1. Introduction

It is a well-known fact in physics and naval architecture that the position of “*Center of Buoyancy*” acting on a ship is equal to the center of the volume of the geometric shape under the water surface.

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The buoyancy taught by Archimedes' principle<sup>(1)</sup> is clearly obtained by the surface integral of the hydrostatic pressure, but the position of the center of buoyancy is described in every textbook (on physics<sup>(2)</sup>, fluid dynamics<sup>(3),(4)</sup>, hydraulics<sup>(5)</sup>, naval architecture<sup>(6),(7),(8),(9),(10),(11)</sup> and nautical mechanics<sup>(12)</sup>, *etc.*) as the center of gravity where the volume under the water surface is replaced by water. There is no explanation that it is the center of pressure due to hydrostatic pressure<sup>(13),(14)</sup>.

Recently, Komatsu<sup>(15)</sup> raised the issue of “the center of buoyancy  $\neq$  the center of pressure?” at 2007 in Japan, and it was actively discussed by Seto<sup>(16),(17)</sup>, Suzuki<sup>(18)</sup>, Yoshimura and Yasukawa<sup>(19)</sup>, Komatsu<sup>(20)</sup>, Yabushita and Watanabe<sup>(21)</sup> and others in research committees and academic meetings of the Japan Society of Naval Architects and Ocean Engineers (hereinafter abbreviated as *JASNAOE*). At the same time, in Europe, the problem was studied in detail by M egel and Kliava<sup>(22),(23)</sup> in terms of potential energy. However, no one was able to solve this issue.

On the other hand, it is also an indisputable fact that the well-known center of buoyancy (*i.e.* the volume center of the underwater portion) is correct from the viewpoint of ship's hydrostatic stability<sup>(24)–(29)</sup> (that is to say, positioning of the metacenter by calculating the metacentric radius<sup>(30)–(33)</sup>  $\overline{BM}$  ).

In response to this unsolved problem, we considered that the reason why the vertical center of pressure could not be determined was because the horizontal forces equilibrated to zero in the upright state. To solve this problem, Hori<sup>(34),(35)</sup> attempted in 2018 to integrate the hydrostatic pressure acting on the ship surface at the inclined state with heel angle  $\theta$ . Then, the forces and moments acting on the ship were calculated with respect to a tilted coordinate system fixed to the ship. In this case, both orthogonal components of the force acting on the ship are not zero. Therefore, it was shown that the center of pressure at the inclined state can be determined. By setting the heel angle  $\theta$  to zero, we proved that the center of hydrostatic pressure coincides with the centroid of cross-sectional area under the water surface in the upright state, *i.e.*, the well-known center of buoyancy. First, a columnar ship with the rectangular cross-section<sup>(34)</sup> was proved and its proof is lectured<sup>(36),(37)</sup> to 2<sup>nd</sup> year students of the naval architectural engineering course<sup>(38),(39)</sup> in the “*Hydrostatics of Floating Bodies*” of the university where one of the authors<sup>(40)</sup> works. And then an arbitrary cross-sectional shape<sup>(35)</sup> was proved and published in the Journal “*NAVIGATION*” of Japan Institute of Navigation (hereinafter abbreviated as *JIN*).

In other way, as many researchers are studying this issue with various approaches<sup>(41)–(45)</sup>, the discussions have deepened in *JASNAOE*. To sublate these discussions, we have illustrated that “the center of buoyancy is equal to the center of pressure” for a semi-submerged circular cylinder<sup>1st half of (46)</sup> and a submerged circular cylinder<sup>(47)</sup> which does not change its shape under the water even if it is inclined, and for a triangular prisms<sup>(48)</sup>, using the same method<sup>(49)</sup>.

In order to put an end to the above discussions, we proved that “the center of buoyancy = the center of pressure” for a submerged body with arbitrary shape<sup>1st half of (50)</sup> using Gauss's integral theorem in 2021. Furthermore, it was published in the same journal “*NAVIGATION*” of *JIN* that it is easier to prove for a floating body with arbitrary shape<sup>2nd half of (50)</sup> than author's previous paper<sup>(35)</sup> by using Gauss's theorem in the same way<sup>(51)</sup>.

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We subsequently summarized the proofs in English for the case of the rectangular cross-section<sup>(34)</sup>, which is the easiest to understand, and for the floating body of arbitrary cross-sectional shape<sup>2nd half of (50)</sup> by applying Gauss's integral theorem. And we published them on this *viXra.org*<sup>(52)</sup> and in the bulletin of our university, *Nagasaki Institute of Applied Science*<sup>(53)</sup>. Furthermore, we showed an extension to the center of buoyancy for a 3-D floating body. More recently, the authors have summarized the above as a new developments for the fundamental theory of hydrostatics of floating body and published it here on *viXra.org*<sup>(54)</sup>.

As noted above, we have already proved this problem for rectangular and arbitrarily shaped cross-sections and published it here on *viXra.org*<sup>(52)</sup> in English. Although the case of a semi-submerged circular cylinder and a triangular prism are also included in the proof of arbitrary shapes<sup>(35),(49)~(54)</sup>, we prove for each shape separately in this 2<sup>nd</sup> report, since they are two typical cross-sectional shapes along with rectangles. However, there is an essential difference in the proof between the two shapes. The reason is why the former does not change its underwater shape when inclined laterally, while the latter, like the rectangle<sup>(34),(36),(37),(49),(52)~(54)</sup>, changes its cross-sectional shape when inclined. The present paper provides clear proofs for both shapes.

We would like to report all of you smart readers about the two proofs.

**2. Positioning of the Center of Hydrostatic Pressure  $C_p$   
Acting on the Semi-Submerged Circular Cylinder**

Fig. 2.1 shows that a cross-section of semi-submerged circular cylinder with radius  $R$  (breadth  $2R$  and draft  $R$ ) inclines laterally with a heel angle  $\theta$  to the starboard side. The origin  $o$  is placed at the center of the still water surface, and the coordinate system fixed in space with the z-axis pointing vertically downward is  $o-yz$ , and that fixed to the inclined circular cylinder is  $o-\eta\zeta$ .

If the argument measured counterclockwise from the  $\zeta$ -axis is  $\phi$  as shown in Fig. 2.1, then the argument of the water surface on the port side  $\phi_L$  and on the starboard side  $\phi_R$  can be written respectively, as follows :

$$\left. \begin{aligned} \phi_L &= -\frac{\pi}{2} + \theta \\ \phi_R &= \frac{\pi}{2} + \theta \end{aligned} \right\} \dots\dots\dots(2.1)$$

Here, the aerial part  $C_{air}$  and the submerged part  $C_{water}$  can be written in terms of argument  $\phi$ , respectively, as follows :

$$\left. \begin{aligned} C_{air} : \quad \phi_R \leq \phi \leq \phi_L + 2\pi &\rightarrow \frac{\pi}{2} + \theta \leq \phi \leq \frac{3\pi}{2} + \theta \\ C_{water} : \quad \phi_L \leq \phi \leq \phi_R &\rightarrow -\frac{\pi}{2} + \theta \leq \phi \leq \frac{\pi}{2} + \theta \end{aligned} \right\} \dots\dots\dots(2.2)$$

The water depth  $z(\phi)$  on the cylinder surface  $(\eta, \zeta) = (R \sin \phi, R \cos \phi)$  is then obtained as :

$$\begin{aligned}
 z(\phi) &= (\zeta + \eta \tan \theta) \cos \theta \\
 &= R(\cos \phi \cos \theta + \sin \phi \sin \theta) \\
 &= R \cos(\phi - \theta) \dots\dots\dots(2.3)
 \end{aligned}$$

Here, the notation in the 3<sup>rd</sup> line of the above equation is evident from Fig. 2.1.

And in the figure, the outward unit normal vector  $n$ , standing on the cylinder surface, can be written using the argument  $\phi$ , as follows :

$$\begin{aligned}
 \mathbf{n} &= n_\eta \mathbf{j} + n_\zeta \mathbf{k} \\
 &= \sin \phi \mathbf{j} + \cos \phi \mathbf{k} \dots\dots\dots(2.4)
 \end{aligned}$$

Here,  $n_\eta$  and  $n_\zeta$  are the directional cosines in the  $\eta$  and  $\zeta$  coordinates fixed to the cylinder, and  $\mathbf{j}$  and  $\mathbf{k}$  are the basic vectors in the  $\eta$  and  $\zeta$  directions, similarly.

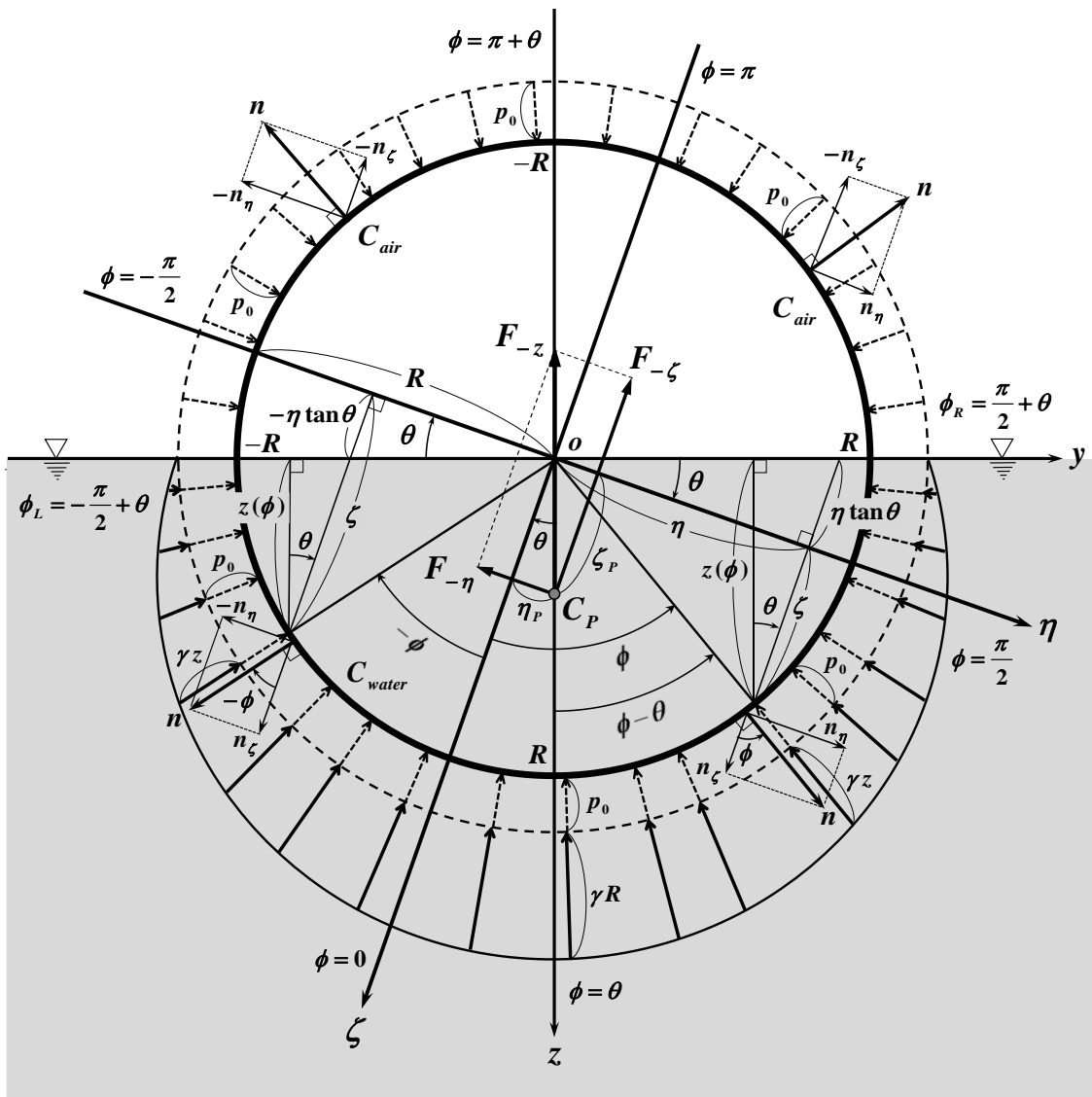


Fig. 2.1 Hydrostatic pressure and the center of pressure acting on the cross-section of an inclined semi-submerged circular cylinder.

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In Fig. 2.1, atmospheric pressure  $p_0$  is shown as a dashed vector and hydrostatic pressure  $\gamma z$  as a solid vector, and all are acting on  $-\mathbf{n}$  direction perpendicular to the cylinder surface. Here,  $\gamma$  is the specific gravity of water.

**2.1 Forces  $F_{-\eta}$  and  $F_{-\zeta}$  due to pressure in the  $-\eta$  and  $-\zeta$  directions acting on the cylinder surface**

The force  $F_{-\eta}$  in  $-\eta$  direction and the force  $F_{-\zeta}$  in the  $-\zeta$  direction due to pressure  $p$  acting on the cylinder surface are the sum of the force due to atmospheric pressure  $p_0$  acting in the aerial part  $C_{air}$  and the force due to hydrostatic pressure  $p_0 + \gamma z$  acting in the submerged part  $C_{water}$  respectively, and are obtained as follows :

$$\left. \begin{aligned} F_{-\eta} &= \oint_{C_{air}+C_{water}} p n_{\eta} d\ell = \int_{C_{air}} p_0 n_{\eta} d\ell + \int_{C_{water}} (p_0 + \gamma z) n_{\eta} d\ell \\ F_{-\zeta} &= \oint_{C_{air}+C_{water}} p n_{\zeta} d\ell = \int_{C_{air}} p_0 n_{\zeta} d\ell + \int_{C_{water}} (p_0 + \gamma z) n_{\zeta} d\ell \end{aligned} \right\} \dots\dots\dots(2.5)$$

Here, on the cylinder surface ( $\sqrt{\eta^2 + \zeta^2} = R$ ), the line element is  $d\ell = R d\phi$ , and the directional cosines in the  $\eta$  and  $\zeta$  directions can be written as  $n_{\eta} = \sin \phi$ ,  $n_{\zeta} = \cos \phi$  according to Eq.(2.4), so that for each part of  $C_{air}$  and  $C_{water}$ , both  $F_{-\eta}$  and  $F_{-\zeta}$  can be expressed by integration with respect to the argument  $\phi$  in the interval of Eq.(2.2).

Therefore,  $F_{-\eta}$  acting in the  $-\eta$  direction is expressed as :

$$\begin{aligned} F_{-\eta} &= \int_{\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} p_0 \sin \phi \cdot R d\phi + \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (p_0 + \gamma z) \sin \phi \cdot R d\phi \\ &= p_0 R \int_{-\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} \sin \phi d\phi + \gamma R \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z \sin \phi d\phi \\ &= \gamma R \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z(\phi) \sin \phi d\phi \quad \dots\dots\dots(2.6) \end{aligned}$$

Similarly,  $F_{-\zeta}$  acting in the  $-\zeta$  direction is expressed as :

$$\begin{aligned} F_{-\zeta} &= \int_{\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} p_0 \cos \phi \cdot R d\phi + \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (p_0 + \gamma z) \cos \phi \cdot R d\phi \\ &= p_0 R \int_{-\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} \cos \phi d\phi + \gamma R \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z \cos \phi d\phi \\ &= \gamma R \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z(\phi) \cos \phi d\phi \quad \dots\dots\dots(2.7) \end{aligned}$$

The results of the above equations for both  $F_{-\eta}$  and  $F_{-\zeta}$  show that the integral over the entire circumference of cylinder with respect to the atmospheric pressure  $p_0$  in the 1<sup>st</sup> term of 2<sup>nd</sup> line is zero and does not contribute to the force. Therefore, we can calculate only the 2<sup>nd</sup> term by using Eq.(2.3) for the water depth  $z(\phi)$ , so that  $F_{-\eta}$  is obtained as :

$$\begin{aligned}
 F_{-\eta} &= \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (\cos \theta \cos \phi + \sin \theta \sin \phi) \sin \phi \, d\phi \\
 &= \gamma R^2 \left\{ \cos \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin \phi \cos \phi \, d\phi + \sin \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin^2 \phi \, d\phi \right\} \\
 &= \frac{1}{2} \gamma R^2 \left\{ \cos \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin 2\phi \, d\phi + \sin \theta \left( \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} d\phi - \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \cos 2\phi \, d\phi \right) \right\} \\
 &= \gamma \frac{\pi R^2}{2} \sin \theta \quad \dots\dots\dots (2.8)
 \end{aligned}$$

And,  $F_{-\zeta}$  is obtained as :

$$\begin{aligned}
 F_{-\zeta} &= \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (\sin \theta \sin \phi + \cos \theta \cos \phi) \cos \phi \, d\phi \\
 &= \gamma R^2 \left\{ \sin \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin \phi \cos \phi \, d\phi + \cos \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \cos^2 \phi \, d\phi \right\} \\
 &= \frac{1}{2} \gamma R^2 \left\{ \sin \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin 2\phi \, d\phi + \cos \theta \left( \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} d\phi + \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \cos 2\phi \, d\phi \right) \right\} \\
 &= \gamma \frac{\pi R^2}{2} \cos \theta \quad \dots\dots\dots (2.9)
 \end{aligned}$$

These are because the integrals of  $\sin 2\phi$  and  $\cos 2\phi$  are zero, and the integral value of 2<sup>nd</sup> term is  $\pi$  in the 3<sup>rd</sup> line of both equations above. Both results indicate that  $F_{-\eta}$  and  $F_{-\zeta}$  are obtained as  $-\eta$  and  $-\zeta$  directional components of the buoyancy  $\gamma \frac{\pi R^2}{2}$ , as shown by  $F_{-z}$  of Eq. (2.10) in the next section.

### 2.2 Forces $F_{-y}$ and $F_{-z}$ converted in the $-y$ and $-z$ directions

By using  $F_{-\eta}$  and  $F_{-\zeta}$  obtained in Eqs. (2.8) and (2.9) of the previous section, the horizontal component  $F_{-y}$  and the vertical component  $F_{-z}$  are converted as follows :

$$\left. \begin{aligned}
 F_{-y} &= F_{-\eta} \cos \theta - F_{-\zeta} \sin \theta \\
 &= \gamma \frac{\pi R^2}{2} (\sin \theta \cdot \cos \theta - \cos \theta \cdot \sin \theta) = 0 \\
 F_{-z} &= F_{-\zeta} \cos \theta + F_{-\eta} \sin \theta \\
 &= \gamma \frac{\pi R^2}{2} (\cos^2 \theta + \sin^2 \theta) = \gamma \frac{\pi R^2}{2}
 \end{aligned} \right\} \dots\dots\dots (2.10)$$

The above results show that the horizontal component  $F_{-y}$  does not act as a combined force due to pressure integration. The vertical component  $F_{-z}$  is the product of the specific gravity  $\gamma$  of water and the area  $\frac{\pi R^2}{2}$  of semicircle below the water surface, and is indeed buoyant force itself acted vertically upward, as Archimedes' principle<sup>(1)</sup> teaches.

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**2.3 Moments  $M_\eta$  and  $M_\zeta$  due to pressure in the  $\eta$  and  $\zeta$  directions acting on the cylinder surface**

The clockwise moment  $M_\eta$  about the origin  $o$  due to the pressure  $p$  in the  $-\eta$  direction acting on the cylinder surface and the counterclockwise moment  $M_\zeta$  due to the pressure in the  $-\zeta$  direction can be obtained by integrating Eq. (2.5) multiplied by  $\zeta$  or  $\eta$  as the lever of the moment respectively, as follows :

$$\left. \begin{aligned} M_\eta &= \oint_{C_{air}+C_{water}} p n_\eta \cdot \zeta d\ell = \int_{C_{air}} p_0 n_\eta \cdot \zeta d\ell + \int_{C_{water}} (p_0 + \gamma z) n_\eta \cdot \zeta d\ell \\ M_\zeta &= \oint_{C_{air}+C_{water}} p n_\zeta \cdot \eta d\ell = \int_{C_{air}} p_0 n_\zeta \cdot \eta d\ell + \int_{C_{water}} (p_0 + \gamma z) n_\zeta \cdot \eta d\ell \end{aligned} \right\} \dots\dots\dots(2.11)$$

Here, if the above moments expressed in terms of integrals with respect to the argument  $\phi$  as in Eqs. (2.6) and (2.7) for  $F_{-\eta}$  and  $F_{-\zeta}$  in the previous section,  $M_\eta$  becomes as :

$$\begin{aligned} M_\eta &= \int_{\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} p_0 \sin \phi \cdot R \cos \phi \cdot R d\phi + \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (p_0 + \gamma z) \sin \phi \cdot R \cos \phi \cdot R d\phi \\ &= \frac{1}{2} p_0 R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} \sin 2\phi d\phi + \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z \sin \phi \cos \phi d\phi \\ &= \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z(\phi) \sin \phi \cos \phi d\phi \quad \dots\dots\dots(2.12) \end{aligned}$$

And,  $M_\zeta$  becomes as :

$$\begin{aligned} M_\zeta &= \int_{\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} p_0 \cos \phi \cdot R \sin \phi \cdot R d\phi + \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (p_0 + \gamma z) \cos \phi \cdot R \sin \phi \cdot R d\phi \\ &= \frac{1}{2} p_0 R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{3\pi}{2}+\theta} \sin 2\phi d\phi + \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z \sin \phi \cos \phi d\phi \\ &= \gamma R^2 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} z(\phi) \sin \phi \cos \phi d\phi \quad \dots\dots\dots(2.13) \end{aligned}$$

The above results show that both equations for  $M_\eta$  and  $M_\zeta$  are equivalent. Thus, the total counterclockwise moment  $M_o$  around the origin  $o$  due to pressure is zero as follows :

$$M_o = M_\zeta - M_\eta = 0 \quad \dots\dots\dots(2.14)$$

This is confirmed by the fact that the pressure acts perpendicular to the cylinder surface, so it is all directed toward the center of the circle.

Then, in both Eqs. (2.12) and (2.13), the integration of  $\sin 2\phi$  with respect to atmospheric pressure  $p_0$  in the 1<sup>st</sup> term of 2<sup>nd</sup> line is zero. Hence, we can calculate only the 2<sup>nd</sup> term by using Eq. (2.3) for the water depth  $z(\phi)$ , as follows :

$$\begin{aligned}
 M_\eta &= M_\zeta \\
 &= \gamma R^3 \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} (\cos \theta \cos \phi + \sin \theta \sin \phi) \sin \phi \cos \phi d\phi \\
 &= \gamma R^3 \left\{ \cos \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin \phi \cos^2 \phi d\phi + \sin \theta \int_{-\frac{\pi}{2}+\theta}^{\frac{\pi}{2}+\theta} \sin^2 \phi \cos \phi d\phi \right\} \dots\dots\dots(2.15)
 \end{aligned}$$

So, if we put  $p = \cos \phi$  for the 1<sup>st</sup> term and  $q = \sin \phi$  for the 2<sup>nd</sup> term and do a substitution integral for each, we obtain by folding the integral interval in half, as follows :

$$\begin{aligned}
 M_\eta &= M_\zeta \\
 &= 2 \gamma R^3 \left( \cos \theta \int_0^{\sin \theta} p^2 dp + \sin \theta \int_0^{\cos \theta} q^2 dq \right) \\
 &= 2 \gamma R^3 \left( \cos \theta \cdot \frac{1}{3} \sin^3 \theta + \sin \theta \cdot \frac{1}{3} \cos^3 \theta \right) \\
 &= \frac{2}{3} \gamma R^3 \sin \theta \cos \theta (\sin^2 \theta + \cos^2 \theta) = \frac{2}{3} \gamma R^3 \sin \theta \cos \theta \dots\dots\dots(2.16)
 \end{aligned}$$

**2.4 Positioning of the center of pressure  $C_p$  for the semi-submerged circular cylinder**

To locate the center of pressure  $C_p$  in  $o-\eta\zeta$  coordinate system fixed to circular cylinder, the hydraulic method used in the authors' previous papers<sup>(34),(35),(46)~(54)</sup> is applied. This method was used by Ohgushi<sup>(9)</sup> for an example problem of the rolling gate.

Since the forces  $F_\eta$  and  $F_{-\zeta}$  due to the hydrostatic pressure obtained in Section 2.1 act on the center of pressure  $C_p (\eta_p, \zeta_p)$ , the moments  $M_\eta$  and  $M_\zeta$  due to the same pressure obtained in Section 2.3 can be expressed respectively, as follows :

$$\left. \begin{aligned}
 M_\eta &= F_{-\eta} \zeta_p \\
 M_\zeta &= F_{-\zeta} \eta_p
 \end{aligned} \right\} \dots\dots\dots(2.17)$$

Therefore, the unknown coordinate  $(\eta_p, \zeta_p)$  of the center of pressure  $C_p$  can be determined by Eq. (2.17). Here, the  $\eta$ -coordinate,  $\eta_p$ , can be calculated by using Eq. (2.9) for  $F_{-\zeta}$  and the Eq. (2.16) for  $M_\zeta$  due to the hydrostatic pressure in the  $-\zeta$  direction, as follows :

$$\begin{aligned}
 \eta_p &= \frac{M_\zeta}{F_{-\zeta}} \\
 &= \frac{\frac{2}{3} \gamma R^3 \sin \theta \cos \theta}{\gamma \frac{\pi R^2}{2} \cos \theta} = \frac{4}{3\pi} R \sin \theta \dots\dots\dots(2.18)
 \end{aligned}$$

Similarly, the  $\zeta$ -coordinate,  $\zeta_p$ , can be calculated by using Eq. (2.8) for  $F_\eta$  and Eq. (2.16) for  $M_\eta$  due to the hydrostatic pressure in the  $-\eta$  direction, as follows :



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$$\begin{aligned} \zeta_p &= \frac{M_\eta}{F_{-\eta}} \\ &= \frac{\frac{2}{3} \gamma R^3 \sin \theta \cos \theta}{\gamma \frac{\pi R^2}{2} \sin \theta} = \frac{4}{3\pi} R \cos \theta \dots\dots\dots(2.19) \end{aligned}$$

Let us consider the above equations. For  $\zeta_p$  in Eq. (2.19), if we assume the upright state  $\theta = 0$  from the beginning,  $\sin \theta$  in the denominator  $F_{-\eta}$  and numerator  $M_\eta$  will be zero, so the fraction becomes indeterminate forms and  $\zeta_p$  cannot be determined. The reason is why we were able to locate the vertical component  $\zeta_p$  of the center of pressure, the semi-submerged cylinder was laterally inclined along with its  $\eta\zeta$  -coordinate axes, even though the shape did not change when inclined.

On the other hand, for  $\eta_p$  in Eq. (2.18), even if the heel angle is  $\theta = 0$  from the beginning, the denominator  $F_{-\zeta}$  can take a finite value because of  $\cos \theta = 1$ , and horizontal component  $\eta_p$  can be determined.

From the results of both equations above, the coordinates  $(\eta_p, \zeta_p)$  of the center of pressure  $C_p$  are determined as :

$$(\eta_p, \zeta_p) = \left( \frac{4}{3\pi} R \sin \theta, \frac{4}{3\pi} R \cos \theta \right) \dots\dots\dots(2.20)$$

The above  $(\eta_p, \zeta_p)$  coordinates fixed to the inclined cylinder are transformed to  $(y_p, z_p)$  coordinates fixed to space, as follows :

$$\left. \begin{aligned} y_p &= \eta_p \cos \theta - \zeta_p \sin \theta \\ &= \frac{4}{3\pi} R (\sin \theta \cdot \cos \theta - \cos \theta \cdot \sin \theta) = 0 \\ z_p &= \zeta_p \cos \theta + \eta_p \sin \theta \\ &= \frac{4}{3\pi} R (\cos^2 \theta + \sin^2 \theta) = \frac{4}{3\pi} R \end{aligned} \right\} \dots\dots\dots(2.21)$$

Therefore, the center of pressure  $C_p$  in the space-fixed coordinate is located as :

$$(y_p, z_p) = \left( 0, \frac{4}{3\pi} R \right) \dots\dots\dots(2.22)$$

This correctly indicates the figure centroid on the centerline (*i.e.*  $z$ -axis) of the semicircle below the water surface. Hence, it is proved that the center of hydrostatic pressure is equal to the well-known center of buoyancy, even for the shape of a semi-submerged circular cylinder.

### 2.5 Considerations

In the case of the semi-submerged circular cylinder in this chapter, the situation differs from that of a rectangle or an arbitrary cross-sectional shape in authors' previous papers<sup>(34),(35),(49)~(54)</sup> and of a triangular prism in the next chapter. The reason is why its geometrical shape under the water surface does not change even when the circular cylinder is inclined laterally. As a result, it is not necessary to determine the center of pressure in the upright position by setting the lateral inclination angle  $\theta$  to zero. So, its position can be computed by coordinate transformation, as shown in Eq. (2.21) of the previous section.

Therefore, it was also found that the center of pressure can be positioned by tilting the coordinate system in a way that it is shifted from the vertical direction, without inclining the floating body as advocated by Yabushita *et al.*<sup>(42)</sup>.

## 3. Positioning of the Center of Hydrostatic Pressure $C_p$ Acting on the Triangular Prism

Fig. 3.1 shows that a cross-section of triangular Prism (breadth  $2b$  draft  $f$ , freeboard  $h$ , vertex angle  $2\phi$ ) inclines laterally with a heel angle  $\theta$  to the starboard side. Here, the half breadth  $b$  of the waterline of the triangular prism in the upright state can be written, using the draft  $f$  and the half vertex angle  $\phi$ , as follows :

$$b = f \tan \phi \quad \dots\dots\dots(3.1)$$

Here, the cross-section of this triangular prism is an isosceles triangle with base (*i.e.* deck length)  $2(f + h)\tan \phi$ , height  $f + h$ , and both sides  $(f + h)\sec \phi$ .

### 3.1 Preparation calculations, including wetted lengths on both port and starboard sides

Let's consider the exposed triangle (port side,  $L$  for short)  $\Delta oE_L T_L$  and the immersed triangle (starboard side,  $R$  for short)  $\Delta oE_R T_R$  near the waterline in Fig. 3. 1. The heights  $q_L = \overline{U_L T_L}$  and  $q_R = \overline{U_R T_R}$  of each triangle can be expressed geometrically in two ways, using  $x_L = \overline{U_L E_L}$  and  $x_R = \overline{U_R E_R}$ , as follows :

$$\left. \begin{aligned} q_L &= (b - x_L) \tan \theta = \frac{x_L}{\tan \phi} \\ q_R &= (b + x_R) \tan \theta = \frac{x_R}{\tan \phi} \end{aligned} \right\} \dots\dots\dots(3.2)$$

Thus, for  $x_L$  and  $x_R$ , the following relations can be obtained respectively as :

$$\left. \begin{aligned} x_L &= (b - x_L) \tan \phi \tan \theta \\ x_R &= (b + x_R) \tan \phi \tan \theta \end{aligned} \right\} \dots\dots\dots(3.3)$$

Therefore,  $x_L$  and  $x_R$  can be solved by using the relation in Eq. (3.1) for the half breadth  $b$  as follows :



The waterline widths  $b_L$  and  $b_R$  on both the port and starboard sides can be obtained by using  $x_L$  and  $x_R$  in Eq. (3.3) as follows :

$$\left. \begin{aligned} b_L &= (b - x_L) \sec \theta = \frac{1}{1 + \varepsilon} f \tan \phi \sec \theta \\ b_R &= (b + x_R) \sec \theta = \frac{1}{1 - \varepsilon} f \tan \phi \sec \theta \end{aligned} \right\} \dots\dots\dots (3.8)$$

Thus, the total waterline breadth is written as :

$$b_L + b_R = \frac{2}{1 - \varepsilon^2} f \tan \phi \sec \theta \quad \dots\dots\dots (3.9)$$

Therefore, the area  $A$  of triangle  $\Delta KT_L T_R$  below the water surface of a triangular prism, which is inclined laterally with heel angle  $\theta$ , is obtained as follows :

$$A = \frac{1}{2} (b_L + b_R) \cdot f \cos \theta = \frac{1}{1 - \varepsilon^2} f^2 \tan \phi \quad \dots\dots\dots (3.10)$$

Since the underwater area  $A_0$  in the upright state ( $\theta = 0$  i.e.  $\varepsilon = 0$ ) is shown below, the underwater area  $A$  in the above inclined state is increased by  $\frac{\varepsilon^2}{1 - \varepsilon^2} A_0$  from the upright state.

$$A_0 \equiv A \Big|_{\theta=0} = f^2 \tan \phi \quad \dots\dots\dots (3.11)$$

### 3.2 Forces due to hydrostatic pressure acting on three surfaces around a triangular prism

Fig. 3.2 shows the pressure distribution and the forces generated by integrating it, acting on the cross-section of the triangular prism drawn in Fig. 3.1. The coordinate systems are  $o - yz$  fixed in space with the  $z$ -axis pointing vertically downward, and  $o - \eta\zeta$  fixed on the prism and tilted, both with the origin  $o$  at the center of still water surface.

The atmospheric pressure is denoted by  $p_0$  and the specific gravity of water is denoted by  $\gamma$ . The atmospheric pressure  $p_0$  is shown as a dashed line, and the hydrostatic pressure  $\gamma z$  as a solid line. The respective pressures are shown as thin vectors, and the forces as thick vectors. Then, all are acting perpendicularly to the surface of the triangular prism.

The depth  $Z_f$  at the vertex  $K$  of the triangle corresponding to the ship's bottom is denoted as :

$$Z_f = f \cos \theta \quad \dots\dots\dots (3.12)$$

The forces  $P_{Left}$  and  $P_{Right}$  acting on the port (subscripts in Left) and starboard (subscripts in Right) sides are obtained by summing the forces  $P_{Left}^{(0)}$ ,  $P_{Right}^{(0)}$  due to uniformly distributed atmospheric pressure acting on the entire port side and the forces  $P_{Left}^{(1)}$ ,  $P_{Right}^{(1)}$  due to the triangularly distributed hydrostatic pressure acting on the submerged part respectively, by using the wetted lengths  $\ell_L$ ,  $\ell_R$  in Eq. (3.7), as follows :

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$$\left. \begin{aligned}
 P_{Left} &= P_{Left}^{(0)} + P_{Left}^{(\gamma)} \\
 &= p_0(f+h)\sec\phi + \frac{1}{2}\gamma Z_f \ell_L \\
 &= p_0(f+h)\sec\phi + \frac{1}{2}\gamma f^2 \frac{\sec\phi \cos\theta}{1+\varepsilon} \\
 P_{Right} &= P_{Right}^{(0)} + P_{Right}^{(\gamma)} \\
 &= p_0(f+h)\sec\phi + \frac{1}{2}\gamma Z_f \ell_R \\
 &= p_0(f+h)\sec\phi + \frac{1}{2}\gamma f^2 \frac{\sec\phi \cos\theta}{1-\varepsilon}
 \end{aligned} \right\} \dots\dots\dots (3.13)$$

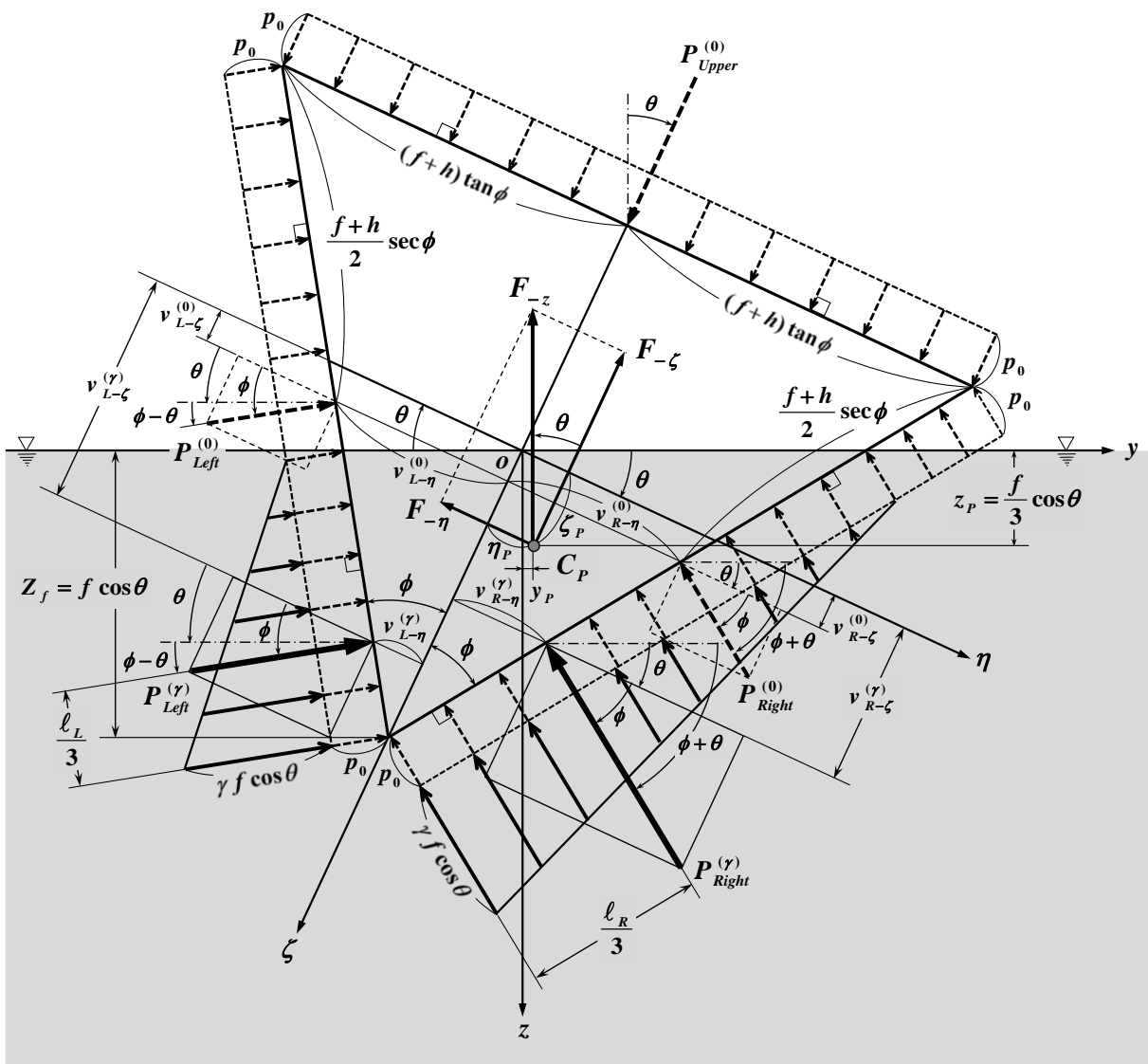


Fig. 3.2 Hydrostatic pressure and the center of pressure acting on the cross-section of an inclined triangular prism.

The force  $P_{Upper}$  acting on the deck (subscripts in Upper) is only  $P_{Upper}^{(0)}$  due to atmospheric pressure, so it is obtained as :

$$\begin{aligned}
 P_{Upper} &= P_{Upper}^{(0)} \\
 &= 2 p_0 (f + h) \tan \phi \dots\dots\dots(3.14)
 \end{aligned}$$

### 3.3 Combined forces $F_{-\eta}$ and $F_{-\zeta}$ in the $-\eta$ and $-\zeta$ directions acting on the prism surface

The combined forces  $F_{-\eta}$  and  $F_{-\zeta}$  acting in the  $-\eta$  and  $-\zeta$  directions fixed to the inclined floating prism are obtained by using  $P_{Left}$ ,  $P_{Right}$  and  $P_{Upper}$  in Eqs. (3.13) and (3.14), as follows :

$$\left. \begin{aligned}
 F_{-\eta} &= P_{Right} \cos \phi - P_{Left} \cos \phi \\
 &= \frac{1}{2} \gamma f^2 \cos \theta \left( \frac{1}{1-\varepsilon} - \frac{1}{1+\varepsilon} \right) + p_0 (f + h) - p_0 (f + h) \\
 &= \gamma f^2 \frac{\varepsilon}{1-\varepsilon^2} \cos \theta = \gamma A \sin \theta \\
 F_{-\zeta} &= P_{Right} \sin \phi + P_{Left} \sin \phi - P_{Upper} \\
 &= \frac{1}{2} \gamma f^2 \tan \phi \cos \theta \left( \frac{1}{1-\varepsilon} + \frac{1}{1+\varepsilon} \right) + 2 p_0 (f + h) \tan \phi - P_{Upper}^{(0)} \\
 &= \gamma f^2 \frac{\tan \phi}{1-\varepsilon^2} \cos \theta = \gamma A \cos \theta
 \end{aligned} \right\} \dots\dots\dots(3.15)$$

Here,  $F_{-\eta}$  and  $F_{-\zeta}$  of the above are obtained as the sine and cosine components of the buoyant force  $\gamma A$ , as shown by  $F_{-z}$  of Eq. (3.17) in the next section, with respect to the heel angle  $\theta$ . This result indicates that the atmospheric pressure  $p_0$  cancels out and does not contribute to the combined forces acting on the floating prism.

### 3.4 Forces $F_{-y}$ and $F_{-z}$ converted in the $-y$ and $-z$ directions

The horizontal component (in the  $-y$  direction)  $F_{-y}$  and the vertical component (in the  $-z$  direction)  $F_{-z}$  are obtained by coordinate transformation of  $F_{-\eta}$  and  $F_{-\zeta}$  in Eq. (3.15) of the previous section.

Then, the horizontal component  $F_{-y}$  is transformed as :

$$\begin{aligned}
 F_{-y} &= F_{-\eta} \cos \theta - F_{-\zeta} \sin \theta \\
 &= \gamma A (\sin \theta \cos \theta - \cos \theta \sin \theta) \\
 &= 0 \dots\dots\dots(3.16)
 \end{aligned}$$

From the above result, the horizontal component of the combined force does not generate even in an left-right asymmetric pressure field due to lateral inclination.

And, the vertical component  $F_{-z}$  is similarly transformed as :

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$$\begin{aligned}
 F_{-z} &= F_{-\zeta} \cos \theta + F_{-\eta} \sin \theta \\
 &= \gamma A (\cos^2 \theta + \sin^2 \theta) \\
 &= \gamma A \quad \dots\dots\dots(3.17)
 \end{aligned}$$

The above result shows that the vertical component is obtained by the product of the specific gravity  $\gamma$  of water and the cross-sectional area  $A$  under the water surface of the triangular prism shown in Eq. (3.10). This indicates that  $F_{-z}$  is the very buoyant force taught by Archimedes' principle<sup>(1)</sup>.

On the other hand, the  $F_{-y}$  and  $F_{-z}$  can also be obtained directly from  $P_{Left}$ ,  $P_{Right}$  and  $P_{Upper}$  in Eqs. (3.13) and (3.14), as follows :

First, the horizontal component  $F_{-y}$  is calculated as :

$$\begin{aligned}
 F_{-y} &= P_{Right} \cos(\phi + \theta) - P_{Left} \cos(\phi - \theta) + P_{Upper} \sin \theta \\
 &= \frac{1}{2} \gamma f^2 \sec \phi \cos \theta \left\{ \frac{\cos(\phi + \theta)}{1 - \varepsilon} - \frac{\cos(\phi - \theta)}{1 + \varepsilon} \right\} \\
 &\quad + p_0(f + h) \left[ \sec \phi \{ \cos(\phi + \theta) - \cos(\phi - \theta) \} + 2 \tan \phi \sin \theta \right] \\
 &= -\gamma f^2 \sec \phi \cos \theta \frac{\sin \phi \sin \theta - \varepsilon \cos \phi \cos \theta}{1 - \varepsilon^2} - 2 p_0(f + h) \sin \theta (\sin \phi \sec \phi - \tan \phi) \\
 &= 0 \quad \dots\dots\dots(3.18)
 \end{aligned}$$

Next, the vertical component  $F_{-z}$  is calculated as :

$$\begin{aligned}
 F_{-z} &= P_{Right} \sin(\phi + \theta) + P_{Left} \sin(\phi - \theta) - P_{Upper} \cos \theta \\
 &= \frac{1}{2} \gamma f^2 \sec \phi \cos \theta \left\{ \frac{\sin(\phi + \theta)}{1 - \varepsilon} + \frac{\sin(\phi - \theta)}{1 + \varepsilon} \right\} \\
 &\quad + p_0(f + h) \left[ \sec \phi \{ \sin(\phi + \theta) + \sin(\phi - \theta) \} - 2 \tan \phi \cos \theta \right] \\
 &= \gamma f^2 \sec \phi \cos \theta \frac{\sin \phi \cos \theta + \varepsilon \cos \phi \sin \theta}{1 - \varepsilon^2} + 2 p_0(f + h) \cos \theta (\sin \phi \sec \phi - \tan \phi) \\
 &= \gamma f^2 \sec \phi \cos \theta \frac{\sin \phi \sec \theta}{1 - \varepsilon^2} = \gamma f^2 \frac{1}{1 - \varepsilon^2} \tan \phi \\
 &= \gamma A \quad \dots\dots\dots(3.19)
 \end{aligned}$$

Both of the above equations cancel out the atmospheric pressure  $p_0$  and are identical to Eqs. (3.16) and (3.17) obtained by transforming the coordinates of  $F_{-\eta}$  and  $F_{-\zeta}$ . This confirms that the forces due to pressure in Section 3.2 have been calculated correctly.

**3.5 Moments  $M_\eta$  and  $M_\zeta$  due to pressure in the  $\eta$  and  $\zeta$  directions acting on the prism surface**

Consider the calculation of the moment  $M_\eta$  about the origin  $o$ , generated by the  $\eta$ -directional components of the forces  $P_{Left}$  and  $P_{Right}$  due to pressure acting perpendicularly on the sides of a triangular prism.

The levers  $v_{L-\zeta}^{(0)}$  and  $v_{R-\zeta}^{(0)}$  parallel to the  $\eta$  - axis on both the port and starboard sides by the atmospheric pressure components<sup>(0)</sup> of the uniform distribution are obtained as the same length on both sides, since the both side lengths including freeboard are  $(f + h) \sec \phi$ , as follows :

$$v_{L-\zeta}^{(0)} = v_{R-\zeta}^{(0)} = f - \frac{(f + h) \sec \phi}{2} \cos \phi = \frac{f - h}{2} \dots\dots\dots(3.20)$$

The levers  $v_{L-\zeta}^{(\gamma)}$  and  $v_{R-\zeta}^{(\gamma)}$  parallel to the  $\zeta$  - axis on both port and starboard sides due to the hydrostatic components<sup>(\gamma)</sup> of the triangular distribution are obtained by using the wetted lengths  $\ell_L$  and  $\ell_R$  in Eq. (3.7) as follows :

$$\left. \begin{aligned} v_{L-\zeta}^{(\gamma)} &= f - \frac{\ell_L}{3} \cos \phi = f - \frac{1}{3(1 + \varepsilon)} f = \frac{2 + 3\varepsilon}{3(1 + \varepsilon)} f \\ v_{R-\zeta}^{(\gamma)} &= f - \frac{\ell_R}{3} \cos \phi = f - \frac{1}{3(1 - \varepsilon)} f = \frac{2 - 3\varepsilon}{3(1 - \varepsilon)} f \end{aligned} \right\} \dots\dots\dots(3.21)$$

By the above two equations, the clockwise moment  $M_\eta$  due to pressure in the  $\eta$  - direction about the origin  $o$  can be obtained independently of the atmospheric pressure  $p_0$ , using Eqs. (3.13), (3.20) and (3.21), as follows :

$$\begin{aligned} M_\eta &= P_{Right}^{(0)} \cos \phi \cdot v_{R-\zeta}^{(0)} + P_{Right}^{(\gamma)} \cos \phi \cdot v_{R-\zeta}^{(\gamma)} \\ &\quad - (P_{Left}^{(0)} \cos \phi \cdot v_{L-\zeta}^{(0)} + P_{Left}^{(\gamma)} \cos \phi \cdot v_{L-\zeta}^{(\gamma)}) \\ &= \frac{1}{6} \gamma f^3 \cos \theta \left\{ \frac{2 - 3\varepsilon}{(1 - \varepsilon)^2} - \frac{2 + 3\varepsilon}{(1 + \varepsilon)^2} \right\} + p_0(f + h) \cdot (v_{R-\zeta}^{(0)} - v_{L-\zeta}^{(0)}) \\ &= \frac{1}{3} \gamma f^3 \frac{\varepsilon(1 - 3\varepsilon^2)}{(1 - \varepsilon^2)^2} \cos \theta = \frac{1}{3} \gamma f A \frac{1 - 3\varepsilon^2}{1 - \varepsilon^2} \sin \theta \dots\dots\dots(3.22) \end{aligned}$$

Next, the moment  $M_\zeta$  around point  $o$ , generated by  $P_{Upper}$  and the  $\zeta$  - directional components of  $P_{Left}$  and  $P_{Right}$ , is calculated.

The levers  $v_{L-\eta}^{(0)}$  and  $v_{R-\eta}^{(0)}$  parallel to the  $\eta$  - axis due to the atmospheric pressure components<sup>(0)</sup> are obtained as :

$$v_{L-\eta}^{(0)} = v_{R-\eta}^{(0)} = \frac{(f + h) \sec \phi}{2} \sin \phi = \frac{f + h}{2} \tan \phi \dots\dots\dots(3.23)$$

Here, the above equation, like Eq. (3.20), has the same length on both sides.

The levers  $v_{L-\eta}^{(\gamma)}$  and  $v_{R-\eta}^{(\gamma)}$  parallel to the  $\eta$  - axis on both the port and starboard sides due to the hydrostatic components<sup>(\gamma)</sup> can be obtained by using  $\ell_L$  and  $\ell_R$  in Eq. (3.7), as follows :

$$\left. \begin{aligned} v_{L-\eta}^{(\gamma)} &= \frac{\ell_L}{3} \sin \phi = \frac{\tan \phi}{3(1 + \varepsilon)} f \\ v_{R-\eta}^{(\gamma)} &= \frac{\ell_R}{3} \sin \phi = \frac{\tan \phi}{3(1 - \varepsilon)} f \end{aligned} \right\} \dots\dots\dots(3.24)$$



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Therefore, the counterclockwise moment  $M_\zeta$  due to pressure in the  $\zeta$  - direction about the origin  $o$  can be calculated by Eqs. (3.13), (3.23) and (3.24), as follows :

$$\begin{aligned}
 M_\zeta &= P_{Right}^{(0)} \sin \phi \cdot v_{R-\eta}^{(0)} + P_{Right}^{(\gamma)} \sin \phi \cdot v_{R-\eta}^{(\gamma)} \\
 &\quad - (P_{Left}^{(0)} \sin \phi \cdot v_{L-\eta}^{(0)} + P_{Left}^{(\gamma)} \sin \phi \cdot v_{L-\eta}^{(\gamma)}) + P_{Upper} \times 0 \\
 &= \frac{1}{6} \gamma f^3 \tan^2 \phi \cos \theta \left\{ \frac{1}{(1-\varepsilon)^2} - \frac{1}{(1+\varepsilon)^2} \right\} + p_0 (f+h) \tan \phi \cdot (v_{R-\eta}^{(0)} - v_{L-\eta}^{(0)}) \\
 &= \frac{2}{3} \gamma f^3 \frac{\varepsilon \tan^2 \phi}{(1-\varepsilon^2)^2} \cos \theta = \frac{2}{3} \gamma f A \frac{\tan^2 \phi}{1-\varepsilon^2} \sin \theta \quad \dots\dots\dots(3.25)
 \end{aligned}$$

Here,  $M_\zeta$ , like  $M_\eta$ , is obtained independently of the atmospheric pressure  $p_0$ .

**3.6 Positioning of the center of pressure  $C_p$   
for the triangular prism at lateral inclination**

The center of pressure  $C_p$  is located in  $o-\eta\zeta$  coordinate system fixed to the inclined triangular prism, as in the case of the semi-submerged circular cylinder in Chapter 2. According to the hydraulic method used by Ohgushi<sup>(9)</sup>, the moments and forces due to pressure are related by Eq. (2.17), assuming the coordinates  $(\eta_p, \zeta_p)$  of center of pressure.

Therefore, the  $\eta$  - coordinate  $\eta_p$  can be determined by the combined force  $F_{-\zeta}$  and moment  $M_\zeta$  due to pressure in the  $-\zeta$  - direction, by using Eq. (3.25) and the latter in Eq. (3.15), as follows :

$$\begin{aligned}
 \eta_p &= \frac{M_\zeta}{F_{-\zeta}} = \frac{\frac{2}{3} \gamma f A \frac{\tan^2 \phi}{1-\varepsilon^2} \sin \theta}{\gamma A \cos \theta} \\
 &= \frac{2}{3} f \frac{\varepsilon \tan \phi}{1-\varepsilon^2} \quad \dots\dots\dots(3.26)
 \end{aligned}$$

And, the  $\zeta$  - coordinate  $\zeta_p$  can be determined by the combined force  $F_{-\eta}$  and moment  $M_\eta$  due to pressure in the  $-\eta$  - direction, by using Eq. (3.22) and the former in Eq. (3.15), as follows :

$$\begin{aligned}
 \zeta_p &= \frac{M_\eta}{F_{-\eta}} = \frac{\frac{1}{3} \gamma f A \frac{1-3\varepsilon^2}{1-\varepsilon^2} \sin \theta}{\gamma A \sin \theta} \\
 &= \frac{1}{3} f \frac{1-3\varepsilon^2}{1-\varepsilon^2} \quad \dots\dots\dots(3.27)
 \end{aligned}$$

Considering the above,  $\zeta_p$  of vertical component can be obtained by offsetting the zero factor  $\sin \theta$  at the heel angle  $\theta \rightarrow 0$  with the denominator and numerator, as shown in Eq. (3.27). Here, if we start the calculation from the beginning as the upright state with  $\theta=0$ , both the denominator  $F_{-\eta}$  and the numerator  $M_\eta$  are in equilibrium and become zero, so the fraction becomes indeterminate forms and  $\zeta_p$  cannot be determined. This is the reason why we were able to determine the position of the center of

pressure in the  $\zeta$  - direction by inclining the floating body laterally.

On the other hand, in the calculation of  $\eta_p$  in Eq. (3.26), even if the heel angle is  $\theta=0$  from the beginning, the numerator  $M_\zeta$  is in equilibrium and zero, but the denominator  $F_{-\zeta}$  takes a finite value as the cosine component of the buoyancy. Therefore, the horizontal component  $\eta_p$  can be determine, even if we start the calculation as the upright state.

Let us now transform the resulting center of pressure  $C_p(\eta_p, \zeta_p)$  in the floating prism-fixed coordinates into the space-fixed coordinate system  $(y_p, z_p)$ .

First,  $y_p$  in the horizontal direction becomes as :

$$\begin{aligned} y_p &= \eta_p \cos \theta - \zeta_p \sin \theta \\ &= \frac{1}{3} f \frac{2 \varepsilon \tan \phi \cos \theta - (1 - 3 \varepsilon^2) \sin \theta}{1 - \varepsilon^2} \\ &= \frac{1}{3} f \frac{2 \tan^2 \phi + 3 \varepsilon^2 - 1}{1 - \varepsilon^2} \sin \theta \dots\dots\dots (3.28) \end{aligned}$$

Next,  $z_p$  in the vertical direction becomes as :

$$\begin{aligned} z_p &= \zeta_p \cos \theta + \eta_p \sin \theta \\ &= \frac{1}{3} f \frac{(1 - 3 \varepsilon^2) \cos \theta + 2 \varepsilon \tan \phi \sin \theta}{1 - \varepsilon^2} \\ &= \frac{1}{3} f \frac{(1 - 3 \varepsilon^2) + 2 \varepsilon^2}{1 - \varepsilon^2} \cos \theta = \frac{1}{3} f \cos \theta \dots\dots\dots (3.29) \end{aligned}$$

From the above results, it is clear that the latter  $z_p$  indicates the vertical position of figure centroid of a triangle of height  $f \cos \theta$ , with the water surface as its base. Hence, we will verify in the next section whether the former  $y_p$  also coincides with the horizontal position of figure centroid of underwater triangle.

### 3.7 Verification by the position of the figure centroid of the triangle below the water surface

Fig. 3.3 shows an extract of the area under the water surface for the cross - section of the triangular prism in Fig. 3.2. Let us divide the triangle  $\Delta K T_L T_R$  into two parts by the  $z'$  - axis connecting the vertex  $K$  of the triangle and the origin  $o'$  taken vertically above the vertex  $K$ .

For the Left triangle  $\Delta K o' T_L$ , the area is  $A_L$  and the base is  $y_L$ , and for the Right triangle  $\Delta K o' T_R$ , the area is  $A_R$  and the base is  $y_R$ . And the height is the common on both left and right triangles,  $o'K = Z_f$ .

In this case, the areas  $A_L$  and  $A_R$  of the left and right triangles respectively, are written as :

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$$\left. \begin{aligned} A_L &= \frac{1}{2} Z_f y_L \\ A_R &= \frac{1}{2} Z_f y_R \end{aligned} \right\} \dots\dots\dots(3.30)$$

The base of the triangle  $\Delta KT_L T_R$  can be written in the following two ways, by using  $y_L$  and  $y_R$  in Fig. 3.3 and  $b_L$  and  $b_R$  in Fig. 3.1.

$$y_L + y_R = b_L + b_R \quad (= \text{base of } \Delta KT_L T_R) \quad \dots\dots\dots(3.31)$$

Therefore, the area  $A$  of  $\Delta KT_L T_R$ , which is the sum of  $A_L$  and  $A_R$  above, is expressed by Eq. (3.10) in Section 3.1, as follows :

$$\begin{aligned} A &= A_L + A_R \\ &= \frac{1}{2} Z_f (y_L + y_R) \\ &= \frac{1}{2} f \cos \theta (b_L + b_R) = \frac{1}{1 - \varepsilon^2} f^2 \tan \phi \quad \dots\dots\dots(3.32) \end{aligned}$$

And,  $y_L$  and  $y_R$ , which correspond to the bases of the two halves of  $\Delta KT_L T_R$ , become respectively, using  $\phi$  and  $\theta$ , as follows :

$$\left. \begin{aligned} y_L &= Z_f \tan(\phi - \theta) = Z_f \frac{\tan \phi - \tan \theta}{1 + \varepsilon} \\ y_R &= Z_f \tan(\phi + \theta) = Z_f \frac{\tan \phi + \tan \theta}{1 - \varepsilon} \end{aligned} \right\} \dots\dots\dots(3.33)$$

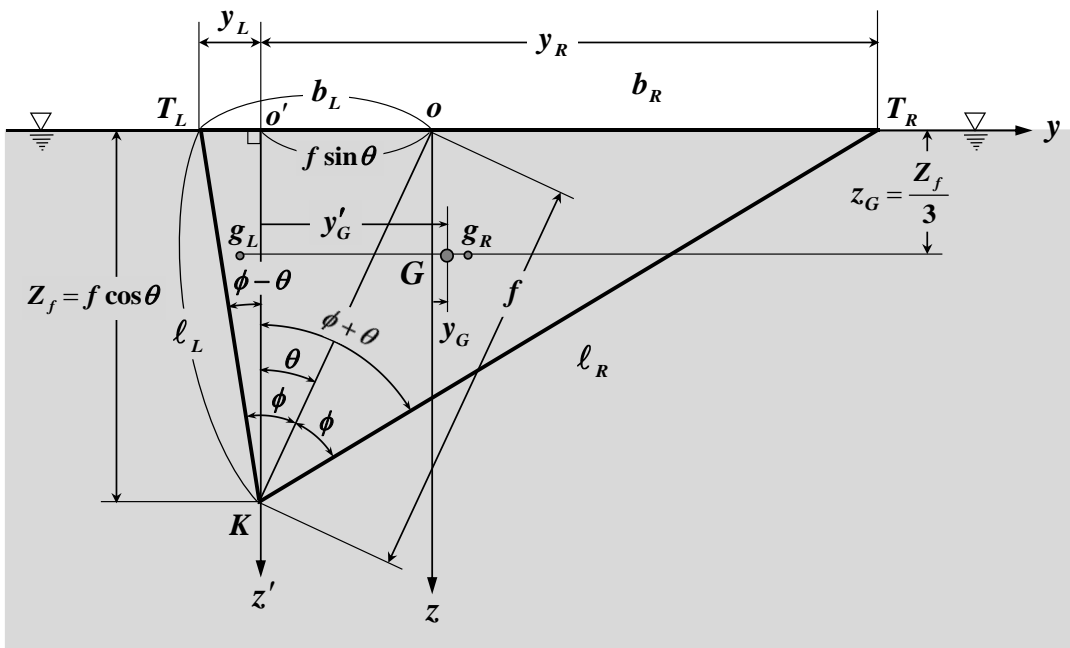


Fig. 3.3 Figure centroid of triangular cross-section below the water surface.

The areal moment  $M_z'$  of triangle  $\Delta KT_L T_R$  about the  $z'$ -axis can be obtained by using Eq. (3.30) for  $A_L$  and  $A_R$  as follows, since the horizontal distance from the  $z'$ -axis to the figure centroids  $g_L$  and  $g_R$  of the divided left and right triangles  $\Delta K o' T_L$  and  $\Delta K o' T_R$  respectively, is the lever of moment.

$$\begin{aligned} M_z' &= A_R \times \frac{y_R}{3} - A_L \times \frac{y_L}{3} \\ &= \frac{1}{6} Z_f (y_R^2 - y_L^2) \dots\dots\dots (3.34) \end{aligned}$$

Proceeding with the calculation, by using Eq. (3.33) for  $y_L$  and  $y_R$ , Eq. (3.12) for  $Z_f$ , and Eq. (3.5) for  $\varepsilon$ , the moment  $M_z'$  can be obtained in terms of  $A$  in Eq. (3.32), as follows :

$$\begin{aligned} M_z' &= \frac{1}{6} Z_f^3 \left\{ \frac{(\tan \phi + \tan \theta)^2}{(1 - \varepsilon)^2} - \frac{(\tan \phi - \tan \theta)^2}{(1 + \varepsilon)^2} \right\} \\ &= \frac{1}{6} Z_f^3 \frac{4 \varepsilon \sec^2 \phi \sec^2 \theta}{(1 - \varepsilon^2)^2} \\ &= \frac{2}{3} f^3 \frac{\tan \phi \sec^2 \phi}{(1 - \varepsilon^2)^2} \sin \theta = \frac{2}{3} f A \frac{\sec^2 \phi}{1 - \varepsilon^2} \sin \theta \dots\dots\dots (3.35) \end{aligned}$$

Therefore, the horizontal distance  $y_G'$  of the figure centroid  $G$  of triangular  $\Delta KT_L T_R$  from the  $z'$ -axis is determined by dividing  $M_z'$  in Eq. (3.35) by the area  $A$  in Eq. (3.32), as follows :

$$\begin{aligned} y_G' &= \frac{M_z'}{A} \\ &= \frac{2}{3} f \frac{\sec^2 \phi}{1 - \varepsilon^2} \sin \theta \dots\dots\dots (3.36) \end{aligned}$$

Finally, consider finding the horizontal distance  $y_G$  of the figure centroid  $G$  from the original  $z$ -axis. Here, the distance  $o'o$  between the two origin points becomes as follows, by using Fig. 3.3 or the former part of Eqs. (3.8) and (3.33).

$$\overline{o'o} = b_L - y_L = f \sin \theta \dots\dots\dots (3.37)$$

Hence,  $y_G$  is calculated by using Eqs. (3.36) and (3.37), as follows :

$$\begin{aligned} y_G &= y_G' - \overline{o'o} \\ &= y_G' - f \sin \theta \\ &= \frac{1}{3} f \frac{2 \sec^2 \phi - 3(1 - \varepsilon^2)}{1 - \varepsilon^2} \sin \theta = \frac{1}{3} f \frac{2 \tan^2 \phi + 3 \varepsilon^2 - 1}{1 - \varepsilon^2} \sin \theta \dots\dots\dots (3.38) \end{aligned}$$

On the other hand, the vertical distance  $z_G$  from the  $y$ -axis to the figure centroid  $G$ , need not be calculated, since  $\Delta KT_L T_R$  is a triangle of height  $Z_f = f \cos \theta$  whose base is the water surface (*i.e.*  $y$ -axis), and is obtained as :

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$$z_G = \frac{1}{3} f \cos \theta \quad \dots\dots\dots (3.39)$$

Thus, by comparing Eqs. (3.28) and (3.38) and Eqs. (3.29) and (3.39), we find as follows :

$$\left. \begin{array}{l} y_P = y_G \\ z_P = z_G \end{array} \right\} \dots\dots\dots (3.40)$$

This result proves that the center of hydrostatic pressure is the well-known position of the center of buoyancy, since it indicates that the center of pressure of the asymmetrical triangular cross - section at lateral inclination coincides with the figure centroid below the water surface.

**3.8 Positioning of the center of pressure  $C_p$  for the upright triangular prism**

In order to clarify the consequences obtained in Eq. (3.40) of the previous section, we find the position of the center of pressure  $C_p$  of the triangular prism in the upright state. As a final step, let us set  $\theta \rightarrow 0$  in the coordinates  $(\eta_p, \zeta_p)$  of the center of pressure obtained for the inclined state.

Here, if the heel angle  $\theta$  tends to zero,  $\varepsilon$  in Eq. (3.5) becomes as :

$$\varepsilon \Big|_{\theta=0} = \tan \phi \tan \theta \Big|_{\theta=0} = 0 \quad \dots\dots\dots (3.41)$$

Thus, by Eqs. (3.26) and (3.27) in Section 3.6,  $C_p$  is determined as follows :

$$C_p(\eta_p, \zeta_p) \Big|_{\theta=0} = \left( 0, \frac{f}{3} \right) \quad \dots\dots\dots (3.42)$$

Alternatively, since the  $o-\eta\zeta$  and  $o-yz$  coordinate systems coincide in the case of  $\theta \rightarrow 0$ , the following conclusion can be obtained by Eqs. (3.28) and (3.29) as well.

$$C_p(y_p, z_p) \Big|_{\theta=0} = \left( 0, \frac{f}{3} \right) \quad \dots\dots\dots (3.43)$$

Since the both of Eqs. (3.42) and (3.43) above clearly show the position of the figure centroid of the isosceles triangle below the water surface, it is proved that the center of hydrostatic pressure is the well-known center of buoyancy for the triangular prism.

**4. Concluding Remarks**

In this 2<sup>nd</sup> report, we proved that the center of hydrostatic pressure is equal to the well-known center of buoyancy (*i.e.* the figure centroid of the underwater area) for the typical cross-sectional shapes of semi-submerged circular cylinder and triangular prism, as in the case of the rectangular shape <sup>1st half of (53)</sup> reported earlier.

Although these two shapes are included in the proof of arbitrary shapes<sup>2nd half of (53)</sup>, there is an essential difference between the two proofs. The reason is why the former does not change its underwater shape when inclined laterally, while the latter, like the rectangle, changes its cross-sectional shape when inclined. The present paper provided clear proofs for both shapes.

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Finally from the 1<sup>st</sup> author<sup>(40)</sup>, let me express the following thanks. I would like to communicate my deepest gratitude to my late teacher, *Pr. Masato KURIHARA*<sup>(58)~(60)</sup>, who cordially taught me the theory of “*Hydrostatics of Ships*” with detailed figures and formulas on the blackboard when I was a 1<sup>st</sup> year undergraduate student and learned my 1<sup>st</sup> specialized subject of naval architecture in *the College of Naval Architecture of Nagasaki* in Japan. Therefore, I am following the appearance of my teacher at that time from more than 40 years ago as an exemplary example, when I currently lecture *Hydrostatics of Floating Bodies* and *Theory of Ship Stability* to 2<sup>nd</sup> year students at my university<sup>(38),(39)</sup>.

### References ‡

- (1) Archimedes : “The Works of Archimedes”, On Floating Bodies, Book I & Book II, Edited and Translated by Heath, T. L., ***Cambridge University Press***, 1897, p.253~300.
- (2) Kinbara, T. : “Fundamental Physics – Upper Volume –” (in Japanese), Chapter 7 : Fluid, Section 7.3 : Buoyancy, *Shouka-bou Publishing*, 1963 (March) 1<sup>st</sup>. Printing, p.144~145.
- (3) Tomochika, S. : “Fluid Dynamics” (in Japanese), Chapter 2 : Hydrostatics, *Gendai-Kougaku-sha Publishing*, 1972 (June) Reprinted Issuance, p.6~35.
- (4) Newman, J. N. : “Marine Hydrodynamics”, Chapter 6 : Waves and Wave Effects, Section 6.16 : Hydrostatics, *The MIT Press* (Cambridge, Massachusetts), 1977, p.290~295.
- (5) Honma, M. : “Standard Hydraulics” (in Japanese), Chapter 2 : Hydrostatics, Section 2.7 : Buoyancy, *Maruzen Publishing*, 1962 (September) 1<sup>st</sup>. Printing, p.22~23.
- (6) Bouguer, P. : “Traité du Navire, de sa Construction, et de ses Mouvements”, (Treatise of the Ship, its Construction and its Movements), Livre II, Section II, ***Edited by Jombert, C.***, Paris, 1746 (January), p.249~324.

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‡ Bold text in the list means that there is a HyperLink.

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(Part 2:) Semi-Submerged Circular Cylinder and Triangular Prism

- (7) Lewis, E. V. (Editor) : “Principles of Naval Architecture (2nd. Revision), Volume I - Stability and Strength”, Chapter 1 (written by Hamlin, N. A.) : Ship Geometry, Section 2. 1 : Archimedes’ Principle, p.16, Chapter 2 (written by Goldberg, L. L.) : Intact Stability, Section 1. 3 : Displacement and Center of Buoyancy, p.64, *The Society of Naval Architects and Marine Engineers*, Jersey City, NJ, 1988 (April) 1st Printing.
- (8) Nishikawa, H. : “Primary Hydrostatics of the ship” (in Japanese), Chapter 4 : Theory of Floating Bodies, Section 4. 2 : Buoyancy, *Kaibun-dou Publishing*, 1964 (July) 1<sup>st</sup> Printing. p.83~85.
- (9) Ohgushi, M. : “Theoretical Naval Architecture (Upper Volume) – New Revision –” (in Japanese), Chapter 1 : Water and Floating Body, Section 1.2 : Hydrostatic Pressure, p.1 ~ 3, Section 1.3 : Buoyancy, Example Problem, p.4~5, *Kaibun-dou Publishing*, 1971 (June) 1<sup>st</sup> Printing
- (10) Matora, S. (Supervisor) : “Kinematics of Ships and Offshore Structures” (in Japanese), Chapter 1 (written by Fujino, M.) : Hydrostatics of Floating Bodies, Section 1. 1 : Buoyancy, Section 1.2 : Static Equilibrium of Floating bodies, *Seizan-dou Publishing*, 1982 (November) 1<sup>st</sup> Printing, p.1~5.
- (11) Ferreiro, L. D. : “Ships and Science – The Birth of Naval Architecture in the Scientific Revolution, 1600-1800–”, Chapter 4 : Inventing the Metacenter, Archimedes and the Stability of Floating Bodies, *The MIT Press* (Cambridge, Massachusetts), 2006 , Hard-Cover, p.207~209.
- (12) Akedo, N. : “Basic Nautical Mechanics” (in Japanese), *Kaibun-dou Publishing*, 1983 (June) 1<sup>st</sup> Printing.  
(a) Chapter 2 : Dynamics of Rigid Bodies, Section 2.3.7 : Center of Buoyancy, p.110~111.
- (13) Hoste, P. : “Théorie de la Construction des Vaisseaux – Qui Contient Plusieurs Traitez, de Mathématique sur des Matières Nouvelles & Curieuses–”, *Chez Anisson & Posuel* (Lyon), 1697, p.1 ~211.
- (14) Nowachi, H. and Ferrero, L. D. : “Historical Roots of the Theory of Hydrostatic Stability of Ships”, 8<sup>th</sup>. International Conference on the Stability of Ships and Ocean Vehicles, *Escuela Técnica Superior de Ingenieros Navales*, 2003, pp.1~30.
- (15) Komatsu, M. : “Considerations on the Center of Action of Buoyancy Acting on a Floating body” (in Japanese), *Boat Engineering*, 2007 (December), No.93, pp.21~25.
- (16) Seto, H. : “Considerations of the Center of Buoyancy – Reexamination of Komatsu’s Considerations on the Center of Action of Buoyancy Acting on a Floating body –” (in Japanese), *Research Committee of Propulsion and Seakeeping Performance* (Japan Society of Naval Architects and Ocean Engineers), 2010 (October), No.14, pp.1~23.
- (17) Seto, H. : “Some Consideration on the Center of Action of Buoyancy” (in Japanese), *Conference proceedings of the Japan Society of Naval Architects and Ocean Engineers*, 2011 (May), Vol.12, No.2011S-G6-22, pp.529~532.
- (18) Suzuki, K. : “The Mythical Theory : the Law of the Center of Buoyancy” (in Japanese), *Private Note*, 2011 (January), pp.1~4.
- (19) Yoshimura, Y. and Yasukawa, H. : “Reconsideration of the Center of Action of Buoyancy and the Stability” (in Japanese), *Research Committee of Propulsion and Seakeeping Performance* (Japan Society of Naval Architects and Ocean Engineers), 2011, No.16.

- (20) Komatsu, M. : “ Considerations on the Center of Action of Buoyancy by means of Coordinate Transformation ” (in Japanese), *Research Committee of Propulsion and Seakeeping Performance* (Japan Society of Naval Architects and Ocean Engineers), 2012, No.19.
- (21) Yabushita, K. and Watanabe, R. : “ Relationship between the Pressure Distribution around Ship Hull and the Center of Buoyancy ” (in Japanese), *Research Committee of Propulsion and Seakeeping Performance* (Japan Society of Naval Architects and Ocean Engineers), 2013, No.21.
- (22) Mégel, J. and Kliava, J. : “ On the Buoyancy Force and the Metacentre ”, 2009 (June), *arXiv : 0906.1112*, Classical Physics, pp.1~33.
- (23) Kliava, J and Mégel, J. : “ Non-Uniqueness of the Point of Application of the Buoyancy Force ”, *European Journal of Physics*, 2010 (July), Volume 31, Number 4, pp.741~762.
- (24) Hori, T. : “ A Typical Example on Ship’s Stability Theorem ” (Exposition in Japanese), *NAVIGATION* (Journal of *Japan Institute of Navigation*), 2021 (July), **No.217**, pp. 39~46.
- (25) Hori, T., Hori, M. : “ Theoretical Treatment on the Hydrostatic Stability of Ships (Part 1:) Stable Conditions for the Upright State ”, 2022 (March), *viXra.org* (*Pre-print Repository*), **viXra:2203.0180** [Ver.4], Classical Physics, pp.1~16.
- (26) Hori, T. : “ Theoretical Procedure on the Hydrostatic Stability of Ships (Part 1:) Stable Conditions for the Upright State ”, *The Bulletin of Nagasaki Institute of Applied Science*, 2022 (December), **Vol.62, No.2**, Research Notes in Mathematical and Physical Science, pp.151~166.
- (27) Hori, T. : “ An Advanced Example on Ship’s Stability Theorem – Solution for Stable Attitude of an Inclined Ship – ” (Exposition in Japanese), *NAVIGATION* (Journal of *Japan Institute of Navigation*), 2021 (October), **No.218**, pp. 58~65.
- (28) Hori, T., Hori, M. : “ Theoretical Treatment on the Hydrostatic Stability of Ships (Part 2:) Stable Attitude in an Inclined State ”, 2023 (January), *viXra.org* (*Pre-print Repository*), **viXra:2301.0159** [Ver.2], Classical Physics, pp.1~15.
- (29) Hori, T. : “ Theoretical Procedure on the Hydrostatic Stability of Ships (Part 2:) Stable Attitude in an Inclined State ”, *The Bulletin of Nagasaki Institute of Applied Science*, 2023 (June), **Vol.63, No.1**, Research Notes in Mathematical and Physical Science, pp.55~69.
- (30) Hori, T. : “ A Consideration on Derivation of Ship’s Transverse Metacentric Radius  $\overline{BM}$  ” (in Japanese), *NAVIGATION* (Journal of *Japan Institute of Navigation*), 2017 (April), **No.200** (First 200th Anniversary Issue), pp.75~79.
- (31) Hori, T. : “ New Developments in the Fundamental Theory for Hydrostatics of Floating Bodies – Part 2 : A Consideration on Derivation of Ship’s Transverse Metacentric Radius  $\overline{BM}$  – ” (in Japanese), *Boat Engineering*, 2018 (December), **No.136**, pp.1~5.
- (32) Hori, T., Hori, M. : “ A New Theory on the Derivation of Metacentric Radius Governing the Stability of Ships ”, 2021 (November), *viXra.org* (*Pre-print Repository*), **viXra:2111.0023** [Ver.4], Classical Physics, pp.1~16.



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- (33) Hori, T. : “A New Theory on the Derivation of Metacentric Radius Governing the Hydrostatic Stability of Ships”, *The Bulletin of Nagasaki Institute of Applied Science*, 2022 (June), **Vol.62, No.1**, Research Notes in Mathematical and Physical Science, pp.51~67.
- (34) Hori, T. : “A Positioning on Ship’s Centre of Buoyancy Derived by Surface Integral of Hydrostatic Pressure –Proof that Centre of Buoyancy is Equal to Centre of Pressure–” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2018 (January), **No.203**, pp.88~92.
- (35) Hori, T. : “A Positioning on Ship’s Centre of Buoyancy Derived by Pressure Integral of Hydrostatic Pressure –Part 2 : In the Case of Arbitrary Sectional Form–” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2018 (July), **No.205**, pp.28~34.
- (36) Hori, T. : “Lecture Video Proving that Center of Buoyancy is Equal to Center of Pressure for the Rectangular Cross-Section (80 minutes in the 1<sup>st</sup> half) ” (in Japanese), 2021 (7 January), Regular Lecture No.13 of “*Hydrostatics of Floating Bodies*” (Specialized Subject of Naval Architectural Engineering Course in *Nagasaki Institute of Applied Science*), <https://youtu.be/Wd7jKMXSghc> .
- (37) Hori, T. : “Lecture Video Proving that Center of Buoyancy is Equal to Center of Pressure for the Rectangular Cross-Section (90 minutes in the 2<sup>nd</sup> half) ” (in Japanese), 2021 (14 January), Regular Lecture No.14 of “*Hydrostatics of Floating Bodies*” (Specialized Subject of Naval Architectural Engineering Course in *Nagasaki Institute of Applied Science*), <https://youtu.be/bniJ6-9vJPI> .
- (38) Hori, T. : “Naval Architecture Course, Department of Engineering , Faculty of Engineering, Nagasaki Institute of Applied Science” (in Japanese), Introduction of Educational and Research Institutes, *NAVIGATION* (Journal of *Japan Institute of Navigation*), 2021 (January), **No.215**, pp.38~45.
- (39) “Naval Architecture Course’s Web Site” (in Japanese), Faculty of Engineering in *Nagasaki Institute of Applied Science*, administrated by Hori, T. , <http://www.ship.nias.ac.jp/> .
- (40) Hori, T. : “Web Page on HORI’s Laboratory of Ship Waves and Hydrostatic Stability ” (in Japanese), Naval Architectural Engineering Course in *Nagasaki Institute of Applied Science*, [http://www2.cncm.ne.jp/~milky-jun\\_0267.h/HORI-Lab/](http://www2.cncm.ne.jp/~milky-jun_0267.h/HORI-Lab/) .
- (41) Yabushita, K. : “The Stability, Resistance and Propulsion of Ships ” (in Japanese), Chapter 4 – Relationship between Center of Buoyancy and Pressure Distribution, Self-made Textbook of Department of Mechanical System Engineering, *National Defense Academy*, 2018 (May), pp.81~96.
- (42) Yabushita, K. , Hibi, S. and Okahata, G. : “Identification of Center of Buoyancy Using Pressure Distribution on an Object ” (in Japanese), *Research Committee of Propulsion and Seakeeping Performance* (Japan Society of Naval Architects and Ocean Engineers), 2018 (June), No.10, SPRC 10-10, pp.1~14.
- (43) Suzuki, K. : “On the Apparent Center of Buoyancy –In Connection with the Hori’s Treatise<sup>(34)</sup>–” (in Japanese), *Private Note*, 2018 (April), pp.1~4.
- (44) Komatsu, M. : “Considerations on the Center of Action of Buoyancy Acting on a Floating Body (Follow-up Report)” (in Japanese), *Boat Engineering*, 2018 (December), No.136, pp.12~18.

- (45) Yabushita, K., Hibi, S. and Okahata, G. : “ Identification of Center of Buoyancy and Relation to Pressure Distributions ” (in Japanese), *Conference proceedings of the Japan Society of Naval Architects and Ocean Engineers*, 2020 (May), Vol.30, No.2020S-GS12-16, pp.629~636.
- (46) Hori, T. : “ A Positioning on Ship’s Centre of Buoyancy Derived by Surface Integral of Hydrostatic Pressure – Part 3 : The Proof for Semi-submerged Circular Cylinder – ” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2019 (April), **No.208**, pp.60~68.
- (47) Hori, T. : “ A Positioning on Ship’s Centre of Buoyancy Derived by Surface Integral of Hydrostatic Pressure – Part 5 : The Proof for Submerged Circular Cylinder – ” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2020 (October), **No.214**, pp.62~67.
- (48) Hori, T. : “ A Positioning on Ship’s Centre of Buoyancy Derived by Surface Integral of Hydrostatic Pressure – Part 4 : The Proof for Triangular Prism – ” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2020 (July), **No.213**, pp.50~58.
- (49) Hori, T. : “ New Developments in the Fundamental Theory for Hydrostatic of Floating Bodies – Part 1 : The Proof that Centre of Buoyancy is Equal to Centre of Pressure – ” (in Japanese), *Boat Engineering*, 2018 (September), **No.135**, pp.1~10.
- (50) Hori, T. : “ A Positioning on Ship’s Center of Buoyancy Derived by Surface Integral of Hydrostatic Pressure – Part 6 : The Proof for Submerged and Floating Body of Arbitrary Form – ” (in Japanese), *NAVIGATION* (Journal of Japan Institute of Navigation), 2021 (January), **No.215**, pp.69~77.
- (51) Hori, T. : “ New Developments in the Fundamental Theory for Hydrostatic of Floating Bodies – Part 3 : The Proof that Center of Buoyancy is Equal to Center of Pressure for Submerged and Floating Body of Arbitrary Form – ” (in Japanese), *Boat Engineering*, 2021 (June), **No.146**, pp.35~43.
- (52) Hori, T. : “ Proof that the Center of Buoyancy is Equal to the Center of Pressure by means of the Surface Integral of Hydrostatic Pressure Acting on the Inclined Ship ”, 2021 (September), *viXra.org* (*Pre-print Repository*), **viXra:2109.0008** [Ver.5], Classical Physics, pp.1~20.
- (53) Hori, T. : “ Proof that the Center of Buoyancy is Equal to the Center of Pressure by means of the Surface Integral of Hydrostatic Pressure Acting on the Inclined Ship ”, *The Bulletin of Nagasaki Institute of Applied Science*, 2022 (January), **Vol.61, No.2**, Research Notes in Mathematical and Physical Science, pp.135~154.
- (54) Hori, T., Hori, M. : “ Theoretical Hydrostatics of Floating Bodies — New Developments on the Center of Buoyancy, the Metacentric Radius and the Hydrostatic Stability — ”, 2023 (July), *viXra.org* (*Pre-print Repository*), **viXra:2307.0154** [Ver.2], Classical Physics, pp.1~102.
- (55) Kobayashi, Y. : “ Tank System of LNG-LH2 — Physical Model and Thermal Flow Analysis by Using CFD — ” (in Japanese), *Seizan-dou Publishing*, 2016 (December) 1<sup>st</sup> Printing, p.1~375.
- (56) Kobayashi, Y. : “ Utilization System of LNG-LH2 at Ultra-Low Temperature and Cold Heat — No Waste Energy System — ” (in Japanese), *Shouwa-dou Publishing*, 2019 (April) 1<sup>st</sup> Printing, p.1~226.
- (57) Kobayashi, Y. : “ LH2 Storage & Transportation Systems — Feed-forward to LNG Systems — ” (in Japanese), *Shouwa-dou Publishing*, 2023 (June) 1<sup>st</sup> Printing, p.1~398.

Proof that the Center of Buoyancy is Equal to the Center of Hydrostatic Pressure  
(Part 2:) Semi-Submerged Circular Cylinder and Triangular Prism

- (58) Kurihara, M. : “On the Rolling Motion of a Buoy in Regular Waves” (in Japanese), *The Bulletin of the College of Naval Architecture of Nagasaki*, 1974 (June), Vol.15, No.1, pp.1~4.
- (59) Kurihara, M. : “On the Motions of a Ringed Buoy in Regular Waves” (in Japanese), *The Bulletin of Nagasaki Institute of Applied Science*, 1978 (October), Vol.19 (Commemorative issue of the name change from former the College of Naval Architecture of Nagasaki), pp. 11~15.
- (60) Kurihara, M. : “On the Motions of a Floating Vertical Cylinder in Regular Waves” (in Japanese), *The Bulletin of Nagasaki Institute of Applied Science* (former the College of Naval Architecture of Nagasaki), 1979 (June), Vol.20, No.1, pp. 1~5.
-