Quantum Gravity Framework: 1.0. A Framework of Principles for Quantum General Relativity with Time and Measurement*

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March 7, 2016

Abstract

The purpose of this article is to outline a framework of concepts and principles to combine quantum mechanics and general relativity so that time and measurement (reduction) are present as integral parts of the basic foundations. First, the problem of time in quantum gravity and the measurement problem in quantum mechanics are briefly reviewed and the popular proposals to tackle these two problems are briefly discussed. Next, on the already known foundations of quantum mechanics, a framework of principles of dynamics is built: 1) Self-Time Evolution - Newtons first law is reinterpreted to define time, 2) Local Measurement by Local Reduction - Quantum diffusion theory is adapted, and 3) Global Evolution by Global Reduction. Ideas on how to apply the framework to study quantum general relativistic physics are discussed. Further, more general and modified forms of some of these principles are also discussed. The theoretical elements in the framework to be made concrete by further theoretical and experimental investigations are listed. Revision information is included.

Please note that this version will be shortly followed up by version 2.0, which is more advanced,better worked out and conceptually somewhat different. We refer to www.qstaf.com for further discussions. Updates regrading this research will be made available at twitter.com/qstaf. The official website for this research is www.qstaf.com. and the author can be contacted through the website.

Contents

[1] Please note that there is an updated version of this paper 2.0 will be made available shortly.

1 Introduction and Brief Review[2]

Quantum mechanics and general relativity are two great theoretical developments of the last century. Both of them have enormous experimental support; Yet, both are incomplete. Each of them leaves open to interpretation one important conceptual foundation in their theoretical framework: measurement (reduction) in quantum mechanics [1] and time in (quantum) general relativity. This has left the quest for unifying quantum mechanics and general relativity in a difficult state. There are many different proposals to tackle these two problems and cure the incompleteness in both the theories. My goal in this article is to propose a framework of concepts and principles for quantum general relativity, such that it 1) has time (dynamics) and measurement (reduction) as integral parts of the basic foundations, 2) is simple and intuitive, 3) has proper physical motivation, 4) is based on simple scientifically established notions and concepts, and 5) makes minimal assumptions.

Primarily this article is intended to address the issues of time and measurement in quantum general relativity. But, I also discuss briefly the established notions in quantum physics in an axiomatic form so that we have a complete framework of principles for the Planck scale physics[3].

We follow the following conventions in this article:

Convention 1: *In any integral, the variables over which the integration is done is same those used in the measure placed in the right most end of the integral, unless explicitly indicated otherwise.*

Convention 2: *Summation is assumed for all repeated Greek indices in the explicit elementary products of the basic variables of the theories discussed.*

Convention 3: *In the differential measures of the integrals, the multiplication over all the suffixes and the prefixes is assumed, for example* $dx^\beta dy_\gamma$ *mean* $\prod\limits_{\beta,\gamma} dx^\beta dy_\gamma$.

Convention 4: *For functions with arguments that have suffixes, prefixes, and parameters: The function depends on all the collection of the arguments for all different values of the suffixes, the prefixes and the parameters. Example:* $f(x^\alpha_\gamma(t), y_\alpha) = f(X)$, *where* $X = \{x^\alpha_\gamma(t), y_\beta, \forall \alpha, \beta, \gamma, t\}$.

Convention 5: *No other summation or multiplication of repeated indices is assumed other than those defined in conventions 2 and 3. Examples: 1) there no summation in* $f_\alpha(x^\alpha, y_\alpha)$, *the three* α*'s are independent, 2)* $(p^\alpha_\beta x_\alpha + f_\beta(x^\alpha, y_\alpha))dx^\alpha dy_\alpha = (\sum\limits_\alpha p^\alpha_\beta x_\alpha + f_\beta(x^\gamma, y_\delta)) \prod_{\eta,\varepsilon} dy_\eta dx^\varepsilon$.

Convention 6: *It is assumed that* $\hbar = c = G = 1$, *unless specified.*

1.1 Quantum Mechanics and Measurement

Let me briefly review the measurement problem in quantum mechanics. Consider a quantum mechanical system S. Let $\hat{q}$ be an observable of the system S to be measured. Let an observing instrument O be setup to measure $\hat{q}$. During measurement, S interacts with the quantum variables of O due to the presence of the interaction terms in the total Hamiltonian of $O + S$. Let $|\Psi^0_S>$ and $|\Psi^0_O>$ are the normalized initial

[2] Please note that there is an updated version of this paper 2.0 will be made available shortly.

[3] Revision comment: This paper has new ideas mixed with old ideas that are well known to physicists. So please be aware that one can easily miss the new ideas. In the next revision of this version of quantum framework this problem will be fixed.

quantum states of S and O respectively. Let $|\Psi_S^0> = \sum_q \Psi_S^0(q)|q>$, where q is an eigenvalue of $\hat{q}$. The initial state of the combined system is

$$|\Psi_S^0> \otimes |\Psi_O^0> = (\sum_q \Psi_S^0(q)|q>) \otimes |\Psi_O^0> .$$

During measurement, the quantum state of $S+O$, $|\Psi>$, evolves according to the time dependent Schrödinger equation:

$$i\hbar \frac{d}{dt}|\Psi> = H_{S+O}|\Psi> . \tag{1}$$

During this evolution, due to the interaction, the initial state is transformed into a superposition of macroscopic states of the form $\sum_q \Psi_S(q)|q> \otimes |\Psi_O^q>$, in which the $|\Psi_O^q>$'s for the different q's are macroscopically distinguishable from each other. The Copenhagen interpretation of quantum mechanics tells that in the final step of the quantum measurement the superposed quantum state collapses to $\Psi_S(q)|q> \otimes |\Psi_O^q>$ with probability $|\Psi_S(q)|^2$. The quantum measurement problem is to explain how the superposed state $\sum_q \Psi_S(q)|q> \otimes |\Psi_O^q>$ randomly collapses to $\Psi_S(q)|q> \otimes |\Psi_O^q>$ an eigenstate of $\hat{q}$ for the eigenvalue q. This is also referred to as the quantum reduction of the superposed state.

To better understand the measurement process, we need to include the conscious observer C who observes the result of the measurement and the rest of the universe $\bar{U}$ (in the Newtonian Sense) with the initial quantum states $|\Psi_C^0>$ and $|\Psi_{\bar{U}}^0>$ respectively. Let U be the entire universe with the quantum state $|\Psi_U>$. The (approximate) initial state of the entire universe $|\Psi_U^0> = |\Psi_S^0> \otimes |\Psi_O^0> \otimes |\Psi_C^0> \otimes |\Psi_{\bar{U}}^0>$ evolves by the Schrödinger equation into a superposed state of the universe $|\Psi_U> = \sum_q \Psi_S(q)|q> \otimes |\Psi_O^q> \otimes |\Psi_C^q> \otimes |\Psi_{\bar{U}}^q>$. In the final step of the measurement (or the human observation) this superposition collapses into $|\Psi_U^q> = \Psi_S(q)|q> \otimes |\Psi_O^q> \otimes |\Psi_C^q> \otimes |\Psi_{\bar{U}}^q>$ with the probability $|\Psi_S(q)|^2$.The ignorance about this quantum reduction process has spawned many proposals. Let me discuss a typical few of the popular proposals, and the good points and the problems with them.

1) The many world theory [2]: In the sum $|\Psi_U> = \sum_q \Psi_S(q)|q> \otimes |\Psi_O^q> \otimes |\Psi_C^q> \otimes |\Psi_{\bar{U}}^q>$, each term for different q is assumed to be representing a different universe (world). So the universe represented by the quantum state $|\Psi_S^0> \otimes |\Psi_O^0> \otimes |\Psi_C^0> \otimes |\Psi_{\bar{U}}^0>$ evolves into a superposition of the many universes corresponding to the different eigenvalues of $\hat{q}$, and the observer jumps into one of these universes with probability $|\Psi_S(q)|^2$. It is assumed that the collapse or reduction of superposition never happens. The critical problems with this theory are: 1) it does not explain exactly when the jump happens and 2) how to split the universe into the observer and the rest of the universe. The many world theory needs to be considered more as a philosophy rather than physics.

2) The GRW collapse theory [5]. In this spontaneous collapse of the wavefunctions of particles is proposed. Even though the probability of each particle to collapse is proposed to be very small, since there are always very large number of particles in any macroscopic quantum system, few of these particles always collapse in any instant. Due to the quantum entanglement with the rest of the system, they collapse the entire system.

3) The orchestrated reduction theory by Penrose and Hameroff [3]: This theory is based on the idea that the presence of the quantum superposition of different macroscopic states of gravity disturbs the flow of time; so the macroscopic superposition collapses itself. An estimate of the time for the collapse or reduction of the superposition is given in terms of the mass of the superposing system. Consciousness is proposed to be related to the collapse phenomenon. These set of ideas are rich in insights, and a systematic theory needs to be developed.

4) Decoherence theories [4], [18]: In these theories the universe is divided into two parts: the quantum system to be studied and the environment. The pure density matrix $|\Psi_U><\Psi_U|$ describing the universe is traced with respect to the environmental degrees of freedom. The resulting mixed density matrix is considered to describe a statistical ensemble of the quantum system. It describes the continuous probabilistic evolution of the quantum system, instead of sudden collapse. Decoherence theories need objective criteria for dividing the universe into a quantum system to be measured and the environment.

5) Quantum diffusion theory [6]: In this theory, the time-dependent Schrödinger equation is modified to include stochastic terms, and terms that remove superposition (both of which I will discuss later in this

article). The wavefunction evolves continuously, but also stochastically, gradually undergoing reduction or semiclassicalization, without any sudden collapse. Presence of these terms makes the evolution of the wavefunction to reproduce the Copenhagen probabilistic reduction.

In this article, I will build on the philosophical and the technical insights in all these ideas, and the dynamical equation in the last proposal.

1.2 Classical General Relativity and Time

The nature of time has been debated right from the antiquity. Great progress has been made in physics in understanding the nature of space and time, but deeper questions still persist. Let me assume that an observer is a finite entity that records events happening around it in space and time.

In Newtonian physics, time and space are absolute. Newton assumes the space is rigid, three-dimensional, Euclidean, and there is a global time parameter t which at a given instant is same at all points of the universe and for all the observers. Space is also absolute, meaning the 3D length between any two points is same as observed by all the observers. Dynamics of the particles or/and the fields on the manifold is described in terms of the absolute space-time coordinates.

In special relativity space and time are merged into a four dimensional pseudo-Euclidean space. There is one $4D$ space-time reference frame corresponding to each observer, and the transformation between any two of them is given by the Lorentz transformation or more generally the Poincare transformation. This makes time observer dependent. Then the reality observed by an observer is just one way to observe the universe and is generally related to that of the other observers by the Poincare transformation. The common physical information between the description of the various reference frames are the invariants such as the pseudo-Euclidian scalar product, the Cassimer Invariants and other conserved quantities.

In general relativity, the time and space split of 4D space-time manifold is highly arbitrary: because of general covariance, any smooth decomposition of the four dimensional space-time manifold into a one parameter family of spatial hypersurfaces is equally acceptable. For example, an observer can choose a one parameter family of hypersurfaces such that he is at rest in it. The parameter plays the role of the time coordinate.

The quantum measurement that we discussed in the last section is observer dependent as the entire measurement theory is formulated on the combined system made of the quantum system whose observable is to be measured, the observing instrument and the observer. The observer dependence in relativity and quantum mechanics has spawned many relational theories such as the relational quantum mechanics [9] and the relational particle mechanics [8]. The relational nature of reality is really common sense: Every entity observes the universe with respect to itself. But this does not explain why all the entities in the universe undergo temporal evolution. Relational theories do not answer this question. More interestingly there are proposals that time itself is not real [10]. But, every one clearly knows that time exists because he can measure it with his wristwatch and feel its flow. The purpose of science is to explain the observational reality and time is real in this context. So any theory that claims time does not exist is a philosophy, or by time they mean something else that is not related to the common sense time.

We need to observe two critical ideas from Einstein's relativity. First, in classical general relativity one can always choose any particular choice of sequences of spatial hypersurfaces to describe physical phenomena. But not all choices are sensible to describe the universe. For example, in cosmology, time is defined based on the flow of the events on the space-time manifold globally such as the expansion of the universe. In this respect, the cosmological time is absolute like in the Newtonian theory, and is objectively based on the physical processes happening on the space-time manifold. Second, an observer doesn't have direct access to the physical processes at a distant point. So the reality observed by him is actually a picture painted to him by the information that traveled through space and time as disturbance in the physical fields. So, it is subjected to the many symmetry properties and other transformations associated to the space-time manifold and the internal space of fields. We can put these two points together. As space-time itself is built on the physical processes associated with the metric field, an objective time is to be based on the physical processes and quantities associated with the metric and the other physical fields living on the underlying 4D manifold. So in this article we will try to formulate a general concept of global flow of time based on the quantum physical processes happening on the $4D$ manifold.

1.3 The Problem of Time in Quantum Gravity

Advanced readers can skip this section.

When one tries to work out the quantum formulation of general relativity, one faces a series theoretical block referred to as the problem of time as a consequence of general covariance (coordinate independence). Let me review this now.

Let S be the total action corresponding to the fields on the space-time manifold $\mathcal{M}$ without including general relativity for now:

$$S = \int L d^4 x,$$

where the L is the corresponding Lagrangian density and the x^a are the space-time coordinates.

To get the Hamiltonian formulation, we assume the topology of the space-time manifold is $\mathsf{R}\otimes\, S$, where S is a smooth $3D$ manifold. The space-time manifold is foliated by a one parameter family of spatial hypersurfaces $\Sigma(t)$ of topology S, parametrized by a (time) parameter t. Now, let x^i be the (spatial) coordinates on the 3D hypersurfaces. We go to the Hamiltonian formulation using the Legendre transformation. For this, usually, formally, we rewrite the Lagrangian and an action as follows:

$$L(p_\alpha, q^\alpha, t) = \int_{\Sigma(t)} (\sum_\alpha p_\alpha \dot{q}^\alpha + H(p_\alpha, q^\alpha, t)) d^3x \qquad , S(p_\alpha, q^\alpha) = \int L dt, \tag{2}$$

where the q^α and p^α are the collection of the configuration variables and the corresponding conjugate momentum variables (one set for each point of the manifold) of the fields and H is the Hamiltonian density. Usually H involves constraint terms C_k,

$$H = H_0(p_\alpha, q^\alpha, t) + \lambda^k C_k(p_\alpha, q^\alpha, t), \tag{3}$$

where the λ^k are the Lagrange multipliers, and the H_0 is the non-constrained part.

Usually the variational analysis leads to the Hamilton equations of motion. For any observable $F(q^\alpha, p^\alpha)$, a function of the variables q^α and p^α, we have,

$$\begin{aligned} \frac{dF}{dt} &= \{F, H\}, \\ C_\alpha &= 0, \end{aligned}$$

where the brackets are the Poisson brackets defined by

$$\{A, B\} = \frac{\partial A}{\partial q^\alpha}\frac{\partial B}{\partial p_\alpha} - \frac{\partial B}{\partial q^\alpha}\frac{\partial A}{\partial p_\alpha}.$$

In particular for q^α and p_α we have,

$$\{q^\alpha(x), p_\beta(x')\} = \delta^\alpha_\beta \delta^3(x - x'),$$

and the equations of motion are

$$\begin{aligned} \frac{dq^\alpha}{dt} &= \frac{\partial H}{\partial p_\alpha}, \\ \frac{dp_\alpha}{dt} &= -\frac{\partial H}{\partial q_\alpha}. \end{aligned} \tag{4}$$

Dirac has formulated the way to go from the classical Hamiltonian formulation to the quantum mechanical formulation [11]:

$$
\begin{aligned}
[\hat{q}^\alpha(x), \hat{p}_\beta(x')] &= i\hbar\delta^\alpha_\beta\delta^3(x-x'), \\
\hat{C}_k \left|\psi\right\rangle &= 0,\ \forall k, \\
[\hat{F}, \hat{C}_k] &= 0,\ \forall k, \\
i\hbar\frac{d}{dt}\left|\psi\right\rangle &= \hat{H}_0\left|\psi\right\rangle, \\
i\hbar\frac{d\hat{F}}{dt} &= [\hat{F}, \hat{H}_0],
\end{aligned}
\tag{5}
$$

where $\hbar$ is the Planck constant. In these equations,

- the classical variables have become linear operators on the Hilbert space of quantum states,
- the Poisson brackets have been replaced by commutators,
- C_k have become quantum constraint operators $\hat{C}_k$,
- the physical states are to be annihilated by $\hat{C}_k$ $\forall k$, and
- the physical observables are supposed to commute with $\hat{C}_k$ $\forall k$.

The C_k are usually the generators of the gauge transformations associated with the theory. The $\hat{C}_k\left|\psi\right\rangle = 0$ and $[\hat{F}, \hat{C}_k] = 0,\ \forall k$ mean that the physical states and operators are gauge invariant.

Let us now add general relativity to the analysis. In the rest of the article we assume $\hbar = c = G = 1$, unless stated. The action can be split into the matter (suffix m) and the gravitational (suffix g) parts:

$$
\begin{aligned}
S &= S_g + S_M = \int L_g d^4x + \int L_m d^4x \\
&= \int \sqrt{-g}\mathcal{R} d^4x + \int L_m d^4x,
\end{aligned}
\tag{6}
$$

where the $\mathcal{R}$ is the space-time curvature. The Hamiltonian formulation of general relativity is given by the ADM formulation [15]. Usually in the theories that do not assume fixed metric the variational analysis leads to the Hamilton equations with clear dynamics, both in the quantum and the classical level. But once we allow full gravitational dynamics, as in the ADM formulation, H_0 becomes zero and H is simply a linear combination of constraint terms:

$$
H = \sum_k \lambda^k C_k. \tag{7}
$$

This is not a problem in case of the classical analysis, the Hamilton equations clearly leads to dynamics. But in case of the quantum formulation, $\hat{H}$ is supposed to be zero on its action on the physical quantum states, as it is a linear combination of $\hat{C}_k$ which are supposed to be zero on their action on them. There is no dynamics! This is called the problem of time in quantum gravity which has lead to numerous conceptual speculations (recently reviewed in ref. [14]).

It is surprising that when we use the Dirac's procedure for quantization for general relativity, time evolution exists in the classical level, while it disappears in the quantum mechanical level. The explanation for this is that at the classical level $\dot{p}_\alpha$ and $\dot{q}^\alpha$ depend on the derivatives of the C_k (with respect to p_α and q^α), and C_k are constrained to zero independent of these dynamical equations. So $\dot{p}_\alpha$ and $\dot{q}^\alpha$ do not disappear. But in quantum mechanics $\hat{H}$ is supposed to be zero in its action on $\left|\psi\right\rangle$. So $\frac{d}{dt}\left|\psi\right\rangle$ is supposed to be zero. And, for any observable $\hat{F}$, we can show that $\frac{d}{dt} < \hat{F} >= 0$. All these suggest that the Dirac quantization procedure leads to no dynamics for quantum gravity.

In the ADM formulation the Hamiltonian density is,

$$H_g = NH_N + N^\alpha D_\alpha, \tag{8}$$

where the H_N is called the Hamiltonian term and D_α, the diffeomorphism terms. These are the generators of space-time diffeomorphisms. Constraining these two to zero has the obvious physical interpretation that the physical state of system must not change under the (small) diffeomorphisms of the space-time coordinates. Since the lapse N relates to time scaling, H_N is called the Hamiltonian term. The problem of time is to extract dynamics from its constraint equation, which is referred as the Wheeler-Dewitt equation [13]:

$$H_N |\psi\rangle = 0. \tag{9}$$

To better understand the problem of time, let me first discuss the path integral formulation of a $0+1$ dimensional theory (a single point and one time dimension) with no constraints. Let p_α and q^α be the conjugate momenta and the configurational variables respectively, and d be the dimension of the configuration space. Then the Lagrangian can written as,

$$L(p_\alpha, q^\alpha) = p_\alpha \dot{q}^\alpha - H. \tag{10}$$

Let me assume that the time variable t is discretized into a sequence of values, with consecutive values separated by a small interval Δt. We can evolve the quantum system through a sequence of steps corresponding to each of these intervals. If $\psi(q^\alpha)$ and $\psi'(q'^\alpha)$ are the wavefunctions corresponding to two consecutive intervals then they are related by the single-step propagator $G(q^\alpha, q'^\alpha; \Delta t)$:

$$\psi'(q'^\alpha) = \int G(q^\alpha, q'^\alpha; \Delta t)\psi(q^\alpha)dq^\alpha.$$

From the path integral formulation of quantum mechanics, the single-step propagator for each interval is,

$$\begin{aligned} G(q^\alpha, q'^\alpha; \Delta t) &= \frac{1}{(2\pi)^d} \int \exp(iL(p_\alpha, q^\alpha)\Delta t)dp_\alpha \\ &= \frac{1}{(2\pi)^d} \int \exp(ip_\alpha \Delta q^\alpha - iH(q^\alpha, p_\alpha)\Delta t)dp_\alpha, \end{aligned} \tag{11a}$$

where $\Delta q^\alpha = q'^\alpha - q^\alpha$.

Now, let us study the Feynman's derivation of the Schrödinger equation from the path integral formulation:

$$\psi'(q'^\alpha) = \frac{1}{(2\pi)^d} \int \exp(ip_\alpha \Delta q^\alpha - iH(q^\alpha, p_\alpha)\Delta t)\psi(q^\alpha)dq^\alpha dp_\alpha. \tag{12}$$

In this $\exp(-iH\Delta t) \approx (1 - iH\Delta t)$ for a finite H and a small Δt,

$$\psi'(q'^\alpha) \approx \frac{1}{(2\pi)^d} \int \exp(ip_\alpha \Delta q^\alpha) \left\{1 - iH(q^\alpha, p_\alpha)\Delta t\right\} \psi(q^\alpha)dq^\alpha dp_\alpha. \tag{13}$$

Let the Hamiltonian be second order in the momenta, and all the q's are to the left of the p's.Then in the above expression, $H(q^\alpha, p_\alpha)$ can be replaced by $H(q_\alpha, \frac{\partial}{i\partial q_\alpha})$ as follows:

$$\psi'(q'^{\alpha}) \approx \frac{1}{(2\pi)^d}\left\{1 - iH(q'^{\alpha}, \frac{\partial}{i\partial q'^{\alpha}})\Delta t\right\} \int \exp(ip_{\alpha}\Delta q^{\alpha})\psi(q^{\alpha})dq^{\alpha}dp_{\alpha} \quad (14)$$

$$= \left\{1 - iH(q'^{\alpha}, \frac{\partial}{i\partial q'^{\alpha}})\Delta t\right\}\psi(q'^{\alpha}). \quad (15)$$

This derivation works straight forward in non-relativistic quantum mechanics leading to the usual Schrödinger equation.

Now consider an action like in quantum gravity in which the Hamiltonian is constrained:

$$L(p_{\alpha}, q^{\alpha}, N) = p_{\alpha}\dot{q}^{\alpha} - NH(p_{\alpha}, q^{\alpha}).$$

Now the path integral also has a sum over lapse,

$$\psi'(q'^{\alpha}) = \frac{1}{(2\pi)^{d+1}}\int \exp(ip_{\alpha}\Delta q^{\alpha} - iNH(p_{\alpha}, q^{\alpha})\Delta t)\psi(q^{\alpha})dN dq^{\alpha}dp_{\alpha}$$

$$= \frac{1}{(2\pi)^d}\int \exp\left(ip_{\alpha}\Delta q^{\alpha}\right)\delta(H(p_{\alpha}, q^{\alpha}))\psi(q^{\alpha})dq^{\alpha}dp_{\alpha}. \quad (16)$$

In quantum gravity the approximation $\exp(-iH\ \Delta t) \approx (1 - iH\Delta t)$ cannot be done, because the exponent $\exp(-iH\ \Delta t)$ is to be replaced by $\delta(H)$ due to the sum over the lapse N. The one-step propagator is

$$G(q^{\alpha}, q'^{\alpha}; \Delta t) = \frac{1}{(2\pi)^d}\int \exp(ip_{\alpha}\Delta q^{\alpha})\delta(H(q^{\alpha}, p_{\alpha}))dp_{\alpha}. \quad (17)$$

We see here that the Δt term is absent in the right hand side. So the propagator is a function of the configurational variables only:

$$G(q^{\alpha}, q'^{\alpha}) = \frac{1}{(2\pi)^d}\int \exp(ip_{\alpha}\Delta q^{\alpha})\delta(H(q^{\alpha}, p_{\alpha}))dp_{\alpha}. \quad (18)$$

This form of the propagator is what is to be expected. This is because t is a coordinate variable, it can be changed by an arbitrary (smooth) rescaling of the lapse (time diffeomorphism). So the physical evolution should not depend on it. Practically we read time by reading observables, for example the position of a clock's needles. The q's are the most basic observables and so the physical time has to be extracted from them, for example, assuming that one of them acts as an internal time variable. But, this way of choosing the time variable is arbitrary and subjective. We need a more objective way to choose the time variable, which I will discuss in next section.

2 A Framework of Principles

In section 1.1, I have reviewed the various existing proposals for solving the measurement problem. In section 2.1, I concluded that an objective time needs to be formulated based on the quantum physical processes happening on the $4D$ manifold. Based on these two, I will propose a framework of three different principles each of which defines a concept, to address the issue of time and measurement (reduction). These principles can be tested experimentally and so are falsifiable. As stated in the last section, we will assume $\hbar = c = G = 1$, unless explicitly specified.

In this section I discuss a list of principles each of which either defines a concept or is defined based on a concept. First I briefly review the basics of statical foundations of quantum theories. Next, I elaborately discuss the principles of dynamics. Later in the article, I also briefly introduce a generalized and modified form of the principles of dynamics as an alternative.

2.1 Principles of Quantum Kinematics: A Very Brief Review

First we define a set of principles so that we have the proper kinematical background to study dynamics. These ideas are simply the basic principles used in studying quantum without the Schrödinger Hamiltonian equation. Let me list the set of three principles. An entity of a reality can be considered as made of a group of interacting elementary quantum system.

Principle 1.1: *To each elementary quantum system of a physical object is associated a physical configuration space.*

Now the physical configuration spaces differs between various theories used in physics. Usually the physical configuration space is defined by a simple kinematic configuration space divided by symmetry groups. In non-relativistic quantum mechanics it is just a vector space of position. In gauge field theory its a complex vector space reduced by gauge symmetries. In quantum gravity it is the space of metric tensor fields reduced by diffeomorphisms.

Principle 1.2: *The quantum state of a physical object is an element of the Hilbert space on the physical configuration space.*

This means the states are complex functions on the configuration space; they are considered as vectors; there are sesquilinear scalar product and Hermitian conjugation defined. The bra and ket notation can be defined as usual.

Principle 1.3: *The quantum variables are operators on the Hilbert Space.*

Most important quantum variables referred to as the observables are the linear operators that are invariant under Hermitian conjugation. Here we do not assume as fundamental the Copenhagen interpretation that observations collapse the state of a system to an eigenstate of the observable to be observed. It will be an emergent concept in our framework, which will be discussed in the next subsection.

Position and momentum quantum variables can be defined as usual in quantum mechanics, which are usually the most basic variables of the theory. Just consider a simple quantum system, with the basic conjugate variables q^α_x and $p_{\alpha,x}$ and the commutator $[q^\alpha_x, p_{\beta,x'}] = \delta^\alpha_\beta \delta(x' - x)$. The theory is built on the Hilbert space of the square integral functions $\psi(q^\alpha_x)$of the variables q^α_x, where the basic variables become operators:

$$\hat{q}^\alpha_x \psi(q'^\alpha_x) = q'^\alpha_x \psi(q^\alpha_x), \qquad \hat{p}_{\alpha,x} \psi(q'^\alpha_x) = \frac{1}{i} \frac{\partial}{\partial q'^\alpha_x} \psi(q'^\alpha_x). \tag{19}$$

Usually reducing the kinematical configuration space by symmetry groups to get the physical configuration is difficult. Formally, the states and the variables can be subjected to projectors to remove the redundancy due to symmetries. They can be directly applied on the kinematical Hilbert space and the operators $F(\hat{q}^\alpha_x, \hat{p}^\alpha_x)$ to get to the physical Hilbert space and the physical operators:

$$\psi_{phxs}(q^\alpha_x) = \hat{O}\psi(q^\alpha_x), \qquad \hat{F}_{phxs} = \hat{O}^\dagger F(\hat{q}^\alpha_x, \hat{p}_{\alpha,x})\hat{O}, \tag{20}$$

$$\hat{O} = \prod_i \hat{O}(\hat{C}_i). \tag{21}$$

where $\hat{O}(\hat{C}_i)$'s are the projectors relating to the constraints $\hat{C}_i$ (excluding the Hamiltonian constraint). For example, the action of $\hat{O}(\hat{C})$ on a state $|\tilde{\psi}>$ for a constraint $\hat{C}$ can be formally defined as follows:

$$|\tilde{\psi}>= \frac{1}{2\pi}\int \exp(ix\hat{C})dx|\psi>, \qquad \hat{O}(\hat{C})|\psi>= \frac{|\tilde{\psi}>}{<\tilde{\psi}|\tilde{\psi}>}. \tag{22}$$

The gauge invariance removes the redundancies due to the internal symmetries of the field equation. The diffeomorphism invariance relates to invariance under the change of spatial coordinates. Physics tells us that reality should be gauge and diffeomorphism invariant.

2.2 Principles of Quantum Dynamics

The principles of quantum dynamics discussed here applies to the theories that are constrained by the Hamiltonian constraint only. The other constraints (gauge and diffeomorphism) are applied on kinematic evolving states using projectors, as discussed in the last subsection. I assume that the metric in the configuration space is positive definite unless specified.

2.2.1 Self-Time Evolution

Consider[4] a simple quantum system which is described by a Hamiltonian constraint only. Let the canonical variables be p_α and q^α, and d is the dimension of the configuration space. Let $s_{\alpha\beta}$, a function of q^α, is the metric in the configuration space. Hereafter I will use $s_{\alpha\beta}$ and its inverse $s^{\alpha\beta}$(assuming it exists), to raise and lower indices. Let me define a scalar product using the metric:

$$< a, b >= a_\alpha b_\beta s^{\alpha\beta}.$$

As I have indicated in the beginning of this section, I will assume $s^{\alpha\beta}$ is positive definite. Let me assume that a typical Hamiltonian is as follows:

$$\begin{aligned} H(p_\alpha, q^\alpha) &= \frac{< p, p >}{2} + V(q^\alpha) = s^{\alpha\beta}p_\alpha p_\beta + V(q^\alpha) \\ &= \frac{p_\alpha p^\alpha}{2} + V(q^\alpha). \end{aligned}$$

Let the Hamiltonian operator $\delta(H(\hat{q}^\alpha, \hat{p}_\alpha))$ be such that all the q's are to the left of the p's. Here after we will assume that this is the case, unless specified explicitly. The propagator $G(q^\alpha, q'^\alpha)$ between two states used in the last section is,

$$\begin{aligned} G(q^\alpha, q'^\alpha) &= < q'^\alpha|\delta(H(\hat{q}^\alpha, \hat{p}_\alpha))|q^\alpha > \\ &= \frac{1}{(2\pi)^d}\int \exp(ip_\alpha \Delta q^\alpha)\delta(H(q^\alpha, p_\alpha))dp_\alpha. \end{aligned}$$

But $G(q^\alpha, q'^\alpha)$ does not specify any dynamics yet because we have not specified the time variable. For this to define time evolution, we need to decide which configuration variable is to be considered as the time

[4]Revision info: I believe, this proposal is interesting. But it may need some modification to work. In version 2.0 of this paper this proposal has been completely modified.

variable. However, we can linearly map the configuration space to a new configuration space through a linear canonical transformation. So we need to specify a time vector at each instant instead of a time variable, whose magnitude and direction defines the rate and the coordinate axis of evolution. (The rate will be more sensible if we define more than one elementary quantum system, this will be done in the third principle of dynamics.) Let v^α be the vector. The norm of v^α is given by $|v| = \sqrt{|v^\alpha v_\alpha|}$, and the unit vector $\bar{v}^\alpha = \frac{v^\alpha}{|v|}$. Let $\Delta q^\alpha = q'^\alpha - q^\alpha$. If we know v^α,we can define the natural time constraint $\sum_i \bar{v}_\alpha \Delta q^\alpha = |v|\Delta t$, where I consider Δt as a measure of physical time interval of the system. This time constraint restricts the coordinate change to be along $\bar{v}_\alpha$ in the transition from q^α to q'^α and the magnitude of change to be $|v|\Delta t$.

Then the single-step propagator for the system to evolve for an interval Δt is defined using the time constraint as[5],

$$G(q^\alpha, q'^\alpha; v_\alpha, \Delta t) = \frac{1}{(2\pi)^{d-1}} \int \exp(ip_\alpha \Delta q^\alpha)\delta(H(p_\alpha, q^\alpha))\delta(\bar{v}_\alpha \Delta q^\alpha - |v|\Delta t)\mu dp_\alpha. \tag{23}$$

The v_α, Δt variables after the semicolon indicate that the propagator depends on these. The μ is a weight, which is deduced so that $G(q^\alpha, q'^\alpha; v_\alpha, 0) = \delta(q^\alpha - q'^\alpha)$, where δ is the Dirac delta function.

Let me put the propagator in an operator form. Let $\hat{M}_v(\Delta t)$ be defined as $\delta(\hat{H})\delta(\bar{v}_\alpha \Delta\hat{q}^\alpha - |v|\Delta t)$, where $\Delta\hat{q}^\alpha = \int (q^\alpha - q'^\alpha)|\acute{q}^\beta >< q^\gamma|\, dq'^\beta dq^\gamma$. Then the time evolution operator $\hat{T}_v(\Delta t) = \hat{M}_v(\Delta t)(\hat{M}_v(0))^{-1}$ is the operator form of the propagator:

$$G(q^\alpha, q'^\alpha; v_\alpha, \Delta t) =< \psi_1|\hat{T}_v(\Delta t)|\psi_2 > \tag{24}$$

The μ in the path integral definition of G are the matrix coefficients of $(\hat{M}_v(0))^{-1}$.

Now the important question is, what is v^α in the configuration space? For this, let us focus on classical physics and consider Newton's first law of motion: Every body continues in its state of rest or of uniform motion. The law states that time flows and a body moves uniformly along the direction of momemtum in an infinitesimal time interval. Since time is seen through movement, inertia is essentially nothing but the unstoppable flow of time associated with the system. We will now reformulate the law slightly such that it defines time itself. The first of Hamilton's equations of motion captures the mathematics of Newton's first law: $\frac{dq^\alpha}{dt} = \{q^\alpha, H\}$. We redefine the newton first law as fundamental definition of time:

Self-Time - *Movement of physical bodies are synonymous with flow of time. The time direction in configuration space is given by $v^\alpha = \{q^\alpha, H\}$.*[6]

In this form Newton's first law defines time itself. This statement focuses on the primary source of the movement itself instead of the nature of the movement like in the Newtons law. I would like to refer to this definition of time as self-time. I do not call it as inertial time because there is a redundancy in the wording, as inertia itself is due to the flow of time. The remaining part of Newton's first law regarding forces is contained in the second Hamilton equation. But in the quantum version discussed next the content of both the equations will be taken into account.

As v^α is given, the location of the system in the configuration space moves along it. Now a time parameter of the evolution can be defined as being a parameter t along the path, which parametrizes evolution. If v^α is zero, then we can replace it with some very small vector. Using equations $\frac{dq^\alpha}{dt} = v^\alpha$, we have, $v_\alpha \Delta q^\alpha = v_\alpha v^\alpha \Delta t$,which is equivalent to the time constraint equation: $\bar{v}_\alpha \Delta q^\alpha = |v|\Delta t$.

[5] Revision Info:The term $\delta(p_\alpha \bar{v}^\alpha - |v|)$ need to be included and integrated over $d|v|$ for the theory to work.

[6] Revision info: v^α will not be referred as velocity vector in this paper to avoid confusion. We will refer to it just as the time direction in the configuration space.

Now, I will formulate the quantum form of the first principle of dynamics. Let $|\psi>$ be an arbitrary initial quantum state of the system. Let me define v^α to be the expectation value of the operator $i[H, q^\alpha]$:

$$v^\alpha = i < \psi|[H, q^\alpha]|\psi > . \tag{25}$$

Principle 2.1: *A quantum system during a time interval Δt evolves by the transition operator $\hat{T}_v(\Delta t)$, where the time vector v^α is the quantum expectation of the operator $i[H, q^\alpha]$, gives the time direction in the configuration space and the rate of evolution.*

Now since v^α depends on the wavefunction itself, the evolution is non-linear in nature. The operator $\hat{T}_v(\Delta t)$ is non-linear.

In classical theory, as a consequence of the Hamilton's first equation of motion, $p_\alpha = s_{\alpha\beta} v^\beta = v_\alpha$. So momentum and velocity are conceptual equivalents. Then, the above principle makes sense if we take into account the quantum nature of a system: It is a superposition of various velocity (momentum) states, each trying to move the system in its own direction. It is natural to expect that the net effect of movement is along its expectation value, similar to conventional quantum mechanics.

Note that v^α is a constant in the phase space at a given instant as defined in equation (25). Now we can define the following in terms of v_α:

$$v_\alpha = s_{\alpha\beta} v^\beta, \qquad |v| = +\sqrt{s^{\alpha\beta} v_\alpha v_\beta} \qquad \bar{v}^\alpha = \frac{v^\alpha}{\sqrt{v^\alpha v_\beta}} \qquad \bar{v}_\alpha = \frac{v_\alpha}{\sqrt{v^\alpha v_\beta}}.$$

These quantities defined are functions of q^α, as $s_{\alpha\beta}$ is a function of q^α. We see that the norm $|v|$ of v^α is function of q^α, but we can also simply use the classical norm calculated using the expectation value of $s^{\alpha\beta}$. So there is some flexibility in defining the norm, and so there is an ambiguity in the theory. But, hereafter, we can simply use $|v|$ in the equations without being explicit about its definition.

Lets go back to the path integral formulation discussed in last section. The time evolution is determined by the path integral sum over phase space with the natural time constraint $\sum_i \bar{v}_\alpha \Delta q^\alpha = |v| \Delta t$. Taking this into account, the new discretized Lagrangian is,

$$L\left(p_\alpha, q^\alpha, N, \lambda; v_\alpha, \Delta t\right) = p_\alpha \Delta q^\alpha - N H(p_\alpha, q^\alpha) \Delta t - \lambda(\bar{v}_\alpha \Delta q^\alpha - |v| \Delta t), \tag{26}$$

where the N and λ are Lagrange multipliers. The new action depends on v^α. The new single-step propagator, after the relevant variables are integrated is,

$$G(q^\alpha, q'^\alpha; v_\alpha, \Delta t) = \frac{1}{(2\pi)^{d-1}} \int \exp(i p_\alpha \Delta q^\alpha) \delta(H(p_\alpha, q^\alpha)) \delta(\bar{v}_\alpha \Delta q^\alpha - |v| \Delta t) \mu dp_\alpha. \tag{27}$$

The v_α, Δt variables after the semicolon indicate that the propagator depends on these. The μ is the weight, which we can deduced later, to make sure that $G(q^\alpha, q'^\alpha; v_\alpha, 0) = \delta(q^\alpha - q'^\alpha)$, where δ is the Dirac delta function. This propagator moves the system through an infinitesimal step,

$$\psi_{t+\Delta t}(q'^\alpha) = \int_q G(q^\alpha, q'^\alpha; \Delta t) \psi_t(q^\alpha) dq^\alpha, \tag{28}$$

which can be repeatedly applied to generate the dynamical evolution of the quantum state. The sequence defines the states of the system at various consecutive instants.

To simplify the illustration of proposal one, let me assume $s^{\alpha\beta} = \delta^{\alpha\beta}$. Now $v, v_\alpha, \bar{v}_\alpha$ and $\bar{v}^\alpha$ are constants in the phase space. Let us do a canonical transformation of the configuration space so that $\bar{v}_\alpha q^\alpha = T$ is one of the coordinates, and the remaining orthogonal coordinates are Q^I, where I varies from 1 to $d-1$. To mathematically codify the transformation, we can define orthogonal unit vectors e_i^α such that i varies from

0 to $d-1$, and $e_0^\alpha = \bar{v}^\alpha$. If e^i_α is the inverse of e_i^α, then $T = e^0_\alpha q^\alpha$ and $Q^I = e^I_\alpha q^\alpha$. Let $E = e_0^\alpha p_\alpha$ be the momentum conjugate to T and $P_I = e_I^\alpha p_\alpha$ be the momentum conjugate to Q^I. Then H becomes a function of E, P_I, T and Q_I:

$$H = \frac{1}{2}(E^2 + \sum_I P_I P_I) + V(T, Q^I),$$

The propagator can be written as follows:

$$G(Q^I, T, Q'^I, T'; v_\alpha, \Delta t) = \frac{1}{(2\pi)^{d-1}} \int \exp(i \sum_I P_I dQ^I - iEdT)\delta(\Delta T - |v|\Delta t)\delta(H)\mu dP_I dE, \tag{29}$$

where the $\Delta T = T' - T$. By integrating over $\delta(H)$, we can get E as a function of P_I, t and Q_I.

$$E = \pm\sqrt{-V(T, Q^I) - \sum_I P_I^2}.$$

Let me assume the sum inside the square is positive, so that E is real. Since the Hamiltonians in physics are usually quadratic in the momenta, E can be positive or negative. So, there are two types of propagators corresponding to the two opposite directions along v^α, both of them need to be added to get the full propagator. But, this would ultimately lead to macroscopic superposition and nature will choose only one of them. For now, let me restrict E and Δt to positive values only, so that time flows along only one direction. (In section four the generalization of the framework to both directions will be discussed.)

Let me denote the resulting positive function E of P_I, T, Q_I of as $H_v(P_I, T, Q_I; v_\alpha)$. Using the value of E, we have

$$H_\nu = \sqrt{-V(T, Q^I) - \sum_I P_I^2}.$$

Now, we have,

$$G(Q^I, T, Q'^I, T'; v_\alpha, \Delta t) = \frac{1}{(2\pi)^{d-1}} \int \exp(i \sum_I P_I \Delta Q^I - iH_v \Delta T)\delta(\Delta T - |v|\Delta t)\frac{\mu}{|H_v|} dP_I. \tag{30}$$

where the additional factor of $|H_v|^{-1}$ is from the integration of $\delta(H)$. It is easy to see that if we set $\mu = |H_v|$, then $G(Q^I, T, Q'^I, T'; v_\alpha, 0) = \delta(Q^I - Q'^I)\delta(T - T')$, as we want it to be. So, hereafter I will set $\mu = |H_v|$. Now the final form of the propagator is,

$$G(Q^I, T, Q'^I, T'; v_\alpha, \Delta t) = \frac{1}{(2\pi)^{d-1}} \int \exp(i \sum_I P_I \Delta Q^I - iH_v \Delta T)\delta(\Delta T - |v|\Delta t) dP_I. \tag{31}$$

Now, $\psi(q^\alpha)$ can be rewritten in the new coordinates as $\psi(Q^I, T)$. Let t be the time parameter (defined right after the classical version of the principle). Consider the single-step evolution of the wavefunction from $\psi_t(T, Q^I)$ to $\psi_{t+\Delta t}(T', Q'^I)$:

$$\begin{aligned}\psi_{t+\Delta t}(T', Q'^I) &= \int G(Q^I, T, Q'^I, T'; v_\alpha, \Delta t)\psi_t(T, Q^I) dQ^I \Delta T \\ &= \frac{1}{(2\pi)^{d-1}} \int \exp(i\sum_I P_I \Delta Q^I - iH_\nu |v|\Delta t) dP_I \delta(\Delta T - |v|\Delta t)\psi_t(T, Q^I) dQ^I \Delta T \\ &= \frac{1}{(2\pi)^{d-1}} \int \exp(i\sum_I P_I \Delta Q^I - iH_\nu |v|\Delta t) dP_I \psi_t(T' - |v|\Delta t, Q^I) dQ^I.\end{aligned}$$

$$\psi_{t+\Delta t}(T', Q'^I) = \frac{1}{(2\pi)^{d-1}} \int \exp(i\sum_I P_I \Delta Q^I - iH_\nu |v|\Delta t) dP_I \psi_t(T' - |v|\Delta t, Q^I) dQ^I. \tag{32}$$

In the Schrödinger differential form the evolution equation is,

$$\begin{aligned}\psi_{t+\Delta t}(T, Q^I) &= \psi_t(T - |v|\Delta t, Q^I) - \hat{H}_\nu \psi_t(T - |v|\Delta t, Q^I)|v|\Delta t \\ &\approx \psi_t(T, Q^I) - |v|\Delta t \partial_T \psi_t(T, Q^I) - H_\nu \psi_t(T, Q^I)|v|\Delta t.\end{aligned} \tag{33}$$

where the higher order terms of $v\Delta t$ from the path integral are ignored. I have placed ν to the right of H_ν. In general, $|\nu|$ is a function of q^α if defined using $|v| = +\sqrt{s^{\alpha\beta} v_\alpha v_\beta}$. So the order between H_ν and ν is important. So there is an ambiguity in the theory as I have indicated before. We can define $|\nu|$ using the expectation value of $s^{\alpha\beta}$, so that there is no ambiguity.

In the appendices A and B, the H_ν's for a general single point system with general $s^{\alpha\beta}$ and canonical quantum gravity are derived. The general differential form of the first principle of dynamics is,

$$d_D|\psi_t >= -iv^\alpha \hat{p}_\alpha |\psi_t > dt - iH_\nu |\psi_t > |v| dt, \qquad v^\alpha = i < \psi_t|[H, q^\alpha]|\psi_t > . \tag{34}$$

where the D denotes the deterministic evolution due to self-time, and $\hat{p}_\alpha = \frac{1}{i}\frac{\partial}{\partial q^\alpha}$.

Let me consider a concrete realization of the system that we discussed with $s^{\alpha\beta} = \delta^{\alpha\beta}$. Let me define H to be,

$$H = \frac{1}{2}\delta^{\alpha\beta} p_\alpha p_\beta + V(\delta^{\alpha\beta} q_\alpha q_\beta).$$

Assume the initial state of the quantum system is a semiclassical state:

$$\psi(q^\alpha) = \frac{1}{(2\pi\sigma)^{\frac{N}{4}}} \exp(iv_\alpha(q^\alpha - q_0^\alpha) - \frac{(q^\alpha - q_0^\alpha)^2}{4\sigma^2}),$$

where v_α, q_0^α and σ are constants that describe the expectation values of the momenta, the positions variables and the spread of the wavefunction. The system has rotational symmetry in the configuration and the momentum space. If we set $T = \bar{v}_\alpha q^\alpha$ and Q^I are the configuration variables along the orthogonal directions to $\bar{v}_\alpha$, we can rewrite the wavefunction as,

$$\psi(T, Q^I) = \frac{1}{(2\pi\sigma)^{\frac{d}{4}}} \exp(i|\nu|(T - T_0) - \frac{(T - T_0)^2 + \sum_I (Q^I - Q_0^I)^2}{4\sigma^2}),$$

where $T_0 = \bar{v}_\alpha q_0^\alpha$ and Q_0^I are the remaining components of q_0^α along the chosen orthogonal directions.

Let the system evolve for an interval Δt. The changes in the expectation values of Q^I and P_I are given by the Hamilton equations:

$$\Delta Q^I = \frac{\partial H_\nu}{\partial P_I}|\nu|\Delta t \qquad \Delta P_I = -\frac{\partial H_\nu}{\partial Q^I}|\nu|\Delta t$$

where the derivatives are calculated at $P_I = 0, Q^I = Q^I_0$ and $T = T_0$. So the new wavefunction is of the form as one would expect from non-relativistic quantum mechanics and equation (32),

$$\begin{aligned}\psi'(T, Q^I) \quad \approx \quad & \frac{1}{(2\pi\sigma)^{\frac{N}{4}}} \exp(i|\nu|(T - T_0 - |\nu|\Delta t) + i\Delta P_I Q^I) \\ & \exp(-\frac{(T - T_0 - |\nu|\Delta t)^2 + \sum_I (Q^I - Q^I_0 - \Delta Q^I)^2}{4\sigma^2}),\end{aligned}$$

which is simply the wavefunction shifted in the position and momentum expectation values as one would expect classically. There would be further small correction terms that depends on the explicit form of V. We need to rewrite the wavefunction in terms of the original coordinates q^α using $q^\alpha = e^\alpha_0 T + e^\alpha_I\ Q^I$, calculate v^α and repeat the same calculations for the next one-step evolution.[7]

2.2.2 Local Quantum Reduction

Consider the single point system discussed in the last subsection. The modified Schrödinger equation (34) derived describes the evolution of the system in the direction ν^α. The equation results in the system evolving into a macroscopic superposition state. To prevent this we need continuous reduction of the system which removes the macroscopic superposition. The general form of continuous measurement of a quantum system is given by the Bloch equations in the Lindblad form [17] governing evolution of density matrix (reviewed in [18]):

$$\dot{\rho} = \frac{-1}{i}[\hat{\rho}, \hat{H}] + \sum_m (2\hat{L}_m \hat{\rho} \hat{L}^+_m - \hat{L}^+_m \hat{L}_m \hat{\rho} - \hat{\rho} \hat{L}^+_m \hat{L}_m), \tag{35}$$

where ρ is the density matrix and L_m are the operators representing observables to be continuously measured. This equation has been extensively studied and has been useful in various experimental situations [19]. It is not the most natural and explicit form to use to describe an individual quantum system. It describes an ensemble of identical quantum systems and does not tell how each individual system evolves. So, we need to consider the equivalent equation, given by Percival, Gisin and Diosi [6], which describes the stochastic motion of the quantum system state $|\psi>$ of a quantum system:

$$\begin{aligned}|d\psi \quad > \quad &= \frac{1}{i}\hat{H}\ dt|\psi> + \sum_m (\hat{L}_m - <\hat{L}_m>)|\psi> dz_m\sqrt{dt} \qquad (36)\\ + \sum_m (2 \quad < \quad & \hat{L}_m > \hat{L}_m - \hat{L}^+_m \hat{L}_m - <\hat{L}^+_m><\hat{L}_m>)|\psi> dt,\end{aligned}$$

where dt is the time interval of evolution in the non-relativistic quantum mechanics. The dz_m are complex numbers representing Gaussian distributed independent random variables. More explicitly, the real and imaginary parts of dz_m are Gaussian random variables such that the statistical expectation values are given by,

$$E(dz_m) = 0, \quad E(dz_m dz_n) = 0, \quad E(dz_m dz^*_n) = 2\delta_{mn}. \tag{37}$$

[7] Revision Info: The term $\delta(p_\alpha \bar{v}^\alpha - |v|)$ need to be included and integrated over $d|v|$ in the path integral for the theory to work as indicated before. There is a possible error in the calculation in this section, which is being investigated and will be clarified in the next resubmission of this paper.

where E refers to the statistical expectation. This equation evolves $|\psi>$ such that its norm is preserved; so a normalized $|\psi>$ remains normalized as it evolves.

Percival applies this to quantum field theory but abandons the analysis reasoning that the resultant theory is non-unitary [7]. But, in case of quantum gravity the universe cannot be described by unitary evolution alone because that would lead to superposition of macroscopic states. Clearly, experimentally, whenever the quantum state of a system evolves into a superposition of macroscopic quantum states it probabilistically evolves to one of the macroscopic states. So, for a macroscopic universe, the quantum evolution must be described by an equation that has three components: a deterministic unitary component, a stochastic component, and a component that prevents macroscopic superposition. The modified Schrödinger equation (36) is the most natural form of it and the three terms in the right hand side of the equation give the necessary components in the respective order.

Let me clarify how the third term works a little bit. Consider that $|\psi>$ is expanded as a superposition of the eigenstates of $\hat{L}_m$. As $|\psi>$ evolves, the third term tends to reduce the amplitude of an eigenstate in the sum to the extent to which its eigenvalue is far away from the expectation value of $<\hat{L}_m>$. Because of this $|\psi>$ evolve such that the amplitudes of the components are peaked close to $<\hat{L}_m>$, a semiclassical state.

In equation (36) the second terms randomizes the system, third term classicalizes the system. These are components of macroscopic quantum reduction. Let me state the second principle of dynamics.

Principle 2.2: Local Quantum Reduction - *The evolution of a quantum state of a single point quantum system along with the Schrödinger type evolution also undergoes semiclassicalization and randomization through quantum state diffusion terms respectively:*

$$|d\psi_t \quad > \quad = |d_D\psi_t> + \sum_m \lambda_m(\hat{L}_m - <\hat{L}_m>)|\psi_t> dz_m\sqrt{|v|\Delta t} \tag{38}$$

$$+\sum_m (2 \quad < \quad \hat{L}_m > \hat{L}_m - \hat{L}_m^+\hat{L}_m - <\hat{L}_m^+><\hat{L}_m>)|\psi_t> |v|\Delta t,$$

Here the $d_D\psi$ and the $|v|\Delta t$ are the modified Schrödinger part (equation (34))and the time measure from the first principle of dynamics, and the operators $\hat{L}_m$ are simple functions of the conjugate variables p_α and q^α to undergo continuous measurement. The λ_m are some functions of of $\hat{L}_m$ and $|\psi_t>$.

The purpose of the function λ_m is to control the random fluctuations in the equation (38). For the quantum diffusion theory to reproduce the Copenhagen probabilistic collapse we need $\lambda_m = 1$. So λ_m needs to be close to one, but close to zero for reducing randomization. The choice of λ_m will be discussed further later.

Usually for the applications of the Bloch equation (35) to study the evolution of the density matrix of a quantum system, the L_m's are to be determined by what are to be measured in the experimental context. But, here in the second principle of dynamics we assume that the L_m's are fundamental quantities in quantum gravity to be determined experimentally. The natural and simplest choice for the L_m's are given by p_α, q^α, or some simple functions of them. The combined quantum system forms a complete reality by itself and there is no outside observer to make measurement. So the system needs to be understood as undergoing continuous reduction by itself instead of being considered as undergoing measurement. In the next section we will discuss, in the quantum general relativistic physics of fields, what is the nature of L_m's, how the equation (38) acts on the non-classical quantum states to promote continuous reduction, and how the phenomenon of quantum measurement in the laboratory experiments can be understood in terms of this process.

Please notice that if $\nu = 0$, then evolution freezes. But we can always set ν to be a very small value instead, so that the system can jump out of any state of unstable equilibrium, under the influence of the stochastic terms in the evolution equation.

2.2.3 Global Quantum Reduction

The first principle of dynamics focused on a quantum system at a single point that evolved according to a single time parameter. In quantum gravity we want to evolve the quantum states from one spatial hypersurface to another spatial hypersurface. In a spatial hypersurface there is infinite number of points, with a quantum system at each point. So let me discuss how to understand time evolution in a many-point quantum system. If there are many interacting fully constrained quantum systems, one pair of conjugate variables $p_{x,\alpha}$, q_x^α, then there is one time parameter t_x for each system. Now, the question is, what is the relative rate at which these time parameters evolve. I will try to answer this question in this subsection.

Let me assume that space is discretized and is made of countable number of points. Let B be the number on points (which can be set to infinity if needed). Assume that the quantum system at each point x is described by an identical Hamiltonian constraint H_x only, and it has an interaction term that involves adjacent quantum systems.

If x is the variable that refers to the quantum system at each point, then the single-step propagator describing the combined systems is,

$$G(q_x^\alpha, q_x'^\alpha; \nu_{x,\alpha}, \Delta t_x) = \frac{1}{(2\pi)^{B(d-1)}} \int \exp(i \sum_x p_{x,\alpha} \Delta q_x^\alpha)) \tag{39}$$

$$\prod_x \{(\delta(H_x)\delta(\bar{v}_{x\alpha} dq_x^\alpha - |v_x|\Delta t_x))\mu_x\} dp_{x,\alpha}, \tag{40}$$

$$\psi_{t+\Delta t}(q_x'^\alpha) = \int_q G(q_{,x}^\alpha, q_{,x}'^\alpha; \nu_{x,\alpha}, \Delta t_x)\psi_t(q_{,x}^\alpha) dq_{,x}^\alpha, \qquad v_x^\alpha = i < \psi_t |[H, q^\alpha]| \psi_t > . \tag{41}$$

where multiplication over α and x is assumed in $dq_{,x}^\alpha$ as per convention indicated in the beginning of the article. Convention 5 applies to G and ψ: G is a function of all $\{q_{,x}^\alpha, q_{,x}'^\alpha; \nu_{x,\alpha}, \Delta t_x\}$ for different α and x; ψ is considered as a function of all $q_{,x}^\alpha$ for different α and x.

Each step of the evolution depends on the values of Δt_x. Let t be a continuous time parameter, which varies from $t = 0$ to $t = T$. Let me define $\Delta t_x = n_x(t)\ \Delta t$, where the $n_x(t)$ are continuous functions of t, one of them for each point x. Now the repeated application of the one-step propagator for infinitesimal $|v_x|\Delta t_x$ creates a smooth evolution of all the systems. The sequence of the quantum states, defines the states of the system at various consecutive instants. As the combined system evolves the classical expectation value of the momentum and the configuration variables $p_{\alpha,x}$ and q_x^α also evolve. Consider the usual propagator defined below for M steps, without the sum over the lapse:

$$\bar{G}(q_I^\alpha, q_F^\alpha, \{N_{x,S}\Delta t_{x,S}\}) = \frac{1}{(2\pi)^{SBd}} \int \exp\left[i \sum_{x,S} \left(p_{\alpha,x,S} \Delta q_{x,S}^\alpha - i H_{x,S} N_{x,S} \Delta t_{x,S} \right) \right] \tag{42}$$

$$dp_{\alpha,x,S} dq_{\alpha,x,S}, \tag{43}$$

where the S indexes the steps in between the initial and the final steps. The integration over the lapses, integrates the integral over various possible foliations. After the summation over the lapses, we get,

$$G(q_I^\alpha, q_F^\alpha) = \frac{1}{(2\pi)^{SBd}} \int \bar{G}(q_I, q_F, \{N_{x,S}\Delta t_{x,S}\}) \prod_{x,S} N_{x,S} d(\Delta t_{x,S}). \tag{44}$$

This is the usual propagator of a fully constrained system.

Now consider the propagator defined in principle 2.1. Let me apply it for S steps. From equation (39) we have:

$$G(q_I^\alpha, q_F^\alpha) = \int \prod_{S,x} (G(q_x, q_F; v_{x,\alpha}, \Delta t_{x,S}) \frac{v_{x,S}}{\mu_x}) d(\Delta t_{x,S}). \tag{45}$$

In this the integrals over the lapses are already performed, the time intervals $\Delta t_{x,S}$ are physical quantities extracted from the internal evolution based on $v_{x,\alpha}$, and assume $v_{x,\alpha}$ are independent of the states just for discussing this sum. So the summation in the net propagator given in this equation is over various possible physical macroscopic evolutions unlike equation (44). Therefore, we can reason that this superposition over this history has to collapse probabilistically, as it usually happens in nature, a global reduction in contrast to the local reduction discussed in last subsection. Now, so let me propose the following:

Principle 2.3: Global Quantum Reduction - *The functions $n_x(t)$ take random but continuous values with the relative probability distribution given by the functional $P(n_x(t)) = \exp(-W(\hat{p}_{\alpha,x}, \hat{q}^{\alpha}_x, \psi_t))$. Here W needs to be found experimentally or theoretically. The W is a scalar functional of the operators $\hat{p}_{\alpha,x}$ and $\hat{q}^{\alpha}_x$, for all x and the sequence of wavefunctions ψ_t between $t = 0$ to $t = T$*[8].

Revision info: This principle is too general and need to be improved.

The $n_x(t)$ essentially are the lapses, and they give the various ways to foliate the manifold of the quantum system whose topology is B point $\otimes$ $1D$. This could be the discretized $4D$ manifold of general relativity. Since now the $n_x(t)$ are random functions, the foliation is a random hypersurface. In this subsection I only assumed that ψ_t evolves by the self-time evolution of the first principle of dynamics. It is straight forward to include the second principle of dynamics, which will be discussed in the next section. This will be discussed in the next section.

Let me discuss now the sample space in which $n_x(t)$ take values. One can come with a simple choice as follows. a) $n_x(t)$ need to be continuous functions on the manifold b) the first derivative is finite but changes only in random steps of values for a random sequence of intervals. This restricts the choice of the sample space, so that the evolution is smooth.

It is important to note that the special case of deterministic $n_x(t)$ is built into the theory. For this to happen $P(n_x(t))$ need to be infinitely peaked for one choice of $n_x(t)$ for each point x.

In the section (2.2), when discussing about time in classical general relativity, I discussed that the physics happening on the space-time manifold is used to define a physically relevant foliation in cosmology. For example, the foliation of the space-time manifold is defined by the scale parameter in big bang cosmology. The principle proposed in this section generalizes this through the use of W functional.

Semiclassical states are usually those whose amplitudes are peaked in and finite near some classical values (wave packets). We may also smear each step of the quantum states using functions that are peaked at $n_x(t)$, for example:

$$\psi_{t+\Delta t}(q'^{\alpha}_x) = \int G(q^{\alpha}_x, q'^{\alpha}_x; \nu_{x,\alpha}, N_x(t)\Delta t) \exp(-a^2 N_x(t)^2)\psi_t(q^{\alpha}_{,x}) dq^{\alpha}_{,x}.$$

where the $n_x(t)$ are determined probabilistically. Here the smearing is done with an exponential factor. It is not clear to me whether this must be done, as this would be over constraining the system, more than what the Hamiltonian constraint has already done.

[8] Revision info: In version 2.0 this principle has been made more specific. The W functions are unspecified in this paper. In version 2.0 concrete choices of W functions are given, where they are referred to as Γ functions.

3 Applying the Framework

3.1 Dynamics

In this section, I discuss how to use the framework of the three conceptual principles in the last section to study physics of quantum fields in general relativity. For this we discretize the spatial manifold into large number of points. Now, we have a multi-point system and at each point the first and the second principle of dynamics describe the stochastic and the self-time evolution. The q^{α}_x and $p_{\alpha,x}$ now stand for the collection of configuration variables and the conjugate momenta respectively of all the quantum fields living at each point x of a spacial hypersurface, such as the spacial metric, the electromagnetic, the Fermionic, the scalar, and other fields like the remaining gauge fields. Let us denote the quantum state of the combined system as $|\psi_t>$. Specify a smooth function $n_x(t)$ for each point. The time increment for each point x is now $n_x(t)\Delta t$. The modified Schrödinger equation of the entire system is now:

$$
\begin{aligned}
|d\psi_t &> = d_D|\psi_t> \qquad (46)\\
+\sum_{m,x}\lambda_{m,x}(\hat{L}_{m,x}- &< \hat{L}_{m,x}>)|\psi_t> dz_{m,x}\sqrt{|n_x(t)||v_x|dt}\\
+\sum_{m,x}(2 &< \hat{L}_{m,x}>\hat{L}_{m,x}-\hat{L}^{+}_{m,x}\hat{L}_{m,x}-<\hat{L}^{+}_{m,x}><\hat{L}_{m,x}>)|\psi_t> n_x(t)|v_x|dt, \qquad (47)
\end{aligned}
$$

$$
\begin{aligned}
d_D|\psi_t &> = -\sum_x n_x(t)idtv^{\alpha}_x\hat{p}_{\alpha,x}|\psi_t> -\frac{1}{i}\sum_x H_{v,x}|\psi_t> n_x(t)|v_x|dt\\
v^{\alpha}_x &= i<\psi_t|[H_x, q^{\alpha}_x]|\psi_t>,
\end{aligned}
$$

where the operator contributions from all the points are added. These equations time evolve the system through a sequence of quantum states. The evolution is continuous but yet stochastic in nature.

Using a choice of the functions $n_x(t)$, we can use these equations to evolve the quantum state Ψ from $t=0$ to $t=T$. Then $exp(-W(\hat{p}_{\alpha,x},\hat{q}^{\alpha}_x))$ gives the relative probability for the $n_x(t)$'s chosen to physically occur. Now we have two sets of random variables the z's and the n's. The z's are Gaussian distributed as discussed in principle 2.2. The probability distribution of the n's depends on the values of the n's themselves and the Gaussian random variables z's, as ψ_t depends on both of them. Essentially, this probability distribution describes the quantum reduction of quantum histories of the combined system. Then for the $n_x(t)$'s chosen to occur, the probability is given by,

$$P(n_x(t)) = \frac{\exp(-W(\hat{p}_x,\hat{q}_x,\psi_t))}{\int\exp(-W(\hat{p}_{\alpha,x},\hat{q}^{\alpha}_x,\psi_t)\Delta t)\prod_t dn_x(t)}. \qquad (48)$$

where the integral is performed over all possible $n_x(t)$ in the interval $t=0$ to $t=T$, at all points x. Consider quantum gravity: The q^{α}_x stand for the metric tensor. Assume q^{α}_x are some linear combination of the components of the spatial metric. For the equations to work we need to proper choice for $\lambda_{m,x}$. They need to set to 1 for the equations to reproduce the quantum probabilistic collapse as described in [6]. The purpose of $\lambda_{m,x}$ is to control the randomization due to the second term in equation (46). We can choose $\lambda_{m,x}$ to be very small so they make the quantum evolution to be mostly deterministic and semiclassical near the space-time singularities such as in the sceneries of big bang theory and black holes. The precise form of $\lambda_{m,x}$ needs to be determined by experimental and theoretical investigation.

In classical general relativity physics is foliation independent. But in the modified quantum theory defined by the framework of the three principles, the quantum evolution is foliation dependent: the evolution described by the first and the second principle of dynamics depend on the foliation which in turn is defined by the third principle of dynamics.

3.2 Choosing time direction

The first principle of dynamics picks the direction of time to be determined by the quantum expectation value of the operators $i[H_x, q_x^\alpha]$ using the wavefunction. But at each point of the space-time manifold there are many fields: the scalar, the vector, the spinorial fields, etc. For Fermions described by the spinorial fields, the expectation values of the conjugate momenta are zero. So they don't contribute to v_x^α. So the direction of time flow is only determined by the scalar, the gravitational and other gauge fields such as the electromagnetic field. But the relative contribution of each of these fields may be different. One can rescale the theory such that all the fundamental constants are unity, and just declare the remaining configurational variables at each point to be of equal footing. Instead, it might be possible that only the scalar fields' momenta give the direction of time flow. But we can see that such a scalar field suitable for time description is only possibly available in the early universe such as in Guth's theory of inflation [12]. In the current universe (away from the exotic objects in the universe such as the black holes) the (Higgs) scalar fields have zero classical conjugate momentum. Also the gauge fields other than the electromagnetic fields have zero conjugate momentum except possibly near particles. So we are left with the gravitational and the electromagnetic fields which can be expected to continuously vary over space-time, contributing non-zero v^α to set the direction of time flow in the internal configuration space of these two fields. Also, these theories can be unified either along the lines of Kaluza-Klein [16], the string theory, or some future theory giving a set of configuration variables which are at equal footing to work with.

3.3 Observables

The operators and variables are supposed to be subjected to the constraints: the Hamiltonian, the diffeomorphism, and the gauge constraints. We have used the Hamiltonian constraint to extract dynamics. The self-time evolution equation (34) is nothing but the Hamiltonian constraint equation rewritten in the Schrödinger equation form. The gauge constraints and the diffeomorphism constraints can be directly applied on the basic Hilbert space and the operators $F(\hat{q}_x^\alpha, \hat{p}_x^\alpha)$ to get to the physical Hilbert space and the physical operators as I have discussed in section (2.1).

We need to apply the $\hat{O} = \prod_i \hat{O}(\hat{C}_i)$ on evolving $|\psi_t>$ described in the last subsection to get the physical states and operators. The projectors are to be applied statically on the combined state $|\psi_t>$ of the all the systems living at all the points on the hyperspace at each instant t.

The gauge invariance removes the redundancies due to the internal symmetries of the field equation. The diffeomorphism constraints relates to invariance under the change of spatial coordinates. Physics tells us that what we physically observe should be gauge and diffeomorphism invariant. But, in the alternative way to tackle the diffeomorphism constraints discussed later in section (3.6), the coordinate invariance could be only an approximation at Planck scale.

3.4 Quantum Reduction and Measurement

In quantum diffusion theory [6] used in principle 2.2, we don't need the Copenhagen interpretation, as it is built into the theory: the quantum diffusion equation (36) takes care of the technical implementation of the interpretation, through a process of continuous quantum reduction, in other words gradual semiclassicalization.

In [6] the quantum diffusion theory is used in a non-relativistic context. Let me discuss how technically the equation works in quantum general relativity with all other fields included. Let me assume the Hamiltonian formulation is discretized, and the full framework is applied. Sometimes in the study of decoherence in non-relativistic quantum mechanics individual particles states, their momentum and position operators are used to define $\hat{L}_m$'s [18]. But in a Fermionic field theory context these are not the proper operators to use, because there is one set of $\hat{L}_m$ for each point x, which we denoted as $\hat{L}_{m,x}$. We discussed in the last subsection that the Fermions don't contribute to v_x^α. Similar to this, they should not contribute to $\hat{L}_{m,x}$'s also, because they are always fully quantum in nature due to Fermi statistics - maximum one particle per point. The expectation values of Fermionic field operators at each point are usually small. The electromagnetic field, metric field, etc. following Bose statistics, with unlimited number of quanta per point, their expectation values can reach to classical values, their contributions to $\hat{L}_{m,x}$ are important. Nevertheless Fermionic particles can under go quantum reduction as we will discuss next.

Assume we have the full general relativistic physics (including all the matter and the gauge fields) discretized on a spatial hypersurface. The quantum state on the hypersurface is made of multiples of the following two types of states:

1) Macroscopic states corresponding to the Bosonic fields: semiclassical states of the gravitational (the spatial metric field), the electromagnetic field states, and possibly the scalar fields. These states are usually not superposed or entangled heavily.

2) Microscopic elementary quantum states of the Fermionic particles and the Bosonic fields: electron, quark, photon, graviton, etc. These states can be superposed and entangled.

So a typical quantum state looks like this: $|\psi_t >= |em, g, s > \otimes \sum\limits_{i,j,k,l,...} |e_i, q_j, em_k, g_l, .. >$, where em stands for electromagnetic, g - spatial metric, e - electron, q - quark, etc., and the indices refers to the various states relating to the different charges, spin and position. The $|em, g, s >$ is of type 1 and the fully quantum sum is of type 2.

Assume the $\hat{L}_{m,x}$'s are derived from the em, g, and s fields. Consider the quantum diffusion equation (46). The purpose of the second and the third terms are to gradually randomize and semiclassicalize $|\psi_t >$ as it evolves. As long there is no superposition and entanglement in the semiclassical states the contribution of the second and third terms are minimal. The contribution is mostly from the first term. But, as $|\psi_t >$ evolves due to the first term, due to the interaction between the configurational variables of the Fermionic and the semiclassical fields (em, g, and s), the superpositions and entanglements in the fully quantum $\sum\limits_{i,j,k,l,...} |e_i, q_j, em_k, g_l, .. >$ passes over to superpositions and entanglements of the semi-classical states $|em, g, s >$. (This is essentially what was discussed in the section (1.1): $\left(\sum_q |\Psi_S(q)|q >\right) \otimes |\Psi^0_O >$ evolves to $\sum_q \Psi_S(q)|q > \otimes |\Psi^q_O >$.) The evolved states are like the superposition of the states of the needle pointer in the measuring instrument, with each needle pointer state having its own semi-classical configuration of the electromagnetic and the metric fields. But since the electromagnetic and the metric fields contribute to $\hat{L}_{m,x}$'s, the second and third terms in the evolution equation (46) remove these macroscopic entanglements and superpositions, performing reduction, exactly in the way it is supposed to happen by Copenhagen interpretation. This reduction process happens continuously as the state evolves. Basically the macroscopic semiclassical states of the metric, the electromagnetic, (and possibly scalar, example in the early universe) keeps measuring the microscopic quantum states of all the quantum fields. This is a deeper and objective way of understanding the quantum measurement by an observer. (This is in the spirit of the gravitational reduction by Roger Penrose [3], but more explicit and generalized.)

Now the relative evolution is determined by $n_x(t)$ at each point. As I have discussed before they choose a random foliation along which the combined hypersurface quantum state evolves. The quantum state is made of superposed and entanglement microscopic states. Now as the state evolves it undergoes continuous quantum reduction, this process correlates the classical and quantum information along different points of the hypersurfaces of the random foliation.

3.5 The W functions

Let me consider a physically interesting choice for the W function. Assume that the spatial manifold is somehow discretized. The q^α_x and $p_{\alpha,x}$ now stand for $h^{\alpha\beta}(x)$ and $\pi_{\alpha\beta}(x)$. Let me evolve the initial state using the path integral in equation (46). Since $n_x(t)$'s govern the relative rate of evolution of the quantum state at each point compared to those of the other points, the different choices of $n_x(t)$'s relate to the different ways to foliate the space-time manifold.

Now, consider the following interesting suggestion:

$$W(\hat{p}_{I,x}, \hat{q}^I_x, \psi_t) = \eta \sum_x \int_t s_{IJ} < \dot{Q}^I_x >_t < \dot{Q}^J_{,x} >_t \, dt_x \tag{49}$$

with η being a physical constant to be determined and $\dot{Q}^I_{\alpha,x}$ are observables orthogonal to $\bar{v}^\alpha_x$ at each point

x described in principle 2.1. The$< (\dot{\hat{Q}^I_{\alpha,x}}) >_t = i < \psi_t|[H_{\nu,x}, \hat{Q}^I_{\alpha,x}]|\psi_t >$, and s_{IJ} is the projected spatial metric into the Q^I_x space. The reason for this suggestion comes from the existing physical sceneries. Consider the evolution of the universe after the big bang singularity. Let λ be the scale parameter. Assume the initial quantum state is such that its $v_{\alpha,x}$ are along the increasing λ direction of the metric configuration space at each point. Then T_x at each point is an increasing function of the scale parameter itself, which is the diagonal value of the spatial metric.

The Q^I_x stand for the perturbations in the diagonal metric. If the $n_x(t)$ are equal for all x's, then the evolution is on the foliation defined by hypersurfaces with constant λ's as usual in cosmology. For this Q^I_x and $\dot{Q}^I_x$ are zero and so the W is zero. Assume the $n_x(t)$ are slightly perturbed. This increases Q^I_x and $\dot{Q}^I_x$ and so the W. So minimality of the W will ensure that the $n_x(t)$ choose a foliation close to the foliation based on the scale parameter. The relative probability being maximum means the W being minimum. So principle 2.3 chooses the functions $n_x(t)$ with high probability such that the foliation is close to that used in big-bang cosmology.

Similarly, in the Schwarzschild case along the temporal killing flow the classical conjugate momenta and the rate of change of the spatial metric are zero. The evolution is described by the Hamiltonian constraints related to the perturbations in metric caused by the long range gravitational waves, the local matter fluctuations, or the movement of bulk matter radially. The W is zero when there is no perturbation, and it increases with increasing perturbation. Smallness of the W will ensure that the evolution happens along the approximate time-like killing vector in these contexts. So again the suggestion for the W suggested is consistent with the Schwarzschild case also[9].

In the last subsection I have discussed that the continuous reduction of the quantum state correlates the classical and quantum information along the hypersurfaces. Now for the W suggested the correlation happens along the random hypersurfaces that are close the physically intuitive ones that we discussed.

3.6 Models and Constraints: Discrete Vs. Continuum

The principles can be easily applied to discrete models. Assume the space is discrete, made of B points and the quantum systems at all the points are coupled to each other. Let q_α and p_α stand for the collection of the configuration variables and the conjugate momenta of all the points. If d is the dimension of the configuration space this mean α varies from 1 to Bd. Let the Hamiltonian constraint at a point I is given by,

$$H_I = \alpha_{I,\alpha\beta} p_\alpha p_\beta + \beta_{I,\alpha\beta} q_\alpha q_\beta, \tag{50}$$

where the sum is over repeated Greek indices. The α and β tells how the quantum systems at the various points interact with each other. Then the commutator $[H_I, H_J]$ is given by

$$[H_I, H_J] = 2\sum_{lm\alpha} \left(\alpha_{J,lm}\beta_{I,m\alpha} - \alpha_{I,lm}\beta_{J,m\alpha}\right)\left(2q_\alpha p_l - \delta_{\alpha l}\right).$$

Let me call this result as C_{IJ}. It can be written in the matrix form as

$$C_{IJ} = 2Tr\left(A_J B_I - A_I B_J\right)\left(2QP^T - \delta\right),$$

where $A_J = \{\alpha_{J,lm}\}, B_I = \{\beta_{I,lm}\}$ with m indexing the rows, and l indexing the columns. Here $Q = \{q_\alpha\}$ and $P = \{p_\alpha\}$ are column matrices[10].

In the framework proposed in the last section, the Hamiltonian constraints are re-interpreted as modified Schrödinger evolution equations in principle 2.1. Assume we discretize the ADM formulation, and assume

[9] Revision info: In version 2.0 more analysis has been done on choices of W functions, where they are referred to as Γ functions.

[10] Revision info: Some possibly erronous analysis located after this in the previous version has been deleted.

that the system evolves under the influence of all the Hamiltonian constraints at each point, as given in the combined propagator in equation (39).

We can simulate quantum evolution of discrete models using the three principles. We need to study whether we can start with a discrete model representing continuum general relativity at certain physical scenario in Planck scale, and under a large number of the points limit, extract sensible quantum physics of the continuum model that satisfies the Hamiltonian and the diffeomorphism constraints. The study is currently under progress and will be reported in the follow-up reports.

3.7 Perturbed Models

In general relativity important scenarios are as follows:

- The big bang singularity and the expanding universe,
- The spherical symmetry and the quasi-static geometry, and
- The black hole singularity and the contracting geometry.

Most scenarios in the real universe are perturbed versions of these. Let me discuss some ideas about how to tackle time in these. Let me consider a general single point model. Assume that the initial wavefunction is peaked around $p^0_\alpha,\ x^\alpha_0$, in the momentum and the configuration space respectively. Let $\dot{q}^\alpha_0$ be the expectation value of $i[H, q^\alpha]$.

Then in the classical case we have,

$$\begin{aligned}
v^\alpha &= \dot{q}^\alpha = \frac{\partial H(p_\alpha, q^\alpha)}{\partial p_\alpha} \\
&= \frac{\partial H(p^0_\alpha + \delta p_\alpha, q^\alpha_0 + \delta q^\alpha)}{\partial (p^0_\alpha + \delta p_\alpha)} \\
&= \frac{\partial H(p^0_\alpha + \delta p_\alpha, q^\alpha_0 + \delta q^\alpha)}{\partial (p^0_\alpha)} \\
&\approx \frac{\partial H(p^0_\alpha, q^\alpha_0)}{\partial p^0_\alpha} \\
&= \dot{q}^\alpha_0 .
\end{aligned}$$

In other words the direction of time in configuration space remains close to the classical value. This can be used to set the initial value of the time direction in both single point and multipoint models. We can use this to identify the time parameters in the three sceneries listed.

In the first scenario in the list, v^α is along increasing λ. In the second scenario, the issue is more involved; nevertheless, the direction of time is given by classical derivatives of the non-angular components of metric coefficients (assuming the electromagnetic field is very weak). If the space-time evolves such as in an expanding star, the time parameter is a function of the classical radial metric parameters. If the space-time is macroscopically static, then the time flow is derived from the momentum generated by the gravitational fluctuations. In the third scenario the time evolution is given by decreasing radial parameters.

We can study the random perturbation around these most probable solutions and analyze the influence of the random perturbations on the physics of the universe, such as in seeding structure formation in cosmology, time evolution in the region of near flat space-times, or physics inside the black-holes.

4 Generalized Principles of Dynamics

The three concepts defined by the three principles that I proposed in section two are physically intuitive and natural. Related to each of these is a more general concept with various different purposes and usefulness, which I will discuss in this section.

4.1 Selective-Time Evolution

Consider the phase space (p_α, q^α) of the single point system discussed in principle 2.1 in section 2. We have the conventional timeless propagator for this as,$G(q^\alpha, q'^\alpha) = \int \exp(ip_\alpha \Delta q^\alpha)\delta(H)dp_\alpha$. Let me introduce a general form of the time step constraint in the phase space as follows:

$$w_\alpha(p_\gamma, q^\beta)\Delta q^\alpha - u\Delta t = 0. \tag{51}$$

Here $w_\alpha(p_\gamma, q^\beta)$ is function of both p_γ and q^α, u is assumed to be independent of p's and q's. Given a Δt, q'^α and p_α, the set of points q^α satisfying this equation gives a surface in the coordinate space that contributes quantum amplitude at q'^α. The new propagator including the general time step constraint is,

$$G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u) = \int \exp(ip_\alpha \Delta q^\alpha)\delta(H)\delta(w_\alpha(p_\gamma, q^\beta)\Delta q^\alpha - u\Delta t)\mu dp_\alpha. \tag{52}$$

I would like to refer to this as the selective-time propagator (or subjective-time propagator) because w_α and u are specified selectively to study the relative evolution of a system with respect to the time step defined in the equation (51) by these two. For example we can choose $w_\alpha(p_\gamma, q^\beta)$ to make quantity of interest such as the scale parameter in cosmology to be the time variable. In appendix C this selective-time propagator is applied to deduce the natural time associated with a simplified system in which a matter-like system interacting with a gravity-like system.

Now I will study the properties of these selective-time propagators. Equation (52) can be integrated to get

$$G(q^\alpha, q'^\alpha) = \int G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u)\frac{u}{\mu}d\Delta t. \tag{53}$$

Let me assume H is a function of q^α and p_α only. Assume in w_α and μ all the p's are placed to the right of the q's. Then using $\hat{p}'^\alpha = \frac{1}{i}\frac{\partial}{\partial q'^\alpha}$, we have,

$$\begin{aligned} G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u) &= \delta(w_\alpha(q^\beta, \hat{p}'_\gamma)\Delta q^\alpha - u\Delta t)\mu(q^\beta, \hat{p}'_\gamma)\int \exp(ip_\alpha \Delta q^\alpha)\delta(H)dp_\alpha \\ &= \delta(w_\alpha(q^\beta, \hat{p}'_\gamma)\Delta q^\alpha - u\Delta t)\mu(q^\beta, \hat{p}'_\gamma)G(q^\alpha, q'^\alpha). \end{aligned} \tag{54}$$

To summarize we have the following:

$$\begin{aligned} G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u) &= \delta(w_\alpha(\hat{p}'_\gamma, q^\beta)\Delta q^\alpha - u\Delta t)\mu(q^\beta, \hat{p}'_\gamma)G(q^\alpha, q'^\alpha) \\ G(q^\alpha, q'^\alpha) &= \int G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u)\frac{u}{\mu}d\Delta t. \end{aligned} \tag{55}$$

If we have two different selective-time propagators parametrized by $\{\tilde{w}, \tilde{u}\}$ and $\{w_\alpha, u\}$ respectively, they can be related to each other using the above two equations as follows:

$$
\begin{aligned}
G_s(q^\alpha, q'^\alpha; \Delta t, w_\alpha, u) &= \delta(w_\alpha(\hat{p}'_\gamma, q^\beta)\Delta q^\alpha - u\Delta t)\mu(q^\beta, \hat{p}'_\gamma)G(q^\gamma, q'^\delta) &(56)\\
&= \delta(w_\alpha(\hat{p}'_\gamma, q^\beta)\Delta q^\alpha - u\Delta t)\mu(q^\beta, \hat{p}'_\gamma) &(57)\\
&\int G_s(q^\alpha, q'^\alpha; \Delta t', \tilde{w}_\alpha, \tilde{u})\frac{\tilde{u}}{\tilde{\mu}} d\Delta t'. &(58)
\end{aligned}
$$

In the regions of space-time, where the evolution is only determined by the first principle of dynamics only, the evolution is determined by the propagators only. We can describe the evolution by many choices of selective propagators. Then the above transformation formally relates these various propagators.

4.2 Bidirectional Time Evolution

The time evolution defined in principles 1 and 2 are actually unidirectional in time. We can generalize this to both the opposite directions along v^α. Consider the momentum operator $\hat{E} = \hat{p}_\alpha v^\alpha$ (assuming v^α is a unit vector in the metric discussed in the previous section). Let $|\psi> = |\psi_+> + |\psi_->$, where $|\psi_+>$ and $|\psi_+>$ are made of the positive and negative eigenvalued eigenvectors of $\hat{E}$ correspondingly. Then we have a more generalized dynamic equation as follows:

$$
\begin{aligned}
|d\psi_t &> = -\hat{p}_\alpha v^\alpha \Delta t|\psi_t> - \frac{1}{i}\hat{H}_v|\psi_{+,t}>|v|\Delta t + \frac{1}{i}\hat{H}_v|\psi_{\_,t}>|v|\Delta t\\
+\sum_k(2 &< \hat{L}_k > \hat{L}_k - \hat{L}_k^+\hat{L}_k - <\hat{L}_k^+><\hat{L}_k>)|\psi_t>|v|\Delta t\\
+\sum_k \lambda_k(\hat{L}_k - &< \hat{L}_k>)|\psi_t> dz_k\sqrt{|v|\Delta t}, &(59)\\
v_\alpha &= i<\psi_t|[H, q_\alpha]|\psi_t>. &(60)
\end{aligned}
$$

The two terms involving H_v evolves the state in the positive and negative direction along v_α. But because of the third summation term in the first equation one of $|\psi_+>$ and $|\psi_+>$ will be fully attenuated eventually. So we only have a unidirectional motion eventually. The above evolutions can be easily included in the framework of section 2.

4.3 Statistical Time Evolution

There is one more alternative to time evolution defined in principles 1 and 3. Consider the many systems one step propagator:

$$
G(q_x^\alpha, q'^\alpha_{,x}; \Delta t_x, v_{\alpha,x}) = \int \exp(i\sum_x p_{x,\alpha}\Delta q_x^\alpha)\delta(H_x)\prod_x \delta(v_{x\alpha}dq_x^\alpha - |v_x|\Delta t)dp_{x,\alpha}. \tag{61}
$$

The $v_{\alpha,x}$, which define the time direction in the phase space, are the expectation values of the operator $i[\hat{H}_x, \hat{q}_x^\alpha]$. In the principle 2.3 we introduced the n_x's to define relative evolution. They are statistical distributed function of t due to statistical selection over sum over histories as reasoned in principle 2.3. The purpose of the relative rate function n_x could be fulfilled by $v_{\alpha,x}$ themselves in all the spirit by modifying principle 2.3 as follows:

Principle 2.3′: Statistical Time - *The continuous functions $v_{\alpha,x}(t)$ takes random values with the relative probability distribution given by $P(v_{\alpha,x}(t)) = \exp(-W(\hat{p}_x, \hat{q}_x, \psi))$. Here W needs to be found by theoretical and experiment investigation.*

This proposal makes the principle 2.1 no longer needed. Now $v_{x\alpha}$ vectors are statistical. The modified principles in this section along with the principles of quantum statics form a complete framework by themselves. We can evolve the combined quantum system using equations(46). The probability distribution of v's depend on values of the Gaussian random variables z's and v's, as W depends on them.

Similar to section (2.3), I suggest $W = \eta \int \sum_x s_{IJ} < \dot{Q}_x^I >< \dot{Q}_x^I > dt_x$, with η being a physical constant and $\dot{Q}_x^S = i[H_{\nu,x}^S, Q_x^S]$. This proposal chooses the values of $v_{x\alpha}$ with a high probability such that they minimize W, so that resulting evolution is based on random foliations approximately close to the conventional foliations in the big bang cosmology and the Schwarzschild case, like we discussed in the last section (3.4).

Even though, this modified proposal is quite interesting, the usefulness of this modified proposal is doubtful. Consider the original proposal 2.3. It is motivated by the fact that the integration of propagator in equation (39) over various $n_x(t)$'s gives the total gravity propagator as given in equation (45). The random choice of $n_x(t)$'s chosen can be interpreted as a quantum projection of the corresponding quantum history from the total gravity propagator (the sum over histories). But this interpretation does not work in the modified proposal because the integration of the modified path integral in equation (61) over the various $v_{\alpha,x}(t)$ does not yield the total propagator. This integration does not look to be physical sensible, in the same way as equation (45), as $v_{\alpha,x}(t)$ have too much degrees of freedom.

5 Discussion

The set of three basic principles discussed in section 2 only lays down a conceptual framework instead of a full concrete proposal. These principles are essential to study quantum dynamics of a full background independent general relativity with time and measurement (reduction). The first principle of dynamics of picks a self-time direction in the configuration space of the quantum system at each point;, the second principle of dynamics introduces spontaneous local quantum reduction for the quantum system at each point, which gives deeper understanding of quantum measurement ; and the third principle of dynamics deals with global evolution by determining the relative rate of time evolution for different points on space by global quantum reduction. These principles embody conceptual foundations but leaves open the concrete implementation to be determined by further theoretical and experimental investigation.

Let me list the various possibilities:

1. This framework is highly abstract and it can be applied to the usual quantum field theory with general relativity using Dirac's method of quantization, or any unified field theory such as the string theory, Kaluza-Klein theory, etc. One needs to discover which is the best theory that works with the framework.

2. The framework is based on discrete model. There are many possible ways to discretize quantum general relativistic physics. The right way to discretize, so that we could extract low energy continuum physics needs to be found.

3. Principle 2.1 proposes time direction to be determined from the expectation value v_x^α of the operator $i[H_x, q_x^\alpha]$. Now there are various possibilities for choosing the configuration variables among scalar, tensor, vector, etc., to calculate v_x^α. Another possibility is using the Kaluza-Klein theory [16], String theory or some other unified where these fields are unified theory as discussed in section 3.3.

4. I have used the time direction v_x^α to be a direction in the configuration space. But we can generalize this, by considering v_x^α to be a direction in the full phase space instead, and create a more general theory.

5. Instead of the self-time propagator, a selective-propagator derived from the modified form of it may be suitable to describe physics.

6. In the first principle of dynamics in its quantum and mathematical form defines ν_x^α as expectation value of $i[\hat{H}_x, \hat{q}_x^\alpha]$. So the quantities $|v_x|$ and $\bar{v}_x^\alpha$, derived from ν_x^α and used in the time constraint depend on the metric which is a function of the configuration variables. But we can use the expectation value of the metric to calculate one or both of these and use them in the constraint, leading to slightly different theories.

7. In the second principle of dynamics we have the $\hat{L}_{m,x}$'s to be determined. The natural choices are the configurational variables q_x^α, which will restrict the quantum evolution to be peaked around their expectation values.

8. In the second principle of dynamics in $\lambda_{m,x}$ is also to be determined. They have serious technical impact by determining the impact of stochastic part of the evolution. It makes evolution semiclassical for small $\lambda_{m,x}$ and stochastic for large $\lambda_{m,x}$. The value needs to set to one for Copenhagen collapse effect to take place. Important scenarios where $\lambda_{m,x}$ are to determined are those related the singularities, such as the black hole or the big bang singularity.

9. In third principle of dynamics W is to be determined. A suggestion has been made in the discussion itself in section (2.3). If possible, better ones need to be found.

10. The sample space of $n_x(t)$ used for calculating probability is also very general. More specific sample space needs to be determined.

11. The modified principle of dynamics $(2.3)'$ defined in the last section along with the principles of quantum statics defines a conceptual framework themselves. It makes self-time principle unnecessary. Whether this framework is more useful than the one discussed in section 2 in describing reality needs to be determined.

12. Now W functionals and $\hat{L}_m$ require two new physical constants to determine their scale. The search of W, $\hat{L}_m$ and the scales may point out to a new fundamental theory.

So the principles presented in this article have a huge choice. The various possible theories relating to the different implementations of the principles have to be studied theoretically and experimentally to come up with the precise details to achieve successful model for quantum general relativistic description of nature, or some or all of them be falsified in the verification process.

6 Conclusion

In this article, I have outlined a conceptual framework and have discussed how to apply this to study physics. Because of highly stochastic nature of the theory we need to use computer simulation, and statistical analysis to get any useful physics out of the theory. Application of the framework to simple models is straight forward. But the complication is, even for simulating simple models, extensive computing power is required. Currently the application of the conceptual framework to some simple models is being studied by the author. The results will be published in the follow-up reports. The framework discussed is quite general and there are wide variety of variations and sceneries. To come up with a specific model that best explains the physics of the entire universe requires exploring as many interesting models as possible.

7 Acknowledgements

This work was motivated by a weekly discussion group (2001) on conceptual problems in quantum mechanics held at University of Pittsburgh, Physics Department, lead by Allen Janis. I like to thank Allen Janis, George Sparling, Martin C Vincent, Jeremy Levy and John Earmen at University of Pittsburgh; Robert Griffith and Leonard Kisslinger at Carnegie Mellon University.

Appendices

A Single Point System

Consider a single point universe made of a simple quantum system with p_α, q^α as the conjugate variables for which:

$$L\left(p_\alpha, q^\alpha, n, \lambda\right) = p_\alpha \delta q^\alpha - N\Delta t H(p_\alpha, q^\alpha). \tag{62}$$

Let me define a scalar product as follows:

$$< a, b >= a_\alpha b_\beta s^{\alpha\beta}, \tag{63}$$

$s^{\alpha\beta}$ are function of q^β. Let the Hamiltonian constraint be,

$$H(p_\alpha, q^\alpha) = \frac{< p, p >}{2} + V(q). \tag{64}$$

The direction $v^\alpha = i < \psi_t|[H, q^\alpha]|\psi_t >$. The norm of v^α is given by $|v| = \sqrt{< v^\alpha, v^\alpha >}$. Using this the normalized v^α is $\bar{v}^\alpha = \frac{v^\alpha}{v}$. Let v_α, Δt are fixed parameters, with $< v, v >= 1$. Then from the first principle of dynamics the modified Lagrangian is as follows:

$$L\left(p_\alpha, q^\alpha, N, \lambda; v_\alpha, \Delta t\right)\Delta t = p_\alpha \delta q^\alpha + \lambda(\bar{v}_\alpha \delta q^\alpha - |v|\Delta t) - N\Delta t H. \tag{65}$$

We can rewrite the Lagrangian as follows:

$$L\left(p_\alpha, q^\alpha, N, \lambda; v_\alpha, \Delta t\right)\Delta t = \left(p_\alpha + \lambda \bar{v}_\alpha\right)\delta q^\alpha - \lambda|v|\Delta t - N\Delta t H. \tag{66}$$

Let me perform a change of variables from p_α to $m_\alpha = p_\alpha + \lambda\bar{v}_\alpha$. This change of variables does not introduce any new weight in the path integral because the integral measure is unchanged $d\lambda dp_\alpha = d\lambda dm_\alpha$. Let me define $\tilde{H}$ as follows using $p_\alpha = m_\alpha - \lambda\bar{v}_\alpha$:

$$\tilde{H}(m_\alpha, q^\alpha, \lambda, N; v_\alpha) = H(m_\alpha - \lambda\bar{v}_\alpha, q^\alpha, \lambda, N; v_\alpha). \tag{67}$$

Then we have,

$$\begin{aligned} \tilde{H}(m_\alpha, q^\alpha, \lambda, N; v_\alpha) &= \left(\frac{1}{2} < m - \lambda\bar{v}, m - \lambda\bar{v} > + V(q)\right) \\ &= \frac{1}{2}(< m, m > + \lambda^2 - 2\lambda < m, \bar{v} > + 2V(q)). \end{aligned} \tag{68}$$

Now the new Lagrangian is given by,

$$\tilde{L}\left(m_\alpha, q^\alpha, \lambda, N; v_\alpha, \Delta t\right)\Delta t = m_\alpha \delta q^\alpha - \lambda|v|\Delta t - N\tilde{H}(m_\alpha, q^\alpha, \lambda; v_\alpha)\Delta t. \tag{69}$$

The propagator is,

$$G(q^\alpha, q'^\alpha; v_\alpha, \Delta t) = \int \exp(m_\alpha \Delta q^\alpha - \lambda|v|\Delta t)\delta(\tilde{H}(m_\alpha, q^\alpha))\mu dm_\alpha d\lambda. \tag{70}$$

Let me integrate over $\delta(\tilde{H})d\lambda$. For this we solve,

$$\begin{aligned}
\tilde{H}(m_\alpha, q^\alpha, \lambda, N; v_\alpha) &= 0, \\
\lambda^2 - 2\lambda &< m, \bar{v} > +2V(q) + < m, m >= 0, \\
\lambda &= < m, \bar{v} > \pm\sqrt{< m, \bar{v} >^2 - (2V(q) + < m, m >)}
\end{aligned} \tag{71}$$

From this we identify,

$$H_v = \sqrt{< m, \bar{v} >^2 - (2V(q) + < m, m >)}. \tag{72}$$

Now the reduced Lagrangian for both directions of time evolution along $\bar{v}^\alpha$is

$$\tilde{L}_\pm\left(m_\alpha, q^\alpha, \lambda, N; v_\alpha, \Delta t\right)\Delta t = m_\alpha \Delta q^\alpha - < m, \hat{v} > |v|\Delta t \mp H_v|v|\Delta t. \tag{73}$$

Integrating $\int \delta(H)d\lambda$ introduces a new weight $\frac{1}{|H_v|}$ in the path integral, but as we discussed in principle 2.1, this is supposed to cancel the weight μ already present in the path integral so that the path integral yields a Dirac delta when $\Delta t = 0$. Now the one step propagator is

$$G_\pm(q^\alpha, q'^\alpha; v_\alpha, \Delta t) = \frac{1}{(2\pi)^d}\int \exp(im_\alpha \Delta q^\alpha - i < m, \bar{v} > |v|\Delta t \mp iH_v|v|\Delta t)dm_\beta \tag{74}$$

Then the evolution of the wavefunction is given by

$$\psi_{\pm,t+\Delta t}(q'^\alpha) = \frac{1}{(2\pi)^d}\int \exp(im_\alpha \Delta q^\alpha - i < m, \bar{v} > |v|\Delta t \pm iH_v|v|\Delta t)\psi_{\pm,t}(q^\alpha)dm_\beta dq^\alpha. \tag{75}$$

From this we have the Schrödinger equation form of evolution as

$$\frac{d\psi_{\pm,t}(q^\alpha)}{dt} = -v^\alpha \partial_\alpha \psi_{\pm,t}(q^\alpha) \mp i\hat{H}_v \psi_{\pm,t}(q^\alpha)|v|. \tag{76}$$

The Schrödinger form of evolution equation is dependent on $v^\alpha = i < \psi_t|[H, q^\alpha]|\psi_t > ..$

B Canonical Quantum Gravity

B.1 Using supermetric

Here I apply principle 2.1 to a theory described by the Hamiltonian constraint of general relativity only. In case of gravity the Lagrangian density (without the diffeomorphism constraints) is

$$\mathcal{L} = -\pi_{\alpha\beta}\dot{h}^{\alpha\beta} - \sqrt{h}N[-R + \frac{\pi^{\alpha\beta}\pi_{\alpha\beta} - \frac{1}{2}\pi^2}{h}], \tag{77}$$

$$\mathcal{H} = [-R + \frac{\pi^{\alpha\beta}\pi_{\alpha\beta} - \frac{1}{2}\pi^2}{h}]. \tag{78}$$

Given two spatial tensors $A^{\alpha\beta}, B^{\alpha\beta}$, I can define the scalar product using a supermetric on second rank spatial tensors as follows:

$$< A, B >= A^{\alpha\beta} B_{\alpha\beta} - \frac{1}{2} AB \qquad A = A_{\alpha\beta} h^{\alpha\beta} \qquad B = B_{\alpha\beta} h^{\alpha\beta}$$

Please note that the scalar product is not positive definite. Let me assume the initial state to be $|\psi >$. Now we can define the following quantities:

$$\begin{aligned} v^{\alpha\beta} &= i < \psi|[H, h^{\alpha\beta}]|\psi >, \\ |v| &= \sqrt{< v, v >} \\ \bar{v}_{\alpha\beta} &= \frac{v_{\alpha\beta}}{|v|} \end{aligned}$$

Using the first principle of dynamics the modified Lagrangian density is given by,

$$\begin{aligned} \mathcal{L}\Delta t &= -\pi_{\alpha\beta}\Delta h^{\alpha\beta} + \lambda(\bar{v}_{\alpha\beta}\Delta h^{\alpha\beta} - |v|\Delta t) - \sqrt{h} N \Delta t[-R + \frac{1}{h} < \pi, \pi >] \\ &= -(\pi_{\alpha\beta} - \lambda\bar{v}_{\alpha\beta})\Delta h^{\alpha\beta} - \lambda|v|\Delta t - \sqrt{h} N \Delta t[-R + \frac{1}{h} < \pi, \pi >] \end{aligned}$$

Let me make a change of variables,

$$\begin{aligned} m_{\alpha\beta} &= \pi_{\alpha\beta} - \lambda\bar{v}_{\alpha\beta} \qquad => \quad \pi_{\alpha\beta} = m_{\alpha\beta} + \lambda\bar{v}_{\alpha\beta}, \\ &< \pi, \pi >=< m, m > +\lambda^2 + 2\lambda < \bar{v}, m > . \end{aligned}$$

Using this in the Lagrangian density we have,

$$\mathcal{L}\Delta t = -\left(m_{\alpha\beta}\right)\delta h^{\alpha\beta} - \lambda|v|\Delta t - \sqrt{h} N\Delta t[-R + \frac{< \pi, \pi >}{h}]. \tag{80}$$

We can expand $\mathcal{H}$ in terms of $m_{\alpha\beta}$ as follows:

$$h\mathcal{H} = -hR + < m, m > +\lambda^2 + 2\lambda < \bar{v}, m >$$

Now we can solve the $H = 0$ for λ :

$$\lambda = - < \bar{v}, m > \pm\sqrt{< \bar{v}, m >^2 - (< m, m > -hR)}.$$

We get the effective Hamiltonian density as,

$$\mathcal{H}_\nu(h^{\alpha\beta}, \pi_{\gamma\delta}) = \pm\sqrt{< \bar{v}, \pi >^2 - (< \pi, \pi > -hR)}. \tag{81}$$

The effective Lagrangian density is

$$\mathcal{L}_\pm\Delta t = -\pi_{\alpha\beta}\Delta h^{\alpha\beta} - < v, \pi > \Delta t \pm H_\nu(h^{\alpha\beta}, \pi_{\gamma\delta})|v|\Delta t \tag{82}$$

The single-step propagator and the modified Schrödinger equation is given as follows:

$$
\begin{aligned}
G_{\pm}(h_x^{\alpha\beta}, h_x'^{\alpha\beta}) &= \int \exp(i \int \mathcal{L}_{\pm,x} \Delta t_x d^3x) d\pi_{\gamma\delta,x}, \qquad (83)\\
d_D|\psi_{\pm,t} &> = i \int \left(< v_x, \hat{\pi}_x > \mp \hat{H}_v(h_x^{\alpha\beta}, \hat{\pi}_{\gamma\delta,x})|v_x| \right) dt_x d^3x |\psi_{\pm,t}>,\\
v_x^{\alpha\beta} &= i < \psi|[H_x, h_x^{\alpha\beta}]|\psi > .
\end{aligned}
$$

In the three equations dependence on location is made explicit. Principle 2.3 describes how dt_x is dealt with.

B.2 Using normal metric

Given two spatial tensors $A^{\alpha\beta}$ and $B^{\alpha\beta}$, I can define the scalar product as follows:

$$
< A, B >= A^{\alpha\beta} B_{\alpha\beta} \qquad (84)
$$

This metric is positive definite. Using the first principle of dynamics the modified Lagrangian density is given by,

$$
\begin{aligned}
\mathcal{L}\Delta t &= -\pi_{\alpha\beta}\Delta h^{\alpha\beta} + \lambda(\bar{v}_{\alpha\beta}\Delta h^{\alpha\beta} - |v|\Delta t) - \sqrt{h} N \Delta t[-R + \frac{1}{h}(< \pi, \pi > - \frac{1}{2} < h, \pi >^2)]\\
&= -(\pi_{\alpha\beta} - \lambda\bar{v}_{\alpha\beta})\Delta h^{\alpha\beta} - \lambda|v|\Delta t - \sqrt{h} N \Delta t[-R + \frac{1}{h}(< \pi, \pi > - \frac{1}{2} < h, \pi >^2)] \qquad (85)
\end{aligned}
$$

and the Hamiltonian constraint is,

$$
\mathcal{H} = \sqrt{h}[-R + \frac{1}{h}(< \pi, \pi > - \frac{1}{2} < h, \pi >^2)] \qquad (86)
$$

Let me make a change of variable,

$$
\begin{aligned}
m_{\alpha\beta} &= \pi_{\alpha\beta} - \lambda v_{\alpha\beta} \qquad => \qquad \pi_{\alpha\beta} = m_{\alpha\beta} + \lambda v_{\alpha\beta},\\
< & \pi, \pi >=< m, m > + \lambda^2 < \bar{v}, \bar{v} > + 2\lambda < \bar{v}, m >,\\
< & h, \pi >=< h, m > + \lambda < h, \bar{v} > .
\end{aligned}
$$

We can expand $\mathcal{H}$ in terms of $m_{\alpha\beta}$ as follows:

$$
\begin{aligned}
\sqrt{h}\mathcal{H} &= -hR + < m, m > - \frac{1}{2} < h, m >^2\\
+\lambda^2(< & \bar{v}, \bar{v} > - \frac{1}{2} < h, \bar{v} >^2) + \lambda(< \bar{v}, m > - < h, \bar{v} >< h, m >)
\end{aligned}
$$

Let us solve the Hamiltonian constraint for λ. We find λ by defining functions a, b and c;

$$
\begin{aligned}
a &= (1 - \frac{1}{2} < h, \bar{v} >^2),\\
b &= (< \bar{v}, m > - < h, \bar{v} >< h, m >),\\
c &= -hR + < m, m > - \frac{1}{2} < h, m >^2,\\
\lambda &= -\frac{< \bar{v}, m > - < h, \bar{v} >< h, m >}{2(1 - \frac{1}{2} < h, \bar{v} >^2)} \pm \frac{\sqrt{b^2 - 4ac}}{2(1 - \frac{1}{2} < h, \bar{v} >^2)}.
\end{aligned}
$$

We get the effective Hamiltonian density as,

$$\mathcal{H}_\nu(h^{\alpha\beta}, m_{\gamma\delta}) = \frac{\sqrt{b^2-4ac}}{2(1-\frac{1}{2}<h,\bar{v}>^2)}. \tag{88}$$

The effective Lagrangian density, the single-step propagator, and the modified Schrödinger equation are given, with dependence on location made explicit, by,

$$\mathcal{L}_{\pm,x}\Delta t_x = -\pi_{x,\alpha\beta}\Delta h_x^{\alpha\beta} - \frac{(<\bar{v}_x,\pi_x> - <h_x,\bar{v}_x><h_x,\pi_x>}{2(1-\frac{1}{2}<h_x,\bar{v}_x>^2)}|v_x|\Delta t_x \mp \mathcal{H}_\nu(h_x^{\alpha\beta},\pi_{\gamma\delta,x})|v_x|\Delta t_x, \tag{89}$$

$$G_\pm(h_x^{\alpha\beta}, h_x'^{\alpha\beta}) = \int \exp(\int \mathcal{L}_{\pm,x}\Delta t_x d^3x) d\pi_{\gamma\delta,x}, \tag{90}$$

$$d_D|\psi_{\pm,t}>= i\int \left(\frac{(<\bar{v}_x,\hat{\pi}_x> - <h_x,\bar{v}_x><h_x,\hat{\pi}_x>}{2(1-\frac{1}{2}<h_x,\bar{v}_x>^2)} \pm \hat{H}_{v,x}(h_x^{\alpha\beta},\hat{\pi}_{\gamma\delta,x})|v_x|\right) dt_x d^3x|\psi_{\pm,t}> . \tag{91}$$

C Gravity-Matter-like Evolution: An Example

In the present universe, at near flat space-times encountered, the local matter does not significantly influence the gravitational field. The gravitational metric gets contributions only in the form of fluctuations. Let me study this using a very simplified example of a single point system. Let the total Hamiltonian H at each point is given by the sum of a background (gravitation-like) term H_g and a matter-like term H_m:

$$H = H_g(q^\alpha, p_\alpha) + H_m(\phi, \pi, q^\alpha). \tag{92}$$

where H_g is finite and $H_m \approx 0$ in the region of the phase space of the theory where the wavefunction of the theory is finite.

To simplify the problem let me assume the H_g is given by a term of the form,

$$H_g = E^2 - h_g(P_I, Q^I), \tag{93}$$

where the E is the momentum along the direction of time flow in configuration space given in the principle 2.1 and $h_g(P_I, Q^I)$ are the remaining contributions. Assume that the matter-like term does not contribute to time direction. The total Hamiltonian is,

$$H = E^2 - h_g + H_m, \tag{94}$$

with $E^2 - h_g >> H_m$. Along with $H = 0$, we get $E^2 \approx h_g >> H_m$. Using $H = 0$ we can solve for positive root of E, which defines H_ν

$$\begin{aligned} E &= +\sqrt{h_g + H_m} \approx \sqrt{h_g} + \frac{1}{2}\frac{H_m}{\sqrt{h_g}}. \\ H_\nu &= \sqrt{h_g} + \frac{1}{2}\frac{H_m}{\sqrt{h_g}}. \end{aligned}$$

We get the matter-like Hamiltonian H_m along with the additive and the scale factors. We assume time evolution only along the $+ve$ direction of v^α. The scale factor can be absorbed by rescaling time definition.

The new Lagrangian is

$$
\begin{aligned}
& P_I \Delta Q^I + \pi\Delta\phi - H_\nu \Delta t \\
= \;& P_I \Delta Q^I + \pi\Delta\phi - \left(\sqrt{h_g} + \frac{1}{2}\frac{H_m}{\sqrt{h_g}}\right)\Delta t \\
= \;& P_I \Delta Q^I + \pi\Delta\phi - (2h_g + H_m)\,\Delta\tilde{T}, \qquad (95)
\end{aligned}
$$

where $\Delta\tilde{T} = \frac{\Delta t}{2\sqrt{h_g}}$. Since the scale factor depends on P_I and Q^I, the scaling must be entered inside the definition of the propagator itself. Let d be the number of q^α's. Consider the propagator defined in the first principle of dynamics:

$$
\begin{aligned}
& G(Q^I, T, \phi, Q'^I, T', \phi'; v_\alpha, \Delta t) \\
= \;& \frac{1}{(2\pi)^d}\int \exp(iP_I\Delta Q^I + i\pi\Delta\phi - iH_\nu\Delta T) d\pi dP_I. \qquad (96)
\end{aligned}
$$

Explicitly,

$$
\begin{aligned}
& G(Q^I, T, \phi, Q'^I, T', \phi'; v_\alpha, \Delta t) \\
= \;& \frac{1}{(2\pi)^d}\int \exp\left\{iP_I\Delta Q^I + i\pi\Delta\phi - i\left(h_g + \frac{1}{2}\frac{H_m}{\sqrt{h_g}}\right)\Delta T\right\}\delta(\Delta T - |v|\Delta t) d\pi dP_I, \qquad (97)
\end{aligned}
$$

The rescaling can be done by modifying $\delta(\Delta T - v\ \Delta t)$ as $\delta(\frac{1}{2\sqrt{h}}\Delta T - v\ \Delta t)$. Basically this defines a new selective-time propagator defined by $w_\alpha = \frac{1}{2\sqrt{h}}\bar{v}_\alpha$ and $|v|$, written formally as

$$
\begin{aligned}
& G_s(q^\alpha, q^{'\alpha}; \Delta t, w_\alpha, |v|) \\
= \;& \frac{1}{(2\pi)^d}\int \exp(ip_\alpha\Delta q^\alpha)\delta(H))\delta(w_\alpha(p, q'^\alpha)\Delta q^\alpha - |v|\Delta t)|v| dp_\alpha \\
= \;& \frac{1}{(2\pi)^d}\int \exp(ip_\alpha\Delta q^\alpha)\delta(H)\delta(\frac{1}{2\sqrt{h_g}}\bar{v}_\alpha\Delta q^\alpha - |v|\Delta t)|v| dp_\alpha. \qquad (98)
\end{aligned}
$$

Making change of variable to Q's and P's we get,

$$
\begin{aligned}
& G_s(Q^I, T, \phi, Q'^I, T', \phi'; \Delta t, w_\alpha, |v|) \\
= \;& \frac{1}{(2\pi)^d}\int \exp\left\{iP_I\Delta Q^I + i\pi\Delta\phi - i\left(h_g + \frac{1}{2}\frac{H_m}{\sqrt{h_g}}\right)\Delta T\right\}\delta(\frac{\Delta T}{2\sqrt{h_g}} - |v|\Delta t)|v| dP_I \\
= \;& \frac{1}{(2\pi)^d}\int \exp\left\{iP_I\Delta Q^I + i\pi\Delta\phi - i\,(2h_g + H_m)\,|v|\Delta t\right\}\delta(\frac{\Delta T}{2\sqrt{h_g}} - |v|\Delta t)|v| dP_I. \qquad (99)
\end{aligned}
$$

Now the selective evolution of the wavefunction,

$$
\begin{aligned}
\psi_{t+\Delta t}(T', Q'^L, \phi') \;=\;& \frac{1}{(2\pi)^d}\int G(Q^I, T, \phi, Q'^I, T', \phi'; w_\alpha, \Delta t)\psi_t(T, Q^I, \phi) dQ^I d\phi \Delta T \\
=\;& \frac{1}{(2\pi)^d}\int \exp(P_L\Delta Q^L + \pi\Delta\phi - (2h_g + H_m)\,\Delta T) \\
& \psi_t(T' - 2\sqrt{h_g}|v|\Delta t, Q^I, \phi) dP_L dQ^I d\phi. \qquad (100)
\end{aligned}
$$

If we assume $\psi_t(T,\ \Delta t, Q^I, \phi)$ to be a product of the form $\psi_t^g(T, Q^I) * \psi_t^m(\ Q^I, \phi)$, where the first is the gravity-like part and second matter-like part,

$$\begin{aligned}\psi_{t+\Delta t}(T', Q'^L, \phi') &= \frac{1}{(2\pi)^d}\int \exp(iP_L\Delta Q^L - 2ih_g|v|\Delta t)\psi_t^g(T' - 2\sqrt{h_g}|v|\Delta t, Q^I)dP_L dQ^I \\ &* \int \exp(\pi\Delta\phi - H_m|v|\Delta t)\psi_t^m(T' - 2\sqrt{h_g}|v|\Delta t, Q^I, \phi)d\pi d\phi. \end{aligned} \quad (101)$$

If H_m and ψ^monly slightly depends on fluctuations in q^α, by substituting the expectation values of Q_I in H_m,we can separate the matter-like and the background-like quantum evolutions:

$$\psi_{t+\Delta t}^g(T', Q'^L) = \frac{1}{(2\pi)^{d-1}}\int \exp(iP_L\Delta Q^L - 2ih_g|v|\Delta t)\psi_t^g(T' - 2\sqrt{h_g}|v|\Delta t, Q^I)dP_L dQ^I, \quad (102)$$

$$\psi_{t+\Delta t}^b(< T >, < Q^I >, \phi') = \frac{1}{2\pi}\int \exp(\sum i\pi\Delta\phi - iH'_m|v|\Delta t)\psi_t^m(< T >, < Q^I >, < T >, \phi)d\pi d\phi. \quad (103)$$

where the H'_m depends on $< q^\alpha >$. This is a very formal result. Knowing explicitly the function h_g can help this make result more concrete.

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