

Quantum Gravity Framework 3: Relative Time Formulation and Simple Applications to derive conventional Hamiltonians.

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Abstract

In this paper we discuss quantum gravity framework 3.0, where we discuss relative time formulations. Applications of relative time formulations are discussed. The conventional Hamiltonian of bulk matter is derived from quantum gravity Hamiltonian. The derivation of Hamiltonians in the contexts of fields is briefly discussed.

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1 Introduction

In this paper¹, I continue the heuristic research of quantum gravity framework project [3] [1], [2]. First, I discuss the quantum gravity formulation 3.0, which is the further update of the quantum gravity formulation 2.0 formulated in [3]. In quantum gravity framework 2.0 I discussed four proposals: self-time evolution, self-time decoherence, global quantum decoherence and a fourth proposal related to scale invariance, determinism and continuum limit. I here discuss the relative nature of the four proposals in quantum gravity formulation 2.0 with respect to choices of observation paths in configuration space and foliation of the space-time, and I discuss the relative formulations as the quantum evolutions. I introduce the relative-time evolution which is the more general form of self-time evolution and its impact on the other proposals. I also introduce the concept of rest frame foliation which I believe is physically significant in understanding global quantum reduction.

Second, I discuss the applications of relative-time formulation. First, I discuss how to get the conventional quantum mechanics from a relativistic Hamiltonian constraint using the relative-time constraint. Then I discuss how the time constraint formulation can be used to derive non-relativistic quantum mechanics in the context of particles. This starts with various basic assumptions, regarding collection of particles interacting with each other by gravitational and gauge fields. I start with Hamiltonian constraint, and apply the relative-time evolution formulation. By systematic deductions, simplifying using flat space approximations, I derive the conventional Hamiltonian including the Newtonian gravity terms. I also briefly discuss deriving the conventional Hamiltonian formulation using the relative-time evolution in the context of field theory.

I apologize for typos and grammatical mistakes in this paper, and the previous papers related to this paper. This paper is only a rough draft of work in progress.

We follow the following conventions in this article:

Convention 1: *In any integral, the variables over which the integration is done is same those used in the measure placed in the right most end of the integral, unless explicitly indicated otherwise.*

Convention 2: *Summation is assumed for all repeated Greek indices in the explicit elementary products of the basic variables of the theories discussed.*

Convention 3: *In the differential measures of the integrals, the multiplication over all the suffixes and the prefixes is assumed, for example $dx^\beta dy_\gamma$ mean $\prod_{\beta,\gamma} dx^\beta dy_\gamma$.*

Convention 4: *For functions with arguments that have suffixes, prefixes, and parameters: The function depends on all the collection of the arguments for all different values of the suffixes, the prefixes and the parameters. Example: $f(x_\gamma^\alpha(t), y_\alpha) = f(X)$, where $X = \{x_\gamma^\alpha(t), y_\beta, \forall \alpha, \beta, \gamma, t\}$.*

Convention 5: *No other summation or multiplication of repeated indices is assumed other than those defined in conventions 2 and 3. Examples: 1) there no summation in $f_\alpha(x^\alpha, y_\alpha)$, the three α 's are independent, 2) $(p_\beta^\alpha x_\alpha + f_\beta(x^\alpha, y_\alpha)) dx^\alpha dy_\alpha = (\sum_\alpha p_\beta^\alpha x_\alpha + f_\beta(x^\gamma, y_\delta)) \prod_{\eta,\varepsilon} dy_\eta dx^\varepsilon$.*

Convention 6: *It is assumed that $\hbar = c = G = 1$, unless specified.*

¹For the latest updates proper discussions, comments and issues, please visit www.qstaf.com. Much of the discussions, updates and supplementary downloadable materials regarding this project will be mostly available on www.qstaf.com, and other websites referred to such as the researchgate: the link for quantum gravity framework project is <https://www.researchgate.net/project/Quantum-Gravity-Framework>. Update information will be provided on social media also (www.qstaf.com/links).

2 Quantum Gravity Framework 3.0

2.1 Single point system

2.1.1 Self-time Constraint and a Naive Path Integral

Let me start by repeating some introductory work discussed in the first version [1] and [2] of the framework. In this subsection I repeat and also clarify the concept of self-time evolution, discuss it from a more general point of view in the next two subsections.

I need to define a set of mathematical concepts so that we can understand time evolution in a fully constrained system. Consider the configuration space of a system described by a Hamiltonian constraint. We can use the understanding of classical physics to describe the flow of time in the quantum Hamiltonian constrained system. The most important issue here is to identify the time variable. For a constrained system it appears that there is no time variable. But I believe proper statement of the problem can tell what the time variable is. Basically, in general if you are an observer moving in space-time, you observe the world using the direction of four momentum as the direction of time. Same thing can be done for a constrained system. It is determined by the initial conjugate momenta that one specifies. Momenta usually is essentially the rate of change of configuration variables. At each instant the direction of conjugate momenta specifies what direction the configuration variables evolve in the configuration space. This direction keeps changing each instant. This is similar to accelerating observer in space-time. Basically, he observes the world in an accelerating reference frame. That is, he observes the universe using sequence of inertial frames, with moving along a time-like direction in space-time. Same thing can be considered to happen in a constrained system, with the time-direction specified by conjugate momenta each instant.

Consider a simple quantum system which is described by a Hamiltonian constraint only. Let the internal configuration space of the quantum system is of dimension d , and is made of canonical variables p_α and q^α . Let $m_{\alpha\beta}$, a function of q^α , is the metric in the internal configuration space. Hereafter I will use $m_{\alpha\beta}$ and its inverse $m^{\alpha\beta}$ (assuming it exists), to raise and lower indices. Usually $m_{\alpha\beta}$ is simply a delta matrix $\delta_{\alpha\beta}$ multiplied by mass m .

Let me define a scalar product using the metric:

$$\langle a, b \rangle = \frac{1}{2} a_\alpha b_\beta m^{\alpha\beta}.$$

I will assume $m^{\alpha\beta}$ is positive definite for now. The Lagrangian is as usual for a simple constrained system, is as follows:

$$\mathcal{L}(p_\alpha, q^\alpha, N) = p_\alpha \dot{q}^\alpha - N \mathcal{H}(p_\beta, q^\gamma).$$

Let me assume that a typical Hamiltonian is as follows (without the Lapse):

$$\mathcal{H}(p_\alpha, q^\alpha) = \frac{\langle p, p \rangle}{2} + V(q^\alpha) = \frac{1}{2} m^{\alpha\beta} p_\alpha p_\beta + V(q^\gamma) \quad (1)$$

$$= \frac{p_\alpha p^\alpha}{2} + V(q^\gamma). \quad (2)$$

We can make the following standard definitions:

$$\begin{aligned} \text{Norm } |p| &= +\sqrt{m^{\alpha\beta} p_\alpha p_\beta} \\ \text{Unit Vector } \bar{p}^\alpha &= \frac{p^\alpha}{|p|} \end{aligned} \quad (3)$$

These quantities defined are functions of q^α , as $m_{\alpha\beta}$ is a function of q^α . In the path integral for a simple fully constrained system,

$$G(q_1^\alpha, q_2^\alpha) = \int \exp(ip_\alpha dq^\alpha) \delta(H) dp_\alpha,$$

where $dq^\alpha = q_2^\alpha - q_1^\alpha$. We need to decide how to derive a time variable from a configuration variable. We can always linearly map the configuration space to a new configuration space through a canonical transformation. Therefore, the right question is, what direction is to be considered as the time direction in the configuration space at each point, defined in an invariant way.

Now, consider Newton's first law of motion: Every body continues in its state of rest or of uniform motion. The law states that time flows, and a body moves uniformly along the direction of its velocity vector in an infinitesimal time interval. Since time is seen through movement, essentially inertia can be considered as effect of unstoppable flow of time associated with the system. I will reformulate the law slightly such that it defines time itself. The first of Hamilton's equations of motion captures the mathematics of Newton's first law:

$$v^\alpha = \frac{dq^\alpha}{d\tau} = [q^\alpha, H] = m^{\alpha\beta} p_\beta = p^\alpha \quad (4)$$

where $d\tau = Ndt$. Please note that momentum and velocity are equivalent in this framework. If q^α is interpreted as position, velocity v^α is simply like the contravariant version, and momentum is like the covariant versions of each other. Here I use $m^{\alpha\beta}$ and its inverse $m_{\alpha\beta}$ to raise and lower. Now

$$p_\alpha dq^\alpha = |p|^2 d\tau$$

We can use

$$\bar{p}_\alpha dq^\alpha - |p| d\tau = 0$$

as time constraint to define a hybrid classical quantum evolution of constrained system as I will discuss now. This time constraint was introduced in the first version [2] and [1].

A naive self-time path integral introduced in [2] and [1] is

$$G(q_1^\alpha, q_2^\alpha; \mathbf{p}_\alpha, d\tau) = \frac{1}{(2\pi)^{d-1}} \int \exp(ip_\alpha dq^\alpha) \delta(H) \delta(\bar{\mathbf{p}}_\alpha dq^\alpha - |\mathbf{p}| d\tau) dp_\alpha.$$

where \mathbf{p}_α is classical momentum of the particle at instant τ . But this path integral is not time oriented.

A naive time-oriented path integral

$$G_+(q_1^\alpha, q_2^\alpha; \mathbf{p}_\alpha, d\tau) = \frac{1}{(2\pi)^{d-1}} \int_{\mathbf{p}_\alpha p^\alpha < 0} \exp(ip_\alpha dq^\alpha) \delta(H) \delta(\bar{\mathbf{p}}_\alpha dq^\alpha - |\mathbf{p}| d\tau) dp_\alpha. \quad (5)$$

which can be used to evolve the quantum state $|\Psi(\tau)\rangle$ of the system. Above the fractured quantities are classical values derived from $\mathbf{p}_\alpha = \langle \Psi(\tau) | \hat{p}_\alpha | \Psi(\tau) \rangle$, and $p^\alpha = m^{\alpha\beta} p_\beta$ at each instant.

In the previous version of this paper I discussed a detailed analysis of this path integral in the configuration space [3]. But this integral is too formal and naively defined. The last two integrals convey the idea, requires proper analysis to put them to use. Next I will discuss how to do that.

2.1.2 Relative Quantum Evolution in Configuration Space

Consider the simple Hamiltonian system described in section (2.1.1). Given any smooth classical path η defined by $\mathbf{q}^\alpha(\tau)$ in the configuration space R^n one can always define the quantum evolution with respect to this path for a simple constraint system described before in section (2.1.1). I also assume the function $\mathbf{q}^\alpha(\tau)$

has smooth first and second order derivatives. In this, evolution is not in space-time, but in configuration space with metric as $m_{\alpha\beta}$. For this let me define the following:

1. Define $\mathbf{v}^\alpha(t) = \dot{\mathbf{q}}^\alpha(\tau)$ and $\mathbf{p}_\alpha = \mathbf{v}^\beta m_{\alpha\beta}(\mathbf{q}^\gamma(\tau))$, where I have assumed $m_{\alpha\beta}$ is a function of \mathbf{q}^γ .
2. Define a one parameter family of hyperplanes $S(\tau)$ isomorphic to R^{n-1} orthogonal to $\mathbf{p}_\alpha(\tau)$ going through $\mathbf{q}^\alpha(\tau)$. If x^α is the points on this plane, then it satisfies $m_{\alpha\beta}(\mathbf{q}^\gamma(\tau))(x^\alpha - \mathbf{q}^\alpha)\mathbf{p}^\beta(\tau) = 0$. We can denote the hyperplanes by $S(\tau) \equiv S(\mathbf{p}_\alpha(\tau), \mathbf{q}^\alpha(\tau))$ as it depends on $\mathbf{q}^\alpha(\tau)$ and $\mathbf{p}_\alpha(\tau)$. $S(\tau)$ describes a foliation of the configuration space if the surfaces don't cross each other.
3. Define quantum states $\psi(q_\perp^\alpha, \tau)$ on $S(\tau)$. Here q_\perp^α takes values in R^n but is restricted to $S(\tau)$.
4. Define a single step path integral from $S(\tau)$ to $S(\tau + d\tau)$

$$\tilde{G}_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, d\tau) = \frac{1}{(2\pi)^{d-1}} \int_{p^\alpha \mathbf{p}_\alpha < 0} \exp(ip_\alpha dq_\perp^\alpha) \delta(H) \mu(p_\alpha, q^\alpha) dp_\alpha. \quad (6)$$

where $p^\alpha \mathbf{p}_\alpha = m^{\alpha\beta} p_\beta \mathbf{p}_\alpha$. Define

$$G_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, d\tau) = \lim_{d\tau' \rightarrow 0} \frac{\tilde{G}_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, d\tau)}{\int \tilde{G}_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, d\tau') dq_{\perp 1}^\alpha} \quad (7)$$

Since G depends on $\eta, \tau, d\tau$, we have $\mathbf{q}^\alpha, \mathbf{p}_\alpha, d\tau$ in the arguments, but separated by a semicolon, as they are not quantum variable. The $\mu(p_\alpha, q^\alpha)$ is a weight, which is deduced so that

$$G_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, 0) = \delta(q^\alpha - q'^\alpha)$$

where δ is the Dirac delta function. Also, the division in equation 7 is done to remove unwanted factors to get the delta function when $d\tau = 0$. Whether $\mu(p_\alpha, q^\alpha)$ and the division is necessary in a physical theory is a good question. I have included them in the definition to make the formalism most general.

The path integral in equation 7 is the relative-time oriented path integral I will use in this section. I refer to this as relative-time oriented because this evolution is in reference to an arbitrary smooth curve to describe time evolution. This evolution has different physical meaning and interpretation in different contexts. If an observer is moving along the curved path η in the configuration space his observations of quantum system will be described by this evolution.

We can use the relative path integral to define the quantum evolution of states on $S(\tau)$ of the configuration space:

$$\psi(q_{\perp 1}^\alpha, \tau + d\tau) = \int G_{s+}(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, \tau, d\tau) \psi(q_{\perp 2}^\alpha, \tau) dq_{\perp 2}^\alpha$$

For this path integral formulation to genuinely describe the evolution of wavefunction we need to have η such that $S(\tau)$ don't intersect other, that is genuinely describe a foliation of the configuration space.

2.1.3 Self-time evolution of a single point system

Let $\bar{\eta}$ is a curve in the configuration space described by the classical expectations of \hat{q}^α ,

$$\bar{q}^\alpha(\tau) = \langle \Psi(\tau) | \hat{q}^\alpha | \Psi(\tau) \rangle .$$

Its momentum in the classical phase space is described by

$$\bar{\mathbf{p}}_\alpha(\tau) = \langle \Psi(\tau) | \hat{p}_\alpha | \Psi(\tau) \rangle .$$

If the path η used to describe the evaluation of wavefunction is same as $\bar{\eta}$, then I can refer to this as **self-time evolution**, which was the original proposal in the first version of the framework [2] and [1], and studied further in detail in [3]. But it is not necessary that η need to be equal to $\bar{\eta}$, but any η that is smooth enough with $m_{\alpha\beta}(\mathbf{q})\dot{q}^\alpha(\tau)\bar{q}^\beta(\tau) > 0$ for each τ is sufficient enough to study the quantum evolution of the quantum system. Self-time evolution is one particular choice of general evolution described in the previous section.

For to properly do self-evolution we need to dynamically evolve η as we evolve the wavefunction. For this I calculate \mathbf{p}_α and \mathbf{q}^α from the classical expectation values of the corresponding quantum system and use this in step (1) in the procedure in the previous subsection, at each instant to evolve the wavefunction to next step after $d\tau$ interval. Then I can use this to calculate the \mathbf{p}_α and \mathbf{q}_α new classical expectation values at $t + d\tau$ and repeat the steps further evolve the wavefunction. In [3], in equation (8), I have derived the propagator for self-evolving the wavefunction using the variables on the self-evolving hypersurfaces. In the same paper in section (2.1.3), I given the algorithm for step by step evolution.

In the second version of the paper [3], in section (3), I have discussed how to apply this to various simple contexts to cosmological reduced model, cosmology with fluctuations, and Newtonian space. In this paper in section 2, I will discuss the relative-time evolution for common situations encountered by physics. There I will discuss time evolution of the extended objects living in 3d space.

2.1.4 Relative Decoherence

In previous versions [3] I discussed inclusion of this using diffusion equation method [10].

We will generalize the formalism to include relative quantum evolution. I define relative quantum decoherence evolution equation as follows: The quantum state undergoes continuous reduction with respect to some fundamental field variables L_m , through semiclassicalization and randomization given by equation below:

$$d|\psi\rangle > = -i\hat{H}_\perp|\dot{\eta}_s^\alpha|d\tau|\psi\rangle + \sum_m(\hat{L}_m - \langle \hat{L}_m \rangle)|\psi\rangle dz^m \sqrt{|\dot{\eta}_s^\alpha|d\tau} \quad (8)$$

$$+ \sum_m(2 \langle \hat{L}_m \rangle \hat{L}_m - \hat{L}_m^+ \hat{L}_m - \langle \hat{L}_m^+ \rangle \langle \hat{L}_m \rangle)|\psi\rangle |\dot{\eta}_s^\alpha|d\tau,$$

where \hat{H}_\perp is the projected Hamiltonian operator, acting defined on the surface of the planes $S(\tau)$. \hat{L}_m are the operators with respect to which reduction is performed. Here the decoherent evolution depends on η . This is the decoherent evolution of the state as seen by the observer moving along η , with the foliation $S(\tau)$ as a curvilinear reference frame. The τ are dummy variables and they can be rescaled. And so I have used $|\dot{\eta}_s^\alpha|d\tau$ instead of $d\tau$ in the above equation, to make time increment invariant.

I am still in the process of properly defining self-time evolution and understanding its relevance. Self-time evolution seems to be arbitrary choice for pure Schrödinger evolution but it becomes essential when we are including Lindblad [25] type decoherence terms. The decoherence evolution is non-linear and so clearly dependent on the time variable used. The self-time evolution is the most natural evolution to include decoherence, because the system evolves with respect to itself in its own momentum direction as time direction at each instant. But the justification for using self-time evolution appears artificial. In the next revision of the framework I will give much more natural definition of relative definition involving of self-time evolution. I also will use path integral approach for decoherent evolution.

2.2 General 4D Curved Space Time

This section let me briefly summarize the ideas in the previous version [3].

In quantum gravity we want to evolve the quantum states from one spatial hypersurface to another spatial hypersurface of a space-time foliation. In a spatial hypersurface there are infinite number of points, with a quantum system at each point. For each point x , there are one set of conjugate variables $p_{x,\alpha}$, q_x^α (D dimensional internal space), with physics identical to the single point system discussed in the previous section. Only major difference is that the Hamiltonian contains interaction terms as functions of the q_x^α of adjacent points. Here I am using simplified version of quantum fields assuming, while in reality it is complex.

I also don't discuss other constraints such as diffeomorphism or gauge constraints. We will see in the next update of quantum gravity framework, all constraints including quantum gravity constraints need not be explicitly needed for formulating dynamics.

To each point we can apply theory discussed for single point fields. Then there will be one classical curve $\eta_x^\alpha(\tau_x)$ for each point, smooth upto second derivative, one parameter family of hyperplanes $S^x(\eta_x^\alpha(\tau_x), \dot{\eta}_x^\alpha(\tau_x))$ in the configuration space of at each point, and one free (dummy) parameter τ_x for each point.

Let me assume that space is discretized for simplicity, and is made of countable number pieces of volume elements such as in cubic lattice. I am assuming this discretization only for simplicity and explanatory purpose.

Let B be the number of lattice points, and for simplicity let us assume B is finite. Let ΔV be the coordinate volume associated to the coordinate volume element associated to each lattice element of the 3D manifold. Assume that the quantum system at each lattice point x is described by an identical Hamiltonian constraint H_x only, and it has an interaction term that involves quantum systems at adjacent lattice points. Each step of the evolution depends on how $\eta_x(\tau_x)$ varies with $d\tau_x$.

2.2.1 Relative-Time Evolution

Now consider the path integral defined in previous section in equation (7). For each system at x , we have one curve η_x assigned. Then we have the combined one step relative path integral as

$$\begin{aligned} & \tilde{G}(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, d\tau_x, \forall x\}) \\ &= \frac{1}{(2\pi)^{BD}} \int_{\mathfrak{p}_{s,x}^\alpha p_{\alpha,x} < 0, \forall x} \prod_x \{\exp(ip_{\alpha,x}(q_{\perp,x}^\alpha - q_{\perp,x}^{\prime\alpha})) \delta(H_x) \mu(p_{\alpha,x}, q_{\perp,x}^\alpha) dp_x^D\}, \end{aligned} \quad (9)$$

To summarize I generalize of relative-time evolution to (3+1) dimensional space-time as follows using equation (9) as I define below.

Relative-Time Evolution in general curved space time. *The propagator for a four dimensional quantum gravity is given by*

$$G(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, d\tau_x, \forall x\}) = \lim_{d\tau'_x \rightarrow 0, \forall x} \frac{\tilde{G}(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, d\tau_x, \forall x\})}{\int \tilde{G}(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, d\tau'_x, \forall x\}) dq_{\perp,x}^\alpha}. \quad (10)$$

The $\mu(p_{\alpha,x}, q_x^\alpha)$ has been introduced to make sure G is a delta function when $d\tau_x = 0$. In the second equation above, the division is done to remove the factors of integration, and make sure we get the proper delta function. The $\mathfrak{p}_{s,x}^\alpha p_{\alpha,x} < 0$ term in the integral restrict evolution to the positive direction of \hat{q}_x^α . The repeated application of the one-step path integral for infinitesimal $\Delta\tau_x$ smoothly evolves all the systems living on the lattice. The sequence of the quantum states, defines the states of the system at various consecutive instants.

Let me define $d\tau_x = n_x(\tau) d\tau$, where the $n_x(\tau)$ are continuous functions of τ , one of them for each lattice point x . The repeated application of the one-step path integral for infinitesimal $d\tau$ evolves the quantum state along the spatial hypersurfaces. The $n_x(\tau)$ functions defines the various ways to foliate the discretized geometry, whose topology is B point $\otimes 1D$. Here $n_x(\tau)$ is essentially is the lapse. Now depending on the choice of $n_x(\tau)$ we will have different foliations of the classical space-time geometry relating to the quantum geometry. We can take the continuous limit by sending the size of the lattice towards zero.

$$G(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, n_x(\tau) d\tau_x, \forall x\}) = \lim_{d\tau'_x \rightarrow 0, \forall x} \frac{\tilde{G}(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, n_x(\tau) d\tau_x, \forall x\})}{\int \tilde{G}(\{q_{\perp,x}^\alpha, q_{\perp,x}^{\prime\beta}; \eta_x, \tau_x, n_x(\tau) d\tau'_x, \forall x\}) dq_{\perp,x}^\alpha}. \quad (11)$$

As the combined system evolves the classical expectation value of the momentum and the configuration variables $p_{\alpha,x}$ and q_x^α also evolve.

2.2.2 Relative-Time Decoherence

Now if we want to include the reduction at each point discussed in proposal 2 of single point system, the evolution of $|\psi_\tau\rangle$ depends on η_x , as the evolution equation is non-linear. Given a foliation described by certain choice of $n_x(\tau)$, we generalize proposal one and two as follows:

Relative-Time Decoherence - *Given a path $\eta_x^\alpha(\tau)$ for each configuration space, and lapse functions, the quantum state of a spatial hypersurface undergoes continuous reduction with respect to observables L_i through semiclassicalization and randomization given by equation below:*

$$d|\psi_\tau\rangle = iH_{s,x}|\psi_\tau\rangle + n_x(\tau)|\dot{\eta}_x^\alpha|d\tau + \sum_{m,x} \beta_{m,x}|\psi_\tau\rangle + dz_{m,x}\sqrt{|\dot{\eta}_x^\alpha|n_x(\tau)\Delta\tau} + (\gamma_x - \sum_{m,x} \beta_{m,x}^+ \beta_{m,x})|\psi_\tau\rangle + n_x(\tau)|\dot{\eta}_x^\alpha|\Delta\tau.$$

where $H_{s,x}$ is from equation (10), $\gamma_x = -\sigma_x + \langle L_{m,x} \rangle L_{m,x}^+ - L_{m,x} \langle L_{m,x}^+ \rangle$, and $\beta_{i,x} = L_{i,x} - \langle L_{i,x} \rangle$, the suffix x indicates the point to which the quantities corresponds. The operators \hat{L}_m are simple functions of the conjugate variables p_I^s and q_s^I to which are the operators that are being continuous measured in this evolution.

Physical relevance of σ will be evident in the fourth proposal. Since the c term is ignored, $|\psi_\tau\rangle$ is not assumed to be normalized. So

$$\langle L_{i,x} \rangle = \frac{\langle \psi_\tau | L_{i,x} | \psi_\tau \rangle}{\langle \psi_\tau | \psi_\tau \rangle}.$$

This evolution heavily depends on $n_x(\tau)$ and η_x , and so is the evolution is relative to these choices. Basically $n_x(\tau)$ chooses a global foliation, otherwise a global reference frame to observe the quantum state. $\eta_s^\alpha(\tau)$ is a local internal reference frame to observe the evolution.

Assume that above Hamiltonian constraint is discretized in a cubic lattice made of B cubes as I discussed in this section.

A demerit of this proposal is that it is not in path integral form. Other three proposals are in path integral form. In the next revision in quantum gravity framework 3.0 decoherence equation will generalized into a path integral form.

2.2.3 Global Quantum Reduction

Let me relative time evolution for F steps. Each step of the evolution depends on the values of $\Delta\tau_x$. Let τ be a continuous time parameter, which varies from $\tau = 0$ to $\tau = T$.

$$G(\{q_x^\alpha, q_x'^\beta; \eta_x, \tau_{x,F}, \tau_{x,0}, \forall x\}) = \int \prod_{k=1}^F [G(\{q_{x,k}^\alpha, q_{x,k-1}^\beta; \eta_x, \tau_{x,k-1}, \forall x\})] \left(\prod_x \prod_{k=1}^{F-1} dq_{x,k} \right).$$

This evolution generates a time dependent quantum state $|\psi_\tau\rangle$ which evolves the initial quantum state. If we express each step in Hamiltonian form we can include relative decoherence discussed also in this evolution. This evolution evolves the initial state $|\psi_\tau\rangle$ continuously to generate an entire quantum space time. But this evolution depends on η_x and $n_x(\tau)$. We need to fix the arbitrariness of this. The best approach is statistical approach.

. Using $|\psi_\tau\rangle$ we can calculate the following: 1) classical metric $g_{\alpha\beta}$ of the corresponding classical geometry 2) hypersurface metric h_{ab} for the hypersurfaces 3) classical conjugate momenta π^{ab} for the hypersurfaces. All these values are dependent on η_x and $n_x(\tau)$. I will propose the following.

Global Quantum Reduction - *The quantum evolution and reduction process occurs along a spatial foliation such that the C^1 smooth functions $n_x(\tau)$ take smooth values, such that relative probability weight is given by $\exp(-c_r \Upsilon)$, where c_r is a fundamental constant, where Υ is defined as function of $\pi^{ab}, h_{ab}, g_{\alpha\beta}$ or a combination of those to be discovered and verified experimentally.*

Above $\pi^{ab}, h_{ab}, g_{\alpha\beta}$, need not be just classical expectation values. They could be directly the quantum variable also. This is equivalent to adding $ic_r\Upsilon$ to the action. In [3] I discussed a various choices for Υ . One the special choice of the Υ which describes a rest frame foliation was discussed later in a subsection. In the next update, in quantum gravity formulation 4.0, I discuss the covariant version of global quantum reduction.

2.2.4 Determinism, Continuum Limit and Scale invariance

The introduction of decoherence makes space-time and fields unstable, and difficult to reach continuum limit. In quantum gravity framework 2.0 ([3]), I introduced a fourth proposal, which I will include as the part of the quantum gravity framework 3.0, without any alteration.

Smoothness Principle: *Every subsystem has several mechanisms built into it explicitly such that the expectation values of quantum variables of nearby or adjacent identical quantum systems are very close to each other. They are such as 1) There are imaginary decay term in the action to keep the quantum variables adjacent to each other, 2) Every system is a collection of large of subsystems each having quantum variables $q_{x,s}^I$ and random variables $z_{m,x}$ attached to it and evolving according the first three principles, and 3) The effective variables of every system is got by weighted averaging of the random and quantum variables of the underlying subsystems; Fundamental Commutators are smoothed as a consequence of this.*

In proposing this I assume that nature is fundamentally discrete, and the continuum limit is due to dynamics rather than kinematics as one would expect in continuum model. Discovering if there is a fundamentally discrete model is a future course of research. There are many choices already available in literature, such as spin foam models.

A simple way to realize the first part of the proposal is to add an extra imaginary term to the action for quantum gravity, for example system,

$$\mathcal{S} \longrightarrow \mathcal{S} + i \sum_{x,s} \left(\frac{1}{2} \sigma_x(q_{y,s}^\alpha) |\dot{Q}_{x,s}^\alpha| n_x(\tau) |\dot{\eta}_x^\alpha| d\tau \Delta V, \right)$$

such that σ_x are

- 1) smooth real functions of the variables $\hat{q}_{x,s}^\beta$ with a lower bound,
- 2) functions of quantum variables at x and adjacent (or nearby) quantum systems to point x , and
- 3) are increasing functions as $|q_x^\alpha - q_x^\alpha| \rightarrow \infty$.

The new imaginary term with σ_x need to be added to Hamiltonian $H_{s,x}$ in the algorithm discussed in the last section to enforce smoothness. For more information I refer to [3].

2.2.5 Rest frame Evolution

Rest Frame Evolution is the study of a system in the coordinates in which it is dynamically at rest. This could be in the internal evolution in the configuration of space of fields or evolution of fields in the (3+1)D space-time geometry. Let me discuss these one by one.

Internal Evolution Rest Frame Evolution of a single point system is described by three things

Proposition 1 *Deterministic Version 1) The evolution is defined by self-time evolution discussed in 2.1.3. That is $\eta(\tau)$ is same as the expectation of \hat{q}^α and is dynamically calculated from the quantum state as the system evolves as explained in 2.1.3. 2) The system moves slow enough that $S(\tau)$ don't intersect at different values of τ , atleast in the regions where the wavefunctions of the quantum state have a finite magnitude.*

This was studied for a simple system in detail in section 2 of [3]. Since the Relative decoherence depends on $\eta(\tau)$ rest frame evolution is one natural possibility of it. It is basically the quantum system observes itself along a reference frame generated by its corresponding classical internal evolution. To understand whether the decoherence occurs along this path we need to do further research and do experimentally test for this proposal.

Proposition 2 *Statistical version:* The choice of $\eta(\tau)$ is statistical in nature. The relative propability of $\eta(\tau)$ is given by $\exp(-\Upsilon_i)$, where $\Upsilon_i = c_i \int (\eta(\tau) - \langle q(\tau) \rangle)^2 d\eta$, and c_i is a constant to be determined experimentally.

This is alternative to the self-time version of the previous proposition. This proposition is statistical in nature and we doesn't need to specify a specific $\eta(\tau)$. The Υ_i is just a possibility. The actual form of it may be different, but if this proposition is valid, it essentially will take of form of some measure of the deviation of $\eta(\tau)$ from the expectation value of $\langle q(\tau) \rangle$.

Space-Time Evolution Now let me introduce the what is I refer to as the rest frame foliation of space-time. For understanding rest frame foliation let me describe what I call as the global quantum reduction I used $n(x)$ for describing various foliations. For a space time foliation as defined by choice of $n_x(\tau)$, I suggested various choices in section (2.3) of previous version [3]. In this paper I will focus on one particular variation among them. The concept of rest frame foliation as the natural foliation as defined below in the canonical version of quantum gravity framework 2.0.

Proposition 3 *Classical Version:* The quantum evolution and reduction process occurs along a spatial foliation which minimizes $\Upsilon_s = \int (\frac{\langle \pi_f^{ab} \pi_{fab} \rangle}{c_g} + \sum_f \frac{1}{2} \vec{E}_f^2) \frac{1}{\sqrt{h}} dx^3$, where π_f^{ab} is the trace free momentum of gravitational field and E_f is the electric part of gauge fields f .

Proposition 4 *Quantum Version:* The quantum evolution and reduction process occurs along a spatial foliation such that the C^1 smooth functions $n_x(\tau)$ take smooth values, such that relative probability weight is given by $\exp(-c_r \Upsilon_s)$, where c_r is a fundamental constant, where $\Upsilon_s = \int (\frac{\langle \pi_f^{ab} \pi_{fab} \rangle}{c_g} + \sum_f \frac{1}{2} \vec{E}_f^2) \frac{1}{\sqrt{h}} dx^3$, where π_f^{ab} is the trace free momentum of gravitational field and E_f is the electric part of gauge fields f .

In the previous version of quantum gravity framework [3], I gave a fundamental statistical role to such foliation defined by Υ_s to do quantum decoherent evolution. Whether that proposal is physically valid or necessary is not clear to me at this moment, but it is the most general way to formulate the theory. In this paper let us just consider the minimal surfaces only, without using the statistical formulation. In this paper I consider such foliation as just as one choice of foliation in which the fields are either perfectly or at least approximately stationary. That is the translational movements of matter particles is reduced or removed. So I will call such evolution as the rest frame foliation.

Above f denotes the various gauge fields. First consider the electromagnetic fields. Consider the field around a charged particle. In a locally flat hypersurface in which the charge is at rest, let \vec{E} be the electric field. We know that $F^{\alpha\beta} F_{\alpha\beta} = \vec{E}^2 - \vec{B}^2$ is invariant. In the rest frame $F^{\alpha\beta} F_{\alpha\beta} = \vec{E}^2$. In other reference frames B is non-zero. So to keep $F^{\alpha\beta} F_{\alpha\beta}$ invariant \vec{E}^2 must increase. Explicitly it is $\gamma^2 \vec{E}^2$, γ is the gamma in the Lorentz transformation. So we have that $\int \frac{1}{2} \vec{E}^2 \frac{1}{\sqrt{h}} dx^3$ is minimal when the hypersurfaces are orthonormal to the 4-velocity of movement of charges in space-time. So at a microscopic level, minimizing $\int \frac{1}{2} \vec{E}^2 \frac{1}{\sqrt{h}} dx^3$ yields a foliation in which the charges are at rest. I assume we can extend this discussion to include longitudinal fields for other gauge fields such as weak and strong forces with same result.

Next consider the gravitational momentum term. Here π_f^{ab} is the trace free part of the conjugate momentum. I have removed the trace of the conjugate momentum to make $\Upsilon_s \geq 0$. For any particle or any spherical celestial object, π_f^{ab} is zero, for example, in case of the Schwarzschild space-time. It is the equivalent of the electric field of a particle. Schwarzschild solution neatly defines a foliation along with the spatial metric doesn't change. In any other foliation π_f^{ab} is non-zero. So we see that Υ_s is minimal identifies spatial hypersurfaces that is orthonormal to direction flow of an object. This approximates the lab reference. In Earth the lab reference frame is at rest with respect to the static Schwarzschild foliation. Similarly, in an inflationary universe, π_f^{ab} is zero if the spatial foliation is determined using the natural time parameter of the Friedmann–Lemaître–Robertson–Walker (FLRW) metric.

Details of rest frame foliation differs depending on the scale of the physics we deal with as follows:

- When dealing with relative motion between celestial bodies, and movement of small objects with respect to celestial bodies, the effective reference foliation is given by minimalizing Υ_s , using gravitational momenta only. This foliation leads to a curved coordinate system with approximates Schwarzschild coordinate system near these bodies.
- When dealing with relative motion between atoms in matter the rest frame foliation can be determined by having electrical field in Υ_s to be minimal. In this the foliations are such that they are orthogonal to the direction of movement of atoms.
- In case of nuclear or sub-atomic particles, the fields used in Υ_s are those of electric field related to higher gauge theories depending on the situation. In the rest frame foliation the nuclear particles including quarks more or less are rest in them. Their respective electric fields I believe they have to minimal in analogy with the electric field of EM theory.

In each of these contexts the translational movement is removed with respect to space-time, and only relative motion between the particles will be dominant. For example, in the first case we only can see relative motion between bodies in Schwarzschild coordinates. In the second, we are dealing with relative motion between atoms in a molecular context. In the third we are dealing with relative motion between sub-atomic particles.

To sum up, these foliations describe pure relative movement between local entities with global movement suppressed. Both self-time evolution and rest-frame foliation provide for natural evolution and conversion of mixed density states to pure states, with respect to conjugate momenta of fields at each configuration space or with respect to foliation defined by their own movement respectively. In quantum gravity framework 4.0 I will discuss how this is related to consciousness.

3 Derivation of Non-Relativistic Hamiltonians

3.1 Conventional Quantum Evolution in (3+1)D

Let us apply the relative-time evolution formulation in the context of conventional quantum mechanics. Let us first consider the conventional quantum mechanics in 3+1 dimensions. Usually in conventional quantum mechanics, we have a R^4 , space-time of four dimensions. One of the dimensions is considered as time. For example, if $q^\alpha = (t, x, y, z)$ are the coordinates, we have t as time variable. Quantum states are defined on a one parameter family of R^3 spaces $S(t)$, defined by $t = \text{constant}$. To make this perfectly 4D I use the relativistic Hamiltonian constraint as follows:

$$H = \eta^{\alpha\beta} p_\alpha p_\beta + m^2 = 0$$

where $\eta_{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$ and assume velocity of light $c=1$.

The naive time-oriented path integral is given by

$$G(q_1^\alpha, q_2^\alpha; \mathbf{p}_\alpha, d\tau) = \frac{1}{(2\pi)^3} \int_{p^\alpha \mathbf{p}_\alpha < 0} \exp(ip_\alpha dq^\alpha) \delta(H) \delta(\bar{\mathbf{p}}_\alpha dq^\alpha - |\mathbf{p}| d\tau) dp_\alpha.$$

This is the naive time-constrained path integral corresponding to equation 5.

Let me properly define the relative-time evolution. Let me denote the points on R^4 that is restricted to $S(t)$ as q_\perp^α . This one parameter family of spaces are hyperplanes orthogonal to the curve η defined by $q^\alpha(\tau) = (0, 0, 0, \tau)$, in the Minkowski metric. $S(\tau)$ are orthogonal to $\mathbf{v}^\alpha = \dot{q}^\alpha(\tau) = (1, 0, 0, 0)$. Assume we are dealing with a particle of mass m . So $\mathbf{p}_\alpha = (-1, 0, 0, 0)$.

Now the relative-time path integral defined with respect to η is

$$G_s(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, d\tau) = \frac{1}{(2\pi)^3} \int_{p^\alpha \mathbf{p}_\alpha < 0} \exp(ip_\alpha dq_\perp^\alpha) \delta(H) dp_\alpha$$

In non-relativistic approximation the Hamiltonian constraint is approximately

$$H \approx m - p_t + \frac{1}{2m} p_\perp^2 = 0$$

Now the relative-time oriented path integral reduces to

$$G_s(q_{\perp 1}^\alpha, q_{\perp 2}^\alpha; \eta, d\tau) = \frac{1}{(2\pi)^3} \int_{p_{\perp \alpha}} \exp(ip_{\perp \alpha} dq_\perp^\alpha - H_\perp d\tau) dp_{\perp \alpha}$$

where $dq_\perp^\alpha = q_{\perp 2}^\alpha - q_{\perp 1}^\alpha$. Here

$$H_\perp = m + \frac{1}{2m} p_\perp^2$$

is the usual Hamiltonian of the particle with the rest-mass included. So I deduced the conventional quantum mechanical path integral using relative-time formulation with respect to η defined by $\mathbf{q}^\alpha(\tau) = (0, 0, 0, \tau)$.

3.2 Bulk Matter

In this section I apply the formalism of relative-time evolution to derive the non-relativistic Hamiltonian to bulk matter in space. Let us consider a group of atoms, such as in a piece of bulk matter, coupled to each other through electromagnetic forces, which are floating in free space. I am going to assume the gravitational field is only due to this matter all the way to infinity. Let us restrict to the case of non-relativistic speeds and mass of order of that of Earth or smaller, so that we can make use of non-relativistic and linear gravity approximation, so that we are in Newtonian regime. To simplify the derivation, I am going to assume the velocity of each particle with respect to the center of mass of the bulk matter is small compared to the velocity of the center of mass with respect to an inertial reference frame which I will be using to study the system.

The Hamiltonian of the bulk matter can be separated into two parts: 1) Hamiltonian relating to the movement of center of mass of the bulk matter and 2) Hamiltonian due to movement of atoms with respect to the center of mass of the bulk matter. By definition the second part will not contribute to the total momentum of the system. So, momentum is mostly due to first part. So the \mathbf{p}_α will be effectively from the first part of the Hamiltonian. The second part of the Hamiltonian will take care of the internal evolution of the system, using the movement of center of mass as time. Consider the Hamiltonian constraints:

$$(\mathcal{H}_{Matter} + \mathcal{H}_{Gauge} + \mathcal{H}_{Gravity}) |\Psi\rangle = 0$$

Here I am assuming that energy from Quintessence field or some other term had cancelled out the energy contributions from vacuum energies and symmetry breaking due to Higgs mechanism. Without such cancellation this entire universe will explode due to expansion of space-time due to these energies.

Let me integrate the above equation assuming flat space approximation over a large volume which contains all the bulk matter at the center of it:

$$\int (\mathcal{H}_{Matter} + \mathcal{H}_{Gauge} + \mathcal{H}_{Gravity}) dV |\Psi\rangle = 0$$

We can ignore all the non-linear term from the metric except those comes from derivatives of the spatial metric. If I restrict the Hilbert space only to few particle states we have the following (please see the appendix for free particle states).

$$\left(\sum_i (m_i c^2 + \frac{1}{2m_i} \hat{p}_i^2) + \hat{H}_{Tem} + \hat{H}_{Lem} + \hat{H}_{Tg} + \hat{H}_{Lg} \right) |\Psi\rangle = 0$$

Where $V(\{x^i\})$ is the EM potential energy between the particles, H_{Tem} and H_{Lem} are Hamiltonians of transverse and longitudinal electromagnetic fields and other gauge fields such as those relating to weak and strong forces, and, H_{Tg} and H_{Lg} are Hamiltonians of transverse and longitudinal field of gravity. That is the Hamiltonian constraint reduces to that for non-relativistic quantum mechanics

$$H(p_\alpha^i, q_i^\alpha) = \sum_i (m_i c^2 + \frac{1}{2m_i} p_i^2) + H_{Tem} + H_{Tg} + H_{Lem} + H_{Lg} = 0 \quad (12)$$

The $H_{Tem} + H_{Tg}$ are the Hamiltonian terms for electromagnetic and gravitational radiation terms. I have included them for generality, but we can neglect them. The longitudinal fields in $H_{Lem} + H_{Lg}$ need to be solved from initial conditions directly from the distribution of matter using first class constraints. For electric field it comes from gauge constraint $div(E) - \varepsilon_0 \rho = 0$, which can be easily solved, by setting $E = -grad(\phi)$:

$$\phi(q^\alpha) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\rho(q^{\alpha'}) dV'}{|q^\alpha - q^{\alpha'}|} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{e}{|q^\alpha - q^{\alpha'}|}$$

where the summation is over all the particles, e is elementary charge and r'_i are the location of the particles. Above the spatial distances can be calculated using flat space approximations. Similar, calculations need to be done for other gauge fields. For gravity, I will discuss it later in this section.

Assuming the Hamiltonian is of the form in equation (12), let us proceed. Assume we have N particle of various mass coupled together. Let (p_α^i, q_i^α) be the phase space variables for each particle i and m_i is its mass. There are about $3N$ configuration and conjugate momentum variables. Assuming the action of external forces on this N particle system is negligible, the center of mass of this particle system moves on a straight line with velocity v_{cm}^α . This simplifies the problem. If we assume v_{cm}^α is much greater than the velocities of motion due to internal interaction between the particles, our η is approximately a straight line for each particle. The center of mass of the system is,

$$Q_{cm}^\alpha = \frac{\sum_i m_i q_i^\alpha}{M},$$

Where $M = \sum_i m_i$.

Lets split the expectation value of velocities of each of the particle into sum of center of mass velocity and a small correction.

$$v_i^\alpha = v_{cm}^\alpha + \delta v_i^\alpha$$

We need to lower the α using the metric for the constrained space $m_i \delta_{\alpha\beta} \delta^{ij}$, where i and j are the particle index, and α and β denote the vector index.

$$v_{i,\alpha} = m_i v_{cm}^\alpha + m_i \delta v_i^\alpha$$

We are going to assume summation over repeated greek indices always in multiplication. Taking the square using the metric $m_i \delta_{\alpha\beta} \delta^{ij}$,

$$p^2 = v^2 = \sum_i m_i \delta_{\alpha\beta} v_i^\alpha v_i^\beta \approx M v_{cm}^2$$

$$\begin{aligned}
\sum_i \mathbf{v}_\alpha^i dq_i^\alpha &= \sum_i (m_i \mathbf{v}_{cm}^\alpha + m_i \delta \mathbf{v}_i^\alpha) dq_i^\alpha \\
&\approx \mathbf{v}_{cm}^\alpha \sum_i m_i (dq_i^\alpha) \\
&= \mathbf{v}_{cm}^\alpha d \left(\sum_i m_i q_i^\alpha \right) \\
&= M \mathbf{v}_{cm}^\alpha dQ_{cm}^\alpha
\end{aligned}$$

I have ignored the $\delta \mathbf{v}_{\alpha,i} dq_i^\alpha$ as the movement is close to the center of mass of the system. Then the time constraint can be expressed as following for the whole system:

$$\begin{aligned}
\sum_i \mathbf{v}_\alpha^i dq_i^\alpha - \mathbf{v}^2 d\tau &= 0 \\
M \mathbf{v}_{cm}^\alpha dQ_{cm}^\alpha - M \mathbf{v}_{cm}^2 d\tau &= 0
\end{aligned}$$

This is the time constraint expressed in velocity or momentum as shown below:

$$\begin{aligned}
\bar{\mathbf{v}}_{cm}^\alpha dQ_{cm}^\alpha - \mathbf{v}_{cm} d\tau &= 0 \\
\bar{\mathbf{p}}_{cm}^\alpha dQ_{cm}^\alpha - \mathbf{p}_{cm} d\tau &= 0
\end{aligned}$$

So the time constraint basically measures the average movement of the entire system along \mathbf{v}_{cm} . So, it would be good, if we re-coordinate the entire system as in conventional quantum textbook analysis, with respect to center of mass, and the center of mass movement as bulk variable. This will be done later.

The action now for the system including the time constraint imposed with Lagrange multiplier λ is as follows:

$$S(p_\alpha^i, q_i^\alpha, N, \lambda; \mathbf{P}_\alpha^{cm}, Q_{cm}^\alpha, d\tau) = \int \left[\sum_{i=1}^N p_\alpha^i dq_i^\alpha - NH - \lambda (\bar{\mathbf{p}}_{cm}^\alpha dQ_{cm}^\alpha - \mathbf{p}_{cm} d\tau) \right], \quad (13)$$

Now we need to make the center of mass coordinates as one of the variables in the action. Let me define

$$\tilde{q}_i^\alpha = q_i^\alpha - Q_{cm}^\alpha.$$

which are the locations of the particles with respect to the center of mass.

Now, I have

$$\sum_{i=1}^N m_i \tilde{q}_i^\alpha = 0$$

using the definition of Q_{cm}^α . So the \tilde{q}_i^α are not independent. So I just choose \tilde{q}_i^α of $N - 1$ particles and Q_{cm}^α as the new free variables. I consider the \tilde{q}_N^α dependent upon these free variables, using the last equation:

$$\tilde{q}_N^\alpha = - \sum_{i=1}^{N-1} \left(\frac{m_i}{m_N} \right) \tilde{q}_i^\alpha$$

Now using this, I have:

$$\sum_{i=1}^N p_{\alpha}^i dq_i^{\alpha} = \sum_{i=1}^{N-1} \tilde{p}_{\alpha}^i d\tilde{q}_i^{\alpha} + P_{\alpha}^{cm} dQ_{cm}^{\alpha}$$

where \tilde{p}_{α}^i and P_{α}^{cm} , are the new conjugate momenta, related to the old variables by

$$\begin{aligned} \tilde{p}_{\alpha}^i &= p_{\alpha}^i - \frac{m_i}{m_N} p_{\alpha}^N \\ P_{\alpha}^{cm} &= \sum_{i=1}^N p_{\alpha}^i. \end{aligned}$$

Now, we have the action simplify as follows:

$$S(\tilde{p}_{\alpha}^i, \tilde{q}_i^{\alpha}, P_{\alpha}^{cm}, Q_{CM}^{\alpha}, N, \lambda; \mathbf{P}_{\alpha}^{cm}, d\tau) = \int \left[\sum_{i=1}^{N-1} \tilde{p}_{\alpha}^i d\tilde{q}_i^{\alpha} + P_{\alpha}^{cm} dQ_{cm}^{\alpha} - NH - \lambda(\tilde{\mathbf{p}}_{cm}^{\alpha} dQ_{cm}^{\alpha} - \mathbf{p}_{cm} d\tau) \right], \quad (14)$$

with the Hamiltonian as a function of new variables

$$H(\tilde{p}_{\alpha}^i, \tilde{q}_i^{\alpha}, P_{\alpha}^{cm}, Q_{cm}^{\alpha}) = \frac{1}{2M} P_{cm}^2 + Mc^2 + \sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} + H_{Lg}$$

where with i takes values between 1 to $N-1$. In deriving this, I have neglected the kinetic energy contribution from the N^{th} particle assuming it is small compared to the total kinetic energy. Also, the potential energies in interaction Hamiltonians now depends on \tilde{q}_i^{α} (gravitational potential energy will be discussed later). Since these the potential energies depend on differences between the q_i , replacing non tilde's by tilde's variables makes no differences.

The dominant momentum contribution comes from the center of mass movement. Assuming there are no external forces this velocity is constant and our analysis simplifies. Since the center of mass is moving at the speed of \bar{v}_{cm}^{α} , I set

$$dQ_{cm}^{\alpha} = v_{cm}^{\alpha} dT$$

Then we have from the time constraint

$$dT = d\tau$$

Solving the time constraint and putting back the result in the action gives

$$S(\tilde{p}_{\alpha}^i, \tilde{q}_i^{\alpha}, P_{\alpha}^{cm}, Q_{CM}^{\alpha}, N; \mathbf{P}_{\alpha}^{cm}, d\tau) = \int \left[\sum_{i=1}^{N-1} \tilde{p}_{\alpha}^i d\tilde{q}_i^{\alpha} + P_{cm} d\tau - NH \right],$$

where

$$P_{cm} = \bar{v}_{cm}^{\alpha} \mathbf{P}_{\alpha cm}$$

Now the Hamiltonian constraint is

$$\frac{1}{2M}P_{cm}^2 + Mc^2 + \sum_{i=1}^{N-1} \frac{1}{2m_i} \hat{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} + H_{Lg} = 0$$

Let us now discuss the expansion of H_{Lg} . The gravitational Hamiltonian constraint without the Kinetic term for longitudinal fields is

$$H_{Lg} = c_g \int R\sqrt{h}d^3x$$

where $c_g = \frac{c^4}{16\pi G}$ in *SI* units.

In this I have dropped the kinetic term for longitudinal fields because its contribution is small for our non-relativistic case with masses such as about the size of Earth or smaller than that. For the spatial metric this approximates as

$$h_{ab} = \delta_{ab} + 2\frac{\phi}{c^2}\delta_{ab}$$

where ϕ is the gravitational potential in *SI* units satisfying

$$\partial^2\phi = 4\pi G\rho$$

where ρ is the mass density equivalent of energy density. The H_{Lg} terms that are first and second order are as follows (using maxima):

$$H_{Lg} = c_g \int \left(-4\frac{1}{c^2}\partial^2\phi + \frac{1}{c^4}2\partial_c\phi\partial^c\phi \right) d^3x$$

For non-relativistic and weak field gravity, we know that

$$\int_{\Omega} \partial^2\phi d^3x = 4\pi GM_{Total}$$

where M_{Total} is the mass equivalent of energy contained within region of integration Ω . The second term in H_{Lg} contains the gravitational potential energy:

$$\begin{aligned} V_g &= \frac{1}{8\pi G} \int \partial_c\phi\partial^c\phi d^3x \\ &\simeq \frac{-1}{8\pi G} \int \phi\partial_c\partial^c\phi d^3x \\ &= \frac{-1}{2} \int \phi\rho d^3x \end{aligned}$$

where ρ is the mass density equivalent of energy density contained in the region, and I have used integration by parts in the derivation in the second step. By including H_{Lg} terms, now I can rewrite the Hamiltonian constraint as follows:

$$\frac{1}{2M}P_{cm}^2 + Mc^2 + \sum_{i=1}^{N-1} \frac{1}{2m_i} \hat{p}_i^2 + V_g + H_{Tem} + H_{Tg} + H_{Lem} = M_{total}c^2$$

The terms on the both sides are positive. Now one can make sense of this equation as the energy conservation equation from any textbook of physics. M_{total} can decrease only due to radiation terms $H_{Tem} + H_{Tg}$, if the radiation leaves the surface of integration. But if we make sure that surface is very far from the system such that radiation stays within the system during our period of analysis, the total on the left-hand side is a constant.

We can express

$$M_{Total}c^2 = Mc^2 + \frac{\mathbf{P}_{cm0}^2}{2M}.$$

Here $\frac{\mathbf{P}_{cm0}^2}{2M}$ is the mean kinetic energy due the center of mass movement. Now I can rewrite the equations as follows:

$$P_{cm}^2 = \mathbf{P}_{cm0}^2 - 2M \left(V_g + \sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} \right)$$

We can solve the Hamiltonian constraint for P_{cm} , assuming the terms in the bracket are very small compared to $\frac{\mathbf{P}_{cm0}^2}{2M}$.

$$\begin{aligned} P_{cm} &= \sqrt{\mathbf{P}_{cm0}^2 - 2M \left(V_g + \sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} \right)} \\ &\approx \mathbf{P}_{cm0} - \frac{M}{\mathbf{P}_{cm0}} \left(V_g + \sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} \right) \end{aligned}$$

Substituting this back in action I get

$$\begin{aligned} &S(\tilde{p}_\alpha^i, \tilde{q}_i^\alpha, P_\alpha^{cm}, Q_{CM}^\alpha, \mathbf{P}_\alpha^{cm}, dT) \\ &= \sum_{i=1}^{N-1} p_\alpha^i dq_i^\alpha + P_{cm} d\tau \\ &= \sum_{i=1}^{N-1} p_\alpha^i dq_i^\alpha - \frac{M}{\mathbf{P}_{cm0}} \left(\sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} + V_g - \frac{P_{cm0}^2}{M} \right) d\tau \\ &= \sum_{i=1}^{N-1} p_\alpha^i dq_i^\alpha - \left(\sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} + V_g - \frac{P_{cm0}^2}{M} \right) dT \end{aligned}$$

from which we have, ignoring the constant terms,

$$H(\tilde{p}_\alpha^i, \tilde{q}_i^\alpha) = \sum_{i=1}^{N-1} \frac{1}{2m_i} \tilde{p}_i^2 + H_{Tem} + H_{Tg} + H_{Lem} + V_g - \frac{P_{cm0}^2}{M} \quad (15)$$

where $dT = \frac{M d\tau}{\mathbf{P}_{cm0}}$ is rescaled time measured in terms of distance of movement of center of mass. The term in the bracket contains the non-relativistic Hamiltonian. Now I have recovered the textbook non-relativistic Hamiltonian, with an extra constant term of $(-\frac{P_{cm0}^2}{M})$.

In this section I assumed that we had one bulk body made of many small particles. We can generalize the analysis to many bulk bodies. Assume we have M bodies. This could be like a planetary system or star system. Similar to the last subsection assume that we have an inertial reference frame to study our entire bulk body system. So now we can do similar calculations like we did in this subsection. Let us assume

there are large distances between the bulk bodies so that the Newtonian gravitational influences are small between each other. In such case we integrate over a separate large region containing each bulk body to get a constraint equation for each of them. But the left-hand side will not be a constant as the bulk bodies will be interacting with each other. To get constant we need integrate over the entire space until infinity as we did before. We can do similar analysis, and effectively reduce the Hamiltonian to a form in equation (15), but with the $\tilde{p}_\alpha^i, \tilde{q}_i^\alpha$ are coordinates of center of mass of the bulk bodies with respect to the center of mass of all bulk bodies. This is a typical calculation for which standard textbook Newtonian analysis is enough.

3.3 Field Theory Dynamics

Let me illustrate the application of the time constraint based path integral in the field theory context. I show how a standard quantum mechanics of a small system can be studied using time variables derived from the macroscopic environment in which it is a part of. Let me split the space into two regions 1) Ω 2) $\bar{\Omega}$. The microsystem is considered as a quantum system. It lives in a small region described by Ω . $\bar{\Omega}$ is the rest of the universe considered as the macroscopic environment. An example of this situation could be a molecule or an atom living in the gravitational field of the Earth.

Let (ϕ, π) are conjugate variables of the microscopic system under study. Let the macroscopic environment is defined by macroscopic field variables (Φ, Π) for simplicity.

Let the Lagrangian density of the system in the region Ω :

$$\mathcal{L}_\Omega dt = \Pi d\Phi + \pi d\phi - NH_\Phi(\Pi, \Phi)dt - NH_\phi(\pi, \phi, \Phi)dt$$

Here I assume Hamiltonian of H_ϕ depends on the macroscopic variable Φ . Since (ϕ, π) is microscopic, it has no impact on (Φ, Π) in the region. Since Ω is small, (Φ, Π) in this region is mostly determined by configuration of the fields in $\bar{\Omega}$, the rest of the universe. In $\bar{\Omega}$, let classical expectation values $(\bar{\Pi}, \bar{\Phi})$ is the approximately constant value of (Π, Φ) , as determined by classical equations in rest of the universe.

Let me assume the field metric for the macroscopic field is simply equal to 1. Then we have

$$\frac{d\Phi}{dt} = N\Pi$$

Assuming $\bar{\Pi}$ is not zero, the momentum of the combined system is dominated by $\bar{\Pi}$. Lets impose the following time constraint:

$$d\Phi = N\bar{\Pi}dt$$

Now

$$Ndt = \frac{d\Phi}{\bar{\Pi}}$$

This constraint will fix the lapse. Now the Langrangian is as follows:

$$\mathcal{L}_\Omega dt = \bar{\Pi}d\Phi + \pi d\phi - H_\Phi(\bar{\Pi}, \bar{\Phi})\frac{d\Phi}{\bar{\Pi}} - H_\phi(\pi, \phi, \bar{\Phi})\frac{d\Phi}{\bar{\Pi}}$$

Now Φ serves as the time variable. To see this explicitly, let us define

$$d\tau = \frac{d\Phi}{\bar{\Pi}} = Ndt$$

Then I have

$$\mathcal{L}_\Omega = \pi d\phi - H_\phi(\pi, \phi, \bar{\Phi})d\tau - H_\Phi(\bar{\Pi}, \bar{\Phi})d\tau + \bar{\Pi}^2 d\tau$$

Here $-H_\Phi(\bar{\Pi}, \bar{\Phi})d\tau + \bar{\Pi}^2 d\tau$ is just a number. The dynamical part is contained in $\pi d\phi - H_\phi(\pi, \phi, \bar{\Phi})d\tau$. The Hamiltonian is

$$H = \int_\Omega (H_\phi(\pi, \phi, \bar{\Phi}) - H_\Phi(\bar{\Pi}, \bar{\Phi}) + \bar{\Pi}^2) dV.$$

We have the Schrödinger equation that controls the evolution of $(\hat{\pi}, \hat{\phi})$ in Ω ,

$$i \frac{\partial}{\partial \tau} |\Psi\rangle = \left[\int_\Omega (H_\phi(\hat{\pi}, \hat{\phi}, \bar{\Phi}) - H_\Phi(\bar{\Pi}, \bar{\Phi}) + \bar{\Pi}^2) dV \right] |\Psi\rangle$$

Let me generalize the formalism to the vectorial case, assuming (ϕ, π) and (Φ, Π) are vectorial, denoted by (ϕ^i, π_i) , (Φ^a, Π_a) . Assume we have a constant classical expectation value $\bar{\Pi}_a$ in the region Ω for Π_a . Then I can split Π^a into two parts: Π_\parallel and $\Pi_{\perp a}$, where Π_\parallel is the momentum along $\bar{\Pi}_a$ and $\Pi_{\perp a}^a$ is momentum in the space orthogonal to $\bar{\Pi}_a$. For doing this calculation, we need to use the supermetric defined in the kinetic part of the Hamiltonian. Similarly I split the Φ^a into two components. Then the new Langrangian density is given by

$$\begin{aligned} \mathcal{L}_\Omega dt &= \Pi_i d\Phi^i + \pi_i d\phi^i - NH_\Phi(\Pi_a, \Phi^a)dt - NH_\phi(\pi_i, \phi^i, \Phi^a)dt \\ &\approx \Pi_\parallel d\Phi_\parallel + \Pi_{\perp a} d\Phi_{\perp}^a + \pi_i d\phi^i - NH_\Phi(\Pi_{\perp a}, \Phi_{\perp}^a, \bar{\Pi}_\parallel, \bar{\Phi}_\parallel)dt - NH_\phi(\pi_i, \phi^i, \bar{\Phi}^a)dt \end{aligned}$$

Here we may have to treat $(\Pi_{\perp a}, \Phi_{\perp}^a)$ as quantum variables, as $\Pi_{\perp a}$ is zero initially and so these variables are small.

Let me assume the field metric for the macroscopic field is simply δ^{ab} . Then I have

$$\frac{d\Phi^a}{dt} = N\Pi^a$$

Imposing the time constraint:

$$d\Phi_\parallel = N\bar{\Pi}_\parallel dt$$

we can reduce the Langrangian density to

$$\mathcal{L}_\Omega dt = \pi_i d\phi^i + \Pi_{\perp a} d\Phi_{\perp}^a - H_\phi(\pi_i, \phi^i, \bar{\Phi}^a)d\tau - H_\Phi(\Pi_{\perp a}, \Phi_{\perp}^a, \bar{\Pi}_\parallel, \bar{\Phi}_\parallel)d\tau + \bar{\Pi}_\parallel^2 d\tau$$

The Hamiltonian is

$$\hat{H} = \int_\Omega (H_\phi(\hat{\pi}_i, \hat{\phi}^i, \hat{\Phi}_{\perp}^a, \bar{\Phi}_\parallel) - H_\Phi(\hat{\Pi}_{\perp a}, \hat{\Phi}_{\perp}^a, \bar{\Pi}_\parallel, \bar{\Phi}_\parallel) + \bar{\Pi}_\parallel^2) dV \quad (16)$$

3.4 Problem Solving Method: Overview

Here is a general idea how to deal with the universe in the relative-time formulation.

1) Right before or after the big bang the scale parameter acts as time as it has the dominant momentum contribution. But the effect of other field variables need to be studied. We can use Hamiltonian formalism deduced from quantum framework 2.0 summarized in this paper. Right at the big bang we cannot use the Hamiltonian formalism as it has an inverse scale factors which is zero. But the Langrangian formalism can

be used. The Langrangian formulation of decoherence is discussed in the next revision: quantum gravity framework 3.0.

2) After the big bang, with formation of bulk matter we can use the movement of matter to measure time, when there is non-relativistic and linear gravity approximation.

3) In case of black holes where there is continuous in-flow of matter, the mass of the blackhole continuously grows. Acts as a clock. In the spherically symmetric case the gravitational metric is determined by mass and distance from center. Here we can use mass to describe the flow of time at a particular distance from the center. This time would be non-linear as the flow of matter into black hole is chaotic. At the singularity we cannot use Hamiltonian formalism as the metric is zero. But we can use the Langrangian formalism (quantum gravity framework 4.0).

3) Even if there is no bulk movement of matter, but the electromagnetic and other quantum fields change, they can be used as time using the modified path integral formalism. This method can be used for the entire universe. The first three cases are special cases of this. This has been described in the last section.

4 Conclusion

In this paper I simply reviewed and updated the quantum gravity framework 2.0, into quantum gravity framework 3.0. The Hamiltonian formulation of quantum diffusion formulation is not consistent with the Langrangian formulation of the other parts of the framework. So further development of the framework will be done in the quantum gravity framework 4.0. We have used the time constraint formalism to get the conventional non-relativistic Hamiltonian. Also, I have discussed the field theory formulation of getting the conventional Hamiltonian. We need to do further development of this approaches to discover new effects that can be experimentally studied. This will give directions for further development of the framework. Quantum gravity framework 4.0 which is a major update of this paper is available along with his paper.

Appendices

A Non Relativistic Quantum Mechanics

Consider a locally flat space-time. Let us derive particle physics from quantum field theory. This derivation is available in many of the textbooks. Let us consider the scalar field theory. The notation used are of standard notations, with a^\dagger and a denoting the rising and lowering operators either in momentum (p) or position space (x) depending on the arguments. Let us first focus on single particle system.

$$\begin{aligned} |\Psi\rangle &= \int \Psi(p_1) |p_1\rangle d^3p_1 \\ &= \int \Psi(p_1) a^\dagger(p_1) d^3p_1 |O\rangle d^3p_1 \end{aligned}$$

where $|O\rangle$ is the ground state. The Hamiltonian for this system

$$\hat{H} = \int (a^\dagger(p)a(p) + \frac{1}{2})\hbar\omega(p)d^3p = \int (a^\dagger(p)a(p))\hbar\omega(p)d^3p + E_0$$

where E_0 is the ground state energy.

$$\begin{aligned}
\hat{H}|\Psi\rangle &= \int ((a^\dagger(p)a(p))a^\dagger(p_1))\hbar\omega(p)\Psi(p_1)d^3p_1|O\rangle d^3p \\
&= \int \hbar\omega(p)\Psi(p)a^\dagger(p)|0\rangle d^3p + E_0|\Psi\rangle \\
&= \int \hbar(m + \frac{1}{2m}p^2)\Psi(p)|p\rangle d^3p + E_0|\Psi\rangle \\
&= \int \frac{1}{2m}p^2\Psi(p)|p\rangle d^3p + (E_0 + m)|\Psi\rangle \\
&= \int \frac{1}{2m}\tilde{p}^2\Psi(x)|x\rangle d^3x + (E_0 + m)|\Psi\rangle
\end{aligned}$$

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