Cosmic gravitational potential

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Abstract. Gravity is a long-range interaction, so the entire causally connected Universe acts gravitationally on a test body. In this investigation, we calculate the cosmic gravitational potential at a fixed location in the Universe. We verified that the most distant bodies are the ones that produce a more intense gravitational action. The Big Bang singularity is responsible for the infinite gravitational potential produced by cosmic radiation, which we avoid by renormalizing the cosmic potential. We calculate the participation of each epoch of the Universe in the cosmic potential and its variation with time.

1. Introduction

The Newtonian theory of gravitation is described by the gravitational potential $\phi\left(x'^{\alpha}, x^{\alpha}, t\right)$, which depends on the position x'^{α} of the source, the position x^{α} of the field point, and the time *t*. From the potential we derive the force acting on a test body of gravitational mass *m*

$$\mathbf{F} = -m\nabla\phi = m\mathbf{E} \tag{1}$$

E is the gravitational field vector or force acting on the unit mass. The derivatives of the nabla operator concerning the coordinates of the field: $\nabla = \left(\partial/\partial x^{\alpha}\right)$.

From Newton's law of gravitation, we find

$$\phi\left(x'^{\alpha}, x^{\alpha}, t\right) = -G\int \frac{\rho\left(x'^{\alpha}, t\right)}{r'\left(x'^{\alpha}, x^{\alpha}\right)} dV' \tag{2}$$

ρ is the gravitational mass density that creates the field, r' is the distance between the source and the test body, and dV' is the volume occupied by the source.

In (2), we arbitrarily choose the sign $-$; therefore, (1) is applicable, and the energy of the test particle is the sum of the kinetic and potential energy (defined by $m\phi$). If (2) were defined with a positive sign, the appropriate modifications would have to be made in (1) and in the expression for mechanical energy.

Another issue related to (2) is that of initial values. We arbitrarily choose zero potential at infinity. The potential is not a measurable magnitude; only the potential difference has a physical meaning. Therefore, a particle's motion equation in a gravitational field must be invariant against recalibration of the potential.

General Relativity describes the gravitational field by the metric tensor g_{ik}, a generalization of the Newtonian potential ϕ. From the metric tensor, we obtain the components of the symbols of Christoffel Γ^{j}_{ik}, a generalization of the gravitational field **E**. The equation of motion is the geodesic equation.

When the field is weak, we expand the component 0,0 of the metric tensor in power series of the inverse of *c*

$$g_{00} = 1 + \frac{2\phi}{c^2} + ...$$

therefore, when we apply the General Relativity (in the weak field case), we must calculate the Newtonian gravitational potential.

The equation of motion of a test particle in second approximation is (Segura 2023)

$$\frac{d\boldsymbol{\sigma}}{d\tau}=-\nabla\phi-4\frac{\partial\mathbf{A}}{\partial\tau}+4\mathbf{w}\wedge\left(\nabla\wedge\mathbf{A}\right)-\frac{1}{c^2}\nabla\psi+\frac{\mathbf{w}}{c^2}\frac{\partial\phi}{\partial\tau}+2\frac{\mathbf{w}}{c^2}\left(\mathbf{w}\cdot\nabla\right)\phi-\frac{w^2}{c^2}\nabla\phi, \tag{3}$$

w is the velocity of the test particle. The first of the terms is of zero order (Newtonian), and the remaining terms are of the second order; they depend on the inverse speed of light squared. The speed, acceleration, and gradient in (3) are calculated with the distance and proper time (σ and τ); **A** is the gravitomagnetic potential, and ψ is the inductive scalar potential. When the test particle is at rest concerning the Universe, (3) reduces to the classical expression

$$\frac{d\boldsymbol{\sigma}}{d\tau}=-\nabla\phi.$$

(3) applies to a weak field; therefore, we can use it to describe cosmic action. Due to the cosmological principle, the cosmic potential is the same throughout the Universe if we calculate it at the same moment and therefore does not produce detectable effects. Even so, the calculations that we will do below are of interest because they give us information on how the gravity of the Universe acts and because the results obtained are applied to a body that is accelerated, when inductive effects arise, which are measurable (Segura 2021).

2. Cosmic potential

In a limited area and far from strong gravitational fields, the space-time geometry is quasi Minkowskian; therefore, we can develop the metric tensor in power series of the inverse of c and consider only the first terms; with this procedure, we find the second-order approximation of the gravitational field and deduce (3). In this reasoning, we assume that the source of the field is nearby objects gravitationally bound to the test body and that they jointly participate in the cosmic expansion *. Let us note that there is a difference between the line element in the small spatial portion considered and the line element of the Universe. That is, with (3), we find the motion of a particle regardless of cosmic evolution.

As in Newtonian theory, in GR, we also study the motion of a particle in a gravitational field disregarding the gravitational action of the entire Universe. In the case of a test particle at rest with respect to the Universe, this approach is justified by the homogeneity and isotropy of space. However, it must be accepted that the entire causally connected Universe acts gravitationally on the test body, although the total action cancels out. The situation changes when the test body is accelerated because then gravitational inductive forces (ie, velocity-dependent) arise.

The line element describing the local space-time geometry does not consider cosmic effects. In contrast, the global geometry of the Universe does not take into account the local gravitational action. If from the cosmic line element (which we will assume to be the Robertson-Walker) we obtain the Lagrangian of a test body, we find that the particle is comoving; that is, it is always at the same point if it was initially at rest, although it would be subject to the Hubble flow.

Without a space-time geometry that describes both local gravitational interaction and cosmic evolution, we will speculatively use a mixture of both. If we accept Mach's Principle, then the Universe as a whole does not produce gravitational action on a test body unless this body is subjected to another force (gravitational or of another type) acquiring acceleration; then, the Universe would exert a reaction force on the test body, that is, a force that opposes the accelerated motion and which is the force of inertia. In our research, we assume that the test body is at rest concerning the Universe, then the reaction force or inertia does not originate.

For nearby sources, that is, gravitationally bound to the test body, we determine the potential ϕ with the weak theory of the gravitational field, finding the expression (2). To find the gravitational potential produced by the Universe as a whole, we assume the validity of (2); that is, we assume that Newton's law of gravitation continues to be valid, regardless of the distance from the source. However, it is necessary to generalize (2), which presents three problems:

* Determine the source of the gravitational field. In General Relativity, in addition to matter, the energy is source of gravity; therefore, the pressure of the cosmic microwave

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* In this reasoning we neglect the effect of cosmic radiation and the cosmological term.

background and the pressure of the vacuum are also sources of gravity.
* Generalize the distance r' that appears in (2).
* Generalize the volume over which we integrate (2).

3. Retarded proper distance, comoving distance, and photonic distance

We assume that the line element that describes the geometry of the Universe is that of Robertson-Walker

$$ds^2 = c^2dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\sin^2\theta d\varphi^2 + r^2 d\theta^2\right)$$

(r,φ,θ) are the spherical coordinates, $R(t)$ is the cosmic scale factor, *k* is the spatial curvature of value +1, 0, or -1, and *t* is the cosmic time. The retarded proper distance from a point in the Universe that emits a light or gravitational signal at the retarded time t_e to the observer who receives the signal at the current moment t_0 is

$$\sigma(t_e) = R(t_e)\int_{t_e}^{t_0}\frac{cdt'}{R(t')} \tag{4}$$

and the corresponding comoving distance is

$$d\chi = -\frac{cdt}{R(t)} \quad \Rightarrow \quad \int_0^{\chi} d\chi = -\int_{t_0}^{t_e}\frac{cdt}{R(t)} \quad \Rightarrow \quad \chi = \int_{t_e}^{t_0}\frac{cdt'}{R(t')}$$

which is independent of time. For a spatially flat Universe $k=0$, that we consider below, $\chi = r$, and then the radial comoving distance is

$$r = \int_{t_e}^{t_0}\frac{cdt'}{R(t')}. \tag{5}$$

We call photonic distance the proper distance traveled by light from its emission at the moment t_e to its reception at t_0

$$d\sigma_P = -cdt \quad \Rightarrow \quad \int_{\sigma_p}^{0} d\sigma'_P = -c\int_{t_e}^{t_0} dt' \quad \Rightarrow \quad \sigma_P = c(t_0 - t_e) = ct_0(1-\xi), \tag{6}$$

we define $\xi = t_e/t_0$.

4. Generalization of the Newtonian potential

Adapting (2) to cosmology requires generalizing the sources of the gravitational field because not only matter but also energy is source of gravity. The inertial mass of a system of bodies bound together is

$$m = \frac{E}{c^2}$$

E is the total energy of the system (energy at rest, kinetic, binding,...) measured at the center of mass. According to (Tolman 1934, 234-235) and (Ohanian 2013)

$$E = \int\left(T_4^4 - T_1^1 - T_2^2 - T_3^3\right)\sqrt{g}\,dxdydz \quad \Rightarrow \quad \rho_T \approx \left(T_4^4 - T_1^1 - T_2^2 - T_3^3\right)\sqrt{1+\frac{2\phi'}{c^2}}$$

T_{ik} is the energy-momentum tensor of «matter» (that is, it does not include the gravitational field), ϕ' is the gravitational binding potential between the bodies that make up the system (that is, it is not the cosmic potential), and ρ_T is the volume density of the inertial mass of the system. According to the weak equivalence principle, the gravitational mass is equivalent to the inertial mass. This means that ρ_T represents the volume density of the gravitational mass, which is responsible for creating the gravitational field. If we assume that the Universe is composed of matter, radiation, and dark energy, then we can consider it a perfect gas, and

$$\rho_T = \rho + 3p/c^2 = \rho_M + 2\rho_R - 2\rho_V \qquad \left(p_M = 0;\quad p_R = 1/3\rho_R c^2;\quad p_V = -\rho_V c^2\right)$$

we despise gravitational self-energy because it is irrelevant at a cosmic level. ρc^2 is the energy density, and *p* is the pressure (from radiation and vacuum). Λ is the cosmological term, the subscript

M is for matter, V for vacuum, R for radiation and the subscript 0 indicates the current time *.

In cosmology, we use a variety of distances; among them are luminosity distances, angular size, proper motion distance, photon flux, etc. These arbitrary definitions have the advantage of being measurable and can be related to cosmological magnitudes. (Vetö 2013) has used the photon flux distance to calculate the cosmic potential. But we understand that none of the above distances, due to their arbitrariness, can be the generalization of the distance r' that appears in (2).

Proper distance (retarded or calculated at any other time) and co-moving distance are physical distances, although we cannot measure it directly. Some authors (Martín, Rañada and Tiemblo 2007) and (Segura 2018) have used the proper distance to calculate the cosmic potential.

In this investigation, we assume that the decrease in gravity with distance is caused by the proper distance that the gravitons travel between the source and the point of the field. Therefore the distance r' of (2) is the photonic distance, different from the retarded proper distance, which is the physical distance at the moment of emission of the gravitational signal, but due to cosmic expansion, the distance between the source and field increases while the gravitons go from one place to another.

To integrate (2), we divide the Universe into spherical shells centered on the observer, with coordinate radius r and thickness dr. The volume dV' is identified with the retarded proper volume

$$dV' = 4\pi R^3\left(t_e\right) r^2 dr$$

which we deduce from the Robertson-Walker line element. We calculate the scale factor at the retarded moment t_e when the gravitational signal is emitted from the source. The signals emitted by the sources inside the spherical shell in t_e reach the observer at present, traveling the distance (6).

5. Cosmic potential in the Einstein-de Sitter Universe

The Einstein-de Sitter Universe is characterized by containing only matter and being spatially flat. By solving the Friedmann equation, we find the cosmic scale factor

$$R(t) = R_0\left(\frac{t}{t_0}\right)^{2/3}; \quad t_0 = \frac{2}{3H_0}; \quad t_0^2 = \frac{1}{6\pi G\rho_0} \tag{7}$$

R_0 and H_0 are the scale factor and the Hubble constant at t_0. Applying (7) in (5), we find that the radial coordinate of the object that emits the signal at t_e and reaches the observer at t_0 is

$$r = \frac{3ct_0^{2/3}}{R_0}\left(t_0^{1/3} - t^{1/3}\right) = \frac{3ct_0}{R_0}\left(1-\xi^{1/3}\right) \quad \Rightarrow \quad dr = \frac{ct_0}{R_0}\xi^{-2/3}d\xi, \tag{8}$$

since r is a decreasing function, dr is negative, but dr represents the width of the spherical shell over which we do the integration, which must be a positive quantity, hence the sign change in (8).

With (7) and (8), we calculate the gravitational potential produced by the entire Universe

$$\phi = -G\int\frac{\rho\left(t_e\right)}{r'}dV' = -G\int\frac{\rho_0\frac{R_0^3}{R^3}}{ct_0\left(1-\xi\right)}4\pi R^3 r^2 dr = -4\pi G\frac{\rho_0}{ct_0}\int\frac{1}{\left(1-\xi\right)}\left[3ct_0\left(1-\xi^{1/3}\right)\right]^2 ct_0\xi^{-2/3}d\xi =$$

$$= -36\pi Gc^2\rho_0 t_0^2\int_0^1\frac{\left(1-\xi^{1/3}\right)^2\xi^{-2/3}}{1-\xi}d\xi = -6c^2\int_0^1\frac{\left(1-\xi^{1/3}\right)^2\xi^{-2/3}}{1-\xi}d\xi,$$

the integral is 1.0728 and the gravitational potential produced by the entire Einstein-de Sitter Universe is

$$\phi = -6.4368c^2.$$

Due to the homogeneity of space, the potential is the same anywhere in the Universe if we calculate

* ρ_T is the gravitational mass density at the centre of mass (where the total momentum is zero). The kinetic energies linked to ρ_T are also a source of gravity, but these terms correspond to the scalar potential ψ, which we do not consider now. The gravitational field source that we are going to use coincides with the source in the linear theory (Weinberg 1972, 209-233).

$\Delta\xi$	% ϕ
0.0 - 0.1	80.25
0.1 - 0.2	8.95
0.2 - 0.3	4.38
0.3 - 0.4	2.55
0.4 - 0.5	1.59
0.5 - 0.6	1.02
0.6 - 0.7	0.64
0.7 - 0.8	0.38
0.8 - 0.8	0.19
0.9 - 1.0	0.06

Table 1.- Percentage of participation in the cosmic potential currently observed for the Einstein-de Sitter Universe as a function of the time interval in which the gravitational interactions that reach the observer at the present moment were emitted.

it at the same moment. Then its gradient and temporal variation are zero and therefore do not produce force on a test body at rest *.

Different cosmic epochs contribute differently to the cosmic potential calculated at the present moment. Table 1 shows the percentage of the potential that corresponds to emission intervals. We verified that the gravitational action of objects that emit the gravitational interaction between the beginning of the Big Bang and the moment $0.1t_0$ is more than 80%; that is to say, almost all the gravitational potential measured at present corresponds to the gravitational action emitted at the beginning of the Universe. This gravitational action was emitted by the objects that are currently furthest away. On the contrary, the objects that emitted the interaction in more recent times and are currently closer to the observer, contribute little to the cosmic potential.

6. Cosmic potential in the spatially flat Universe with cosmological term

Now we consider a more realistic Universe containing matter, dark energy, radiation, and spatially flat $(k=0)$. We consider a simplified model in the sense that there is no variation of cosmic matter; therefore, we simplify the Big Bang, considering the content of matter and radiation unalterable; that is, we do not consider the lepton, hadron or quarks epochs or the inflationary period. The density parameters and the Hubble constant at present is

$$\begin{gathered}\Omega_{0M} = \frac{8\pi G}{3H_0^2}\rho_{0M} = 0.315; \quad \Omega_{0V} = \frac{\Lambda c^2}{3H_0^2} = 0.685; \\ \Omega_{0R} = \frac{8\pi G}{3H_0^2}\rho_{0R} = 8.35\cdot 10^{-5}; \quad H_0 = 67.3\,\mathrm{km\,s^{-1}Mpc^{-1}},\end{gathered} \tag{9}$$

from (9) we deduce the parameters

$$\begin{gathered}\Lambda = 1.088\cdot 10^{-52}\,\mathrm{m}^{-2}; \quad H_0 = 2.181\cdot 10^{-18}\,\mathrm{s}^{-1}; \quad t_\Lambda = 3.693\cdot 10^{17}\,\mathrm{s}; \quad \rho_{0V} = 5.833\cdot 10^{-27}\,\mathrm{kgm^{-3}}; \\ t_0 = 4.3586\cdot 10^{17}\,\mathrm{s}; \quad \alpha_0 = 1.1802; \quad \rho_{0M} = 2.681\cdot 10^{-27}\,\mathrm{kgm^{-3}}; \quad \rho_{0R} = 7.108\cdot 10^{-31}\,\mathrm{kgm^{-3}},\end{gathered} \tag{10}$$

$t_\Lambda = 2\big/\left(c\sqrt{3\Lambda}\right)$ and $\alpha_0 = t_0/t_\Lambda$.

The cosmic scale factor $R(t)$ is determined by the Friedman equation

* In the Einstein-de Sitter Universe there is a singularity in the Big Bang, although there is no singularity in the cosmic potential.

$$\xi = \frac{1}{t_0 H_0} \int_0^a \frac{da'}{\sqrt{\frac{\Omega_{0M}}{a'} + \frac{\Omega_{0R}}{a'^2} + \Omega_{0V} a'^2}}, \tag{11}$$

$\xi = t/t_0$ and $a = R(t)/R_0$ which we call the relative cosmic scale factor. We solve (11) numerically. From (11), we find the age of the Universe, since when $a = 1$, then $\xi = 1$, and from (11), we calculate t_0.

The retardation function or radial comoving distance from where the gravitational interaction comes

$$r = \int_{t_e}^{t_0} \frac{cdt'}{R(t')} = \frac{ct_0}{R_0} \int_{\xi}^{1} \frac{d\xi'}{a(\xi')} \tag{12}$$

by integrating (12) numerically we find the function $\xi = \xi(R_0 r)$.

The potential is

$$\phi = -G \int \frac{\rho_M + 2\rho_R - 2\rho_V}{r'} dV', \tag{13}$$

note that the density of matter and radiation vary with cosmic expansion, but the energy density of the vacuum remains constant. We calculate the energy densities at the moment t_e of the emission of the interaction that arrives at t_0 to the observer; that is, they are retarted values. To facilitate the calculations, we divide (13) into three integrals

$$\phi_M = -G \int \frac{\rho_M}{r'} dV'; \qquad \phi_R = -2G \int \frac{\rho_R}{r'} dV'; \qquad \phi_V = 2G \int \frac{\rho_V}{r'} dV'.$$

The first potential is

$$\phi_{0M} = -G \int \frac{\rho_{0M} \frac{R_0^3}{R^3}}{ct_0 (1-\xi)} 4\pi R^3 r^2 dr = -\frac{4\pi G \rho_{0M}}{ct_0} \int_0^{\max R_0 r} \frac{(R_0 r)^2}{1-\xi(R_0 r)} d(R_0 r), \tag{14}$$

the integral (14) is with respect to $R_0 r$; therefore, its limits must be the maximum and minimum value of $R_0 r$ deduced from (12). The second potential is

$$\phi_{0R} = -2G \int \frac{\rho_{0R} \frac{R_0^4}{R^4}}{ct_0 (1-\xi)} 4\pi R^3 r^2 dr = -\frac{8\pi G \rho_{0R}}{ct_0} \int_0^{\max R_0 r} \frac{(R_0 r)^2}{1-\xi(R_0 r)} \frac{1}{a[\xi(R_0 r)]} d(R_0 r). \tag{15}$$

And the third potential is

$$\phi_{0V} = 2G \int \frac{\rho_V}{ct_0 (1-\xi)} 4\pi R^3 r^2 dr = \frac{8\pi G \rho_V}{ct_0} \int_0^{\max R_0 r} \frac{(R_0 r)^2}{1-\xi(R_0 r)} a^3 [\xi(R_0 r)] d(R_0 r). \tag{16}$$

7. Cosmic potential produced by radiation

We will verify that the potential originated by radiation has a singularity at the beginning of the Big Bang. To verify it, we assume the Universe with only radiation $\Omega_{0R} = 1$, then (11) is

$$\xi = \frac{1}{t_0 H_0} \int_0^a a' da' \quad \Rightarrow \quad a = \sqrt{2 t_0 H_0} \sqrt{\xi} = \gamma \sqrt{\xi},$$

the age of the Universe is now $t_0 = 1/(2H_0)$ and H_0 is a characteristic parameter of this Universe.

The comoving distance from where the signal leaves at ξ and arrives at $t = \beta t_0$ is by (12)

$$rR_0 = ct_0 \int_{\xi}^{\beta} \frac{d\xi'}{\gamma \sqrt{\xi'}} = \frac{2ct_0}{\gamma} \left(\beta^{1/2} - \xi^{1/2} \right) \quad \Rightarrow \quad \xi = \left(\beta^{1/2} - \frac{\gamma}{2ct_0} rR_0 \right)^2$$
$$a[\xi(rR_0)] = \gamma \left(\beta^{1/2} - \frac{\gamma}{2ct_0} rR_0 \right), \tag{17}$$

the maximum value of $R_0 r$ corresponds to $a = 0$

$$\max(rR_0) = \frac{2ct_0}{\gamma} \beta^{1/2}.$$

ξ	$rR_0\,(\times 10^{25})$	$\frac{(rR_0)^2}{1-\xi}\left(\times 10^{50}\right)$	$\frac{(rR_0)^2}{1-\xi}a^3\left(\times 10^{50}\right)$
3.1595 10^{-6}	43.865	1924.127	0
0.001	39.985	1600.361	0.002
0.02	32.188	1057.225	0.589
0.04	28.980	874.846	1.951
0.06	26.730	760.116	3.818
0.08	24.940	676.091	6.045
0.1	23.429	609.899	8.535
0.2	17.961	403.256	22.888
0.3	14.149	286.008	37.377
0.4	11.143	206.929	49.645
0.5	8.634	149.102	58.223
0.6	6.475	104.801	61.943
0.7	4.578	69.854	59.559
0.8	2.889	41.743	49.670
0.9	1.372	18.835	30.546
0.94	0.807	10.862	19.836
0.98	0.264	3-482	7.143
1	0	0	0

Table 2. Values to numerically calculate the cosmic potential at the present moment.

With the previous results, we apply (15), for which we make the change of variable

$$\chi = rR_0\frac{\gamma}{2ct_0}; \qquad d\left(rR_0\right) = \frac{2ct_0}{\gamma}d\chi; \qquad \max\ \chi = \beta^{1/2},$$

then the potential at the moment t_0 is

$$\phi_{0R} = -64\pi G\rho_{0R}\frac{c^2t_0^2}{\gamma^4}\int_0^1 \frac{\chi^2}{\left(1-\chi\right)-\left(1-\chi\right)^3}d\chi. \tag{18}$$

(18) is the gravitational potential produced at time t_0 in the Universe where there is only radiation. The integral (18) is divergent; therefore, the cosmic potential produced by radiation is infinite. To circumvent this singularity that originated in the Big Bang, we proceed in several ways (see below), including renormalization. The physical equations (for example, the equation of motion) are invariant against recalibration of the potential; that is to say, adding to the potential any fixed amount, we return to find an equally applicable potential.

To avoid divergence, we choose a cut-off point that, for example, coincides with a moment $\xi = \xi_\varepsilon$, in which radiation is no longer significant in the Universe. Therefore, we only have to consider matter and dark energy. The cosmic potential produced up to the moment ξ_ε is a fixed quantity ϕ_ε (renormalization term) that is infinite and that, when subtracted from ϕ, will give us a finite renormalized potential $\hat{\phi}$

$$\hat{\phi} = \phi - \phi_\varepsilon. \tag{19}$$

8. Renormalized Cosmic Potential

Our method for finding the renormalized potential involves dividing cosmic evolution into two parts. The first part, from $\xi = 0$ to $\xi = \xi_\varepsilon$, is dominated by radiation. After $\xi = \xi_\varepsilon$, we assume that only matter and dark energy exists in the Universe. While this model differs from the real Universe, it is acceptable as we are primarily interested in qualitative results. We will keep the parameters (10) in this simplified model.

We characterize the moment ξ_ε by the parameter ε, which relates the density of matter and radiation

$$\rho_R = \varepsilon \rho_M$$

ε represents a numerical coefficient that we arbitrarily assign the value 1. It defines the time when the energy densities of radiation and matter become equal.

The changes in density with the cosmic scale factor are

$$\rho_M = \rho_{0M} \frac{R_0^3}{R^3}; \qquad \rho_R = \rho_{0R} \frac{R_0^4}{R^4},$$

we deduce

$$a_\varepsilon = \frac{1}{\varepsilon} \frac{\rho_{0R}}{\rho_{0M}} = \frac{1}{\varepsilon} \frac{\Omega_{0R}}{\Omega_{0M}},$$

a_ε is the relative scale factor from which we compute (14) and (16) and neglect (15). Then

$$\varepsilon = 1; \qquad a_\varepsilon = 2.651 \cdot 10^{-4}; \qquad \xi_\varepsilon = 3.1595 \cdot 10^{-6}.$$

The renormalization term ϕ_ε is the potential created in the first phase of the Universe, we calculate it by (15), making the integral between the moments $\xi = 0$ and $\xi = \xi_\varepsilon$, and assuming

$$\frac{\Omega_{0R}}{a^2} >> \frac{\Omega_{0M}}{a} \quad \text{and} \quad \frac{\Omega_{0R}}{a^2} >> \Omega_{0V} a^2.$$

The photonic distance travelled by the gravitational signal is

$$r' = ct - ct_e = ct_0 \left(\frac{t}{t_0} - \frac{t_e}{t_0} \right) = ct_0 \left(\beta - \xi \right)$$

then

$$\begin{aligned} \phi_\varepsilon(\beta) &= -\frac{8\pi G \rho_{0R}}{ct_0} \int_0^{\xi_\varepsilon} \frac{(R_0 r)^2}{\beta - \xi(R_0 r)} \frac{1}{a\left[\xi(R_0 r)\right]} d(R_0 r) = \\ &= -32\pi G \rho_{0R} \frac{c^2 t_0^2}{\gamma^4} \int_0^{\xi_\varepsilon} \frac{\left(\beta^{1/2} - \xi^{1/2}\right)^2}{(\beta - \xi)\xi} d\xi = -32\pi G \rho_{0R} \frac{c^2 t_0^2}{\gamma^4} \int_0^{\xi_\varepsilon/\beta} \frac{\left(1 - \eta^{1/2}\right)^2}{(1-\eta)\eta} d\eta; \quad \left(\eta = \frac{\xi'}{\beta} \right), \end{aligned} \tag{20}$$

the renormalization term depends on the moment we calculate the cosmic potential. The parameter γ is now defined by

$$\gamma = \sqrt{2 t_0 H_0 \sqrt{\Omega_{0R}}} = 0.1318$$

the values of t_0 and H_0 are in (10).

In a Universe consisting only of matter and dark energy, and with zero spatial curvature, the scale factor is

$$R(t) = R_0 \sqrt[3]{\frac{\Omega_{0M}}{\Omega_{0V}}} \sinh^{2/3}\left(t/t_\Lambda\right) = R_0 \sqrt[3]{\frac{\Omega_{0M}}{\Omega_{0V}}} \sinh^{2/3}\left(\alpha_0 \xi\right). \tag{21}$$

The comoving radial distance from the place where emitting a signal at t_e reaches the observer at $t = \beta t_0$ is by (11)

$$rR_0 = ct_0 \int_\xi^\beta \frac{d\xi'}{a(\xi')} = ct_0 \sqrt[3]{\frac{\Omega_{0V}}{\Omega_{0M}}} \int_\xi^\beta \frac{d\xi'}{\sinh^{2/3}\left(\alpha_0 \xi'\right)}; \quad \xi \in \left(\xi_\varepsilon, \beta\right), \tag{22}$$

from which we deduce the function $\xi = \xi(R_0 r)$, which allows us to calculate numerically the integrals (14) and (16) (Table 2), that correspond to the renormalized potential, which is cosmic potential from which the renormalization term has been subtracted (20). For the potential at the

$\Delta\xi$	$\%\,\hat{\phi}$
3.16 10^{-6}	85.62
0.1-0.2	9.17
0.2-0.3	3.82
0.3-0.4	1.69
0.4-0.5	0.62
0.5-0.6	0.046
0.6-0.7	- 0.24
0.7-0.8	- 0.33
0.8-0.9	- 0.28
0.9-1	- 0.11

Table 3.- Percentage of participation in the cosmic potential as a function of the time interval in which the gravitational interactions that reach the observer at the present moment were emitted. We verify that in recent times the potential produced by the vacuum is superior to that of matter. The table displays the renormalized potential, which is the combined potential from matter and dark energy since the end of the radiation dominance epoch.

present moment $\beta = 1$, we find

$$\hat{\phi}_M = -5.76c^2; \quad \hat{\phi}_V = 0.35c^2; \quad \hat{\phi}_R \approx 0 \quad \Rightarrow \quad \hat{\phi} = -5.41c^2. \tag{23}$$

According to the cosmological principle, the potential is equal across the entire Universe when calculated at the same time.

9. Participation of the epochs of the Universe in the cosmic potential

The cosmic potential at time t_0 is the result of the entire Universe's actions that are causally related to the observer. Interactions from far distances happened much earlier than the present, and as a result, the cosmic potential is a result of the interaction of the entire observable Universe.

To determine the renormalized potential, we need to perform a decomposition of the integral with limits 0 and ξ_ε.

$$\hat{\phi}(\xi_\varepsilon - 1) = \hat{\phi}(\xi_\varepsilon - 0.1) + \hat{\phi}(0.1 - 0.2) + \hat{\phi}(0.3 - 0.4) + ... + \hat{\phi}(0.9 - 1)$$

which is the sum of the cosmic potentials produced in different cosmic epochs (Table 3).

Note that the percentages calculated in Table 3 depend on the cut factor ε, that is, of the renormalization term ϕ_ε. In Table 3, we notice that the potential produced by the vacuum is the predominant one in recent times. We also verified that the first epochs of the Universe contributed to a greater extent to the cosmic potential.

10. The change in cosmic potential over time

The potential of the cosmos is not influenced by location but rather by cosmic time. Also, the renormalization term ϕ_ε depends on the moment we calculate the potential. The difference in cosmic potential at two different times is

$$\phi(\beta) - \phi(\beta') = \hat{\phi}(\beta) - \hat{\phi}(\beta') + \left[\phi_\varepsilon(\beta) - \phi_\varepsilon(\beta')\right], \tag{24}$$

by (20)

$$\phi_\varepsilon(\beta) - \phi_\varepsilon(\beta') = -32\pi G \rho_{0R} \frac{c^2 t_0^2}{\gamma^4} \left[\int_0^{\xi_\varepsilon/\beta} \frac{\left(1-\eta^{1/2}\right)^2}{(1-\eta)\eta} d\eta - \int_0^{\xi_\varepsilon/\beta'} \frac{\left(1-\eta^{1/2}\right)^2}{(1-\eta)\eta} d\eta \right] =$$

$\beta = t/t_0$	$\hat{\phi}$ $\left(\times c^2\right)$	$\phi_\varepsilon - \phi_\varepsilon(\beta=1)$ $\left(\times c^2\right)$	$\phi - \phi(\beta=1)$ $\left(\times c^2\right)$
0.2	- 5.96	- 4.78	- 5.33
0.4	- 5.95	- 2.74	- 3.28
0.6	- 5.84	- 1.53	- 1.96
0.8	- 5.66	- 0.67	- 0.92
1	- 5.41	0	0
1.2	- 5.12	0.55	0.84

Table 4.- We see the cosmic potential changing over time. We refer the potential to the current moment $\beta = 1$. We verify that with time, the cosmic potential decreases; that is, the gravitational action of the Universe decreases.

$$
\begin{aligned}
&= -32\pi G\rho_{0R}\frac{c^2t_0^2}{\gamma^4}\int_{\xi_\varepsilon/\beta'}^{\xi_\varepsilon/\beta}\frac{\left(1-\eta^{1/2}\right)^2}{(1-\eta)\eta}d\eta = -32\pi G\rho_{0R}\frac{c^2t_0^2}{\gamma^4}\left[\ln\eta - 4\ln\left(\sqrt{\eta}+1\right)\right]_{\xi_\varepsilon/\beta'}^{\xi_\varepsilon/\beta} = \\
&= -32\pi G\rho_{0R}\frac{c^2t_0^2}{\gamma^4}\left[\ln\frac{\beta'}{\beta} + 4\ln\frac{\sqrt{\xi_\varepsilon/\beta'}+1}{\sqrt{\xi_\varepsilon/\beta}+1}\right].
\end{aligned}
\tag{25}
$$

We compare the cosmic potential at two different moments, using (24) and (25). Our reference is the cosmic potential at t_0. The results are in Table 4, where we verified that the absolute value of the cosmic potential decreases with time

We remember that the potential calculated in Table 4 does not produce any measurable effect since the potential is the same everywhere. However, the cosmic potential in Table 4 indicates the intensity of the gravitational action as a function of cosmic time.

11. Proposals to circumvent the Big Bang singularity

In the simplified model of the Universe that we consider, we find that there is an infinite potential whose origin is radiation and is caused by the Big Bang singularity; this is an unsatisfactory result, which we have circumvented by renormalizing the potential, which we can do because the gravitational potential is invariant against its recalibration.

A hypothesis that could solve the problem of infinite potential is to consider a Yukawa-type term

$$
\phi = -G\int\frac{\rho\left(x'^\alpha,t\right)}{(1+\delta)r'\left(x'^\alpha,x^\alpha\right)}\left(1+\delta e^{-r'/\lambda}\right)dV' \quad \text{or} \quad \phi = -G\int\frac{\rho\left(x'^\alpha,t\right)}{r'\left(x'^\alpha,x^\alpha\right)}e^{-\mu r'}dV'
$$

δ is a measure of the intensity of the Yukawa correction, and λ and μ are scale factors (Schmid 2006).

We can also assume that there is gravitational absorption

$$
\phi = -G\int\frac{\rho\left(x'^\alpha,t\right)}{r'\left(x'^\alpha,x^\alpha\right)}e^{-kr'}dV'
$$

k is the gravity absorption coefficient, which depends on the absorbent medium (Assis 1992).

The Yukawa term dampens gravity over distance; absorption reduces gravitation when passing through an absorbing medium. With both procedures the infinity of the potential would be eliminated.

There is an additional factor that could affect the calculation in this investigation, which is Mach's principle, according to which the gravitational action of the Universe produces the inertial mass, therefore

$$
m_i(t) = \chi(t)m_g
$$

$\chi(t)$ is the coefficient of inertia or proportionality factor between the inertial mass m_i and the gravitational mass m_g. We assume that the gravitational mass of a body or a system of bodies is a characteristic quantity and, therefore, invariable. $\chi(t)$ is a factor that depends on time; however, it is the same everywhere in the Universe for the same moment. We modify the universal gravitational constant to ensure that inertial and gravitational masses currently possess identical values: $\chi(t_0)=1$.

In Special Relativity and General Relativity the energy at the center of mass is

$$E = m_i c^2 \tag{26}$$

It's important to remember that the mass equivalent to energy is the inertial mass, not the gravitational mass. (26) is valid for both a particle and a system of interacting particles. As the inertial mass varies with time, the energy associated with it will also be variable, keeping the gravitational mass invariable.

Consider a system of particles whose energy at the moment *t* is $E(t)$, then its inertial mass at the same moment is $m_i(t) = E(t)/c^2$. The associated gravitational mass, which is independent of time, is

$$m_g = \frac{1}{\chi(t)} m_i(t) = \frac{1}{\chi(t)} \frac{E(t)}{c^2} = \frac{E(t_0)}{c^2}$$

$E(t_0)$ is the energy of the system, which we assume to be invariable, measured at the current time t_0. Note, therefore, that if we calculate the gravitational mass by the energy value at time *t*, we have to divide the result by $\chi(t)$ to find the correct value of the gravitational mass. This result would alter the calculations we have made before, in which assuming that the inertial mass (and its associated energy) is unalterable and maintains the same relationship with the gravitational mass.

12. Conclusions

We generalized the Newtonian potential to calculate the gravitational potential of the entire Universe, taking into account the singularity in the Big Bang, which produces an infinite potential from cosmic radiation. However, the singularity in matter density at the Big Bang does not cause an infinite potential. We renormalized the cosmic potential and obtained finite values to obtain physical results.

The early stages of the Universe are the main source of potential. The gravitational action in recent times, produced by nearby objects, is small compared to the gravitation produced in the first epoch of the cosmos.

We found that the gravitational potential produced by dark energy increases with time, becoming dominant in recent times.

Due to the cosmological principle, the cosmic potential is the same everywhere as long as it is calculated at the same moment; therefore, it does not produce a gravitational force on a body at rest. However, the cosmic potential decreases with cosmic time, indicating that the gravitational action of the Universe is decreasing.

Radiation's infinite potential is problematic, which is why we've suggested various solutions to avoid this issue.

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