Theoretical value for the Hubble Constant

Stergios Pellis
sterpellis@gmail.com
ORCID iD: 0000-0002-7363-8254
Greece
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Abstract

In this paper we will study the Hubble constant. From the dimensionless unification of the fundamental interactions will be calculated the theoretical value of the Hubble Constant $H_0=72.69$ (km/s)/Mpc. This value is very close to the last experimental measurements. Also will be presented the formulas of the Hubble constant.

Keywords

Hubble constant , Dimensionless unification of the fundamental interactions , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Cosmological parameters , Cosmological constant , Poincaré dodecahedral space

1. Introduction

Hubble's law is the observation in physical cosmology that galaxies are moving away from Earth at speeds proportional to their distance. In other words, the farther they are, the faster they are moving away from Earth. The velocity of the galaxies has been determined by their redshift, a shift of the light they emit toward the red end of the visible spectrum. Hubble's law is considered the first observational basis for the expansion of the universe, and today it serves as one of the pieces of evidence most often cited in support of the Big Bang model. The motion of astronomical objects due solely to this expansion is known as the Hubble flow. It is described by the equation $v=H_0 \cdot D$, with H_0 the constant of proportionality,the Hubble constant,between the "proper distance" D to a galaxy, which can change over time, unlike the comoving distance, and its speed of separation v, i.e. the derivative of proper distance with respect to the cosmological time coordinate. The Hubble constant is most frequently quoted in (km/s)/Mpc, thus giving the speed in km/s of a galaxy 1 megaparsec away, and its value is about 70 (km/s)/Mpc. However, crossing out units reveals that H_0 is a unit of frequency and the reciprocal of H_0 is known as the Hubble time. The Hubble constant can also be interpreted as the relative rate of expansion. In this form H_0= 7%/Gyr, meaning that at the current rate of expansion it takes a billion years for an unbound structure to grow by 7%. Although widely attributed to Edwin Hubble, the notion of the universe expanding at a calculable rate was first derived from general relativity equations in 1922 by Alexander Friedmann. Friedmann published a set of equations, now known as the Friedmann equations, showing that the universe might be expanding, and presenting the expansion speed if that were the case. Then Georges Lemaître, in a 1927 article, independently derived that the universe might be expanding,observed the proportionality between recessional velocity of, and distance to, distant bodies, and suggested an estimated value for the proportionality constant; this constant, when Edwin Hubble confirmed the existence of cosmic expansion and determined a more accurate value for it two years later, came to be known by his name as the Hubble constant. Hubble inferred the recession velocity of the objects from their redshifts, many of which were earlier measured and related to velocity by Vesto Slipher in 1917. Though the Hubble constant H_0 is constant at any given moment in time, the Hubble parameter H, of which the Hubble constant is the current value, varies with time, so the term constant is sometimes thought of as somewhat of a misnomer. The Hubble constant H_0 is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies,examine the same properties in more distant galaxies and galaxy clusters,infer the amount of dark matter in the universe,and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929,American astronomer Edwin Hubble announced his discovery that galaxies,in all directions,seemed to be moving away from us and have greater displacement for attenuated galaxies. The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is $H_0=67.66\pm0,42$ (km/s)/Mpc= $(2.1927664\pm0.0136)\times10^{-18}$ s^{-1}. H_0 is commonly called the "Hubble constant", but that is a misnomer since it is constant in space only at a fixed time; it varies with time in nearly all cosmological models, and all observations of far distant objects are also observations into the distant past, when the “constant” had a different value. "Hubble parameter" is a more correct term, with H_0 denoting the present-day value.

More recent measurements from the Planck mission published in 2018 indicate a lower value of 67.66±0.42 (km/s)/Mpc, although, even more recently, in March 2019, a higher value of 74.03±1.42 (km/s)/Mpc has been determined using an improved procedure involving the Hubble Space Telescope. The two measurements disagree at the 4.4σ level, beyond a plausible level of chance. The resolution to this disagreement is an ongoing area of active research. In October 2018, scientists presented a new third way (two earlier methods, one based on redshifts and another on the cosmic distance ladder, gave results that do not agree), using information from gravitational wave events (especially those involving the merger of neutron stars, like GW170817), of determining the Hubble constant. In July 2019, astronomers reported that a new method to determine the Hubble constant, and resolve the discrepancy of earlier methods, has been proposed based on the mergers of pairs of neutron stars, following the detection of the neutron star merger of GW170817, an event known as a dark siren. Their measurement of the Hubble constant is 73.3±5.3 (km/s)/Mpc. Also in July 2019, astronomers reported another new method, using data from the Hubble Space Telescope and based on distances to red giant stars calculated using the tip of the red-giant branch (TRGB) distance indicator. Their measurement of the Hubble constant is 69.8±1.9 (km/s)/Mpc. In February 2020, the Megamaser Cosmology Project published independent results that confirmed the distance ladder results and differed from the early-universe results at a statistical significance level of 95%. In July 2020, measurements of the cosmic background radiation by the Atacama Cosmology Telescope predict that the Universe should be expanding more slowly than is currently observed [1].

2. Dimensionless unification of the fundamental interactions

In [2] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. New interpretation and very accurate values of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden radio. We propose in [3] , [4] and [5] the exact formula for the fine-structure constant α with the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}=137.035999164... \tag{1}$$

Also we propose in [5] , [6] and [7] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2=137.035999078... \tag{2}$$

We propose in [8] the exact mathematical expressions for the proton to electron mass ratio μ:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \Rightarrow \mu=1836.15267343... \tag{3}$$

$$7\cdot\mu^3=165^3\cdot\ln^{11}10 \Rightarrow \mu=1836.15267392... \tag{4}$$

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}=1836.15267343... \tag{5}$$

Also in [8] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42) \tag{6}$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12 \tag{7}$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231 \tag{8}$$

$$\mu-807\cdot\alpha=1205\cdot\pi-518\cdot\varphi-411\cdot e \tag{9}$$

In [9] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot10^2\cdot\cos(\mu\cdot\alpha^{-1})+13^2=0 \tag{10}$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \tag{11}$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \tag{12}$$

It was presented in [10] the mathematical formulas that connects the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1 \tag{13}$$

$$\mu^2=4\cdot e^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{14}$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A^2 \tag{15}$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=1 \tag{16}$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{17}$$

$$\mu^2=4\cdot e^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \tag{18}$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{19}$$

In [11] we presented the recommended value for the strong coupling constant:

$$\alpha_s=\frac{Euler'\ number}{Gerford's\ constant}=\frac{e}{e^{\pi}}=e^{1-\pi}=0,11748.. \tag{20}$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. In the papers [12] , [13] , [14] and [15] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant α_s and the weak coupling constant α_w. We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s=10^7\cdot\alpha_w \tag{21}$$

$$\alpha_s^2=i^{2i}\cdot10^7\cdot\alpha_w \tag{22}$$

Resulting the unity formulas that connects the strong coupling constant α_s and the fine-structure constant α:

$$\alpha_s\cdot\cos\alpha^{-1}=i^{2i} \tag{23}$$

$$\cos\alpha^{-1}=\frac{\alpha_s^{-1}}{e^{\pi}} \tag{24}$$

The figure 1 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^{π}.

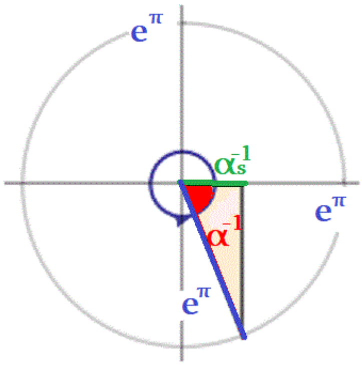

Figure 1. *The angle in α^{-1} radians.The rotation vector moves in a circle of radius e^{π}.*

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^{\pi}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2 \tag{25}$$

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i} \tag{26}$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

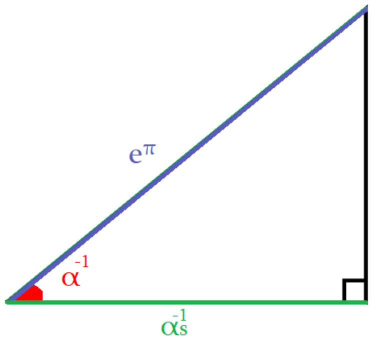

Figure 2. *Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.*

The electroweak theory,in physics,is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^{7}\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i} \tag{27}$$

The figure 3 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^{7}\cdot e^{\pi-1}$.

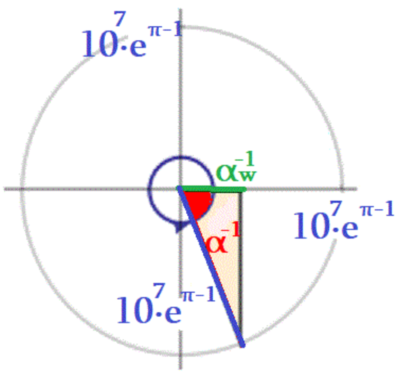

Figure 3. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^{7}\cdot e^{\pi-1}$.*

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.

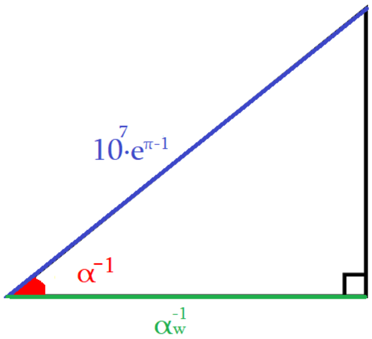

Figure 4. *Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions*

Resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = \alpha_s \tag{28}$$

$$\cos\alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7} \tag{29}$$

The figure 5 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7.

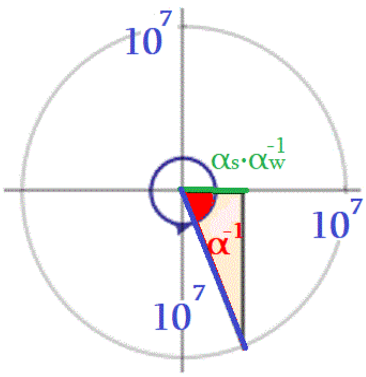

Figure 5. *The angle in α^{-1} radians.The rotation vector moves in a circle of radius 10^7.*

The figure 6 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.

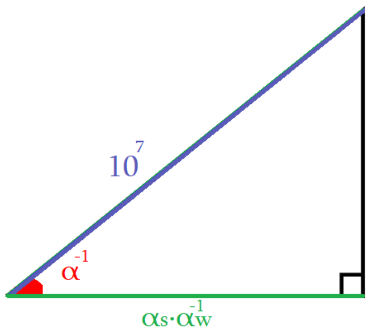

Figure 6. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.*

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha}+e^{-i/\alpha})=2 \cdot \alpha_s \quad (30)$$

Resulting the unity formula that connects the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1 \quad (31)$$

$$\alpha^{-2} \cdot \cos^2\alpha^{-1}=4 \cdot \alpha_G \cdot N_A^2 \quad (32)$$

The figure 7 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

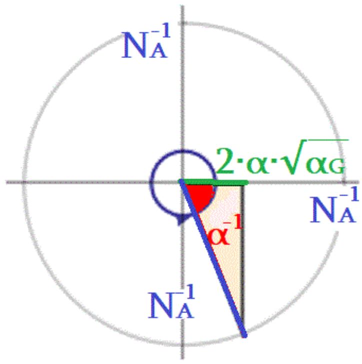

Figure 7. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.*

The figures 8 and 9 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

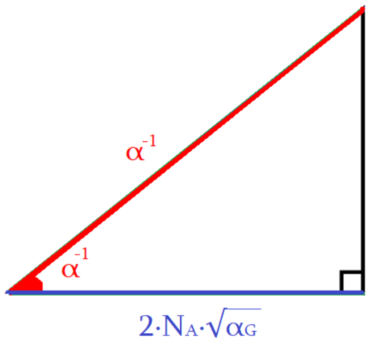

Figure 8. *First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions*

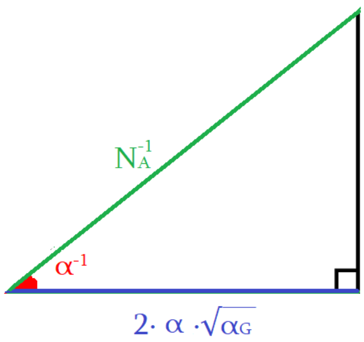

Figure 9. *Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1 \quad (33)$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/\alpha} + e^{-i/\alpha})^2 \quad (34)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \quad (35)$$

$$\alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha}) \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (36)$$

The figure 10 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.

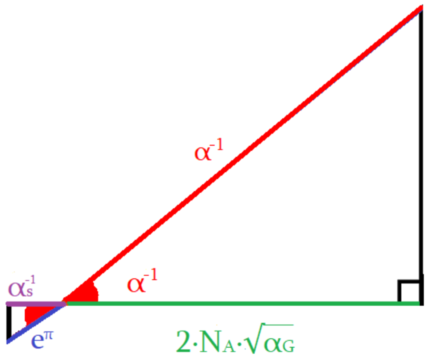

Figure 10. *Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions*

We reached the conclusion of the dimensionless unification of the weak nuclear,the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (37)$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i} \quad (38)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions.

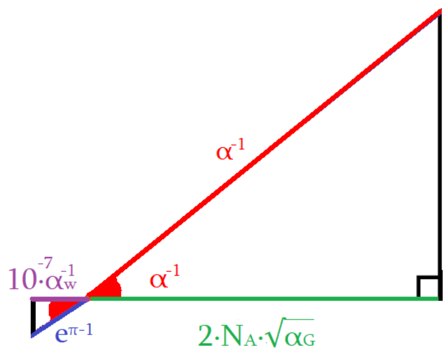

Figure 11. *Geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions*

Resulting the unity formula that connect the strong coupling constant α_s,the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant $\alpha_{G(p)}$ for the proton:

$$4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} = \mu^2 \cdot \alpha_s^2 \quad (39)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2 \tag{40}$$

$$8\cdot 10^7\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G=\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha}) \tag{41}$$

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions.

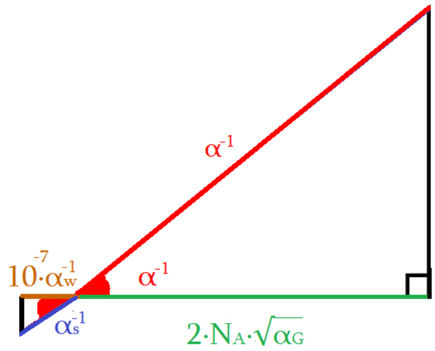

Figure 12. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions*

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron,the gravitational coupling constant of the proton $\alpha_{G(p)}$,the strong coupling constant α_s and the weak coupling constant α_w:

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2 \tag{42}$$

$$\mu^2\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_{G(p)}\cdot N_A^2 \tag{43}$$

$$\mu\cdot N_1\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^3\cdot N_A^2 \tag{44}$$

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1 \tag{45}$$

$$\mu^3\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{46}$$

$$\mu\cdot \alpha_s=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2 \tag{47}$$

$$\mu\cdot \alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha\cdot \alpha_G\cdot \alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{48}$$

These equations are applicable for all energy scales. The figure 13 and 14 below shows the geometric representation of the unification of the fundamental interactions.

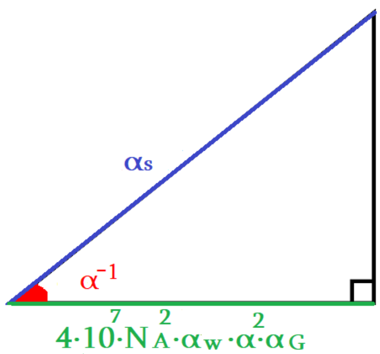

Figure 13. *Geometric representation of the unification of the fundamental interactions.*

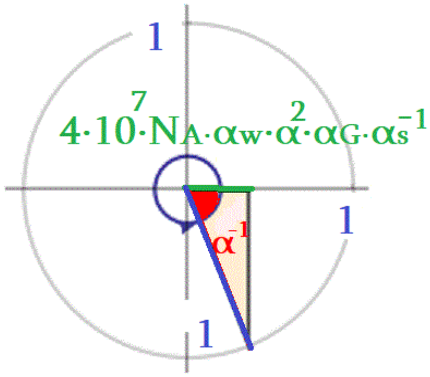

Figure 14. *The unification of the fundamental interactions.*

In [16] and [17] we found the expressions for the gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \tag{49}$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \tag{50}$$

$$G = i^{4i} e^2 \left(2\, 10^7 \alpha_w \alpha N_A\right)^{-2} \frac{\hbar c}{m_e^2} \tag{51}$$

$$G = \alpha_s^2 \left(2\, 10^7 \alpha_w \alpha N_A\right)^{-2} \frac{\hbar c}{m_e^2} \tag{52}$$

It presented the theoretical value of the Gravitational constant $G=6.67448\times10^{-11}$ $m^3/kg\cdot s^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring 6.674184×10^{-11} $m^3/kg\cdot s^2$ and 6.674484×10^{-11} $m^3/kg\cdot s^2$-of-swinging and angular acceleration methods, respectively.

3. Dimensionless unification of atomic physics and cosmology

In [18] and [19] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant α,which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant α_g. It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant α_g is an equivalent way to express the biggest issue in theoretical physics. The mysterious value of the gravitational fine-structure constant α_g is an equivalent way to express the biggest issue in theoretical physics. The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3} = \sqrt{\frac{\alpha_G^3}{\alpha^6}} = 1.886837 \times 10^{-61} \tag{53}$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5} \times 10^{-60} = 1{,}886837 \times 10^{-61} \tag{54}$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3} \tag{55}$$

$$\alpha_g=i^{6i}\cdot(2\cdot \alpha_s\cdot \alpha^2\cdot N_A)^{-3} \tag{56}$$

$$\alpha_g=i^{6i}\cdot e^3\cdot(2\cdot 10^7\cdot \alpha_w\cdot \alpha^3\cdot N_A)^{-3} \tag{57}$$

$$\alpha_g=(10^7\cdot \alpha_w\cdot \alpha_G^{1/2}\cdot e^{-1}\cdot \alpha_s^{-1}\cdot \alpha^{-1})^3 \tag{58}$$

$$\alpha_g^2=(10^{14}\cdot \alpha_w^2\cdot \alpha_G\cdot e^{-2}\cdot \alpha_s^{-2}\cdot \alpha^{-2})^3 \tag{59}$$

$$\alpha_g=10^{21}\cdot i^{6i}\cdot \alpha_w^3\cdot \alpha_G^{3/2}\cdot \alpha_s^{-6}\cdot \alpha^{-3} \tag{60}$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g^2=10^{42}\cdot i^{12i}\cdot \alpha_w^6\cdot \alpha_G^3\cdot \alpha_s^{-12}\cdot \alpha^{-6} \tag{61}$$

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$l_{pl}^2\cdot\Lambda=(2\cdot e\cdot \alpha^2\cdot N_A)^{-6} \tag{62}$$

$$l_{pl}^2\cdot\Lambda=i^{12i}\cdot(2\cdot \alpha_s\cdot \alpha^2\cdot N_A)^{-6} \tag{63}$$

$$l_{pl}^2\cdot\Lambda=i^{12i}\cdot e^6\cdot(2\cdot 10^7\cdot \alpha_w\cdot \alpha^3\cdot N_A)^{-6} \tag{64}$$

$$e^6\cdot \alpha_s^6\cdot \alpha^6\cdot l_{pl}^2\cdot\Lambda=10^{42}\cdot \alpha_G^3\cdot \alpha_w^6 \tag{65}$$

$$\alpha_s^{12}\cdot \alpha^6\cdot l_{pl}^2\cdot\Lambda=10^{42}\cdot i^{12i}\cdot \alpha_G^3\cdot \alpha_w^6 \tag{66}$$

For the cosmological constant Λ equals:

$$\Lambda=\left(2e\alpha^2\mathrm{N_A}\right)^{-6}\frac{c^3}{G\hbar} \tag{67}$$

$$\Lambda=i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{68}$$

$$\Lambda=i^{12i}e^6\left(2\,10^7\alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{69}$$

$$\Lambda=10^{42}\left(\frac{\alpha_G\alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{70}$$

$$\Lambda=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{71}$$

In [20] we found the Equations of the Universe:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \tag{72}$$

$$e^{6\pi} \frac{\Lambda G\hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \tag{73}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120} \tag{74}$$

In [21] , [22] and [23] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy. The sum of the contributions to the total density parameter Ω_0 at the current time is $\Omega_0=1.02\pm0.02$. Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda=0.73, \Omega_D=0.23$ and $\Omega_B=0.04$. The figure 15 shows the Geometric representation of the density parameter for the baryonic matter.

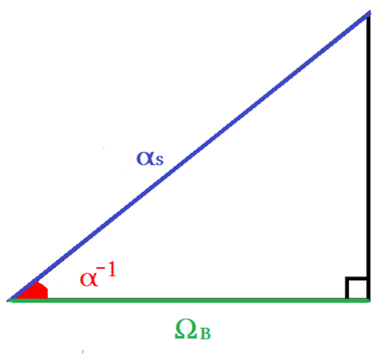

Figure 15. *Geometric representation of the the density parameter for the baryonic matter*

The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B=0.044\pm0.004$. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\% \tag{75}$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^{i}+1=0 \tag{76}$$

$$\Omega_B^{i}=i^2 \tag{77}$$

$$\Omega_B^{2i}=1 \tag{78}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B=e^{-1}\cdot\alpha_s \tag{79}$$

$$\Omega_B=\alpha_w^{-1}\cdot\alpha_s^2\cdot10^{-7} \tag{80}$$

$$\Omega_B=2^{-1}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha}) \tag{81}$$

$$\Omega_B=2\cdot N_A\cdot\alpha_s\cdot\alpha\cdot\alpha_G^{1/2} \tag{82}$$

$$\Omega_B=2^{-1}\cdot e^{-1}\cdot 10^{7}\cdot \alpha_W\cdot (e^{i/\alpha}+e^{-i/\alpha}) \tag{83}$$

$$\Omega_B=2\cdot 10^{7}\cdot N_A\cdot e^{-1}\cdot \alpha_W\cdot \alpha\cdot \alpha_G^{1/2} \tag{84}$$

$$\Omega_B=10^{-7}\cdot \alpha_g^{1/3}\cdot \alpha_s^{2}\cdot \alpha\cdot \alpha_W^{-1}\cdot \alpha_G^{-1/2} \tag{85}$$

In [24] we presented the solution for the Density Parameter of Dark Energy. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot e^{-1}=0.73576=73.57\% \tag{86}$$

So apply:

$$2\cdot R_d^2=e\cdot L_H^2 \tag{87}$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot \cos\alpha^{-1} \tag{88}$$

So apply the expression:

$$\cos\alpha^{-1}=\frac{\Omega_\Lambda}{2} \tag{89}$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda=e^{i/\alpha}+e^{-i/\alpha} \tag{90}$$

The figure 16 and 17 below shows the geometric representation of the solution for the Density Parameter of Dark Energy.

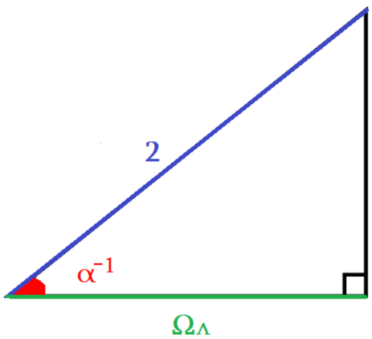

Figure 16. *Geometric representation of the the density parameter for the dark energy*

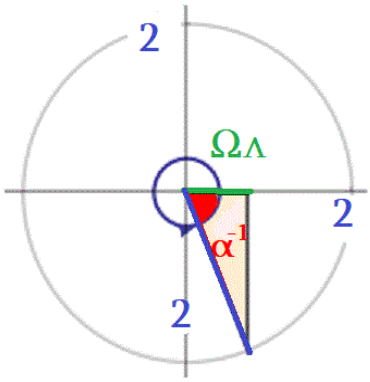

Figure 17. *The solution for the Density Parameter of Dark Energy.*

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_{\mathrm{H}}^2}{2R_d^2} \tag{91}$$

The figure 18 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

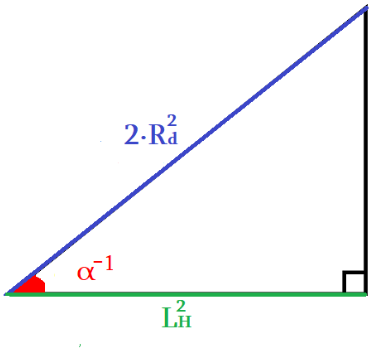

Figure 18. *Geometric representation of the relationship between the de Sitter radius and the Hubble length*

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot 10^{-7} \alpha_s \cdot \alpha_w^{-1} \tag{92}$$

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot \alpha_s^{-1} \tag{93}$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot \alpha_w^{-1} \tag{94}$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1} \tag{95}$$

$$\Omega_\Lambda = 4 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \tag{96}$$

$$\Omega_\Lambda = i^{8i} \cdot \alpha^{-2} \cdot \alpha_s^{-4} \cdot \alpha_G^{-1} \cdot N_A^{-2} \tag{97}$$

$$\Omega_\Lambda = 10^7 \cdot i^{4i} \cdot \alpha^{-1} \cdot \alpha_w^{-1} \cdot \alpha_G^{-1/2} \cdot N_A^{-1} \tag{98}$$

$$\Omega_\Lambda = 8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_s^{-1} \tag{99}$$

The figure 19 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

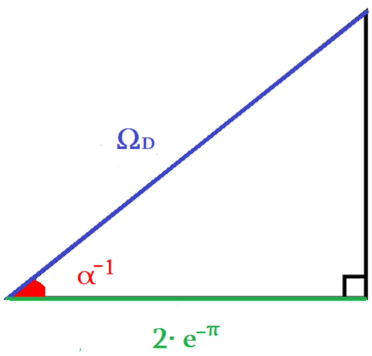

Figure 19. *Geometric representation of the density parameter of dark matter.*

The figure 20 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.

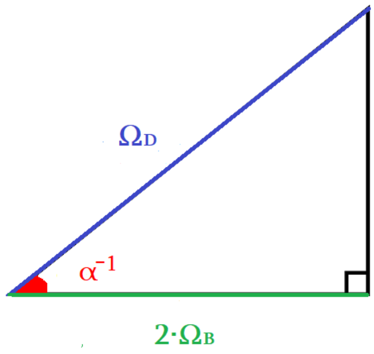

Figure 20. *Geometric representation of the relationship between the density parameter of dark and baryonic matter.*

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D=0.23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2\cdot e^{1-\pi}=2\cdot e\cdot i^{2i}=0.2349=23.49\% \tag{100}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2\cdot a_S \tag{101}$$

$$\Omega_D=2\cdot 10^7\cdot e^{-1}\cdot a_W \tag{102}$$

$$\Omega_D=2\cdot(i^{2i}\cdot 10^7\cdot a_W)^{1/2} \tag{103}$$

$$\Omega_D=4\cdot i^{2i}\cdot(e^{i/a}+e^{-i/a})^{-1} \tag{104}$$

$$\Omega_D=10^7\cdot a_W\cdot(e^{i/a}+e^{-i/a}) \tag{105}$$

$$\Omega_D=4\cdot 10^7\cdot a_W\cdot a\cdot a_G^{1/2}\cdot N_A \tag{106}$$

$$\Omega_D=16\cdot 10^7\cdot N_A^2\cdot a_W\cdot a^2\cdot a_G\cdot(e^{i/a}+e^{-i/a})^{-1} \tag{107}$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D=2\cdot e\cdot\Omega_B \tag{108}$$

The relationship between the density parameter of dark energy,dark matter and baryonic matter is:

$$\Omega_D\cdot\Omega_\Lambda=4\cdot\Omega_B \tag{109}$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_\Lambda=e^{-\pi}+2\cdot e^{1-\pi}+2\cdot e^{-1}=1.0139 \tag{110}$$

In [25] we proposed a possible solution for the Equation of state in cosmology. From the dimensionless unification of the fundamental interactions the state equation w has value:

$$w=-24\cdot e^{-\pi}=-24\cdot i^{2i}=-1.037134 \tag{111}$$

4. Unification of the Microcosm and the Macrocosm

In [26] , [27] and [28] we presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities.This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length l,time t,speed υ and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{\max}} = \frac{t_{min}}{t_{\max}} = \frac{\upsilon_{min}}{\upsilon_{\max}} = \frac{\mathrm{T}_{min}}{\mathrm{T}_{max}} \tag{112}$$

Energy E, mass M, action A,momentum P and entropy S have another min/max ratio,which is the square of α_g:

$$\alpha_g^2 = \frac{\mathrm{E}_{min}}{\mathrm{E}_{\max}} = \frac{\mathrm{M}_{min}}{\mathrm{M}_{\max}} = \frac{\mathrm{A}_{min}}{\mathrm{A}_{\max}} = \frac{\mathrm{P}_{min}}{\mathrm{P}_{\max}} = \frac{S_{min}}{S_{max}} \tag{113}$$

Force F has min/max ratio which is α_g^4:

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \tag{114}$$

Mass density has min/max ratio which is α_g^5:

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \tag{115}$$

Also apply the expressions:

$$\frac{l_{max}}{\mathrm{M}_{\max}} = \frac{l_{pl}}{m_{pl}} \tag{116}$$

$$\left(\frac{l_{max}}{l_{\min}}\right)^2 = \frac{\mathrm{M}_{max}}{m_{min}} \tag{117}$$

In [29] we presented the Unification of the Microcosm and the Macrocosm. For the minimum mass M_{min} apply:

$$\mathrm{M}_{\min} = \frac{m_{pl}^2}{\mathrm{M}_{\max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \tag{118}$$

$$M_{min}=(2\cdot e\cdot N_A)^{-2}\cdot\alpha^{-1}\cdot m_e=4.06578\times10^{-69}\ \mathrm{kg} \tag{119}$$

The expressions for the mass of the observable universe M_U are:

$$M_U=\alpha^{-1}\cdot\alpha_g^{-4/3}\cdot m_e=\alpha^3\cdot\alpha_G^{-2}\cdot m_e=(2\cdot e\cdot\alpha^2\cdot N_A)^2\cdot N_1\cdot m_p=\mu\cdot\alpha\cdot N_1^2\cdot m_p \tag{120}$$

For the value of the mass of the observable universe M_U apply $M_U=1.153482\times10^{53}$ kg. The expressions who calculate the number of protons in the observable universe are:

$$\mathrm{N}_{Edd} = \frac{\mathrm{M}_U}{m_p} = \mu\alpha\mathrm{N}_1^2 = \frac{\mathrm{N}_1}{\alpha_g^{\frac{2}{3}}} = \left(2e\alpha^2\mathrm{N}_\mathrm{A}\right)^2\mathrm{N}_1 = \left(\frac{r_e}{l_{pl}}\right)^2\mathrm{N}_1 = 6.9\times10^{79} \tag{121}$$

In [30] and [31] we presented the Dimensionless theory of everything. The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \tag{122}$$

The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}}\frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}} \tag{123}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2\alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha\frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha\alpha_0}\right)^2 \tag{124}$$

A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length l_{pl}. The smallest components will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length $2\cdot e\cdot l_{pl}$. Perhaps for the minimum distance l_{min} apply:

$$l_{min}=2\cdot e\cdot l_{pl} \tag{125}$$

$$l_{min}=2\cdot e^{\pi}\cdot \alpha_s\cdot l_{pl} \tag{126}$$

From expressions apply:

$$\cos\alpha^{-1}=e^{-1}$$

$$\cos\alpha^{-1}\cdot l_{min}=2\cdot l_{pl}$$

$$\cos\alpha^{-1} = \frac{2l_{pl}}{l_{min}} \tag{127}$$

The figures 21 below show the geometric representation of the fundamental unit of length.

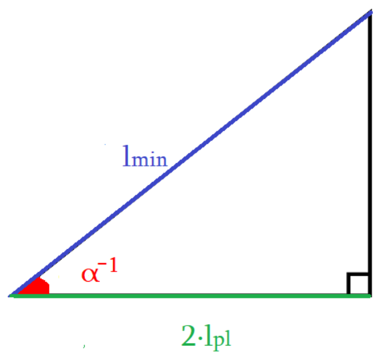

Figure 21. *Geometric representation of the fundamental unit of length.*

For the Bohr radius α_0 apply:

$$\alpha_0=N_A\cdot l_{min} \tag{128}$$

$$\alpha_0=2\cdot e\cdot N_A\cdot l_{pl} \tag{129}$$

The figures 22 below show the geometric representation of the relationship between the Bohr radius and the Planck length.

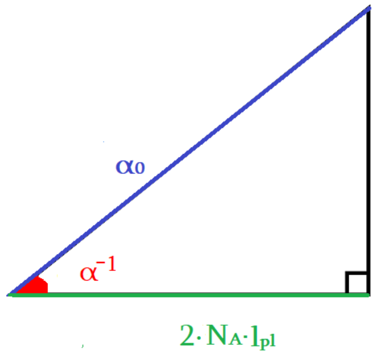

Figure 22. Geometric representation of the relationship between the Bohr radius and the Planck length.

We will use this expression and the new formula for the Planck length l_{pl} to resulting the unity formula that connects the fine-structure constant α and the gravitational coupling constant α_G:

$$\alpha_0=2\cdot e\cdot N_A\cdot l_{pl}$$

$$\alpha_0 = 2eN_A\alpha\sqrt{\alpha_G}\alpha_0$$

$$2eN_A\alpha\sqrt{\alpha_G} = 1$$

Therefore the unity formula that connect the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A is:

$$4\cdot e^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=1 \tag{130}$$

The unity formula is equally valid:

$$\alpha^2\cdot \alpha_G=(2\cdot e\cdot N_A)^{-2} \tag{131}$$

Length l has the max/min ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \tag{132}$$

The maximum distance l_{max} corresponds to the distance of the universe:

$$l_{max}==\alpha_g^{-1}\cdot l_{min}=4.657\times10^{26}\ \mathrm{m} \tag{133}$$

The figure 23 shows the geometric representation of the relationship between the maximum distance and the Planck length.

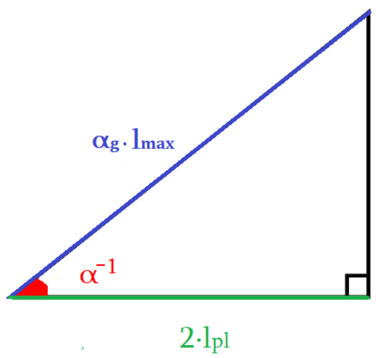

Figure 23. Geometric representation of the relationship between the maximum distance and the Planck length.

The figure 24 shows the geometric representation of the relationship between the radius of the universe with the Planck length.

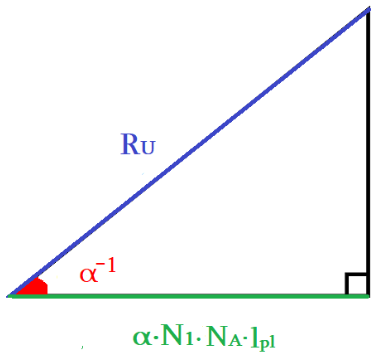

Figure 24. *Geometric representation of the relationship between the radius of the universe with the Planck length*

In [32] we presented the New Large Number Hypothesis of the universe. The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2\cdot R_U = N_1 \cdot \lambdabar_c \tag{134}$$

So apply the expression:

$$R_U = e\cdot\alpha\cdot N_1\cdot N_A\cdot l_{pl} \tag{135}$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2}\alpha_0 = \frac{N_1}{2\alpha}r_e = \frac{1}{2\mu\alpha_G}r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2Gm_e m_p} = \frac{\alpha\hbar}{2Gm_e^2 m_p} \tag{136}$$

We Found the value of the radius of the universe $R_U = 4.38\times10^{26}$ m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha\hbar}{2cGm_e^2 m_p} = \frac{\hbar r_e}{2Gm_e m_p} \tag{137}$$

For the value of the age of the universe apply $T_U = 1.46\times10^{18}$ s. The expressions for the relationship between the mass of the observable universe M_U with the radius of the universe R_U are:

$$\frac{M_U}{R_U^2} = 4\alpha\mu^2\frac{m_e}{r_e^2} \tag{138}$$

$$\frac{M_U}{m_p} = \alpha\mu\left(\frac{2R_U}{r_e}\right)^2 \tag{139}$$

The gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. The time quantum in the brain t_B,the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \tag{140}$$

For the minimum distance l_{min} apply $l_{min}=2\cdot e\cdot l_{pl}$. So for the minimum time t_{min} apply:

$$t_{\min} = \frac{l_{min}}{c} = \frac{2el_{pl}}{c} = 2et_{pl} \quad (141)$$

From expressions apply:

$$\cos\alpha^{-1}=e^{-1}$$

$$\cos\alpha^{-1}\cdot t_{min}=2\cdot t_{pl}$$

$$\cos\alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (142)$$

The figures 25 below show the geometric representation of the fundamental unit of time.

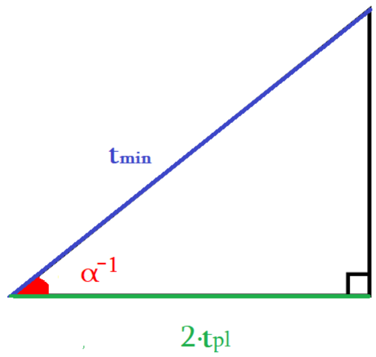

Figure 25. *Geometric representation of the fundamental unit of time.*

5. The value for the Hubble constant

The Planck time t_{pl} is the time required for light to travel a distance of 1 Planck length in vacuum. No current physical theory can describe timescales shorter than the Planck time, such as the earliest events after the Big Bang. Some conjecture that the structure of time need not remain smooth on intervals comparable to the Planck time. Today it plays a tantalizing role in our understanding of the Big Bang and the search for a theory of quantum gravity. All scientific experiments and human experiences occur on time scales that are many orders of magnitude larger than the Planck time t_{pl},making events on the Planck time t_{pl} undetectable with current scientific technology. The Planck time t_{pl} is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl}c^2}$$

The 2018 CODATA recommended value of the Planck time is $t_{pl}=5.391247\text{x}10^{-44}$ s with standard uncertainty $0.000060\text{x}10^{-44}$ s and relative standard uncertainty 1.1×10^{-5}.

Hubble length or Hubble distance is a unit of distance in cosmology,defined as the speed of light multiplied by Hubble time $L_H=c\cdot H_0^{-1}$. This distance is equivalent to 4.550 million parsecs,or 14.4 billion light-years,13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light,as shown by the substitution $r=c\cdot H_0^{-1}$ in the equation for Hubble's law,$u=H_0\cdot r$. The maximum time period t_{max} is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe $t_U=H_0^{-1}$. Therefore $t_{max}=t_U=H_0^{-1}$. It is equivalent to 4.420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting $D=c\cdot H_0^{-1}$ into the equation for Hubble's law, $u=H_0^{-1}\cdot D$. The Hubble length L_H or Hubble distance is a unit of distance in cosmology, defined as $L_H=c\cdot H_0^{-1}$.

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. The cosmological constant is the simplest realization of dark energy,which is the more generic name given to the unknown cause of the acceleration of the universe. The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Laurent Nottale in [33] assumed a large-number relation:

$$\alpha\frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}}\right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g:

$$\alpha\frac{m_{pl}}{m_e} = \left(l_{pl}\sqrt{\Lambda}\right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl}\sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G\hbar\Lambda}{c^3}}$$

The density parameter for dark energy is defined as:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3\mathrm{H}_0^2}$$

Also for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{c^2}{R_d^2\mathrm{H}_0^2}$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \left(\frac{L_\mathrm{H}}{R_d}\right)^2 = \frac{L_\mathrm{H}^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is $\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\%$. So from this expression apply:

$$2 \cdot Rd^2 = e \cdot L_H^2 \quad (143)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (144)$$

The figure 26 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

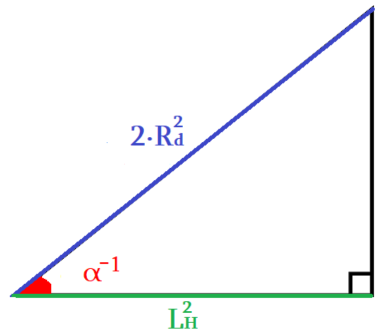

Figure 26. *Geometric representation of the relationship between the de Sitter radius and the Hubble length.*

For the cosmological constant Λ equals:

$$\Lambda = \frac{6H_0^2}{ec^2} \quad (145)$$

So apply the expressions:

$$\frac{6H_0^2}{\Lambda c^2} = e \quad (146)$$

$$\frac{H_0^2}{\Lambda c^2} = \frac{e}{6} \quad (147)$$

$$\cos \alpha^{-1} = \frac{\Lambda c^2}{6H_0^2} \quad (148)$$

The figure 27 and 28 shows the geometric representation of the relationship between the Hubble constant and the cosmological constant.

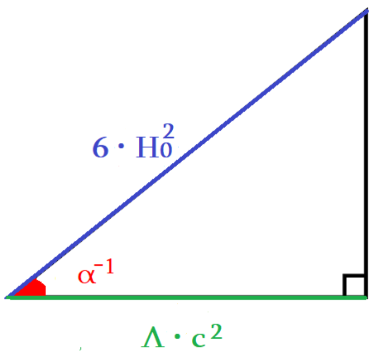

Figure 27. *Geometric representation of the relationship between the Hubble constant and the cosmological constant.*

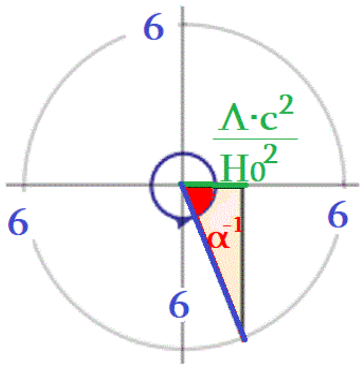

Figure 28. *The relationship between the Hubble constant and the cosmological constant.*

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = c\sqrt{\frac{e}{6}\Lambda} \tag{149}$$

For the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \frac{\alpha_g}{t_{pl}}\sqrt{\frac{e}{6}} \tag{150}$$

$$\mathrm{H}_0 = \frac{\alpha_g c}{l_{pl}}\sqrt{\frac{e}{6}} \tag{151}$$

Also apply the expression:

$$(\mathrm{H}_0 t_{pl})^2 = \frac{e}{6}\alpha_g^2 \tag{152}$$

These equations calculate the theoretical value of the Hubble Constant:

$$\mathrm{H}_0 = 2.355683\times10^{-18}\ \mathrm{s}^{-1} \tag{153}$$

$$\mathrm{H}_0 = 72.69\ (\mathrm{km/s})/\mathrm{Mpc} \tag{154}$$

The cosmological constant Λ equals:

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{c l_{pl}^2}{r_e^2}\sqrt{\frac{e}{6}} \tag{155}$$

Also the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \alpha_g \sqrt{\frac{ec^5}{6G\hbar}} \tag{156}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{e}{6}\alpha_g^2 \tag{157}$$

The cosmological constant Λ equals:

$$\Lambda = \frac{G}{\hbar^4}\left(\frac{m_e}{a}\right)^6$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{cm_e^3}{\alpha^3\hbar^2}\sqrt{\frac{eG}{6}} \tag{158}$$

From the dimensionless unification of the atomic physics and the cosmology apply:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=(2\cdot e\cdot \alpha^2\cdot N_A)^{-6} \tag{159}$$

$$(2\cdot e\cdot \alpha^2\cdot N_A)^6\cdot l_{pl}^2\cdot\Lambda=1 \tag{160}$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 \mathrm{N_A}\right)^{-6}\frac{c^3}{G\hbar} \tag{161}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = \frac{1}{\left(2e\alpha^2 \mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{162}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{1}{6e^5\left(2\alpha^2 \mathrm{N_A}\right)^6} \tag{163}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g=i^{6i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=i^{12i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-6} \tag{164}$$

$$(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^6\cdot l_{pl}^2\cdot\Lambda=i^{12i} \tag{165}$$

For the cosmological constant equals:

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{166}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \frac{i^{6i}}{\left(2\alpha_s \alpha^2 \mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{167}$$

$$\mathrm{H}_0 = \frac{1}{\left(2e^{\pi}\alpha_s \alpha^2 \mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{168}$$

Also apply the expression:

$$\frac{G\hbar \mathrm{H}_0^2}{c^5} = \frac{e}{48\left(e^{\pi}\alpha_s \alpha^2 \mathrm{N_A}\right)^3} \tag{169}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_G = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-6} \tag{170}$$

$$(2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \tag{171}$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 \left(2\, 10^7 \alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{172}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \frac{i^{6i} e^3}{\left(2\, 10^7 \alpha_w \alpha^3 \mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}} \tag{173}$$

$$\mathrm{H}_0 = \frac{1}{\left(2\, 10^7 e^{\pi-1}\alpha_w \alpha^3 \mathrm{N_A}\right)^3}\sqrt{\frac{ec^3}{6G\hbar}} \tag{174}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2 = 10^{42}\left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \tag{175}$$

$$l_{pl}^2 \Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \tag{176}$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{177}$$

For the cosmological constant equals:

$$\Lambda = 10^{42}\left(\frac{\alpha_G\alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{178}$$

So the formula for the Hubble Constant is:

$$\mathrm{H}_0 = 10^{21}\left(\frac{\alpha_w\sqrt{\alpha_G}}{e\alpha_s\alpha}\right)^3\sqrt{\frac{ec^5}{6G\hbar}} \tag{179}$$

Also apply the expression:

$$\frac{G\hbar\mathrm{H}_0^2}{c^5} = \frac{10^{42}}{6e^5}\left(\frac{\alpha_w^2\alpha_G}{\alpha_s^2\alpha^2}\right)^3 \tag{180}$$

From the dimensionless unification of atomic physics and cosmology apply:

$$\alpha_g^2 = 10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{181}$$

$$l_{pl}^2\Lambda = 10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{182}$$

$$\alpha s^{12}\cdot\alpha^6\cdot lpl^2\cdot\Lambda = 10^{42}\cdot i^{12i}\cdot\alpha G^3\cdot\alpha w^6 \tag{183}$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{184}$$

So the formulas for the Hubble Constant are:

$$\mathrm{H}_0 = \left(\frac{10^7 i^{2i}\alpha_w\sqrt{\alpha_G}}{\alpha\alpha_s^2}\right)^3\sqrt{\frac{ec^5}{6G\hbar}} \tag{185}$$

$$\mathrm{H}_0 = \left(\frac{10^7\alpha_w\sqrt{\alpha_G}}{e^{\pi}\alpha\alpha_s^2}\right)^3\sqrt{\frac{ec^5}{6G\hbar}} \tag{186}$$

Also apply the expression:

$$\frac{6G\hbar\mathrm{H}_0^2}{ec^5} = \left(\frac{10^{14}\alpha_w^2\alpha_G}{e^{2\pi}\alpha^2\alpha_s^4}\right)^3 \tag{187}$$

$$6e^{6\pi}\frac{G\hbar\mathrm{H}_0^2}{c^5} = e\left(\frac{10^{14}\alpha_w^2\alpha_G}{\alpha^2\alpha_s^4}\right)^3 \tag{188}$$

$$6e^{5\pi}\frac{G\hbar H_0^2}{c^5}=\frac{1}{\alpha_s^{11}}\left(\frac{10^{14}\alpha_w^2\alpha_G}{\alpha^2}\right)^3 \tag{189}$$

So the Equations of the Universe are:

$$6e^{5\pi}\frac{G\hbar H_0^2}{c^5}=10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{11}} \tag{190}$$

$$e^{7\pi}\frac{G\hbar\Lambda^2}{cH_0^2}=6\cdot 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{13}} \tag{191}$$

6. Conclusions

In the last decade a significant gap has emerged between different methods of measuring it, some anchored in the nearby universe, others at cosmological distances. The SH0ES team has found $H_0=73.2\pm1.3$ (km/s)/Mpc locally. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot e^{-1}=0.73576=73.57\%$$

So apply:

$$2\cdot R_d^2=e\cdot L_H^2$$

Best values of H_0 from the distance ladder lie in the range 72–75 (km/s)/Mpc. From the dimensionless unification of the fundamental interactions we calculated the theoretical value of the Hubble Constant:

$$H_0=72.69 \text{ (km/s)/Mpc}$$

Also we presented the formulas of the Hubble constant:

$$H_0=c\sqrt{\frac{e}{6}\Lambda}$$

$$H_0=\frac{\alpha_g}{t_{pl}}\sqrt{\frac{e}{6}}$$

$$H_0=\frac{\alpha_g c}{l_{pl}}\sqrt{\frac{e}{6}}$$

$$H_0=\frac{c l_{pl}^2}{r_e^2}\sqrt{\frac{e}{6}}$$

$$H_0=\alpha_g\sqrt{\frac{ec^5}{6G\hbar}}$$

$$H_0=\frac{cm_e^3}{\alpha^3\hbar^2}\sqrt{\frac{eG}{6}}$$

$$\mathrm{H_0} = \frac{1}{\left(2e\alpha^2\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \frac{i^{6i}}{\left(2\alpha_s\alpha^2\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \frac{1}{\left(2e^{\pi}\alpha_s\alpha^2\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \frac{i^{6i}e^3}{\left(2\,10^7\alpha_w\alpha^3\mathrm{N_A}\right)^3}\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \frac{1}{\left(2\,10^7e^{\pi-1}\alpha_w\alpha^3\mathrm{N_A}\right)^3}\sqrt{\frac{ec^3}{6G\hbar}}$$

$$\mathrm{H_0} = 10^{21}\left(\frac{\alpha_w\sqrt{\alpha_G}}{e\alpha_s\alpha}\right)^3\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \left(\frac{10^7 i^{2i}\alpha_w\sqrt{\alpha_G}}{\alpha\alpha_s^2}\right)^3\sqrt{\frac{ec^5}{6G\hbar}}$$

$$\mathrm{H_0} = \left(\frac{10^7\alpha_w\sqrt{\alpha_G}}{e^{\pi}\alpha\alpha_s^2}\right)^3\sqrt{\frac{ec^5}{6G\hbar}}$$

The Equations of the Universe are:

$$e^{6\pi}\frac{\Lambda G\hbar}{c^3} = 10^{42}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3$$

$$6e^{5\pi}\frac{G\hbar\mathrm{H}_0^2}{c^5} = 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{11}}$$

$$e^{7\pi}\frac{G\hbar\Lambda^2}{c\mathrm{H}_0^2} = 6\cdot 10^{42}\frac{\alpha_w^6\alpha_G^3}{\alpha^6\alpha_s^{13}}$$

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