The Third Attempted Proof of Riemann Hypothesis[1]

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There have been two previously published attempts at the proof of Riemann hypothesis.[3] The author learned from the papers but also found some errors therein. This paper presents the third attempt at the proof. It may contain some errors as well, but we will let our future generations of mathematicians correct them if any.

Prologue

Hello everyone, thank you for your kind and generous readership //:-D This is rather a serious research paper, but I will keep it as entertaining as possible. Please enjoy-

1. Riemann Zeta Function

Actually it was Euler who discovered the function, so it is a misnomer. But, we'll just forgive these erroneous mathematicians who renamed the function after rather the overrated man. Any ways, it looks like this:

$Z(x) = \frac{1}{1^x} + \frac{1}{2^x} + \frac{1}{3^x} + \cdots$, where $x > 1$.

[1] This paper is dedicated to the People in the world who support this author's 2024 US Presidential campaign: his social media and internet Friends (in DailyMotion, YouTube, Facebook, Instagram, SSRN, and VIXRA and other websites), his past and current in-person Friends, and his Family in Korea. Started being written on 8/11/2023. He's a secular-religious, politically independent, and a private academic. The author is running for the US President in 2024 as an independent thinker.

[2] A lawyer by trade, a scientist by hobby, a humanologist by mission, a U.S. Army veteran by record, a former computer programmer, a prior PhD candidate in computational biology (withdrawn after 2 years without a degree), a former actor/writer/director/indie-filmmaker/background-music-composer. Born in the USA, 1978. Grew up in Seoul, South Korea as a child and a teenager. Returned to America as a college student. Still growing up in America as a person //!-)

[3] See https://arxiv.org/abs/2201.06601 ; https://mail.vixra.org/abs/2304.0087 .

Basically, the infinite sum above, known as "series" in math, is known to converge to a positive number, when x is bigger than 1. In particular, when x is 2, the series converges to pi squared over six and that is known as "Basel Problem" and Euler solved it back in 1700s.

Now, let us generalize the function above to use a complex number:

Z(z) = $\frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \cdots$, where Re(z) > 1.

Our next step is what's known as "analytic continuation" of the zeta function above. It means that we want to find a function that has bigger domain than the one above, while not changing the result (codomain or image) of the small domain above.[4] And the more generalized zeta function looks like this. Well, actually it's for the other half, i.e., for inputs where Re(z) < 1:

$$Z(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z) Z(1-z)$$

2. <u>Riemann Hypothesis</u>

In the equation above, we have sine function. When z is 2k, where k is a negative integer, sine function becomes zero and the whole zeta function becomes zero. These are known as "trivial zeros" of zeta function. Now, Mr. Riemann hypothesized that non-trivial zeros of the zeta function would have to be a complex number with its real part being a one half, i.e., ½.

3. <u>A Proof of Riemann Hypothesis</u>

We will use the methodology of "divide and conquer" and "process of elimination" in this proof.

[4] See https://math.stackexchange.com/questions/437883/what-is-the-analytic-continuation-of-the-riemann-zeta-function .

$$Z(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z) Z(1-z)$$

z = a + bi

$$Z(z) = Z(a+bi) = 2^{a+bi}\pi^{a-1+b} \sin\left(\frac{\pi a}{2} + \frac{\pi bi}{2}\right) \Gamma(1-a-bi) Z(1-a-bi)$$

$$Z(z) = Z(a+bi) = 2^{a+bi}\pi^{a-1+b} * \sin\left(\frac{\pi a}{2} + \frac{\pi bi}{2}\right) * \Gamma(1-a-bi) * Z(1-a-bi) = 0$$

A * B * C * D = 0

Above, we have four factors. At least one of them should be zero. Now, let us look at the first part, A:

$$2^{a+bi}\pi^{a-1+bi} = 2^a \pi^{a-1} * (2\pi)^{bi} = \frac{(2\pi)^a}{\pi} * (2\pi)^{bi}$$

A1 * A2

We know that an exponential function is always a positive number and cannot be a zero. So the first half of the above "far right hand side" equations, A1, cannot be a zero. Let's look at the second half of that, A2:

$$(2\pi)^{bi} = e^\wedge \ln\left((2\pi)^{bi}\right) = e^{bi*\ln(2\pi)} = \cos(b*ln(2\pi)) + i\sin(b*\ln(2\pi))$$

The above cannot be zero because there is no angle theta that can make both sine and cosine zeros at the same time.

Next, let's see if the B factor can be zero:

$$\sin\left(\frac{\pi a}{2}+\frac{\pi bi}{2}\right)=\sin\left(\frac{\pi a}{2}\right)\cos\left(\frac{\pi bi}{2}\right)+\cos\left(\frac{\pi a}{2}\right)\sin\left(\frac{\pi bi}{2}\right)$$

$$\sin\left(\frac{\pi a}{2}\right)\cos\left(\frac{\pi bi}{2}\right)+\cos\left(\frac{\pi a}{2}\right)\sin\left(\frac{\pi bi}{2}\right)=\sin\left(\frac{\pi a}{2}\right)\cosh\left(\frac{\pi b}{2}\right)+\mathrm{i}\cos\left(\frac{\pi a}{2}\right)\sinh\left(\frac{\pi b}{2}\right)$$

For B to be zero, the following must be true:

$$\sin\left(\frac{\pi a}{2}\right)\cosh\left(\frac{\pi b}{2}\right)=0\quad AND\quad \cos\left(\frac{\pi a}{2}\right)\sinh\left(\frac{\pi b}{2}\right)=0$$

B1 = 0 AND B2 = 0

Let's look at B1:

$$\sin\left(\frac{\pi a}{2}\right)\cosh\left(\frac{\pi b}{2}\right)=\sin\left(\frac{\pi a}{2}\right)*\frac{e\wedge\left(\frac{\pi b}{2}\right)+e^{-\left(\frac{\pi b}{2}\right)}}{2}=0$$

B11 * B12 = 0

We know that cosh is a positive number and cannot be zero. So B12 cannot be zero. Then B11 should be zero. Then 'a' must be an even integer.

Next, let's look at B2:

$$\cos\left(\frac{\pi a}{2}\right)\sinh\left(\frac{\pi b}{2}\right) = \cos\left(\frac{\pi a}{2}\right) * (\mathrm{e}\text{^}\left(\frac{\pi b}{2}\right) - e^{\left(\frac{-\pi b}{2}\right)})/2 = 0$$

B21 * B22 = 0

We established that 'a' must be an even integer. But when 'a' is an even integer, B21 would be either pi or minus pi, which is not zero. So B22 must be zero. And B22 is zero only when 'b' is zero. 'a' being an even integer and 'b' being zero, that is the case of 'trivial zeros' of zeta function. We are not looking for that. We are looking for 'non-trivial zeros' of zeta function. So we can count out this case of B being zero.

Next, let's look at the C part. C part is gamma function, which is a generalization (and also analytic continuation) of factorial function. It is known that gamma function can never be zero. So we can count out the case of C being zero.

Then, the only factor left is D. D must be zero.

$$Z(a+bi) = Z(1-a-bi) = 0$$

The question is, is zeta function a one-to-one function? One-to-one function is like this:

$x1 \neq x2 \rightarrow f(x1) \neq f(x2)$

Which also means:

$f(x1) = f(x2) \rightarrow x1 = x2$

It may be an error to assume that zeta is a one-to-one function, as this author does not know the answer to that question. But, for now, let's hypothetically assume that it is. Then, we have:

$$a + bi = 1 - a - bi$$

$$a = 1 - a \quad ; \quad b = -b$$

$$a = \tfrac{1}{2} \quad ; \quad b = 0$$

So, what does this all mean? Well, we kinda proved that 'a', the real part of the non-trivial root of zeta function, should be ½. Which is kinda a proof of Riemann hypothesis. But, 'b' being zero is kinda icky part here.

Well, our goal was to share with the world the discovery we have made so far. This may not be the most perfect proof of Riemann hypothesis, but it is indeed a definite improvement over what we had previously. Let us let our present and future generations of mathematicians continue the endeavor //:-)

Epilogue[5]

Hello everyone, thank you for your kind and generous readership //:-D We hope you enjoyed the show. Our next article to write and publish will be titled, “Recent Development in Humanology”. There, we’ll introduce some interesting concepts in science and religion and anything in between.[6]

Thank you for your time and see you later, kind and generous ladies and gentlemen //:-)

[5] This paper was started being written on 8/11/2023. It was finished being written on 8/11/2023 //:-)
[6] See https://en.wikipedia.org/wiki/The_Road_Not_Taken .