Quantum Graviy Framework 2.0 : A Complete Dynamical Framework of Principles for Quantization of General Relativity

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Abstract

Developing Planck scale physics requires addressing problem of time, quantum reduction, determinism and continuum limit. In this article on the already known foundations of quantum mechanics, a set of proposals of dynamics is built on fully constrained discrete models: 1) Self- Evolution - Flow of time in the phase space in a single point system, 2) Local Measurement by Local Reduction through quantum diffusion theory, quantum diffusion equation is rederived with different assumptions, 3) Evolution of a multipoint discrete manifold of Systems through a foliation chosen dynamically, and 4) Continuum limit, and determinism are enforced by adding terms and averaging to the action. The proposals are applied to the various physical scenarios such as: 1) Minisuperspace reduced cosmology of isotropic and homogenous universe with scalar field, 2) Expanding universe with perturbation, and 3) Newtonian Universe. Ways to experimentally test the theory is discussed.

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1 Overview

1.1 The problem

The purpose of this paper is to address problems related to combining quantum mechanics and general relativity. They are as follows:

1. Problem of time: Physics is used to make predictions. Knowing the past, physics helps predict the future. Without time subject of physics is useless. In classical gravity time-evolution clearly is present. The Wheeler-Dewitt equation, which is the fundamental equation for quantum gravity, contains time evolution internally, and we need a way to extract time evolution from it in a quantum general relativistic context.

2. Since general relativity is a description of the macroscopic universe, quantum general relativity requires addressing the quantum reduction problem. So developing a theory of quantum gravity requires including quantum reduction theory. Usually the difficulty maintaining superposition closely relates to the mass of the quantum system. Higher the energy of a particle or photon more it acts with particle nature. For example a proton, which is massive, is a smaller wave-packet than an electron, even though they have equal charge. This suggests strength of gravitational field may be related to the size of a wave-packet. And so the gravitational field associated need to be linked to appearance of decoherence. For example, Roger Penrose [3] have already promoting gravity as a quantum reduction. A systematic theory for quantum reduction using gravity is not yet available and it needs to be developed.

3. Quantum States are not covariant objects, and they depend on the space-time foliation. So quantum reduction depends on foliation. We need a way to objectively describe the quantum reductive evolution and the issue of foliation on the manifold on which the process depends.

4. Because of non-perturbative nature of quantum gravity one may need to approach it like lattice gauge theory. This leads to discretizing space-time and fields. Also quantum reduction leads to randomness. These leads to problems in recovering back the continuum physics. So we need way to recover back continuum classical physics addressing the issues of smoothness and determinism.

The goal of this article is to outline a framework of proposals for Planck scale physics, such that it 1) has time (dynamics), quantum reduction and continuum limit as integral parts of the basic foundations, 2) is simple and intuitive, 3) has proper physical motivation, 4) is based on simple scientifically established notions and concepts, and 5) makes minimal assumptions.

Revision info: This paper is a modified version of the Quantum Gravity Framework1.0 [24]. This paper is somewhat conceptually different from that of the previous version. It is more advanced overall with more calculations, and sample applications of the basic principles.

1.2 Outline of the paper and Assumptions

In section 2, I elaborately discuss the proposals for dynamics, which are conceptual in nature. The proposals discussed there apply to the discrete models that are constrained by the Hamiltonian constraint only. This means, we assume, we have already solved the gauge constraints and diffeomorphism constraints, by restricting the kinematical Hilbert space. First a proposal for quantum evolution of a single point system with quantum variables is discussed. Then I discuss the inclusion of quantum reduction to the single point universe through Bloch (Lindblad) equations and quantum diffusion theory. Then I discuss multipoint universe such as physics on a manifold. Quantum reduction using quantum diffusion or Bloch equation requires a preferred foliation. So a proposal is introduced to obtain a preferred foliation through minimizing a functional (chosen experimentally) which depend on the dynamical variables of gravitational fields. Various possible choices for the functional is given. Finally, a proposal is introduced to enforce smoothness and determinism by adding an extra term and averaging to the action. Various possible choices for the extra term is given.

In section 3, I discuss how to put together all the four proposals. In section 4, I apply the proposals to various physical scenarios such as: 1) Minisuperspace reduced cosmology of Isotropic and homogenous universe with scalar field, 2) Expanding universe with perturbation, 3) Newtonian Universe. In section 5, ways to experimentally test the theory is discussed.

This article aims to develop a framework of proposals, which are more of guidelines, to develop Planck scale physics with time and reduction. It is precisely not quantum general relativity, because some aspects of the conceptual proposals of quantum mechanics and general relativity are not subsumed, such as unitarity and foliation independence. Primarily this article was intended to address the issues of time and reduction, by developing a theory of Planck scale process. But since the framework is built on discrete models, a fourth proposal is added to impose continuum limit. But the framework aims to reproduce general relativity and quantum mechanics consistent with the experiments at the appropriate scales. Not all the elements of the framework is new, but some are already well known, such as Bloch and quantum diffusion equations. But I want to embed them in a proper framework so that they can be used to develop Planck scale physics.

I assume that the metric in the internal configuration space is positive definite unless specified. We follow the following conventions in this article: 1) It is assumed that $\hbar = c = G = 1$, unless specified 2) Einstein's summation convention is assumed over indices of internal spaces.

2 A Framework of Proposals

2.1 Self Evolution in a single point universe

2.1.1 The Theory

Consider a single point universe, with one simple quantum system living on it. Assume that simple quantum system is described by a Hamiltonian constraint only. Let the internal configuration space of the quantum system is of dimension d, and is made of canonical variables p_α and q^α. Let $m_{\alpha\beta}$, a function of q^α, is the metric in the internal configuration space. Hereafter I will use $m_{\alpha\beta}$ and its inverse $m^{\alpha\beta}$(assuming it exists), to raise and lower indices. Let me define a scalar product using the metric:

$$< a, b >= a_\alpha b_\beta m^{\alpha\beta}.$$

I will assume $m^{\alpha\beta}$ is positive definite for now. The Langrangian as usual is

$$\mathcal{L}(p_\alpha, q^\alpha, N) = p_\alpha \dot{q}^\alpha - N\mathcal{H}(p_\beta, q^\gamma).$$

Let me assume that a typical Hamiltonian is as follows (without the Lapse):

$$\mathcal{H}(p_\alpha, q^\alpha) = \frac{< p, p >}{2} + V(q^\alpha) = m^{\alpha\beta} p_\alpha p_\beta + V(q^\gamma) \quad (1)$$

$$= \frac{p_\alpha p^\alpha}{2} + V(q^\gamma). \quad (2)$$

In this case the classical dynamic equations are

$$\frac{dq^\alpha}{dt} = N\{\mathcal{H}, q^\alpha\} = N\frac{\partial\mathcal{H}}{\partial p_\alpha}, \quad (3a)$$

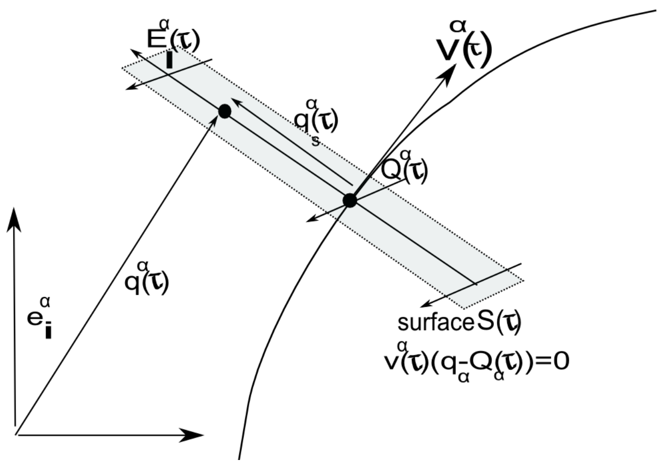
$$\frac{dp_\alpha}{dt} = N\{\mathcal{H}, p^\alpha\} = -N\frac{\partial\mathcal{H}}{\partial q_\alpha}. \quad (3b)$$

The propagator is

$$G(q^\alpha, q'^\alpha; \Delta t) = \frac{1}{(2\pi)^d}\int \exp(ip_\alpha \Delta q^\alpha)\delta(\mathcal{H}(q^\alpha, p_\alpha))dp^D.$$

We see here that the Δt term is absent in the right hand side. So the propagator is a function of the configurational variables only:

$$G(q^\alpha, q'^\alpha) = \frac{1}{(2\pi)^d}\int \exp(ip_\alpha \Delta q^\alpha)\delta(\mathcal{H}(q^\alpha, p_\alpha))dp^D. \quad (4)$$



This form of the propagator is what is to be expected. This is because t is a coordinate variable, it can be changed by an arbitrary (smooth) rescaling of the lapse (temporal diffeomorphism). So the physical evolution should not depend on it. Practically we read time by reading observables, for example the position of a clock's needles. The q's are the most basic observables and so the physical time may have to be extracted from them, for example, assuming that one of them acts as an internal time variable. But, this way of choosing the time variable is arbitrary and subjective. We need a more objective way to choose the time variable.

We need to define a set of concepts so that we understand time evolution in a fully constrained system. Consider the configuration space. We need to use the understanding of classical physics to describe the flow of time in the Hamiltonian constrained system. Various solutions to the problem of time was reviewed by Isham [15]. In the schemes described there, our approach will be to identify time after quantization. Usually in the semi-classical approach using the Wheeler-Dewitt equation, WKB wave-function is assumed to exist. Then using the Hamilton-Jacobi equation derived from it, the phase of the wave-function is related to time (for example [16], [17]). Our proposal is closely related this. But assumptions of validity of WKB approximation is too strict a requirement. Here we will make no such assumptions, and try to develop time evolution by linking the classical trajectory given by equations (3a) and quantum path integral in equation (4), only assuming that both of them exist.

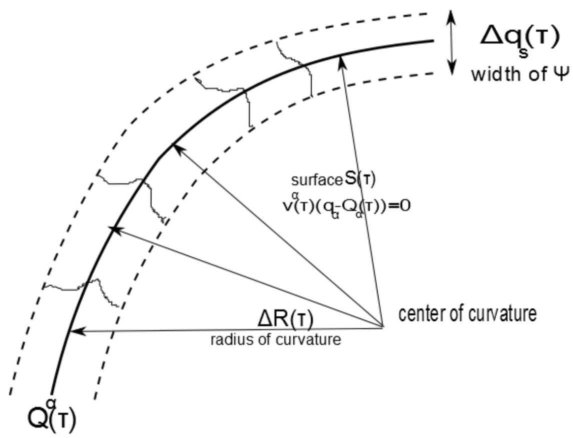
In classical analysis doing the constrained system analysis is straight forward. It is a trajectory $(q^\alpha(\tau), p_\alpha(\tau))$ in the phase space given by equations (3a). The configuration space is just enough to formulate the proposals. Given $\dot{q}^\alpha(0), q^\alpha(0)$, we usually have the curve $q^\alpha(\tau)$ as a solution satisfying the initial conditions. The tangent to each point is $\dot{q}^\alpha(\tau)$.

Consider a smooth trajectory $Q^\alpha(\tau)$. Consider a $D-1$ dimensional one parameter family of flat hyperplane S_τ in the configuration space going through a point $Q^\alpha(\tau)$ and the normal be $v^\alpha(\tau) = \dot{Q}^\alpha(\tau)$. Let me denote this hyperplane by $S(v^\alpha(\tau), Q^\alpha(\tau))$ or in short $S(\tau)$. Let $q^\alpha = (t, q^I_s)$,where t is the coordinate along the normal $v^\alpha_s(\tau)$, and q^I_s are $D-1$ coordinates on the hyperplane orthogonal to it. We can use these coordinates to study physics on these hyperplanes $S(v^\alpha(\tau), Q^\alpha(\tau))$ in the neighborhood of the region near the point. Let $\psi_\tau(q^I_s)$ be a wavefunction on this hyperplane. Then we can derive a propagator for evolving this wavefunction with respect to the one parameter family of hyperplane as I will discuss next.

If $q^\alpha \in S(\tau)$, from the equation for the hyperplane we have,

$$v^\alpha(\tau)(q^\alpha - Q^\alpha(\tau)) = 0.$$

Let e^α_i be the global unit vectors, where i varies from 0 to $D-1$, with $e^\alpha_0 v_\alpha(\tau) > 0$. Let e^i_α be the inverse of e^α_i. Let E^α_I are the unit vectors on the surface $S = S(\tau)$ where I varies from 1 to $D-1$. The metric on the hyperplane is $\sigma_{\alpha\beta} = (m_{\alpha\beta} - \bar{v}_\beta \bar{v}_\alpha)$. We can get E^α_I by projecting e^β_I to S, $E^\alpha_I = \sigma^\alpha_\beta e^\beta_I$. Let $Q^\alpha(\tau)$ be the origin on S, then the hyperplane coordinate of point in the e^β_I basis is $q^\alpha_s = Q^\alpha(\tau) - q^\alpha$, implying $q^\alpha_s v_\alpha = 0$.In the E^α_I basis the hyperplane coordinates are $q^I_s = E^I_\alpha q^\alpha_s$,where $E^I_\beta = \sigma^\alpha_\beta e^I_\alpha$.



Let us associate a normalized wavefunction $\psi(q_s^I, \tau)$ to the surface $S(\tau)$. If $q'^\alpha \in S' = S(\tau + d\tau)$, the propagator between S to S' is given by,

$$\tilde{G}(q^\alpha, q'^\alpha; S, S') = \frac{1}{(2\pi)^d} \int_{v_{s,\alpha} p^\alpha < 0} \exp(ip_\alpha(q^\alpha - q'^\alpha))\delta(H) dp^D. \tag{5a}$$

I have used the condition $v_{s,\alpha} p^\alpha < 0$ to define a casual propagator, such that the time evolution along v^α.

Let me assume the width of the wavefunction $\Delta q_s(\tau)$, defined by

$$< \quad q_s^I(\tau) >= \int q_s^I |\psi(q_s^J, \tau)|^2 dq_s^{D-1}, \tag{6a}$$

$$\Delta q_s(\tau)^2 \quad = \quad \int (q_s^I(\tau) - < q_s^I(\tau) >)(q_{s,I}(\tau) - < q_{s,I}(\tau) >) |\psi(q_s^J, \tau)|^2 dq_s^{D-1}, \tag{6b}$$

remains much smaller than the radius of curvature of $Q(\tau)$. Then q_s^I are good coordinates to study the evolution of the $\psi(q_s^I, \tau)$.

$\tilde{G}(S, S')$ be the operator form of $\tilde{G}(q^I, q'^I; S, S')$ in the q^I, q'^I coordinates. The propagator needs to be normalized as follows:

$$G(S, S') = \lim_{S'' -> S} \tilde{G}(S, S')\tilde{G}(S, S^{''})^{-1}. \tag{7}$$

This renormalization removes factors of integration from $\delta(H)$, so that $\lim_{S' -> S} G(q^I, q'^I; S, S') = \delta(q^I - q'^I)$.

Proposal 1: Self-Evolution: *The temporal flow of time in a quantum Hamiltonian constrained system is described by 1) Wavefunction $\psi_\tau(q_s^I)$ on the one parameter family of hyperplanes $S(\tau) = S_\tau(\dot{Q}^\alpha(\tau), Q^\alpha(\tau))$, for a given $Q^\alpha(\tau)$, 2) The propagator between the wavefunctions on the hyperplane given by equation (7), 3) The path $Q^\alpha(\tau)$ considered as a C^1 smooth function of τ. 4) Given an arbitrary path $Q^\alpha(\tau)$, one can evolve the wavefunction $\psi_\tau(q_s^I)$ normal to the hyperplane $S(\dot{Q}^\alpha(\tau), Q^\alpha(\tau))$.at each instant τ.The physical value of $Q^\alpha(\tau)$ is such that $< \psi_\tau | \hat{p}_s^I \hat{p}_{sI} + \hat{q}_s^I \hat{q}_{sI} | \psi_\tau >$ is a minimum for all possible of $q_s^\alpha(\tau)$*[1].

The $S(\tau) = S_\tau(\dot{Q}^\alpha(\tau), Q^\alpha(\tau))$ represents the classical information contained, which can be derived from the Euler-Langrange Equations or Hamilton equations, while $\psi_\tau(q_s^I)$ represents the quantum information.

[1] This principle is different from that of the first proposal in Quantum Gravity Framework [24]. There the quantum state of system is defined on the configuration space. Time is defined as extra parameter associated with the state. Time evolution happens as the quantum state amplitudes simultaneously change at each point of the configuration space. But in version 2.0 time is considered as an internal coordinate. The evolution happens on a one parameter sequence of cross-sections of the configuration space, similar to conventional relativistic quantum mechanics. It is not clear which version is correct regarding the quantum state. Comments are welcome as www.qstaf.org/skm-article2.

Then α_s is very large, the $Q^\alpha(\tau)$ evolve close to its classical expected value given by Hamilton equations. Because of this Ω becomes close to zero. Now this is just a generalization of what a person experiences in relativistic quantum mechanics. He observes the physics around himself as it happens on the hyperplane orthonormal to the direction along which he travels, with he being the center.

Given an arbitrary path $Q^\alpha(\tau)$,one can evolve the wavefunction along $\psi_\tau(q_s^I)$ along the hyperplane $S_\tau(\dot{Q}^\alpha(\tau), Q^\alpha(\tau))$.The physical value of $Q^\alpha(\tau)$ is such that Ω is a minimum for all possible $Q^\alpha(\tau)$.But there could be more than one solution for this. Also if $\Delta q_s(\tau)$ is two high compared to norm of $< q_s(\tau) >$ (equation (6a)) then it is not sensible to think $Q^\alpha(\tau)$ as the unique classical information associated with the quantum evolution. Proposal 2 that we will discuss in the next section will improve this situation.

Using this proposal 1 as a guideline, let me derive the Hamiltonian associated to the continuous evolution of the wavefunction on a parameter family of hyperplanes S_τ. Let me calculate the Hamiltonian for the self-evolution, from surface $S = S(\tau)$ to $S' = S(\tau + d\tau)$. The details of the calculation is given in appendix A. The propagator for the evolution of the wave function from $S(\tau)$ to $S' = S(\tau + d\tau)$ is,

$$G(q_{s'}^{'I}, q_s^I, \Delta\tau) = \int_{p_\alpha \bar{v}^\alpha < 0} \exp(i(p_I^s \delta q_s^I - H_s(p_I^s, q_{s'}^{'I}, Q^\alpha(\tau))\Delta\tau)) dq_s^I, \tag{8}$$

where

$$H_s(p_I^s, q_s^I, Q^\alpha(\tau)) = p_I^s q_s^J h_J^I - p_v(|v| + q_s^I \frac{d\bar{E}_I^\alpha}{d\tau} \bar{v}_\alpha), \tag{9}$$

$$|v| = |\frac{dQ^\alpha}{d\tau}|,$$

and

$$h_J^I = \bar{E}_\alpha^I \frac{d\bar{E}_J^\alpha}{d\tau}.$$

Assuming $m^{\alpha\beta} = \delta^{\alpha\beta}$. Using

$$p_v = \sqrt{-p_s^I p_I^s - 2V(Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha)},$$

we can calculate the effective classical equation of motion for the Hamiltonian H_s

$$\dot{q}_s^I = q_s^J h_J^I - \frac{\partial p_v}{\partial p_I^s}(|v| + q_s^K \frac{d\bar{E}_K^\alpha}{d\tau} \bar{v}_\alpha), \tag{10}$$

$$\dot{p}_J^s = -p_I^s h_J^I + \frac{\partial p_v}{\partial q_s^J}(|v| + q_s^K \frac{d\bar{E}_K^\alpha}{d\tau} \bar{v}_\alpha) + p_v(\frac{d\bar{E}_J^\alpha}{d\tau} \bar{v}_\alpha). \tag{11}$$

Examples of application of these equations is given in Appendix A.

2.1.2 Recovering the usual Classical motion

Let me show that we can recover the classical Hamilton motion from the equations (10) and (11) with the condition of minimality of Ω. Let $\sigma^{\alpha\beta} = \delta^{\alpha\beta}$. Let initially $q_s^I = 0$ and $p_I^s = 0$.

$$p_v = \sqrt{-p_s^I p_I^s - 2V(Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha)},$$

$$\begin{aligned}
\frac{\partial p_v}{\partial p_I^s} &= -\frac{p_s^I}{p_v}, \\
\frac{\partial p_v}{\partial q_s^J} &= -\frac{1}{p_v}\frac{\partial V(Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha)}{\partial q_s^J}.
\end{aligned}$$

Using the initial conditions $q_s^I = 0$ and $p_I^s = 0$, we need to solve for $\dot{q}_s^I = 0$ and $\dot{p}_J^s = 0$ to get the equations of motion for $Q^\alpha(\tau)$. Using these,

$$\begin{aligned}
p_v &= \sqrt{-2V(Q^\alpha(\tau))}, \\
\dot{q}_s^I &= 0 \implies -\frac{\partial p_v}{\partial p_I}(|v|) = 0, \\
&\implies p_s^I = 0.
\end{aligned}$$

$\dot{p}_J^s = 0$ implies,

$$\begin{aligned}
\frac{\partial p_v}{\partial q_s^J}(|v|) + p_v(\frac{d\bar{E}_J^\alpha}{d\tau}\bar{v}_\alpha) &= 0, \\
-\frac{1}{p_v}\frac{\partial V(Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha)}{\partial q_s^J}(|v|) + p_v(\frac{d\bar{E}_J^\alpha}{d\tau}\bar{v}_\alpha) &= 0, \\
-\frac{1}{p_v}\frac{\partial V(Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha)}{\partial q_s^J}(|v|^2) - p_v(\bar{E}_J^\alpha \frac{dv_\alpha}{d\tau}) &= 0.
\end{aligned}$$

Using initially $\bar{E}_J^\alpha = \delta_J^\alpha$ for $\alpha \neq 0$,

$$-\frac{|v|^2}{(p_v)^2}\frac{\partial V(Q^\alpha)}{\partial Q^\alpha} - (\frac{d^2 Q_\alpha}{d\tau^2}) = 0.$$

Since τ is a arbitrary parameter v is defined upto an arbitrary scale. So by setting $|v| = p_v$, we get,

$$\frac{d^2 Q_\alpha}{d\tau^2} = -\frac{\partial V(Q^\alpha)}{\partial Q^\alpha}, \qquad \alpha \neq 0.$$

Evolution of Q_0 is determined by the condition $|v| = p_v$, which is equivalent to the Hamiltonian constraint. This and the above equation both are equivalent to equations (3a).

2.1.3 Algorithm for evolution

Let me write out the algorithm for evolving the wavefunction using proposal 1:

1. First a wavefunction $\psi(q_s^I, \tau)$ is given on initial hyperplane $S(\tau)$ with normal vector $\nu^\alpha(\tau)$. Let the expectation value of the wavefunction be $Q^\alpha(\tau)$ in the global coordinates. Let $Q^\alpha(\tau)$ be the origin of the coordinates q_s^I in $S(\tau)$. Let $< p_I^s >$ be p_α in global coordinates. Let me assume that the norm of p_α is small compared that of $\nu^\alpha(\tau)$.

2. Evolve the wavefunction $\psi(q_s^I, \tau)$ along $\nu^\alpha(\tau)$ to a new hyperplane $S(\tau + d\tau)$ with normal $\nu^\alpha(\tau + d\tau) = m^{\alpha\beta} p_\alpha$ going through point $Q^\alpha(\tau) + d\tau\nu^\alpha(\tau)$. Set $Q^\alpha(\tau + d\tau) = Q^\alpha(\tau) + d\tau\nu^\alpha(\tau)$ as origin of coordinates

on the new hyperplane. The Propagator and Hamiltonian for evolution was given in equation (8) and equation (9). Using equation (9), the modified Schrodinger equation is

$$d\psi(q_s^I, \tau) = iH_s(p_I^s, q_s^I, Q^\alpha(\tau))d\tau. \tag{12}$$

3. Repeat steps 1 and 2.

The above evolution ensures minimization of $< \psi_\tau | \hat{p}_s^I \hat{p}_{sI} + \hat{q}_s^I \hat{q}_{sI} | \psi_\tau >$ at each step.

2.1.4 Application to Physics

Let me apply discuss the application of the theory to general relativity coupled to electromagnetic field and scalar field. The canonical coordinates are (h_{ab}, π^{ab}) for gravity, (A_a, E^a) of Electromagnetic field, and (ϕ, χ) for the scalar field. Since we are studying a single point system, without the interaction terms, the Hamiltonian constraint is

$$H_{non-int} = +\frac{1}{2}\left(\chi^2 + m^2 h\phi^2\right) + \frac{1}{2}(E^2) - \frac{1}{c_g}(\pi^2 - \pi^{ab}\pi_{ab}),$$

where c_g is the gravitational coupling constant. Now our basis is made of tensors and scalars. Our basis e_i^α is actually made of collection different tensor bases: $dx^a \otimes dx^b$, dx^a and 1, where 1 is the basis for scalar field. They belong to different spaces. But it does physically make sense in unified theories such as in Kaluza-Klein theory, where tensor, vector and scalar fields becomes components of higher dimensional tensor, and in string theory the fields are just various string components. Nevertheless to apply the method discussed in this section, we need to name the 10 pairs of conjugate variables, as q^α and p^α. For example:

$$\begin{aligned} q^0 &= \phi, q^2 = A_1, q^3 = A_2, q^4 = A_3, \\ q^5 &= h_{11}, q^6 = h_{22}, q^7 = h_{33}, q^8 = h_{12}, q^9 = h_{23}, q^{10} = h_{32}. \end{aligned}$$

$$\begin{aligned} p_0 &= \phi, p_2 = E^1, p_3 = E^2, p_4 = E^3, \\ p_5 &= \pi^{11}, p_6 = \pi^{22}, p_7 = \pi^{33}, p_8 = \pi^{12}, p_9 = \pi^{23}, p_{10} = \pi^{32}. \end{aligned}$$

Some of these terms are redundant because the diffeomorphism constraint, and gauge constraints need to be imposed, which is another separate problem. In section four, I solve these constraints explicitly assumming small curvature approximation.

The Hamiltonian constraint can be rewritten in terms of the p and q variables and the theory we developed in this section can be applied. The kinetic term in term of $p's$ and $q's$ would be too complicated for display, unless we use some unified theory of fields. So I don't explicitly show it here. Computer simulation to analyze the behavior of such a system, where all the relevant variables can be directly entered into the program.

2.2 Quantum Reduction for single point system

Consider the single point system discussed in the last subsection. The modified Schrödinger equation (12) derived describes the evolution of the system in the direction v^α. The equation results in the system evolving into a macroscopic superposition state. To prevent this we need continuous reduction of the system which removes the macroscopic superposition. The general form of continuous reduction of a quantum system is given by the Bloch equations in the Lindblad form [21] governing evolution of density matrix (reviewed in [22]):

$$\dot{\rho} = i[\hat{\rho}, \hat{H}] + \sum_m (2\hat{L}_m \hat{\rho} \hat{L}_m^+ - \hat{L}_m^+ \hat{L}_m \hat{\rho} - \hat{\rho} \hat{L}_m^+ \hat{L}_m), \tag{13}$$

where ρ is the density matrix and L_m are the operators representing observables to be continuously measured. This equation has been extensively studied and has been useful in various experimental situations [23]. It describes an ensemble of identical quantum systems and does not tell how each individual system evolves. It is not the most natural and explicit form to use to describe an individual quantum system. So, we need to consider the equivalent equation, given by Percival, Gisin and Diosi [6], which describes the stochastic motion of the quantum system state $|\psi>$ of a quantum system:

$$\begin{aligned} d|\psi &> = -i\hat{H} d\tau |\psi> + \sum_m (\hat{L}_m - <\hat{L}_m>)|\psi> dz^m \sqrt{d\tau} \\ + \sum_m (2 &< \hat{L}_m > \hat{L}_m - \hat{L}_m^+ \hat{L}_m - <\hat{L}_m^+><\hat{L}_m>)|\psi> d\tau, \end{aligned} \tag{14}$$

where $d\tau$ is the time interval of evolution in the non-relativistic quantum mechanics. The dz_m are complex numbers representing Gaussian distributed independent random variables. More explicitly, the real and imaginary parts of dz^m are Gaussian random variables such that the statistical expectation values are given by,

$$M(dz^m) = 0, \quad M(dz^m dz^n) = 0, \quad M(dz^m dz^{*n}) = 2\delta^{mn}, \tag{15}$$

where M refers to the statistical mean.

Let me clarify how the third term works a little bit. Consider that $|\psi>$ is expanded as a superposition of the eigenstates of $\hat{L}_m$. As $|\psi>$ evolves, the third term tends to reduce the amplitude of an eigenstate in the sum to the extent to which its eigenvalue is far away from the expectation value of $<\hat{L}_m>$. Because of this $|\psi>$ evolve such that the amplitudes of the components are peaked close to $<\hat{L}_m>$, a semiclassical state. In equation (14) the second terms randomizes the system, third term classicalizes the system. These are natural components of macroscopic quantum reduction.

Percival applies this to quantum field theory and indicates that the resultant theory is non-unitary [7]. But, in case of quantum gravity the universe cannot be described by unitary evolution alone because that would lead to superposition of macroscopic states. Clearly, experimentally, whenever the quantum state of a system evolves into a superposition of macroscopic quantum states it probabilistically evolves to one of the macroscopic states. So, for a macroscopic universe, the quantum evolution must be described by an equation that has three components: a deterministic unitary component, a stochastic component, and a component that prevents macroscopic superposition. The modified Schrödinger equation (14) is the most natural form of it and the three terms in the right hand side of the equation give the necessary components in the respective order.

In general the stochastic evolution can defined upto a norm of $|\psi_\tau>$. The norm of $|\psi_\tau>$ is not physically relevant. The physical interpretation of the theory comes through its relation to density matrix which doesn't depend on the norm of $|\psi_\tau>$

$$\rho = \frac{M(|\psi_\tau><\psi_\tau|)}{<\psi_\tau|\psi_\tau>},$$

which evolves by the Lindblad equation. The M is the statistical mean with respect to the random variables z. The eigenstates and eigenvalues of ρ gives the possible physical states $|\psi_\tau>$ of the quantum system and probabilities for observing them respectively.

In the appendix C a general stochastic evolution equation motivated by equation (14) is derived. The derivation is based on [8], but different in details. Let me briefly summarize the derivation in the appendix C. A general stochastic evolution equation is

$$|d\psi>=\alpha|\psi>dt+\beta_m|\psi>z^m\sqrt{dt},$$

where α and β are operators on $|\psi>$, and z^mobeys equation (15). Summation over repeated indices is assumed.

Let me solve $d(<\psi|\psi>)=0$, assuming that dt and z^m are free variables, and, α and β are independent of dt and z^m. As shown in the appendix C, the constraints are too strong that they will eliminate the quantum diffusion equation (14) itself.

We need to take a different route. First let me solve $Md(<\psi|\psi>)=0$, assuming α and β are independent of dt and z^m.The general solution for α is

$$\alpha=iH+\gamma-<\gamma>-\beta_m^+\beta_m, \tag{16}$$

assuming α and β are independent of dt and z^m. Here H is a Hermitian operator, and γ is an arbitrary operator.

Now let me for solve $d(<\psi|\psi>)=0$ assuming, dt are the free variables, α,β are independent of dt. Further a solution for α can be obtained by adding a real number to equation (16) to keep the norm constant. To summarize we have the final form of the dynamics equations are

$$|d\psi_\tau>=\alpha|\psi>dt+\beta_m|\psi>z^m\sqrt{dt}, \tag{17}$$

where

$$\begin{aligned}
\alpha &= iH-\gamma-\beta_m^+\beta_m+c, && (18a)\\
\beta_m &= L_m-<L_m>, && (18b)\\
c &= -\frac{1}{2}<\beta_m^+\beta_n>(z^m\bar{z}^{\ n}-2\delta^{mn})+<\gamma>-<2\alpha^+\beta_m>z^m\sqrt{dt}. && (18c)
\end{aligned}$$

The c is a $c-$number and is a random function of z. The c is also dependent on $\sqrt{dt}$. I will assume that $z^m\sqrt{dt}$ can be neglected hereafter, unless specified otherwise.

We can derive the evolution equation for $\rho=M(|\hat{\psi}><\hat{\psi}|)$, from equation (18a) (without neglecting $\sqrt{dt}$ term). The evolution equation of ρ is

$$\begin{aligned}
\frac{d\rho}{dt} &= \rho(\tilde{\alpha}^+)+(\tilde{\alpha})\rho+\beta_m\rho\beta_m^+, && (19)\\
\tilde{\alpha} &= iH+\gamma-<\gamma>-\beta_m^+\beta_m.
\end{aligned}$$

Here the $\sqrt{dt}$ term does not show up.

To get the quantum diffusion equation and the Lindblad equation we need to set [8],

$$\begin{aligned}
\beta_m &= L_m-<L_m>,\\
\gamma &= <L_m>L_m^+-L_m<L_m^+>.
\end{aligned}$$

In this case, using the evolution equation (19) it easy see that the system in long term approaches the expected probability distribution for Copenhagen interpretation.

Let me state the second proposal of dynamics.

Proposal 2: Local Quantum Reduction - *Given a path $Q^\alpha(\tau)$, the quantum state of a single point quantum system along with the self-evolution also undergoes continuos reduction with respect to observables L_i through semiclassicalization and randomization given by equation below.*

$$\begin{aligned} d|\psi_\tau &> = iH_s|\psi_\tau > d\tau + \sum_m \beta_m|\psi_\tau > dz_m\sqrt{|\dot{Q}^\alpha|\Delta\tau} + (\gamma - \sum_m \beta^+_m\beta_m)|\psi_\tau > |\dot{Q}^\alpha|\Delta\tau, \qquad (20) \\ \gamma &= -\sigma + < L_m > L^+_m - L_m < L^+_m > . \end{aligned}$$

with H_s is from equation (9), $\beta_i = L_i - < L_i >$, with σ is Hermitian operator to subject the evolution to further reduction.

Physical relevance of σ will be evident in the fourth proposal. Since the c term is ignored, $|\psi_\tau >$ is not assumed to be normalized. So

$$< L_i > = \frac{< \psi_\tau|L_i|\psi_\tau >}{< \psi_\tau|\psi_\tau >}.$$

The $iH_s|\psi_\tau >$ term is self-time Hamiltonian derived from proposal 1 in equation (12). The $\Delta\tau$ is the time measure from the first proposal of dynamics, and the operators $\hat{L}_m$ are simple functions of the conjugate variables p^s_I and q^I_s to undergo continuous reduction.

Usually for the applications of the Bloch equation (13) to study the evolution of the density matrix of a quantum system, the L_m's are to be determined by what are to be measured in the experimental context. But, here in the second proposal of dynamics we assume that the L_m's are fundamental quantities in quantum gravity to be determined experimentally. The natural and simplest choice for the L_m's are given by p^s_I and q^I_s, or some simple functions of them. These observables need to be gauge invariant and diffeomorphism invariant as expected by the theory studied. Introducing diffeomorphism invariance requires studying multipoint system, which will be discussed in the next to subsections.

The fermionic fields have zero expectation values. So these cannot contribute to the L_m's. The fermionic particles can be measured by measuring the bosonic fields they generate. For example, superposition of a particle wavefunction at different points, results in superposition of fields generated by it such as gravitational and electromagetic fields. Continous reduction of these fields with L_m's, reduces fermionic fields.

The combined quantum system forms a complete reality by itself and there is no outside observer to make reduction. The system needs to be understood as undergoing continuous reduction by itself instead of being considered as undergoing measurement.

There is no necessity that one needs the quantum diffusion theory. It might be simple to just use the Bloch equations (13). A detailed study of the model might help whether one can just restrict to the density matrix formalism of the theory. Also since the reference frame with respect to which the quantum diffusion occurs keeps changing according to proposal one. This might interfere with the reduction process and make quantum diffusion theory problematic for reduction process. But if the reduction occurs faster than the curving of $Q^\alpha(\tau)$, which determines the reference frame, then the theory will remain sensible.

The new modified Hamiltonian can be directly included in the algorithm discussed in the last section. This can randomize the evolution of $Q^\alpha(\tau)$ and quantum state. This randomness can reduced and smooth evolution can be reproduced by introducing many-body interaction and continuity conditions in the next two proposals.

2.3 Quantum Evolution and Reduction for Multipoint Universe

The first proposal of dynamics focused on a quantum system at a single point that evolved according to a single time parameter. In quantum gravity we want to evolve the quantum states from one spatial hypersurface to another spatial hypersurface. In a spatial hypersurface there is infinite number of points, with a quantum system at each point. So let me discuss how to understand time evolution in a many-point quantum system. If there are many interacting fully constrained quantum systems, then for each point x, there are one set of conjugate variables $p_{x,\alpha}$, q_x^α (D dimensional space internal space). To each point we can apply theory discussed in proposal one, then there will one classical curve $Q_x^\alpha(\tau_x)$ for each point, one hyperplance $S_\tau^x(\dot{Q}_x^\alpha(\tau_x), Q_x^\alpha(\tau_x)$, and one free (dummy) parameter τ_x for each point.

Let me assume that space is discretized and is made of countable number of points. Let B be the number of points, and for simplicity let us assume B is finite. Assume that the quantum system at each point x is described by an identical Hamiltonian constraint $\mathcal{H}_x$ only, and it has an interaction term that involves adjacent quantum systems. Each step of the evolution depends on how $Q_\alpha(\tau_x)$ varies with $\Delta\tau_x$. Now consider the propagator defined by proposal 1 in equation (7). For each system at x, we have one curve $Q_x^\alpha(\tau)$ assigned. Then the combined one step propagator is

$$\begin{aligned}\tilde{G}(\{q_{s,x}^\alpha, q_{s',x}'^\beta; S_x, S_x', \forall x\};) &= \frac{1}{(2\pi)^{BD}}\int_{v_{s,x}^\alpha p_{\alpha,x}<0,\forall x}\prod_x\{\exp(ip_{\alpha,x}(q_x^\alpha - q_x'^\alpha))\delta(H_x)dp_x^D\}, \\ G(\{S_x, S_x', \forall x\}) &= \lim_{S_x''->S_x,\forall x}\frac{\tilde{G}(\{S_x, S_x', \forall x\})}{\tilde{G}(\{S_x, S_x'', \forall x\})}.\end{aligned} \tag{21}$$

The repeated application of the one-step propagator for infinitesimal $\Delta\tau_x$ smoothly evolves all the systems. The sequence of the quantum states, defines the states of the system at various consecutive instants. As the combined system evolves the classical expectation value of the momentum and the configuration variables $p_{\alpha,x}$ and q_x^α also evolve. Let me apply it for F steps. From equation (21) we have:

$$\begin{aligned}G(\{q_{x,0}^\alpha, q_{x,F}^\beta, \forall x\}) &= \int\prod_{k=1}^{F}[G(\{q_{x,k}^\alpha, q_{x,k-1}^\beta; S_x(\tau_k), S_x(\tau_{k-1}), \forall x\})] \\ &\quad (\prod_x\prod_{k=I+1}^{F-1} dq_{x,k}).\end{aligned} \tag{22}$$

Here $Q_x^\alpha(\tau_x)$ is a free variable at each point of the discretized spacial surface. Apply the principal 1 extremal proposal yields a curve $Q_x^\alpha(\tau_x)$ for each point. But since τ_x is a dummy variable we can fix the arbitrariness. This done by using the condition $|\dot{Q}_x^\alpha(\tau_x)| = p_{v,x}$, where $p_{v,x}$ is the classical momentum along $\dot{Q}_x^\alpha(\tau_x)$. Then $\Delta\tau_x$ physically represents the proper time.

Now let me apply this to physics with gravity, scalar field and vector field. The Hamiltonian constraint with interaction term is

$$H_T = +\frac{1}{2}\left(hh^{ab}\partial_a\phi\partial_b\phi + \chi^2 + m^2h\phi^2\right) + \frac{1}{2}(E^2+B^2) - \left(c_g hR + \frac{1}{c_g}(\frac{1}{2}\pi^2 - \pi^{ab}\pi_{ab})\right).$$

As discussed in section 2.1.2 variables can be rewritten in terms of p_α and q^α. The ultimate expression for the above Hamiltonian constraint will be cumbersome. Important thing to note here is that there will one time parameter τ_x for each point. Assume that above Hamiltonian constraint is discretized in a cubic lattice made of B cubes. Then we can apply the above theory.

Each step of the evolution depends on the values of $\Delta\tau_x$. Let τ be a continuous time parameter, which varies from $\tau = 0$ to $\tau = T$. Let me define $\Delta\tau_x = n_x(\tau)\ \Delta\tau$, where the $n_x(\tau)$ are continuous functions of τ, one of them for each point x. The repeated application of the one-step propagator for infinitesimal $\Delta\tau$ evolves the quantum state. The $n_x(\tau)$ functions defines the various ways to foliate the discrete geometry, whose topology is B point $\otimes$ $1D$, described by the above Hamiltonian constraints. $n_x(\tau)$ is essentially is the lapse.

Now if we want to include the reduction at each point discussed in proposal 2, it depends on the foliation, as it is not covariant. Given a foliation described by certain choice of $n_x(\tau)$,we generalize proposal one and two, given a path $q^{\alpha}_{s,x}(\tau)$ for each point, the combined quantum state of all points of the manifold can made to undergo continuos reduction with respect to fundamental field variables $L_{i,x}$ at each point, through semiclassicalization and randomization given by equation below:

$$d|\psi_\tau >= iH_{s,x}|\psi_\tau > n_x(\tau)d\tau + \sum_{m,x}\beta_{m,x}|\psi_\tau > dz_{m,x}\sqrt{|\dot{Q}^{\alpha}_s|n_x(\tau)\Delta\tau} + (\gamma_x - \sum_{m,x}\beta^{+}_{m,x}\beta_{m,x})|\psi_\tau > n_x(\tau)|\dot{Q}^{\alpha}_s|\Delta\tau. \tag{23}$$

where $H_{s,x}$ is from equation (9), $\gamma_x = -\sigma_x + < L_{m,x} > L^{+}_{m,x} - L_{m,x} < L^{+}_{m,x} >$, and $\beta_{i,x} = L_{i,x} - < L_{i,x} >$, the suffix x indicates the point to which the quantities corresponds.

Now we need to pick a foliation that is be relevant to do the reduction process. Now there are three questions to be addressed: 1) whether the reduction process occurs along a preferred foliation, 2) what is the choice of the foliation along which the reduction occurs, and 3) whether this can be addressed as experimental questions. If reduction happens along a preferred foliation, this process will correlate information along the hypersurfaces of this foliation. This correlation is a physically measurable effect.

If the answer to the first question is yes, we can try to guess what could be the most natural foliation along which the reduction might occur. For this, consider the set up used for studying canonical general relativity. Consider a space time with metric $g_{\alpha\beta}$ and one parameter spacial foliation $\mathcal{S}_t$, where $\mathcal{S}_t$ is the spacial hypersurface for a given t. This can foliation can be specified by function $t(x)$, x is a point on space time, with $t =$ constant describes the surface $\mathcal{S}_t$. We can choose t to be the time coordinate. Consider the vector field, $T^{\gamma} = (\frac{\partial}{\partial_t})^{\gamma}$. T^{γ} generates a one parameter family of space-time diffeomorphism, such that a given initial surface $\mathcal{S}_{t^1}$ is mapped to a different surface $\mathcal{S}_{t^2}$ of the foliation. So specifying T^{γ} is another way to define the foliation. The universe is described by combination of ideal space times such as 1) Homogenous and Isotropic: Robertsen-Walker metric, 2)Static: Schwarzschild metric, 3) Stationary type: Kerr-Metric, and Reisnesser-Nordstrom metric, listed below:

$$ds^2 = a(t)^2(\ dt^2 - dx^2 + dy^2 + dz^2), \tag{24}$$

$$ds^2 = N(r)\ dt^2 - R(r)dr^2 - r^2 d\Omega, \tag{25}$$

$$ds^2 = N(r,\theta,\phi)dt^2 - h(r,\theta,\phi)_{ab}dx^a dx^b - 2N^a dx^a dt. \tag{26}$$

Space-Time Type->	Homogenous and Isotropic	Static type	Stationary type
Metric	equation(24)	equation(25)	equation(26)
Most Natural Foliation	$t =$ constant	$t =$ constant	$t =$ constant
$\frac{\partial}{\partial t}$	conformal killing	conformal killing	conformal killing
$\frac{d}{dt}\bar{h}_{ab}$	0	0	0
R	0	0	$\neq 0$
$\bar{\pi}^{ab}$	0	0	$\neq 0$

The most natural foliation for each case is given in the table above.

Consider the static case - Schwarzschild space time. The physical information is contained in the distribution of matter and the gravitational field around it. All this information is transferred unchanged along the foliation in which it is static. Our quantum measurement experiments are usually done along the time parameter along the time-like killing vector $(\frac{\partial}{\partial t})^{\gamma}$. Any motion of the measurement instruments or earth itself is too non-relativistic to alter the direction of flow compared to $(\frac{\partial}{\partial t})^{\gamma}$. Also consider the linear gravity. To first order the gravity is described by gravitational potential. On the Schwarzschild case the gradients are parallel to the hypersurfaces of the static foliation t =constant. To first order these static surfaces are the directions along which gravitational forces act. So these hypersurfaces are unique in this way.

In case of the Robertson Walker metric, similar to Schwarzschild metric, most calculations quantum or semiclassical is done along the foliation given by scale factor as cosmological time. Long distance correlation in Cosmic Microwave background (CMB) has been derived using quantum correlations during inflation using scale factor as time. Until now all the observations of Cosmic microwave background is consistent with such a theoretical analysis.

From the above two case we can seen that the foliation defined by the conformal killing vector $(\frac{\partial}{\partial t})^{\gamma}$ is appears to a good candidate. Let me define tensor $C_{\alpha\beta}$ defined as a function of space-time metric $g_{\mu\nu}$ by

$$C_{\alpha\beta}(g_{\mu\nu}, T^{\eta}) = \pounds_T(g_{\alpha\beta}) - \frac{1}{4}(g^{\gamma\delta}\pounds_T(g_{\gamma\delta}))g_{\alpha\beta},$$

where $\pounds_T$ is the lie derivative along T^{α}.For a vector T^{α} to be conformal killing , $C_{\alpha\beta}$ is to be zero.

One can see from the table that there are many ways to identify the natural foliations. This are listed below:

1. Trace free momentum $\bar{\pi}_{ab}$ is zero,
2. The scalar curvature of the hypersurfaces R is zero,
3. Trace free transverse component of h_{ab} or π_{ab} is zero,
4. Hypersurface volume $V = \int < \sqrt{h} > dx^3$ is maximum (not mentioned in the table),
5. If $\bar{h}_{ab} = h^{-\frac{1}{3}}h_{ab}$, with $h = \det(h_{ab})$,then $\frac{d\bar{h}_{ab}}{dt} = 0$,and
6. $C_{\alpha\beta}(g_{\mu\nu}, T^{\eta})$ is zero.

The first four are clearly true for spherical static case and cosmological case. The last two are also true for these two space times and also for stationary types such as rotating and/or charged case. The real physical space-time, is a combination of many types of metric and the six conditions hold only approximately. So we need to consider a physical choice of foliation such that it fits very closely to the natural time-like hypersurfaces associated to them. Instead of considering the tensors to be zero, we need specify a norm like functional on these tensors, to measure how small they are. Let me consider them on by one:

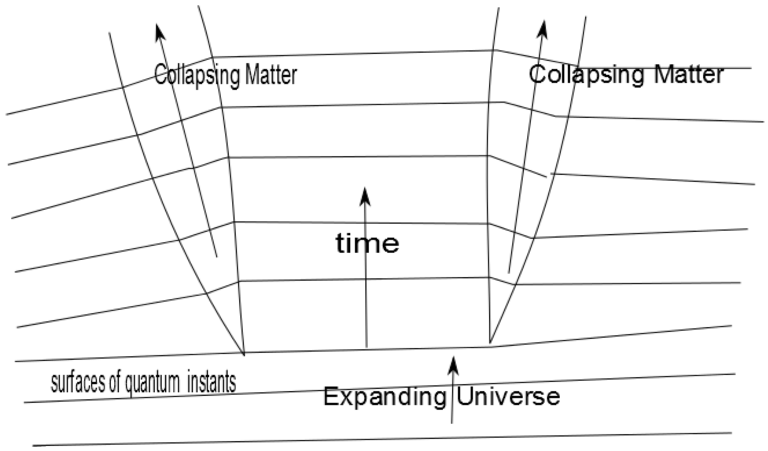
1. $\Upsilon(\pi^{ab}) = \int \frac{\bar{\pi}_{ab}\pi^{ab}}{\sqrt{h}} dx^3$.
2. $\Upsilon(h_{ab}) = \int R\sqrt{h} dx^3$.
3. $\Upsilon(\pi^{ab}) = \int \frac{\pi^{TT}_{ab}\pi^{ab}_{TT}}{\sqrt{h}} dx^3$, TT stands for trace free transverse component.
4. $\Upsilon(h_{ab}) = \int h^{\frac{2}{3}} \frac{d\bar{h}_{ab}}{dt} \frac{d\bar{h}_{ab}}{dt} \sqrt{h} dx^3$.
5. For measuring the smallness of $C_{\alpha\beta}$,consider the most obvious norm:

   $$\int C_{\alpha\beta}C^{\gamma\delta}\sqrt{g}d^4x = \int g^{\alpha\gamma}g^{\beta\delta}C_{\alpha\beta}(g_{\mu\nu}, T^{\eta})C_{\gamma\delta}(g_{\mu\nu}, T^{\eta})\sqrt{g}d^4x$$

   The second line makes the depends on $g^{\alpha\gamma}$ and T^{η} to be explicit. Since the metric is Lorentzian, the measure is not positive definite. So the smallness of $\int C_{\alpha\beta}C^{\gamma\delta}\sqrt{g}d^4x$ does not imply smallness of components of $C_{\alpha\beta}$. To surmount this, metric can be Euclideanized so that the norm is positive definite.

   $$\Upsilon(g_{\mu\nu}, T^{\eta}) = \int g_E^{\alpha\gamma}g_E^{\beta\delta}C_{\alpha\beta}(g^E_{\mu\nu}, T^{\eta})C_{\gamma\delta}(g^E_{\mu\nu}, T^{\eta})\sqrt{g_E}d^4x$$

   where $g^E_{\mu\nu}$ is the Euclidean version of the Lorentzian metric $g_{\mu\nu}$, and $g_E^{\mu\nu}$ is the inverse of $g^E_{\mu\nu}$.



6. $\Upsilon(h_{ab}) = -\int < \sqrt{h} > dx^3$, smallness of this norm measures the largeness of the volume.
7. Also one can consider using the full canonical momentum of gravitational field to define Υ :

$$\begin{aligned} \Upsilon &= \int (\frac{\pi^{ab}\pi_{ab}}{c_g h})\sqrt{h} dx^3 \\ &= \int (c_g \frac{K^{ab}K_{ab}}{h})\sqrt{h} dx^3. \end{aligned}$$

8. One also need investigate a more general way to define Υ using the kinetic terms of the integral spin fields, without including Dirac fields (as the field expectation value are zero):

$$\Upsilon = \int (\pi_\phi^2 + \frac{1}{2}E^2 + \frac{\bar{\pi}^{ab}\bar{\pi}_{ab}}{c_g h})\frac{1}{\sqrt{h}} dx^3$$

where $K_{\alpha\beta}$ is the full extrinsic curvature. But, in most cases away from singularities, the gravitational term dominates because, c_g is very large.

Now let me propose the following

Proposal 3: Global Quantum Reduction - *The quantum evolution and reduction process occurs along a spatial foliation such that the C^1smooth functions $n_x(\tau)$ take smooth values, such that relative probability weight is given by* $\exp(-c_r\Upsilon)$, *where c_r is a fundamental constant, where Υ is one of the measures in the above list, to be discovered and verified experimentally.*

One can assume Υ being minimum is sufficient to determine the foliation. But minimality of Υ may not necessarily give a unique foliation. That is why I have chosen a statistical form for proposal three. There are various possible candidates for describing the foliation of the three types of geometries: The first five choices fits with the canonical form of dynamics. The last choice is covariant.

When there are more than mixture bodies evolving in expanding universe, the minimal foliation is made merging of various types of foliations. This is illustrated in the figure below. Close to the cosmic celestial bodies such planets or stars, the foliation is determined by purely Schwarzschild metric, the minimal surfaces are normal to time-like vector along with the body moves. Between the celestial these surfaces deform slowly through intermediate foliation, whose normals are some weighted average of the velocity vectors of the planets depending the position and the masses of the bodies. In between galaxies we have surfaces described by constant scale factor.

Particles such as atoms or elementary particles do not disturb appreciably the gravitational field determined by large celestial bodies. So they evolve and decohere along these special foliations as I have discussed in proposal 3. One might think that proposal 3 is not in the spirit of General relativity as it depends on the foliation. But this not true, because we have not chosen the foliation kinematically. The foliation is chosen here dynamically based on values of gravitional canonical field variables.

The various alternative proposals for Υ need to be investigated theoretical and experimentally to look for a precise theory. The continuous reduction of the quantum state correlates the classical and quantum information along the hypersurfaces. Now for the $\Upsilon's$ suggested the correlation happens along the slightly random hypersurfaces that are close to the physically intuitive ones. This may be physically observable effects, as mentioned before, that may have useful consequences. So this effect needs to be studied more theoretically and/or experimentally.

2.3.1 Interpretation

The τ is defined as a global evolution parameter. The evolution along τ could physically mean two different things:

1. 3D Evolving block model: This is the Newtonian way of interpretation: τ is a global time parameter along which a curved 3D semi-classical universe undergoes quantum evolution. The past is semiclassical. Future is non existent. The present evolves along a unique foliation of 4D metric, which is probabilistically chosen from many possible foliations from proposal 3. Consciousness of all observers evolve along τ.

2. 4D Block Universe Model: τ is simply a global foliation parameter of a 4D block universe model. Observers physical time is a proper time parameter along his world line as usually defined in relativity. But, this interpretation does not explain why observer time flows unlike the first interpretation.

Which one of this may be right physical interpretation can only be decided by if possible by experimental study.

2.3.2 Algorithm for evolution

Let me discuss the algorithm for evolving a piece of a Planck scale sized spatial slice of reality.

1) Discretize the region into cubes of size L^3 in coordinate units. Discretize the Hamiltonian constraint. To each cube x assign quantities discussed in algorithm 2.1.2.All the quantities have suffix x.The Hamiltonian constraint is a function of near by points of $x's$.To the effective Hamiltonian $H_{s,x}$ add the quantum diffusion term.

2) First wavefunction $\psi(\{q^{\alpha}_{x,s}, \forall x\}, \tau)$ is given on the product of initial hypersurfaces $S_x(\tau)$ with normal vector $\nu^{\alpha}_x(\tau)$.Let the expectation value of the wavefunction be $Q^{\alpha}_x(\tau)$ in the global coordinates. The $Q^{\alpha}_x(\tau)$ needs to the origin of the coordinates $q^I_{x,s}$ in $S_x(\tau)$. Let $< p^I_{x,s} >$ be $p_{x,\alpha}$ in global coordinates. Let me assume the norm of $< p_{x,\alpha,s} >$ is small compared that of $\nu^{\alpha}_x(\tau)$.

3) Choose values for $n_x(\tau)$,and evolve each the global wavefunction wavefunction $\psi(\{q^{\alpha}_{x,s}, \forall x\}, \tau)$.To to this at each x evolve $\psi(\{q^{\alpha}_{x,s}, \forall x\}, \tau)$ from $Q^{\alpha}_x(\tau)$ along $\nu^{\alpha}_x(\tau)$ to a new hypersurface $S_x(\tau + d\tau)$ with normal $\nu^{\alpha}_x(\tau+d\tau) = m^{\alpha\beta}_x p_{x,\alpha}$ going through point $Q^{\alpha}_x(\tau)+n_x(\tau)d\tau\nu^{\alpha}_x(\tau)$. Set $Q^{\alpha}_x(\tau+d\tau) = Q^{\alpha\backslash}_x(\tau)+n_x(\tau)d\tau\nu^{\alpha}(\tau)$ as origin of coordinates on the new hypersurface. The Propagator and Hamiltonian for evolution was given in equation (8) and equation (9).

4) Calculate $\Upsilon(\tau)$,where Υ is defined by one proposal three, or one of the alternatives discussed before.

5) Change value of $n_x(\tau)$. Repeat steps 3 and 4.The probability $n_x(\tau)$ of values is given by $\exp(-c_r\Upsilon)$ The most probable $n_x(\tau)$ is for which $\Upsilon(\tau)$ is minimum.

6) The $\psi(\{q^{\alpha}_{x,s}, \forall x\}, \tau + d\tau)$ for which $\Upsilon(\tau)$ is minimum is the most probable new initial wavefunction. Now start over from step 2.

If c_r is large enough, then the evolution happens such that Υ is minimum. To study evolution of quantum particles in Schwarzschild or Cosmological case, the most probable $n_x(\tau)$ can be chosen easily. These cases will be later studied in this article.

2.4 Deterministic, Continuum Limit and Scale invariance

Let me assume that nature is made of large number of interacting identical discrete quantum systems at the Planck scale. The stochastic evolution in proposal three in the many body system, results in random evolution of the system, as the classical expectation values evolve randomly. Whatever discrete quantum model one proposes at a microscopic scale (atomic, nuclear or Planck), the model need to have a proper relation to the macroscopic classical world. One of the important aspects of this is the continuum limit. For this theories has to provide smoothness and deterministic evolution in the macroscopic limit.

Achieving continuum and deterministic limit for a discrete model is always a difficult problem. But nature seems to be continuous and deterministic at macroscopic scale. One simple way to solve this problem is given by the following proposal.

Proposal 4: *Every subsystem has several mechanisms built into it explicitly such that the expectation values of quantum variables of nearby or adjacent identical quantum systems are very close to each other. They are such as 1) There are imaginary decay term in the action to keep the quantum variables adjacent to each other, 2) Every system is a collection of large of subsystems each having quantum variables $q^I_{x,s}$ and random variables $z_{m,\ x}$ attached to it and evolving according the first three principles, and 3) The effective variables of every system is got by weighted averaging of the random and quantum variables of the underlying subsystems; Fundamental Commutators are smoothened as a consequence of this.*

Let me discuss the three parts of the proposal 4 one by one. First let me discuss the first part: A simple way to realize the first part of the proposal is to add an extra imaginary term to the action (20)

$$\mathcal{S} \longrightarrow \mathcal{S} + i \sum_{x,s} (\frac{1}{2}\sigma_x(q^\alpha_{y,s})|\dot{Q}^\alpha_{x,s}|)n_x(\tau)\Delta\tau,$$

such that σ_x are

1) smooth real functions of the variables $\hat{q}^\beta_{x,s}$ with a lower bound,

2) functions of quantum variables at x and adjacent (or nearby) quantum systems to point x, and

3) are increasing functions as $|q^\alpha_x - q^\alpha_{x'}| -> \infty$.

Now the new single-step propagator (without the quantum diffusion and global reduction) is

$$\begin{aligned} & \tilde{G}(\{q^\alpha_x, q'^\beta_x; S_x, S'_x, \forall x\};) \\ = & \frac{1}{(2\pi)^{BD}} \int_{v^\alpha_{s,x} p_{\alpha,x} > 0, \forall x} \prod_x \{\exp(ip_{\alpha,x}(q^\alpha_x - q'^\alpha_x) - \sum_x \sigma_x(q^\alpha_{y,s})|\dot{q}^\alpha_{x,s}|n_x(\tau)\Delta\tau)\delta(H_x)dp^D_x\}. \end{aligned}$$

The new term with σ_x need to be added to Hamiltonian $H_{s,x}$ in the algorithm discussed in the last section to enforce smoothness. The negative sign of σ_x in the evolution equation makes sure that $|\psi_\tau >$ wavefunction weights $|q^\alpha_x - q^\alpha_{x'}|$, $\forall\alpha$, for every pair of quantum systems adjacent to each other in the discrete model.

A simple choice for σ_x is

$$\sigma_x(q^\alpha_y) = \sum_{\alpha, \text{Adjacent } x'} |q^\alpha_x - q^\alpha_{x'}|^2.$$

But the problem with this function is that it will suppress the differences between q^α_x and $q^\alpha_{x'}$, erasing out the physics in long term. So alternative choices for $\hat{\sigma}$ are the following functions with minimums for non-zero $(q^\alpha_x - q^\alpha_{x'})^2$.

$$
\begin{aligned}
1)\ \ \sigma_x(q_y^\alpha) &= \sum_{\alpha,\text{Adjacent } x'} A\frac{\exp(B*|q_x^\alpha - q_{x'}^\alpha|^2)}{|q_x^\alpha - q_{x'}^\alpha|^2}, \\
2)\ \ \sigma_x(q_y^\alpha) &= \sum_{\alpha,\text{Adjacent } x'} \frac{A}{|q_x^\alpha - q_{x'}^\alpha|^2} + B|q_x^\alpha - q_{x'}^\alpha|^2,
\end{aligned}
$$

where A, B are real constants and positive. In this $\hat{\sigma}$ goes towards infinity for both $|q_x^\alpha - q_{x'}^\alpha| -> 0$and $|q_x^\alpha - q_{x'}^\alpha| -> \infty$. So this restricts the evolution of quantum state such that $|q_x^\alpha - q_{x'}^\alpha|$ are finite. For large expectation values of $|q_x^\alpha| >> 0$ (in quantum units), which corresponds to macroscopic case, $\frac{<|q_x^\alpha - q_{x'}^\alpha|>}{<q_x^\alpha>}$ are infinitesimal. This will help reproduce the continuum limit.

Another advantage of the $\hat{\sigma}$ operator is that it limits randomness in the fields. The quantum reduction in the last two proposals introduces randomness, and it can build to large values. The $\hat{\sigma}$ operator can reduce this randomness, and help fields to be smooth in the continuum limit. An extra term can be added to keep σ terms from disturbing the norm of the wavefunction.

Let me discuss the scale invariance of dynamical equations effectively. Let me study the large number limit of the stochastic evolution equation for a many body system in a finite neighborhood Ω made of finite number of systems. Let me assume the Ω is considered to made many cubes of volume Δv, with n lattice points in each orthonormal directions in three dimensions. Therefore Ω is made of n^3 cubes. Let the total volume $\Delta V = n^3 \Delta v$.

The evolution equation for a combined state is as follows:

$$
\begin{aligned}
|d\psi_\tau &> = (\sum_x \alpha_x \Delta V n_x(\tau) dt + \beta_m^x z_x^m \sqrt{\Delta V n_x(\tau) dt})|\psi_\tau>, \\
\alpha_x &= iH_x - \gamma_x - \sigma_x - \beta_m^{x+}\beta_m^x.
\end{aligned}
$$

where x indicate different points, c is ignored, σ is the operator in proposal four, and n_x are set to be equal to 1. Let me define the following averages:

$$
\begin{aligned}
\bar{\alpha} &= \frac{\sum_x \alpha_x}{n^3},\ \bar{\beta}_m = \frac{\sum_x \beta_m^x}{n^3}, \bar{z}^m = \frac{\sum_x z_x^m}{n^3}, \\
\bar{H} &= \frac{\sum_x H_x}{n^3},\ \bar{\gamma} = \frac{\sum_x \gamma_x}{n^3},\ \bar{\sigma} = \frac{\sum_x \sigma_x}{n^3}.
\end{aligned}
$$

For $\bar{z}^m$, we have $M(\bar{z}^m \bar{z}^{*n}) = \frac{2\delta^{mn}}{n^3}$. So we define $\tilde{z}^m = \bar{z}^m\sqrt{n^3}$. Then we have $M(\tilde{z}^m \tilde{z}^{*n}) = 2\delta^{mn}$.

Presence of σ_x makes quantum amplitude $< \{q_x^\alpha, \forall x\} \, |\psi_\tau>$ non-zero for the values of q_x^α close to each other. Then we can approximate the quantum diffusion equation by an macroscopic averaged equation,

$$
\begin{aligned}
|d\psi_\tau &> = \sum_x \bar{\alpha} \Delta V n_x(\tau) dt + \bar{\beta}_m \tilde{z}^m \sqrt{\Delta V n_x(\tau) dt})|\psi_\tau>, \\
\bar{\alpha} &= i\bar{H} - \bar{\gamma} - \bar{\sigma} - \bar{\beta}_m^+ \bar{\beta}_m.
\end{aligned}
$$

We find that the dynamical equations are scale invariant. But since the multiplying factor $\sqrt{\Delta V}$ of $\tilde{z}^m$ is larger, the system undergoes semiclassicalization rapidly in terms of the averaged values, than the each subsystem.

Let me now focus on the commutators. Consider the scalar field ϕ.

$$
\phi(x) = \int (a(k)\exp(ik.x) + a^\dagger(k)\exp(-ik.x))\frac{d^3k}{(2\pi)^{3/2}\sqrt{\omega}},
$$

Consider the expectation value of $\phi(x)^2$ in the ground state $|0>$:

$$< \phi(x)^2 >= \int < 0|a(k)a^\dagger(k')|0 > \exp(-ix.(k-k'))\frac{d^3kd^3k'}{(2\pi)^3\omega}.$$

This needs to be calculated using the commutator of the field:

$$[a(k), a^\dagger(k')] = \delta(k-k'),$$

which is a consequence of the fundamental commutator $[\phi(x), \pi(x')] = \delta(x-x')$. The result is divergent:

$$< \phi(x)^2 >= \int \frac{d^3k}{\omega} = \infty.$$

It is essential that $< \phi(x)^2 >$ to be finite, so that there is clear semiclassical nature for the ground state. To achieve this commutator need to be smoothened:

$$[a(k), a^\dagger(k')] = F(k)\delta(k-k'),$$

resulting in

$$< \phi(x)^2 >= \int F(k)\frac{d^3k}{\omega}.$$

If $F(k)$ sufficiently falls of as $k->\infty$, $< \phi(x)^2 >$ becomes finite. Now the new fundamental commutator of the field is

$$[\phi(x), \pi(x')] = f(x-x').$$

where $f(x)$ is the Fourier transform of $F(k)$.This commutator can be achieved by considering ϕ and π weighted averaging of fundamental fields $\tilde{\phi}(x), \tilde{\pi}(x)$ satisfying $[\tilde{\phi}(x), \tilde{\pi}(x')] = \delta(x-x')$:

$$\phi(x) = \int \tilde{\phi}(\tilde{x})\eta(x-\tilde{x})d\tilde{x}^3, \tag{27a}$$

$$\pi(x) = \int \tilde{\pi}(\tilde{x})\eta(x-\tilde{x})d\tilde{x}^3, \tag{27b}$$

where $\eta(x)$ is suitable weighting function. Now we have,

$$[\phi(x), \pi(x')] = \int \eta(x-\tilde{x})\eta(\tilde{x}-x')d\tilde{x}^3.$$

If we choose $\eta(x)$ to be the Gaussian function $\frac{1}{(d\sqrt{2\pi})^3}\exp(-\frac{x^2}{d^2})$, we have

$$\begin{aligned} [\phi(x), \pi(x')] &= \exp[-\frac{(x-x')^2}{d^2}], \\ [a(k), a^\dagger(k')] &= \frac{1}{(d\sqrt{2\pi})^3}\exp[-d^2k^2]\delta(k-k'), \\ < \phi(x)^2 > &= \frac{4\pi}{d^2}. \end{aligned}$$

Now the expectation value is made finite and the commutator $[\phi(x), \pi(x')]$ has been smoothened. The equations (27a) can be written as discrete sum over large number of subsystems, to make this analysis combatible with the second part of the fourth proposal.

3 Applications

In this section I discuss simple applications of the four principles proposed. I will first discuss the simple minisuperspace homogenous and isotropic expanding cosmological model in which the scalar field is coupled to the gravitational field. Then I will discuss the extension of this model with perturbations added to these field. Next I will discuss the model of celestial gravitational objects (small curvature) moving and perturbing the flat space.

3.1 Cosmological Reduced Model

Consider the expanding perfectly homogenous and isotropic cosmological model with a scalar field. Let the metric be given by,

$$ds^2 = N^2 dt^2 - \frac{1}{3}A(dx^2 + dy^2 + dz^2),$$

and the scalar field is ϕ. The Langrangian and Hamiltonian is

$$\begin{aligned} L &= \Delta V \left\{ \frac{6c_g \dot{a}^2}{\sqrt{A}N^2} - a^{\frac{3}{2}}\frac{\dot{\phi}^2}{2} - a^{\frac{3}{2}}U(\phi) - a^{\frac{3}{2}}N\Lambda \right\}, \\ H &= \left\{ \frac{\sqrt{a}}{24c_g\Delta V}\pi_a^2 - \frac{\pi_\phi^2}{2\Delta V a^{\frac{3}{2}}} + \Delta V a^{\frac{3}{2}}U(\phi) + \Lambda\Delta V a^{\frac{3}{2}} \right\}, \end{aligned}$$

where Λ is the cosmological constant and c_g is the gravitational coupling constant, $U(\phi)$ is scalar pontential, ΔV is the volume the region universe we are studying.

Let me apply the first principle. The path corresponding the minimum of $< \psi_\tau|\hat{p}_s^I \hat{p}_{sI} + \hat{q}_s^I \hat{q}_{sI}|\psi_\tau >$ is described be the classical equations of motion. Let the capitalized variables describe the evolution of classical configurational and conjugate variables of this path.

$$\begin{aligned} \dot{\Pi}_\phi &= \Delta V a^{\frac{3}{2}}U'(\phi), \\ \dot{\Pi}_A &= -\frac{\pi_A^2}{48c_g\Delta V\sqrt{A}} + \frac{3\pi_\phi^2}{4VA^{\frac{5}{2}}} - \frac{3}{2}\Delta V A^{\frac{1}{2}}U(\phi) - \frac{3}{2}\Lambda\Delta V A^{\frac{1}{2}}, \end{aligned}$$

$$\begin{aligned} \dot{\Phi} &= -\frac{\Pi_\phi}{VA^{\frac{3}{2}}}, \\ \dot{A} &= \frac{\Pi_A}{3c_g V\sqrt{A}}. \end{aligned}$$

Now let me discuss the quantum evolution of this model. To calculate the effective Hamiltonian in proposal one,

$$H_s(p_I, q_{s'}^{\prime I}, Q^\alpha(\tau)) = p_I q_{s'}^{\prime J} h_J^I - p_V|(|V|\eta^{vv} + q_{s'}^{\prime I}\frac{dE_I^\alpha}{d\tau}\bar{V}_\alpha)|.$$

we need to calculate the following quantities.

$$
\begin{aligned}
h^I_J d\tau &= -E^\alpha_J dE^I_\alpha, \\
p_V &= \sqrt{-\eta^{vv}\sigma^{\alpha\beta}p^s_\alpha p^s_\beta - 2\eta^{vv}U(Q^\alpha(\tau+d\tau)+q'^I_{s'}E'^\alpha_I)}, \\
\sigma_{\alpha\beta} &= m_{\alpha\beta} - \eta^{vv}\bar{V}_\alpha\bar{V}_\beta,
\end{aligned}
$$

where $m_{\alpha\beta}$ is the full metric on the internal space and $m_{\alpha\beta}$ is the projected metric on the surfaces $S(V^\alpha(\tau), Q^\alpha(\tau))$. Let me assume $e^\alpha_0 V_\alpha!$ is finite. We have

$$
\begin{aligned}
Q^\alpha &= (A, \Phi). \\
V^\alpha &= (\dot{A}, \dot{\Phi}).
\end{aligned}
$$

$$
m^{\alpha\beta} = \begin{bmatrix} \frac{\sqrt{A}}{12\Delta V c_g} & 0 \\ 0 & \frac{-1}{\Delta V A^{3/2}} \end{bmatrix} \quad , \quad m_{\alpha\beta} = \begin{bmatrix} \frac{12\Delta V c_g}{\sqrt{A}} & 0 \\ 0 & -\Delta V A^{3/2} \end{bmatrix}.
$$

Let me assume $\dot{A} >> \dot{\Phi}$. Results of calculations of various relevant quantities are

$$
h_1 = -E^\alpha_a \frac{dE^a_\alpha}{d\tau} = -\frac{3\,A\left(\dot{A}\right)c\left(\dot{\phi}\right)\left(3\,A\left(\dot{A}\right)\left(\ddot{\Phi}\right) - 3\,A\left(\ddot{A}\right)\left(\dot{\Phi}\right) + 4\left(\dot{A}\right)^2\left(\dot{\Phi}\right)\right)}{\left(A^2\left(\dot{\Phi}\right)^2 - 12\left(\dot{A}\right)^2 c\right)\left(A^2\left(\dot{\Phi}\right)^2 - 3\left(\dot{A}\right)^2 c\right)},
$$

$$
h_2 = \frac{dE^\alpha_I}{d\tau}\bar{V}_\alpha = \frac{12\sqrt{A}\left(\dot{A}\right)c\left(A\left(\dot{A}\right)\left(\ddot{\Phi}\right) - A\left(\ddot{A}\right)\left(\dot{\Phi}\right) + 2\left(\dot{A}\right)^2\left(\dot{\Phi}\right)\right)v}{\left(A^2\left(\dot{\Phi}\right)^2 - 12\left(\dot{A}\right)^2 c\right)\sqrt{-\frac{\left(A^2\left(\dot{\Phi}\right)^2 - 12\left(\dot{A}\right)^2 c\right)v}{\sqrt{A}}}},
$$

$$
M^{-1} = -\frac{A^2\left(\dot{\Phi}\right)^2 - 12\left(\dot{A}\right)^2 c}{4\,A^{\frac{3}{2}}\left(A^2\left(\dot{\Phi}\right)^2 - 3\left(\dot{A}\right)^2 c\right)v},
$$

$$
|V| = \sqrt{\Delta V\left\{\frac{12c_g\dot{A}^2}{\sqrt{A}N^2} - A^{\frac{3}{2}}\dot{\Phi}^2\right\}}.
$$

Let p and q be the free conjugate momentum and configurational variables. Now the reduced Hamiltonian is

$$
H_s(p, q, Q^\alpha(\tau)) = pqh_1 - p_V|(|V| + qh_2)|
$$

where $p_V = \sqrt{M^{-1}p^2 + 2\eta^{vv}U(Q^\alpha(\tau+d\tau)+qE'^\alpha_I)}$.

3.2 Cosmology with Fluctuations

Let me add small perturbations η_{ab} to the spatial metric of the last model,

$$h_{ab} = (\frac{1}{3}A\delta^{ab} + \eta_{ab}),$$

where $\int \eta^{ab} dV \approx 0$, with integral done over volume of finite size much greater than Planck scale

Let the conjugate momentum be

$$\Pi^{ab} = \pi^{ab} + Pg^{ab},$$

where π^{ab} is the perturbation, and $\int \pi^{ab} dV = 0$ with integral done over the same size as the corresponding integral for the metric. Now the Hamiltonian constraint is

$$\frac{1}{6}A^2P^2 - \frac{1}{3}AP\eta^{ab}\pi_{ab} - P^2\eta^{ab}\eta_{ab} - \frac{1}{9}A^2\pi^{ab}\pi_{ab} = V,$$

where $V = h(R + h_m), h_m$ is the contribution from matter terms. Let me assume that the matter field is just the scalar field.

$$\begin{aligned} & \int (\pi^{ab} + P\delta^{ab})d(\frac{A}{3}\delta_{ab} + \eta_{ab}) + \pi d\phi \\ = & \int PdA + \pi^{ab}d\eta_{ab} + \pi d\phi. \end{aligned}$$

In the similar way we write the potential as,

$$V = V_0 + v,$$

where v is the perturbation, with $\int v dx^3 = 0$ as before.

We can rewrite the metric as follows:

$$\left(\frac{1}{6}A^2 - \eta^{ab}\eta_{ab}\right)P^2 - \frac{1}{3}AP\eta^{ab}\pi_{ab} - \frac{1}{9}A^2\pi^{ab}\pi_{ab} - V_0 - v = 0.$$

We can solve this equation for P, which is the dominant momentum component along which the internal time flows.

$$P = \pm\left(\frac{\sqrt{6V}}{A} + \frac{v\sqrt{6}}{2\sqrt{V}A} + \frac{(6)^{\frac{3}{2}}\eta^{ab}\eta_{ab}\sqrt{V}}{2A^3} + \frac{A\pi^{ab}\pi_{ab}\sqrt{6}}{18\sqrt{V}}\right) + \frac{\eta^{ab}\pi_{ab}}{A}.$$

The sign is decided by initial conditions. Now the effective Hamiltonian is

$$H = \pm\left(\frac{\sqrt{6V}}{A} + \frac{v\sqrt{6}}{2\sqrt{V}A} + \frac{(6)^{\frac{3}{2}}\eta^{ab}\eta_{ab}\sqrt{V}}{2A^3} + \frac{A\pi^{ab}\pi_{ab}\sqrt{6}}{18\sqrt{V}}\right) + \frac{\eta^{ab}\pi_{ab}}{A}.$$

We can split the η and π into orthogonal components as described in appendix E.

The TTf and Tr components of η_{ab} don't change with diffeomorphism. So they are dynamical quantum variables. But LL and the TL components change by small diffeomorphism. They can be set to zero by appropriate diffeomorphism. Let me assume below conditions as gauge choice.

$$\eta_{LL} = 0, \eta^{ab}_{TL} = 0.$$

Now we have the action as

$$\begin{aligned} & \int HdA + \pi^{ab} d\eta_{ab} + \pi d\phi \\ = & \int \pi^{ab}_{TTf} d\eta^{TTf}_{ab} + \pi^{ab}_{Tr} d\eta^{Tr}_{ab} + \pi d\phi + HdA. \end{aligned}$$

Using the diffeomorphism constraint in equation (41) (appendix F), we can solve for π_{LL} and π^{ab}_{TL} using the matter distribution, ignoring the second order transverse terms.

$$\pi_{LL} = \pi_{LL}(\phi, \eta^{T}_{ab}), \tag{28a}$$

$$\pi^{ab}_{TL} = \pi^{ab}_{TL}(\phi, \eta^{T}_{ab}). \tag{28b}$$

We can also include EM field. Like before for the EM field, using the gauge freedom, we can set the Longitudinal component A_L to be zero. The conjugate momentum E_L can be solved from initial conditions using $div(E_L) = \rho$, where ρ is the charge density. For this paper let me assume that there is no charge in the theory. Then we can assume that,

$$A_L = 0. \tag{29}$$

$$E_L = 0. \tag{30}$$

The state of the wavefunction is

$$|\psi >= \sum_{\{x_m\}} \int \psi(\eta^{TTf}_{ab}, \eta^{Tr}_{ab}, A^T, \phi) |\eta^{TTf}_{ab}, \eta^{Tr}_{ab}, A^T, \phi_m; \pi_{LL}, \pi^{ab}_{TL}, A^L = 0 > .$$

The variables after the semi-colon are non-dynamical solved using equations (29) and (28). Now the action is

$$\begin{aligned} & \int HdA + \pi^{ab} d\eta_{ab} + E_T dA^T + \pi_m d\phi_m \\ = & \int \pi^{ab}_{TTf} d\eta^{TTf}_{ab} + \pi^{ab}_{Tr} d\eta^{Tr}_{ab} + E_T dA^T + \pi_m d\phi_m + HdA. \end{aligned}$$

Now H is the schrodinger part and we need add appropiate L_m's in terms of η^{TTf}_{ab}, η^{Tr}_{ab}, A^T and ϕ_m (and possibly their conjugate momentas) to get the quantum diffusion equation (23). In the spirt of the third principle, constant value of A gives appropriate physical hypersurfaces along which the quantum state evolves. We need to further discretize the theory and apply fourth principle.

3.3 Newtonian Space

Consider that A and P are small or same order as the perturbations, everything else same as before. This represents the current universe, whose expansion rate has slowed down. Let me assume the matter is lumped in spherical shells. We can use the same simplification for longitudinals as was in the last subsection. Now consider the phase space integral for the system,

$$
\begin{aligned}
& \int (\pi^{ab})d(h_{ab}) + E_T dA^T + \pi_m d\phi_m \\
= & \int (\pi^{ab}_{TT} d\eta^{TT}_{aB} + E_T dA^T + \pi_m d\phi_m + 2\pi_T d\eta^T) \\
= & \int (\pi^{ab}_{TT} d\eta^{TT}_{aB} + E_T dA^T + \pi_m d\phi_m - 2\nabla^2 \eta^T_{ab} d\left(\nabla^{-2}\pi^{ab}_T\right)) \\
= & \int (\pi^{ab}_{TT} d\eta^{TT}_{aB} + E_T dA^T + \pi_m d\phi_m - 2\nabla^2 \eta^T_{ab} d\tau)
\end{aligned}
$$

where $\tau = \nabla^{-2}\pi^{ab}_T$. Now from the Hamiltonian constraint (upto a factor),

$$
\nabla^2 \eta^T_{ab} = \pi^{ab}\pi_{ab} - \frac{\pi^2}{2} + h_m + h_g \tag{32}
$$

where h_m is the matter and EM field Hamiltonian, h_g is the second order terms of η_{ab} from hR, and $\pi_{ab} = \pi^L_{ab} + \pi^{TT}_{ab} + \pi^{Tt}_{ab}$. π^L_{ab} is a non-dynamical field to be solved using diffeomorphism constraint as in the last section.

Using momentum as time variable is described in appendix (G) in the context of a single point system in WKB approximation. The same can generalized to multipoint system described by the above action, if we consider the gravitational degrees of freedom is semiclassical in η^T, with existence of WKB approximation. Now the wavefunction is (using appendix (G)),

$$
\psi(\eta^{TT}_{aB}, A^T, \phi_m, \eta^{Tt})) = \exp\left[i(\pi^{ab}_{TT}(\eta^{TT}_{aB})\eta^{TT}_{aB} + E_T(A^T)A^T + \pi_m(\phi_m)\phi_m)\right]\tilde{\psi}(\eta^{TT}_{aB}, A^T, \phi_m, \pi_{Tt}(\eta^{Tt})),
$$

where we can consider the $\tilde{\psi}$ as function of π_{Tt}. The momentum functions the exponent are slow functions of the variables. Now from equations (31), the time variable is the field $\tau = \nabla^{-2}\pi^{ab}_{Tt}$. The wavefunctions can be rewritten as

$$
\psi(\eta^{TT}_{aB}, A^T, \phi_m, \tau)) = \exp\left[i(\pi^{ab}_{TT}(\eta^{TT}_{aB})\eta^{TT}_{aB} + E_T(A^T)A^T + \pi_m(\phi_m)\phi_m)\right]\tilde{\psi}(\eta^{TT}_{aB}, A^T, \phi_m, \tau),
$$

where τ is a field and it has a different value at different point. The $\tilde{\psi}$ has the same physical information as ψ. Its evolution is described by the Hamiltonian density (upto a factor of -2),

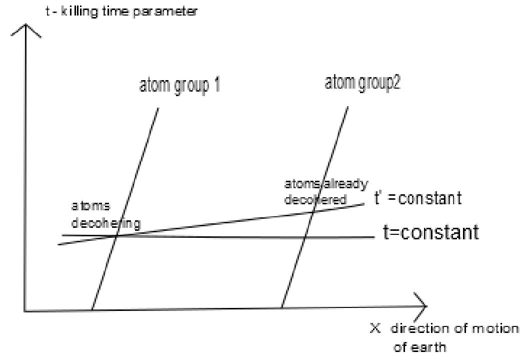
$$
\begin{aligned}
H_s &= \nabla^2 \eta^T_{ab} \\
&= \pi^{ab}\pi_{ab} - \frac{\pi^2}{2} + h_m + h_g
\end{aligned}
$$

The discrete version of this in combination with principles second, third and fourth principle can give our evolving model. The quantum diffusion equation (23) has H_s from the above equation. Each point has an unique n_x and L_m's derived from η^{TT}_{aB}, A^T and ϕ_m (and possibly their conjugate momentas). Appropriate Υ need to be determined experimentally, to evolve the system.

4 Experimental Test

For the proposals to be experimentally tested we need to apply them to specific experimental situation. For this to happen the applications of the proposals needs to be further developed. Nevertheless, let me discuss some simple ideas how to test the ideas.

Consider the application of the first proposal to the expanding universe during big bang expansion as discussed in section 3.1: the time parameter is a combination of the scale parameter and the scalar field. This must have an impact on perturbation in the cosmic matter and radiation fields. We need to find the



signature of this in the CMB and galactic matter perturbations. Also using CMB perturbations one may try to test the third principle as following: One need to look for correlations between CMB perturbations at different wavelength. Each wavelength was freed from matter at different cosmological scale parameter value. If there are more correlations within the same wavelength than any other wavelengths, the third principle may be confirmed. The wavelengths need be to as small as possible so that the perturbations has small components directly related to quantum reduction process, and not due to classical collision processes between particles. This may be difficult analysis to perform.

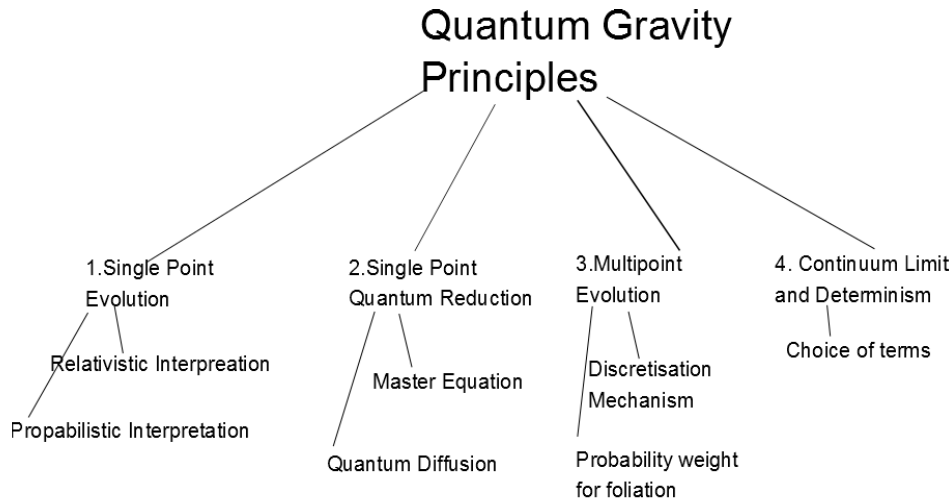
The reduction process of the fields need to be observed to study the second proposal. But it depends on the time scale of the process. For example, if the reduction time is in the order of Planck time (5.39056×10^{-44} s), it is difficult to be observed in the lab. But if it is not, if there is hope of observing the reduction process. One possibility is the study of the magnetic field of the spin of atoms. Consider the magnetic field due to trapped ions that are popularly utilized in quantum computers. If we have sufficient number of ions sufficiently close two each other, their magnetic fields due to their spin may add up to measurable level. Let me assume this is experimentally realizable. Assume the system is put in the macroscopic superposition of their spins (example: all up + all down). Assume they slowly decohere as the magnetic field is measured spontaneously as proposed in the second principle. Then if the time scale is within our experimental capabilities we can observe the process. This will give various we ways to study and test one or more of the more principles.

Let me discuss a test of the third proposal. Consider the following apparatus which has 1) two group of atoms distant from each other, 2) instruments for measuring the magnetic fields of these two groups. According the third proposal time flows along the temporal killing field of Earth, i.e. the frame in which earth is at rest is the unique reference frame along which continuos quantum reduction occurs. Assume we make the apparatus move along the direction in which the earth moves. Let me assume that earth move along the x axis and t is the time in its rest frame. The apparatus move with velocity β along the direction the earth ($c = 1$). Let the primed coordinates denote the apparatus coordinates.

$$\begin{aligned} x &= \gamma(x' + \beta t'), \\ t &= \gamma(t' + \beta x'). \end{aligned}$$

According to proposal three the measuring process occurs along $t =$ constant hypersurfaces. Thus in the rest frame of the apparatus, two measuring instruments will see asymmetrical measurement output, with one group decohering before the other one. Assume we switch on the instruments for measuring the magnetic field simultaneously in the rest frame of the apparatus. One group of atoms will decohere spontaneously before other one, ultimately collapsing the other one. As a consequence, measuring the magnetic field of first group of atom will show slow continuos evolution starting from the time at the measurement apparatus is switched on. The measurement in the second measuring apparatus will show a magnetic field which has already decohered due to entanglement of the second group with the first group. If this discrepancy between the measurement of the two magnetic field can be seen, this will demonstrate proposal 3. By modifying this experiment many aspects of the four proposals can be studied.

The fourth principle is essential for restoring continuum and deterministic limit. Applications of the ideas to experimental context need to be developed, to test the various parts of the principle.



5 Discussion

The set of four basic proposals discussed in section 2 only lays down a conceptual framework instead of a full concrete proposal. The first proposal of dynamics of picks a self evolution direction in the configuration space of the quantum system at each point;, the second proposal of dynamics introduces spontaneous local quantum reduction for the quantum system at each point, the third proposal of dynamics deals with global evolution by determining the relative rate of time evolution for different points on space by global quantum reduction, and the fourth proposal enforces continuum limit. These proposals embody conceptual foundations but leaves open the concrete implementation to be determined by further theoretical and experimental investigation.

Let me list the various possibilities:

1. This framework is highly abstract and it can be applied to the usual quantum field theory with general relativity using Dirac's method of quantization, or any unified field theory such as the string theory, Kaluza-Klein theory, etc. One needs to find out the best theory that works with the framework that could reproduce physics.

2. One needs to find ways to impose the diffeomorphism and gauge constraints on the kinematical Hilbert space.

3. The framework is based on discrete model. There are many possible ways to discretize quantum general relativistic physics. The right way to discretize, so that we could extract low energy continuum physics needs to be found.

4. The first proposal depends on the configuration space. Now there are various possibilities for choosing the configuration variables, for example using the Kaluza-Klein theory [20], string theory or loop quantum gravity, or some other theory where these fields are unified as discussed in section 3.3. The proper choice needs to found out.

5. In the second proposal of dynamics we have the $\hat{L}_{m,x}$'s to be determined. The natural choices are gauge invariant and diffeomorphism invariant quantities, such as scalar curvature, square of the extrinsic curvature, that are related to the gravitational field, etc.

6. In third proposal of dynamics various possible candidate for the Υ function has been suggested. If possible right one need to be found.

7. The σ function in the fourth proposal that specifies continuum functional needs to be determined.

8. Now σ, Υ functionals and $\hat{L}_m$ require three new physical constants to determine their scale. The search of σ, Υ, $\hat{L}_m$ and the scales may point out to a new fundamental theory.

So the proposals presented in this article have a huge choice. The various possible theories relating to the different implementations of the proposals have to be studied theoretically and experimentally to come up with the precise details to achieve successful model for Planck scale description of nature, or some or all of them be falsified in the verification process[2].

6 Conclusion

In this article, I have outlined a conceptual framework and have briefly discussed how to apply this to study physics. Because of highly stochastic nature of the theory we need to use computer simulation, and statistical analysis to get any useful physics out of the theory, and verify whether it reproduces quantum mechanics and general relativity consistent with experiments in the low energy limit. Application of the framework to simple models is straight forward. But the complication is, even for simulating simple models, extensive computing power is required. Currently the application of the conceptual framework to some simple models is being studied by the author. The results and algorithm will be published in the follow-up reports. The computer code for simulation will be made available publicly. The framework discussed is quite general and there are wide variety of variations and sceneries. To come up with a specific model that best explains the physics of the entire universe requires exploring as many interesting models as possible.

7 Acknowledgements

This work was motivated by a weekly discussion group (2001) on conceptual problems in quantum mechanics held at University of Pittsburgh, Physics Department, lead by Allen Janis.

Appendices

A Derivation Self-Evolution Hamiltonian

Here let me calculate the Hamiltonian for the self-evolution described in proposal 1, from surface $S = S(\tau)$ to $S' = S(\tau + d\tau)$. In terms of the propagator, the evolution of the wavefunction is:

$$\psi'(q_{s'}^{'I}, \tau) = \int_{p_\alpha \bar{v}^\alpha < 0} G(q_{s'}'^{\alpha}, q_s^\alpha)\psi(q_s^I, \tau) dq_s^I.$$

Let me split the global coordinates q^α and q'^α corresponding to the surface coordinates q_s^α and $q_{s'}'^\alpha$ into two parts along the normal and tangential directions.

$$\begin{aligned} q^\alpha &= Q^\alpha(\tau) + q_s^I \bar{E}_I^\alpha, \\ q'^\alpha &= Q^\alpha(\tau + d\tau) + q_{s'}'^I \bar{E}_I'^\alpha, \\ &= Q^\alpha(\tau) + \bar{v}^\alpha \Delta t_s + q_{s'}'^I \bar{E}_I^\alpha. \end{aligned}$$

The components are calculated as follows:

$$\begin{aligned} dQ^\alpha(\tau) &= Q^\alpha(\tau + d\tau) - Q^\alpha(\tau), \\ (dQ^\alpha(\tau) + q_s'^I \bar{E}_I'^\alpha)\bar{v}_\alpha &= \Delta t_s, \\ (dQ^\alpha(\tau) + q_{s'}'^I \bar{E}_I'^\alpha)\bar{E}_\alpha^I &= q_s'^I. \end{aligned}$$

Now let me calculate Δq^α:

[2]Revion Info: As I have discussed before, version 2.0 has been published. It has some improvements, slightly different concepts (particularly proposal 1), and more applications. Further revisions are in progress.

$$\begin{aligned}\Delta q^\alpha &= \bar{v}^\alpha \Delta t_s + \Delta q_s^I E_I^\alpha, \\ \Delta q_s^I &= q_s'^I - q_s^I.\end{aligned}$$

Δq_s^I can be further analyzed as

$$\Delta q_s^I = (dQ^\alpha(\tau)\bar{E}_\alpha^I + q_{s'}'^J \bar{E}_J'^\alpha \bar{E}_\alpha^I) - q_s^I.$$

Let me define: $\bar{E}_J'^\alpha \bar{E}_\alpha^I = H_J^I = \delta_J^I + h_J^I d\tau$. Then we have

$$\begin{aligned}\Delta q_s^I &= (dQ^\alpha(\tau)\bar{E}_\alpha^I + q_{s'}'^J H_J^I) - q_s^I \\ &= (dQ^\alpha(\tau)\bar{E}_\alpha^I + q_{s'}'^J h_J^I) + q_{s'}'^J - q_s^I.\end{aligned}$$

Defining $\delta q_s^J = q_{s'}'^J - q_s^I$ and using $dQ^\alpha(\tau)\bar{E}_\alpha^I = 0$,

$$\Delta q_s^I = (q_{s'}'^J h_J^I) d\tau + \delta q_s^I,$$

$$\begin{aligned}\Delta t_s &= (dQ^\alpha(\tau) + q_s'^I \bar{E}_I'^\alpha)\bar{v}_\alpha \\ &= (\frac{dQ^\alpha(\tau)}{d\tau}\bar{v}_\alpha + q_s'^I \frac{d\bar{E}_I^\alpha}{d\tau}\bar{v}_\alpha) d\tau \\ &= (|v| + q_s'^I \frac{d\bar{E}_I^\alpha}{d\tau}\bar{v}_\alpha)\Delta\tau.\end{aligned}$$

The propagator from $S = S(\tau)$ to $S' = S(\tau + d\tau)$ in terms of the surface coordinates is

$$\begin{aligned}& G(q_{s'}'^\alpha, q_s^\alpha) \\ =\ & G(q'^\alpha, q^\alpha) \\ =\ & \int_{p_\alpha \bar{v}^\alpha < 0} \exp(i[p_I \Delta q_s^I + p_\alpha \bar{v}^\alpha \Delta t])\delta(H) dp^D.\end{aligned}$$

Now H can be split using into parallel and normal components to the Surface S, assuming the signature is + along normal direction.

$$\begin{aligned}H &= \frac{1}{2}p^\alpha p_\alpha + V(q'^\alpha) \\ &= \frac{1}{2}(p_v)^2 + \frac{1}{2}p^I p_I + V(q'^\alpha),\end{aligned}$$

where $p_v = p_\alpha \bar{v}^\alpha$. Therefore

$$\begin{aligned}H &= 0 => \\ p_v &= \pm\sqrt{-p^I p_I - 2V(q'^\alpha)} \\ &= \pm\sqrt{-p^I p_I - 2V(Q^\alpha(\tau + d\tau) + q_s'^I \bar{E}_I'^\alpha)}.\end{aligned}$$

Now the propagator is

$$G(q'^{\alpha}, q^{\alpha}) = \int \exp(i[p_I \Delta q_s^I - p_v \Delta t^s]) dp^{D-1},$$

$$\begin{aligned} & p_I \Delta q_s^I - p_v \Delta t^s \\ = \quad & p_I \delta q_s^I + p_I q'^J_{s'} h^I_J \Delta\tau - p_v(|v| + q'^I_s \frac{d\bar{E}^{\alpha}_I}{d\tau} \bar{v}_{\alpha})\Delta\tau. \end{aligned}$$

The effective Hamiltonian $H_s(p_I, q'^I_{s'}, Q^{\alpha}(\tau))$ in terms of the surface coordinates and $\Delta\tau$ as time parameter is

$$\begin{aligned} & p_I \Delta q_s^I - p_v \Delta t^s \\ = \quad & p_I \delta q_s^I + H_s(p_I, q'^I_{s'}, Q^{\alpha}(\tau))\Delta\tau. \end{aligned}$$

From the previous calculation,

$$H_s(p_I, q_s^I, Q^{\alpha}(\tau)) = p_I q_s^J h^I_J - p_v(|v| + q_s^I \frac{d\bar{E}^{\alpha}_I}{d\tau} \bar{v}_{\alpha}),$$

where

$$|v| = |\frac{dQ^{\alpha}}{d\tau}|,$$

and

$$\begin{aligned} h^I_J d\tau &= H^I_J - \delta^I_J \\ &= \bar{E}'^{\alpha}_J \bar{E}^I_{\alpha} - \delta^I_J \\ &= \bar{E}^I_{\alpha} d\bar{E}^{\alpha}_J, \end{aligned}$$

implying

$$h^I_J = \bar{E}^I_{\alpha} \frac{d\bar{E}^{\alpha}_J}{d\tau}.$$

Using

$$p_v = \sqrt{-p^I p_I - 2U(Q^{\alpha}(\tau) + q_s^I \bar{E}^{\alpha}_I)},$$

we can calculate the effective classical equation of motion for the Hamiltonian H_s

$$\dot{q}_s^I = q_s^J h^I_J - \frac{\partial p_v}{\partial p_I}(|v| + q_s^K \frac{d\bar{E}^{\alpha}_K}{d\tau} \bar{v}_{\alpha}), \tag{33}$$

$$\dot{p}_J = -p_I h^I_J + \frac{\partial p_v}{\partial q_s^J}(|v| + q_s^K \frac{d\bar{E}^{\alpha}_K}{d\tau} \bar{v}_{\alpha}) + p_v(\frac{d\bar{E}^{\alpha}_J}{d\tau} \bar{v}_{\alpha}). \tag{34}$$

B An Application of Self-Evolution Equations

Let $U = \frac{q^\alpha q_\alpha}{2} - m^2$ and $m^{\alpha\beta} = \delta^{\alpha\beta}$

Let me set $Q^I(\tau) = r(\cos\tau, \sin\tau)$. Then $\bar{\nu}(\tau) = (-\sin\tau, \cos\tau)$ and $E(\tau) = (\cos\tau, \sin\tau)$.

Also have $h_1^1 = \bar{E}^I_\alpha \frac{d\bar{E}^\alpha_J}{d\tau} = 0$, and $\frac{d\bar{E}^\alpha_1}{d\tau}\bar{v}_\alpha = 1$.

The the evolution equations are

$$\begin{aligned}
\dot{q} &= -\frac{\partial p_v}{\partial p_I}(r+q), \\
\dot{p} &= +\frac{\partial p_v}{\partial q_s^J}(r+q) + p_v.
\end{aligned}$$

$$\begin{aligned}
p_v &= \sqrt{-p^2 + 2m^2 - (r+q)^2}, \\
\frac{\partial p_v}{\partial p_I} &= -\frac{p}{p_v}, \\
\frac{\partial p_v}{\partial q_s^J} &= \frac{-(r+q)}{p_v}.
\end{aligned}$$

Let initially $q = 0$ and $p = 0$, then,

$$\dot{q} = \frac{p}{p_v}(r+q) = 0,$$

$$\begin{aligned}
\dot{p} &= \frac{-(r+q)}{p_v}(r+q) + p_v \\
&= \frac{2m^2 - 2r^2}{p_v}.
\end{aligned}$$

For $\dot{p} = 0$ we need to have $m^2 = r^2$. This is just circular motion around the origin.

C A Derivation of Quantum Diffusion Equations

Let me derive a general stochastic evolution equation motivated by equation (14). Our analysis is based on [8], but different in details. A general stochastic evolution equation is

$$|d\psi >= \alpha|\psi > dt + \beta_m|\psi > z^m\sqrt{dt},$$

where α and β are operators on $|\psi>$ .A summation over repeated indices is assumed. Let M denote the averaging over all z^m, and let me assume the following definitions:

$$\begin{aligned}
M(z^m\bar{z}^n) &= 2\delta^{mn}, \\
M(z^m) &= 0, \\
Hr(\alpha) &= (\alpha + \alpha^+)/2, \\
Ar(\alpha) &= (\alpha - \alpha^+)/2, \\
E(\alpha) &= <\psi|\alpha|\psi> .
\end{aligned}$$

Then we have,

$$
\begin{aligned}
d(&< \psi|\psi >) =< \psi|d\psi > + < d\psi|\psi > + < d\psi|d\psi > \\
&= 2Hr < \alpha > dt + < \beta_m^+\beta_n > z^m\bar{z}^n dt + 2Hr < \beta_m > z^m\sqrt{dt} + 2Hr < \alpha^+\beta_m > z^m dt\sqrt{dt}.
\end{aligned}
$$

Let me solve $d(< \psi|\psi >) = 0$, assuming that dt and z^m are free variables, and, α and β are independent of dt and z^m. Then we have the following conditions.

$$
\begin{aligned}
Hr &< \alpha >= 0, \\
&< \beta_m^+\beta_n >= 0, \\
&< \beta_m >= 0, \\
&< \alpha^+\beta_m >= 0.
\end{aligned}
$$

These constraints are too strong that, these will eliminate the quantum diffusion equation (14) itself zzz check.

Let me take a different route. First let me solve $Md(< \psi|\psi >) = 0$, assuming α and β are independent of dt and z^m.

$$
Md(< \psi|\psi >) = 2Hr < \alpha > dt + 2 < \beta_m^+\beta_m > dt.
$$

Then $Md(< \psi|\psi >) = 0$ implies

$$
Hr < \alpha > + < \beta_m^+\beta_m >= 0.
$$

The general solution for this equation is

$$
\alpha = iH + \gamma - < \gamma > - \beta_m^+\beta_m. \tag{35}
$$

assuming α and β are independent of dt and z^m. Here H is a Hermitian operator, and γ is an arbitrary operator.

Now let me solve $d(< \psi|\psi >) = 0$, assuming, dt as the independent free parameters, α, β are independent of dt, and α (or β) may depend on second order terms of z^m. The we have following conditions:

$$
\begin{aligned}
Hr &< \alpha > + \frac{1}{2} < \beta_m^+\beta_n > z^m\bar{z}^{\ n} = 0, \\
&< \beta_m >= 0, \\
&< \alpha^+\beta_m >= 0.
\end{aligned}
$$

A solution for the first equation can be obtained by adding a real number to equation (35)

$$
\alpha = iH + \gamma - < \gamma > - \beta_m^+\beta_m - \frac{1}{2} < \beta_m^+\beta_n > (z^m\bar{z}^n - 2\delta^{mn}), \tag{36}
$$

The general solution for the second equation is

$$
\beta_m = L_m - < L_m > .
$$

The third equation can ignored in $d(< \psi|\psi >) = 0$, as $dt\sqrt{dt}$ is too small. But if we don't want to neglect the $2Hr < \alpha^+\beta_m > z^m dt\sqrt{dt}$ term in $d(< \psi|\psi >) = 0$ then we can modify α to

$$\alpha = iH + \gamma - <\gamma> - \beta_m^+\beta_m - \frac{1}{2} <\beta_m^+\beta_n> (z^m \bar{z}^{\ n} - 2\delta^{mn}) - <2\alpha^+\beta_m> z^m\sqrt{dt}, \tag{37}$$

which makes α dependent on $\sqrt{dt}$ also. I will assume that $z^m\sqrt{dt}$ can be neglected hereafter, unless specified otherwise.

To summarize we have the final form the dynamics equations

$$|d\psi_\tau >= \alpha|\psi > dt + \beta_m|\psi > z^m\sqrt{dt}, \tag{38}$$

where

$$\begin{aligned} \alpha &= iH - \gamma - \beta_m^+\beta_m + c, \\ \beta_m &= L_m - <L_m>, \\ c &= -\frac{1}{2} <\beta_m^+\beta_n> (z^m \bar{z}^{\ n} - 2\delta^{mn}) + <\gamma> . \end{aligned}$$

The c is suggested is $c-$number and is a random function of z. We can derive the evolution equation for $\rho = M(|\hat{\psi} >< \hat{\psi}|)$

$$d(\rho) = M|d\psi >< \psi| + M|\psi >< d\psi| + M|d\psi >< d\psi|,$$

$$M|d\psi >< \psi| = \rho M(\alpha) dt,$$

$$M|d\psi >< d\psi| = \beta_m\rho\beta_m^+ dt + 2He\{\beta_m\rho M(\alpha^+ z^m)\} dt\sqrt{dt}.$$

From equation (37) (without neglecting $\sqrt{dt}$ term),

$$\begin{aligned} M(\alpha) &= iH + \gamma - <\gamma> - \beta_m^+\beta_m, \\ M(\alpha^+ z^m) &= 0. \end{aligned}$$

So the evolution equation of ρ is

$$\begin{aligned} \frac{d\rho}{dt} &= \rho(\tilde{\alpha}^+) + (\tilde{\alpha})\rho + \beta_m\rho\beta_m^+, \\ \tilde{\alpha} &= iH + \gamma - <\gamma> - \beta_m^+\beta_m. \end{aligned}$$

It is easy to see that $\sqrt{dt}$ term does not show up.

To get the quantum diffusion equation and the Lindblad equation we need to set,

$$\begin{aligned} \beta_m &= L_m - <L_m>, \\ \gamma &= <L_m> L_m^+ - L_m <L_m^+> . \end{aligned}$$

D Bidirectional Time Evolution

The time evolution defined in proposals 1 and 2 are actually unidirectional in time. We can generalize this to both the opposite directions along v^α. Consider the momentum operator $\hat{E} = \hat{p}_\alpha \bar{v}^\alpha$ (assuming $\bar{v}^\alpha$ is a unit vector in the metric discussed). Let $|\psi> = |\psi_+> + |\psi_->$, where $|\psi_+>$ and $|\psi_+>$ are made of the positive and negative eigenvalued eigenvectors of $\hat{E}$ correspondingly. Then we have a more generalized dynamic equation as follows:

$$\begin{aligned} |d\psi_\tau &> = -i\hat{H}_{s,v}|\psi_{+,\tau}>|v|n_x(\tau)\Delta\tau + i\hat{H}_{s,v}|\psi_{\_,\tau}>|\dot{q}^\alpha_{x,s}|n_x(\tau)\Delta\tau \\ -\sum_x(\hat{\sigma}_x - &< \hat{\sigma}_x>)|\psi_\tau>|\dot{q}^\alpha_{x,s}|n_x(\tau)d\tau \qquad (39) \\ +\sum_k(2 &< \hat{L}_k>\hat{L}_k - \hat{L}^+_k\hat{L}_k - <\hat{L}^+_k><\hat{L}_k>)|\psi_\tau>|\dot{q}^\alpha_{x,s}|n_x(\tau)\Delta\tau, \\ +\sum_k \lambda_k(\hat{L}_k - &< \hat{L}_k>)|\psi_\tau> dz_k\sqrt{|v|n_x(\tau)\Delta\tau} - \frac{\hat{\sigma}_x}{2}|\psi_\tau>|\dot{q}^\alpha_{x,s}|n_x(\tau)d\tau. \qquad (40) \end{aligned}$$

The two terms involving $\mathcal{H}_v$ evolves the state in the positive and negative direction along v_α. But because of the third summation term in the first equation one of $|\psi_+>$ and $|\psi_+>$ will be fully attenuated eventually. So we only have a unidirectional motion eventually.

E Transverse-Longitudinal split of metric

Consider that a tensor field Q_{ab}. Let me fourier transform it to the momentum space k^a. Assume Q_{ab} does not have $k = 0$ fourier term. Let the R^a_b be the transverse projector and P^a_b is the longitudinal.

$$\begin{aligned} R^a_b &= \delta^a_b - \frac{k^a k^b}{k^2} \\ P^a_b &= \frac{k^a k^b}{k^2} \\ R^a_a &= 2 \\ P^a_a &= 1 \end{aligned}$$

Let tilde on indices indicates transverse projected component, bar on index indicates longitudinal projected, for example

$$v^a = R^{\bar{a}}_b v^b + P^{\tilde{a}}_b v^b = v^{\tilde{a}} + v^{\bar{a}}$$

We can split Q into various components:

$$Q_{ab} = Q_{\tilde{a}\tilde{b}} + Q_{\tilde{a}\bar{b}} + Q_{\bar{a}\tilde{b}} + Q_{\bar{a}\bar{b}},$$

$$Q_{aa} = Q_{\tilde{a}\tilde{a}} + Q_{\bar{a}\bar{a}}.$$

The transverse component is given by

$$
\begin{aligned}
Q^T_{ab} &= Q_{\tilde{a}\tilde{b}} = Q_{cd}(\delta^c_a - \frac{k^c k_a}{k^2})(\delta^d_b - \frac{k^d k_b}{k^2}) \\
&= Q_{ab} - \frac{Q_{k(a}k_{b)}}{k^2} + Q_{kk}\frac{k_a k_b}{k^4}
\end{aligned}
$$

Also

$$
Q^T_{aa} = Q_{ab}R^{ab} = Q_{aa} - Q_{kk}\frac{1}{k^2},
$$

Let me calculate calculate the transverse trace-free part indicated by TT. For we need to calculate X in $Q^{TT}_{ab} = Q^T_{ab} + XR_{ab}$, such that $Q^{TT}_{aa} = 0$. Then we have $X = -\frac{Q^T_{aa}}{2}$.
The transverse trace part is

$$
\begin{aligned}
Q^{Tt}_{ab} &= R_{ab}\frac{Q^T_{aa}}{2} \\
&= \frac{1}{2}\left(\delta_{ab} - \frac{k^a k^b}{k^2}\right)\left(Q_{cc} - Q_{kk}\frac{1}{k^2}\right)
\end{aligned}
$$

The transverse trace free part is

$$
\begin{aligned}
Q^{TT}_{ab} &= Q^T_{ab} - R_{ab}\frac{Q^T_{aa}}{2} \\
&= Q_{ab} - \frac{Q_{k(a}k_{b)}}{k^2} - \frac{1}{2}\left(\delta_{ab} - \frac{k^a k^b}{k^2}\right)Q_{cc} + \frac{1}{2}\delta_{ab}\frac{Q_{kk}}{k^2} + \frac{1}{2}Q_{kk}\frac{k_a k_b}{k^4}
\end{aligned}
$$

The various combinations of longitudinal components are

$$
\begin{aligned}
Q^L_{ab} &= \left(Q_{\tilde{a}\bar{b}} + Q_{\bar{a}\tilde{b}} + Q_{\bar{a}\bar{b}}\right) \\
Q^{TL}_{ab} &= Q_{\tilde{a}\bar{b}} + Q_{\bar{a}\tilde{b}} \\
Q^{LL}_{ab} &= Q_{\bar{a}\bar{b}}
\end{aligned}
$$

The longitudinal component is

$$
\begin{aligned}
Q^L_{ab} &= Q_{ab} - Q^T_{ab} \\
&= Q_{ab} - \left(Q_{ab} - \frac{Q_{k(a}k_{b)}}{k^2} + Q_{kk}\frac{k_a k_b}{k^4}\right) \\
&= \frac{Q_{k(a}k_{b)}}{k^2} - Q_{kk}\frac{k_a k_b}{k^4}
\end{aligned}
$$

To summarize, we have

$$
Q_{ab} = Q^L_{ab} + Q^{TT}_{ab} + Q^{Tt}_{ab},
$$

and in the momentum space,

$$
\begin{aligned}
Q^L_{ab} &= \frac{Q_{k(a}k_{b)}}{k^2} - Q_{kk}\frac{k_a k_b}{k^4}, \\
Q^{TT}_{ab} &= Q_{ab} - \frac{Q_{k(a}k_{b)}}{k^2} - \frac{1}{2}\left(\delta_{ab} - \frac{k^a k^b}{k^2}\right) Q_{cc} + \frac{1}{2}\delta_{ab}\frac{Q_{kk}}{k^2} + \frac{1}{2}Q_{kk}\frac{k_a k_b}{k^4}, \\
Q^{Tt}_{ab} &= \frac{1}{2}\left(\delta_{ab} - \frac{k^a k^b}{k^2}\right)\left(Q_{cc} - Q_{kk}\frac{1}{k^2}\right).
\end{aligned}
$$

and in the position space,

$$
\begin{aligned}
Q^L_{ab} &= \frac{1}{\nabla}\partial^c\partial_{(a}Q_{b)c} - \frac{\partial_a\partial_b}{\nabla^2}\partial^c\partial^d Q_{cd}, \\
Q^{TT}_{ab} &= Q_{ab} - \frac{1}{\nabla}\partial^c\partial_{(a}Q_{b)c} - \frac{1}{2}\left(\delta_{ab} - \frac{\partial^a\partial^b}{\nabla}\right) Q_{cc} + \frac{1}{2}\delta_{ab}\frac{\partial^c\partial^d}{\nabla}Q_{cd} + \frac{1}{2}\frac{\partial_a\partial_b}{\nabla^2}\partial^c\partial^d Q_{cd}, \\
Q^{Tt}_{ab} &= \frac{1}{2}\left(\delta_{ab} - \frac{\partial^a\partial^b}{\nabla}\right)\left(Q_{cc} - \frac{\partial^c\partial^d}{\nabla}Q_{cd}\right).
\end{aligned}
$$

F Linearised Constraints

In this section, let me summarize the linearized version of kinetic and ponential parts of gravitational Hamiltonian constraint. Let me rewrite the gravitational canonical conjugate momentum π^{ab} and the metric h_{ab} as

$$
\begin{aligned}
\pi^{ab}- &> \epsilon\Pi^{ab} + P\delta^{ab}, \\
h^{ab}- &> \epsilon\eta^{ab} + \delta^{ab} \text{ zzz}
\end{aligned}
$$

where ϵ is a small value.

$$
\begin{aligned}
K.E &\approx -\frac{1}{2}\,\epsilon^2\Pi^a{}_a\Pi^b{}_b - \epsilon P\Pi_a{}^a - \frac{3}{2}\,P^2 + \epsilon^2\Pi^{ab}\Pi_{ab} - 3\,\epsilon^2 P\Pi_{ab}\eta^{ab} \\
&\quad -\epsilon^2 P\Pi^b{}_b\eta_a{}^a - \epsilon P^2\eta_a{}^a + 4\,\epsilon^2 P\Pi_b{}^a\eta_a{}^b - \frac{1}{2}\,\epsilon^2 P^2\eta_a{}^a\eta_b{}^b + \epsilon^2 P^2\eta_{ab}\eta^{ab}.
\end{aligned}
$$

If $P = 0$,

$$
K.E \approx -\frac{1}{2}\,\epsilon^2\Pi^a{}_a\Pi^b{}_b + \epsilon^2\Pi^{ab}\Pi_{ab}.
$$

Also have

$$
\begin{aligned}
R\sqrt{h} &\approx -\epsilon\partial^a{}_a\eta^b{}_b + \epsilon\partial^{ab}\eta_{a\ b} - \frac{1}{2}\,\epsilon\epsilon\eta^a{}_a\partial^b{}_b\eta^c{}_c + \frac{1}{2}\,\epsilon\epsilon\eta^a{}_a\partial^{bc}\eta_{b\ c} + \epsilon\epsilon\eta^{ab}\partial_{ab}\eta^c{}_c \\
&\quad -2\,\epsilon\epsilon\eta^{ab}\partial_a{}^c\eta_{b\ c} + \epsilon\epsilon\eta^{ab}\partial^c{}_c\eta_{a\ b} + \epsilon\epsilon\partial^a\eta_a{}^b\,\partial_b\eta^c{}_c - \epsilon\epsilon\partial^a\eta_a{}^b\,\partial^c\eta_{b\ c} - \frac{1}{4}\,\epsilon\epsilon\partial^a\eta^b{}_b\,\partial_a\eta^c{}_c \\
&\quad +\frac{3}{4}\,\epsilon\epsilon\partial^a\eta^{b\ c}\,\partial_a\eta_{b\ c} - \frac{1}{2}\,\epsilon\epsilon\partial^a\eta^{b\ c}\,\partial_b\eta_{a\ c}.
\end{aligned}
$$

$$
\begin{aligned}
Rh \;\approx\; & -\epsilon\partial^a{}_a\eta^d{}_d + \epsilon\partial^{ad}\eta_{a\ d} - 2\,\epsilon^2\eta^{ad}\partial_a{}^c\eta_{d\ c} + \epsilon^2\eta^{ad}\partial_{ad}\eta^c{}_c + \epsilon^2\eta^{ad}\partial^c{}_c\eta_{a\ d} \\
& -\epsilon^2\eta^b{}_b\partial^a{}_a\eta^d{}_d + \epsilon^2\eta^b{}_b\partial^{ad}\eta_{a\ d} - \epsilon^2\partial^a\eta_a{}^d\,\partial^c\eta_{d\ c} + \epsilon^2\partial^a\eta_a{}^d\,\partial_d\eta^c{}_c \\
& -\frac{1}{4}\,\epsilon^2\partial_a\eta^c{}_c\,\partial^a\eta^d{}_d + \frac{3}{4}\,\epsilon^2\partial^a\eta^{d\ c}\,\partial_a\eta_{d\ c} - \frac{1}{2}\,\epsilon^2\partial^a\eta^{d\ c}\,\partial_d\eta_{a\ c}\,;
\end{aligned}
$$

The linearized diffeomorphism constraint D^b is

$$
\begin{aligned}
\partial_a\Pi_L^{ab} \;\approx\; & \frac{1}{2}\Pi\partial_a\phi + \frac{1}{2}\,P\epsilon\partial^b\eta_c{}^c - P\epsilon\partial_c\eta^{b\ c} - \epsilon^2\Pi^{ac}\partial_a\eta^b{}_c + \frac{1}{2}\,\epsilon^2\Pi^{ac}\partial^b\eta_{a\ c} - \frac{1}{2}\,P\epsilon^2\eta^a{}_a\partial^b\eta^d{}_d \\
& +\frac{1}{2}\,P\epsilon^2\eta^{ac}\partial^b\eta_{a\ c} - \frac{1}{2}\,P\epsilon^2\eta^{ba}\partial_a\eta_d{}^d + P\epsilon^2\eta^{ba}\partial_d\eta_a{}^d\,.
\end{aligned}
\tag{41}
$$

G WKB Approximation and momentum time

Consider a single point system with variables x,ϕ and their conjugate momenta p,π. Assume the wavefunction $\Psi(\phi,x)$ of the system has a WKB approximation in terms of ϕ :

$$\Psi(\phi,x) = e^{i\pi(\phi)\phi}\tilde{\psi}(\phi,x)$$

where $\pi(\phi)$ is a large classical momentum as a function of ϕ.

Let me make the following necessary assumption:

$$|\frac{\partial\tilde{\psi}}{\partial\phi}| << |\pi(\phi)|$$

Also let me assume that $\tilde{\psi}$ depends on ϕ through $\pi(\phi)$:

$$\psi(\phi,x) = e^{i\pi(\phi)\phi}\tilde{\psi}(\pi(\phi),x)$$

Let the Hamiltonan constraint be

$$\phi + h(x,p) = 0.$$

Then using ϕ as time variable, the path integral evolution of the wavefunction as follows:

$$\acute{\psi}(\pi(\acute{\phi}),\acute{x}) = \int \delta(\phi + h(x,p))\exp(i(\pi\delta\phi + p\delta x))\psi(\pi(\phi),x)dpdx,$$

This can be rewritten as

$$
\begin{aligned}
e^{i\pi'(\phi')\phi'}\tilde{\psi}'(\pi(\acute{\phi}),x') &= \int \delta(\phi + h(x,p))\exp(i(\pi\delta\phi + p\delta x))e^{i\pi(\phi)\phi}\tilde{\psi}(\pi(\phi),x)dpdx, \\
\tilde{\psi}'(\pi(\acute{\phi}),x') &= \int \delta(\phi + h(x,p))\exp(i(-\phi\delta\pi + p\delta x))\tilde{\psi}(\pi(\phi),x)dpdx, \\
\tilde{\psi}'(\pi',x') &= \int \exp(i(p\delta x + h(x,p)\delta\pi))\tilde{\psi}(\pi,x)dpdx.
\end{aligned}
$$

In the last line I have considered π as independent variables. Since ψ and $\tilde{\psi}$ both have same probability information they describe physically equivalent quantum evolutions. So now the projected Hamiltonian is $h(x, p)$, with π as time. We can add to this decoherence terms described in the second principle.

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