


# Invalidation and Proof of the Mass Gap and Viability of The Standard Model on a Discrete Spacetime

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## Abstract

*The Yang Mills Mass gap is often considered as phenomenologically solved, after all QCD does not halt, but mathematically, it remains an open problem, that must be theoretically proven, in order to axiomatically reconstruct Yang Mills, and prove its suitability. The mathematical problem has been elevated to one of the seven Millennium Prize Problems defined by the Clay Mathematics Institute, which has offered a prize of USD1,000,000 for its resolution.*

*In past papers, we argued its resolution in a multi-fold universe with its discrete spacetime, relying on existing proofs in Lattice QCD (Quantum Chromodynamic). It works for discrete spacetime, but was not definitive in the continuous spacetime limit. In this paper, we revisit the standard model on lattices, in particular in the context of the Nielsen–Ninomiya theorem that identifies challenges for modeling both the Weak interaction and QCD on lattices, and thus the Standard Model (SM).*

*Instead of aiming at solving what happens in the continuous limits, we rather treat the continuous case as a larger scale approximation of SM on a discrete spacetime. The paper studies Yang Mills fields and SM viability on discrete spacetimes besides continuous spacetime, and that includes the mass gap. Indeed, in previous papers we showed that General Relativity (GR) prescribes a discrete spacetime. Therefore, the mass gap and the Nielsen–Ninomiya theorem challenges must be addressed for SM in GR-based universes, which, a priori, includes our real universe.*

*Doing so, the paper derives properties like asymptotic freedom, confinement and chiral symmetry breaking in non-abelian 4D Yang Mills theories. It also provides a new understanding of the link between confinement, and the chiral symmetry breaking, which typically occur around when confinement occurs, and explains the differences for abelian fields. These results are valid for discrete and continuous spacetime.*

*In addition, the paper invalidates the mass gap conjecture and its Millennium prize formulation for 4D continuous spacetime by explaining that it can't guarantee a mass gap. With reformulation, the mass gap Millennium Prize Problem is generally correct only in a 4D discrete spacetime, or in a multi-fold universe.*

*SM<sub>G</sub> denotes the SM with gravity effects are not negligible at its scales. The present work gives new arguments for SM<sub>G</sub>: Yang Mills, QCD, and SM. The results also reinforce arguments for a (fundamental) particle desert above the electroweak scale, where the Ultimate Unification (UU) is encountered, and GUTs do not exist. The mass gap is generically proven for 2D random walk multi-fold universes, which characterize all of Physics.*

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## 1. Introduction

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In this paper, we start by recalling result previously obtained with Lattice QCD [1-4]. As Lattice QCD, is by definition on a discrete spacetime, we explored the possibility that our real universe is actually discrete. In [1] we showed that the discrete spacetime can still be Lorentz invariant, 2D, fractal at very small scales and noncommutative. Discreteness of spacetime was already expected from the work of many, despite its detractors, as for example discussed in the section motivating multi-fold spacetime reconstruction [1], as well as in [5]. In [6], we showed that GR implies a discrete spacetime at Planck scales also with Lorentz invariant, 2D fractal at very small scales and noncommutative properties.

So far, in [1,2], we assume that the standard Model (SM) must exist, and be consistent on a lattice, possibly amorphous, which includes fractal. We did not really care to study limits where the lattice's cells, or its minimal distance, goes to zero, as for lattice QCD, but we needed to make sure that proofs, the mass gap in the context of [1,2], remain stable when the cell size goes towards Planck scales. That is not yet something that lattice QCD has absolutely confirmed.

This paper reviews the implications of the Nielsen–Ninomiya theorem for discrete spacetime, that a priori forbids the SM on a lattice [7-11]. If [6] is correct, this is a major consistency issue<sup>2</sup>. Resolving this includes addressing the need for an even number of chiral fermion species, the weak interaction conundrum (it can't exist on a lattice because it interacts differently with left and right chirality, i.e. interact only with left-handed fermions), and the QCD Dirac fermion doubling problem that seems to also make lattice QCD incompatible with reality.

In order to ensure consistency of the SM on a lattice, we explore, how to invalidate applicability of the Nielsen–Ninomiya theorem, something achieved by spontaneous chiral symmetry breaking, and managing the consequences for regimes where the Nielsen–Ninomiya theorem would still apply, typically when the chiral symmetry is not broken, or is restored. In the process, we link the chiral symmetry breaking, appearing under many forms in QCD [11], and often associated to confinement [11,12], to the gravity electroweak symmetry breaking [13-17]. The electroweak scales is defined as in [26,28].

And so, for example, in order to handle Fermion doubling, we observe that the best approach so far, in our view at least, is based on using the 5D domain wall and Wilson fermions approach [11]. Interestingly this approach directly relates to the derivation of SM in multi-fold universes based on 7D, ~ 5D, multi-fold space time matter induction and scattering [18-25,119,153,165].

Consistency with the Nielsen–Ninomiya theorem, at energy scales above the gravity electroweak symmetry breaking, implies the need for entering asymptotic freedom regimes as the chiral symmetry breaking fades away. It implies that the chiral sector of the electroweak interaction and the strong interaction will fade away progressively in above the electroweak energy scales. This shows that above the energy scales of the gravity electroweak symmetry breaking, we encounter the Ultimate Unification (UU) proposed in [1,27,119,153,165] based on radically different reasonings, as well as a new fundamental particle desert, unless if additional interactions were to appear [28-30]. Alternatively, it sets up the conditions for the weak gravity conjecture (WGC) [31], that we do not strictly support [1]. *Note added on July 23, 2023: references in italic were added on July 23, 2023.*

In any case, we do not expect to encounter Grand Unification (GUT) with a unification of the strong and electroweak interactions. We have a different proposal here. It has the benefit to provide an explanation of where these interactions went to follow [1,27,47,119,153,165]. There were many reasons already for us to argue that

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<sup>2</sup> It was therefore a key issues for multi-fold theories that derived also a discrete spacetime before obtaining the results of [6]. [35] focuses on the need to have the same number of left-handed and right-handed chiral fermions to address the gravitational anomalies and chiral symmetry breaking due to gravity to handwave consistency of the weak interaction as well as lattice QCD methods to address Fermion doubling. The truth is that the arguments in [35] are incomplete to address these last two challenges, including what happens at energy scales way larger than the electroweak symmetry breaking. The present paper fills these gaps.

GUTs couldn't physically exist [1,27,40,47,52,53,153], based on the issues<sup>3</sup> with the absence of magnetic monopoles<sup>4</sup> [1,36], absence of proton decay [1,37], and absence of supersymmetry and supersymmetric partners [38-40,41,47,52,53,119,153].

We also review, and justify, the Yang Mills properties of asymptotic freedom, confinement, de-confinement phases and the links to chiral symmetry breaking, with and without Fermions and chiral symmetry restoration, and explain the link between confinement, chiral symmetry breaking and mass gap.

At the end, we are able to conclude that the mass gap problem, as formulated for the Millennium prize, is satisfied on a discrete spacetime and in multi-fold universes. On the other hand, it could be invalidated on a continuous spacetime, something aligned with the undecidability of the mass gap [118]. Such results are of paramount importance. We also review reasons to believe that our real universe is discrete<sup>5</sup>.

The approach of the last two paragraphs are abstract and pictorial, yet they constitute rigorous derivations that we haven't encountered in the literature. This is also why we claim that we explain and demonstrate the properties and behavior that we claim.

Most of the paper does not involve a priori the multi-fold hypothesis, even if inspired by it. When it does we mention it explicitly. At the end, we briefly recast all the results obtained in the context of a multi-fold universe<sup>6</sup>.  
*Note added on July 23, 2023: As already mentioned, following [184,185] and [6], it looks like our real universe is multi-fold.*

## 2. The mass gap and the Millennium Problem

The Yang Mills mass gap problem is considered essential for rigorously proving, or constructing, the existence and viability of Yang Mills field [11,32-34]. [34] provides a succinct summary of the problem statement: *Yang–Mills Existence and Mass Gap*. *Prove that for any compact simple gauge group  $G$ , a non-trivial quantum Yang–Mills theory exists on  $\mathbb{R}^4$  and has a mass gap  $\Delta > 0$ . Existence includes establishing axiomatic properties at least as strong as those cited in (the references in [34]).* Per [34], this would also include proving the existence confinement, and chiral symmetry breaking.

As discussed in [11], the problem is really only interesting for  $G = \text{SU}(3)$ , at best also  $\text{SU}(2)$ , but it has been generically formulated for any suitable group  $G$ . Covering all (compact) groups, without having to handle them case by case and praying for correct inferred generalization, requires a looser approach, focused on key features of the groups and interactions than detailed mathematical computations per group.

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<sup>3</sup> While derived in multi-fold universe, the related papers are not limited to such universe, as long as some of the underlying principles required for the proofs are encountered: non-negligible gravity effects at the SM scales, discrete spacetime and/or 2D gravity process at small scales, as well as asymptotically safe gravity. We explicitly prove the latter for GR-based gravity in [41]. *Note added on July 23, 2023: [184,185] show that in presence of gravity, SM, i.e.,  $SM_G$ , is multi-fold.*

<sup>4</sup> Not to be confused with magnetic monopole modeled in condensed matter physics, or in QCD [1,11].

<sup>5</sup> No matter what all the nay-sayers are saying. As already mentioned, there are many other signs pointing to it as discussed in [1,5,6] and references therein.

<sup>6</sup> Of course, [6,184,185] make the distinction a bit artificial.

$\mathbb{R}^4$  is the term on which we will disagree later in the paper. Spacetime has to be discrete instead like  $\sim\mathbb{Z}^4$ , although rather with a fractal/random sprinkling, generated by random walks, on it. This is required to ensure Lorentz invariance [1,5,6,73].

It is probably worth understanding why the mass gap matters, as none of the cited references seem to clearly explain it. Typically, the mass gap is only investigated at low energies, and therefore it can in general focus on Yang Mills field in the absence of fermions, i.e., quarks for QCD. Let us consider QCD. Yang Mills interactions are carried over by gluons, which are doubly, i.e., carrying two color charge, colored bosons. If it takes arbitrarily low energy to bind two or multiple gluons, any (virtual) gluon risks to becoming a neutral glueball, even with higher order color (multi-)pole, therefore rendering impossible / weakened any propagation of Yang Mills interactions or dynamics.

The same applies for the weak interaction with the weak isospin, playing the roles of color. There is no equivalent for the electromagnetism as the underlying group is abelian, and the photon carries no charge, which means always an infinite mass gap as no ball of photons can be created. Electromagnetism is always viable and consistent. We knew that with QED.

### 3. The mass gap resolution in a multi-fold universe

[1,2] provide an initial proof of the mass gap in a multi-fold universe based on lattice QCD proofs. It is done a bit tongue in cheek: in a multi-fold universe, we have a discrete spacetime, and we can see this as being due to gravity. Lattice QCD is on a discrete spacetime. Therefore the mass gap is satisfied for multi-fold universes. Of course the assertion was with a question mark as confirmation of the proof would require that we can also ensure that the proof remains valid as the lattice cell size decrease towards Planck lengths. It is what does not exactly happen though.

The present paper adds more details to the analysis, by explaining why a mass gap must be encountered on a lattice, and doing so, illustrating the problems with a continuous spacetime. But the proof of [1,2] was essentially correct already. The main missing ingredients were probably:

- a) is our real universe, instead of a multi-fold one, really discrete because on gravity
- b) SM must be consistent on a lattice
- c) we can say a bit more about confinement, spontaneous chiral symmetry and the mass gap.

This is what the rest of the paper strives to achieve.

*Note added on July 23, 2023: By now, we have stronger arguments that our real universe has a discrete spacetime following [5,6,163,184,185], and all the qualitative answers that multi-folds and  $SM_G$  brings to open issues with SM and the Standard Cosmological model ( $\Lambda$ CDM) [1,2,5,6,13,18,22,27,28,35-45,47-52,66,67,72,73,84,85,92-105,108-139,153-186,190-193,200].*

### 4. Multi-fold Lattice viability and the Nielsen–Ninomiya theorem

Following our analysis of multi-fold universes, with their discrete spacetimes, we were bound to check what other challenges could be raised against discrete spacetime.

The main remaining one was the apparent incompatibility of the standard model with a lattice. [1,2] did not consider all the consistency issues with a discrete spacetime as raised by the Nielsen–Ninomiya theorem and its implications [7-11]. We hadn't explore the potential challenges raised by the theorem when we published [1,2]. Ensuring consistency is needed to complete the arguments discussed [1,2]. [35] addressed some of these challenge, but not all of them when it comes to the electroweak interaction and QCD.

## 4.1 Same number of left and right species of chiral Fermion

[35] addresses the issues of the even amount of chiral fermion species required on the lattice. It only briefly touches the weak interaction compatibility with the lattice. Also, it did not discuss in detail the QCD aspects, only mentioning that the lattice QCD methods and the (spontaneous) chiral symmetry breaking should resolve the problem of Dirac fermion doubling in QCD, which is correct, but probably not detailed enough<sup>7</sup>.

Key features of the multi-fold solution discussed in [35] include:

- Discrete spacetime, which requires symmetry in the number of species of chiral fermions something that gravity induced chirality flips, or spacetime orientation flips, handle by ensuring the presence of non-interacting right-handed neutrinos and their anti-particles [1,43,35]. Subsequently additional roles were found for the right-handed neutrinos and their anti-particles, as hidden behind Higgs bosons at the edge of spacetime and multi-folds, in fact at their entry and exit, and mapping, points [1,43-45]. It also ensures traversability of wormholes that, then, may implements multi-folds in a GR-based spacetime.
- Chiral symmetry breaking, due to gravity [13], and gravity-induced chirality switches [1,43], which ensures consistency of the weak interaction on a discrete spacetime, invalidates the applicability of the Nielsen–Ninomiya theorem [7-11], besides rending possible the existence of non-interacting, in-flight, right-handed neutrinos, as discussed in the previous bullet.
- Conventional methods to address fermion doubling, as in the next section, along with the arguments that chiral symmetry breaking is also induced by gravity, reduce the issue.

## 4.2. Fermion doubling

Related to the derivation of the Nielsen–Ninomiya theorem, on a lattice, fermion doubling occurs when just putting fermionic fields on it, resulting in more fermionic states than expected. For discretized Dirac fermions in  $D$  Euclidean dimensions, each fermionic field results in  $2^D$  fermion (tastes) that remain as  $2^D$  Dirac fermions in the continuous limit. It renders QCD simulations in the presence of quarks impractical [11,46].

Solutions for suitable numerical computations have been proposed. See for example [11,64]. Furthermore, chiral symmetry breaking proper to QCD and gravity reduces the problem. More on this later in section 6.4.

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<sup>7</sup> To be fair, [35] announced this as future work, which is concretized by the present paper.

## 5. GR-based Universes

[1,6] shows how GR at Planck scales reduces to a discrete spacetime: at very small scales, Planck scales, the spacetime of a GR-based universe is discrete, initially 2D with fractal random walk behavior, non-commutative and Lorentz symmetric. These latter properties answer the main objections to a discrete spacetime [1,5]: it is still possible to recover Lorentz invariance, and the Weyl tile argument can be addressed [1,187], and, incidentally, in [187] it is done relying on quantum random walks as in [1,6].

[1,47] also illustrated how Yang Mills and GR are included in their respective actions, e.g., via the double copy duality (See [48] and references therein), that describes GR-based gravity, at least at high enough energies above the energy scale of the gravity electroweak symmetry breaking<sup>8</sup>, or the AdS/CFT<sub>YM+2CG</sub> correspondence conjecture [49], that shows how Yang Mills and gravity, including superstrings and M-theory relate by dualities, and hint at multi-folds. Superstrings also recover GR and Yang Mills through the Hilbert Einstein action and conformance, as discussed in [47] and references therein.

These results do not originate from multi-fold principles. These analysis can be repeated without ever introducing multi-folds<sup>9</sup>, but, of course, [6,184,185] make such statements a bit ambiguous.

## 6. Lattice SM<sub>G</sub> Physics

Let us assume a universe with a GR-based spacetime, that is therefore discrete per [1,6].

### 6.1 Gravity electroweak symmetry breaking and spontaneous chiral symmetry breaking

[1,7-10-13,52] show spacetime symmetry breaking between gravity and electroweak.

Optionally, following the reasoning of [52-63], without bringing necessarily the microscopic model<sup>10</sup> of particles that we detailed in [52] for a multi-fold universe. The gravity electroweak symmetry breaking, with or without these assumptions, relies on the orientation of spacetime, which can be seen as spinning (a first) initially massless particle{s}, that then become massive by interacting with the Higgs bosons, or locally when massive particles are created as quantum fluctuations.

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<sup>8</sup> At lower energy scales, references in [48] show that the modeled gravity also include scalars. That is fine, Yang Mills field and gravity originate in a massless world, where the dual copy gravity is pure GR-based. *Note added on July 23, 2023: in [48,179], we show that the dilaton associated to the dynamics of the multi-folds, and associated to the multi-fold space time matter induction and scattering [18,22,52,143], can be seen as the massless Higgs boson and justify its presence also in the multi-folds. With this understanding, even at lower energy, we encounter pure Einstein GR, even below the electroweak symmetry breaking.*

<sup>9</sup> Even if per [6,50], they are obviously present in GR-based universes, or even in quantum ones per [51].

<sup>10</sup> It includes 7D, dominantly 5D, space time matter induction and scattering and microscopic black holes solitons as Higgs condensation in Qballs. These were detailed in a multi-fold universe, but they both have equivalent, or original, introduction without bringing any multi-fold principles or considerations. We can assume, but don't have to, such a microscopic model for the rest of the analysis.

In other words, [1, 7-10] implies that at the same time, spacetime is oriented and particles acquire mass. This is at the difference of most models where electroweak symmetry breaking takes place independently of gravity considerations: they are not modeling gravity with SM; a known short coming.

The orientation of spacetime is an example of spontaneous symmetry breaking. So we can assume that Yang Mills in SM are always co-existing with chiral symmetry breaking below the energy scale of gravity electroweak symmetry breaking. As we will see with QCD additional effects contribute also to chirality symmetry breaking.

In other words, along with the (gravity) Electroweak symmetry breaking, the spacetime chiral symmetry is broken, which can be understood as a spontaneous symmetry breaking of the vacuum.

## 6.2 Validity of the Nielsen–Ninomiya theorem

Whenever chiral symmetry is broken, the consequences of the Nielsen–Ninomiya theorem do not apply on a discrete spacetime<sup>11</sup>.

## 6.3 The weak interaction on a lattice

### 6.3.1. Below the gravity electroweak symmetry breaking

Multiple mechanisms are involved to break the chiral symmetry including those described in [1], in presence of Yang Mills, (massive) fermions and Higgs mechanisms. QCD itself may also contribute to the electroweak symmetry breaking.

Per sections 6.1 and 6.2, the Nielsen–Ninomiya theorem does not apply. There are no issues for the weak interaction to be modeled on a discrete spacetime. A priori, there is even no need to have an even number of chiral fermion species, but that is still required at and above the gravity electroweak scale.

However, handling all the anomalies encountered in the SM [11] also benefits from having the same amount of chiral fermion species [7-11], even on a continuous spacetime, including the need to not have odd number of Weyl fermion [11]. So, this is still expected to be a requirement, but indirectly driven from the Nielsen–Ninomiya theorem that already linked their analysis to the gravity anomaly [11].

Without the benefits of the mechanisms proposed<sup>12</sup> in [1,43], one would still need to address the need for a right-handed neutrino, or, maybe, a suitable sterile neutrino [11].

### 6.3.2. Above the gravity electroweak symmetry breaking

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<sup>11</sup> Non regular lattice is not the way to escape the issues of the no-go statements of the Nielsen–Ninomiya theorem, including QCD fermion doubling [8].

<sup>12</sup> These also ensure conservation of B-L (Baryon minus Lepton) numbers [1,37] by eliminating the anomalies behind B and L.

Above gravity electroweak symmetry breaking, spacetime is not globally oriented and all fermions, and bosons, are massless. Chiral symmetry breaking only occurs locally, due to gravity flips, allowing notions of chiral fermions<sup>13</sup>. The Nielsen–Ninomiya theorem essentially applies, and now requires an even number of chiral fermion species. It has the benefit of continuing to cancel all the anomalies.

However, beyond the phase transition, where chirality flip and smearing can be invoked, we now have a chiral interaction, the electroweak interaction, on a lattice. Equal number of chiral fermion species is not sufficient. As described in [7-11], we may still have a potential consistency problem.

### 6.3.3. Asymptotic freedom to the rescue

Or do we?

As we now know that the spacetime of the real universe must be discrete, and we know about the electroweak as a (partially) chiral interaction, there must be a consistent solution, unless, of course, a mistake was made somewhere.

It will take us a few more pages to cover all what we want to cover about Yang Mills. To that extent, we will explain here how asymptotic freedom can come to the rescue. We note that other schemes may exist, but the present one has the merit to work, a bit like the Higgs mechanism when it was proposed but not yet experimentally validated.

When energy scales increase sufficiently above the energy scale of the gravity electroweak symmetry breaking, so that the phase transition has well completed, and spurious spacetime orientations are fading away, we would traditionally expect that traditionally we cannot have asymptotic freedom, mainly because [65]<sup>14</sup>:

- $U(1)_Y$  is abelian, and renormalizable abelian gauge fields do not have asymptotic freedom. Also, see [68].
- The presence of scalars contributing to a spontaneous mass acquisition with a Higgs mechanism precludes in general asymptotic freedom for a renormalizable gauge theory [69].
- The presence of scalars threatens asymptotic freedom.
- The more fermions are involved, the less likely it is to encounter asymptotic freedom. This, and the previous bullet, can be i) mathematically seen when writing Lagrangian examples [11,65], as in section 8.2, or ii) assumed from the fact that fermions or scalar introduces regions where the reasoning of section 7.2 (and figure 4) is screened, so that effect weakens.
- All pure Yang Mills without abelian factors are asymptotically free, but none have ever been encountered with asymptotic freedom and abelian factors.

We will propose an alternative solution as a loophole to the list above<sup>15</sup>. Let us enumerate, or postulate:

- i) Above the gravity electroweak symmetry breaking scale of energy, the Higgs potential goes to zero

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<sup>13</sup> This matches the model proposed in [18], aligned with the resolution to fermion doubling later discussed in section 6.4.

<sup>14</sup> These statements come from the compilation of analyses and computations across a large set of Yang Mills fields with different groups, actions and particles. We provide an abstract generic proof of them in section 8.

<sup>15</sup> After all, we know that some of the factor clause is not exactly correct: we have QCD asymptotic freedom despite having an abelian factor in the SM. It looks like there the argument was the difference of strength between weak and strong interaction. That is a way. We will do something similar.



- ii) At that moment, the Yukawa coupling between the Higgs scalar and the fermions also disappears: the Yukawa coupling constants go to zero. They are no more constant.

(i) is factual. (ii) is inspired and motivated by the microscopic explanation from [52-63,66,67], but as formulated in non-multi-fold universes, with the intuition of particles as microscopic black-hole like Qball solution of Higgs (superconductor) condensates: with these microscopic models, mass acquisition results from Higgs condensation, i.e. acquiring mass, and being literally swallowed by massive particles, something modeled by the Yukawa interactions and couplings.

As a result, now, it is possible to encounter a form of electroweak asymptotic freedom, while not running afoul of the rules above. It goes as follow, with the (i) and (ii):

- asymptotic freedom can be encountered for the  $W^+$  and  $W^-$  bosons, as they involve non-abelian  $SU(2)$  interactions.
- Asymptotic freedom is not possible for the  $\gamma$  and  $Z^0$ . Just as is the case for QED. It just is now with the weak hypercharge instead of the electromagnetic charge [65,68].

It is possible because there are actually two distinct sectors in the electroweak Lagrangian density between say leptons<sup>16</sup> and bosons. See for example equation (6.14.16) in [70]. So one sector can become asymptotically free, while the other isn't.

Interestingly doing so, equation (6.14.16) in [70] implies that the effects of the chiral interaction disappears at such energy scales and only the weak hypercharge a la QED remains. This way, we have achieved two successful outcomes:

- Without a chiral interaction, there are no inconsistencies on a lattice. The SM can remain consistent.
- With just  $U(1)_Y$  remaining strong, we are ready to present arguments like the Weak Gravity Conjecture (WGC)<sup>17</sup>, or like UU – they are not the same [1,27,108,153]. However going to 3D then 2D dominant spacetime processes, the electromagnetic effects weakens at small scales due to charge effect dilution inside the random walk patterns of solitons (*Note added on April 16, 2023, see [92], especially figure 1 in [92]*), and at longer ranges (of the scale of (massless) fermions) due to confinement of QED (3D then 2D, see also section 8.1 hereafter for references). Note that chiral anomalies remain at the larger scales where fermions are presents.

## 6.4 Viability of lattice QCD

There is another consequence of the Nielsen–Ninomiya theorem: fermion doubling for lattice QCD. It is detailed in [7-11,46,64]. We believe that one of the best summary can be found at [11]. This is also our recommendation to understand lattice QCD, besides [64].

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<sup>16</sup> Quarks are similarly handled: look at [71].

<sup>17</sup> It is to be noted that we do not exactly align with the WGC formulation. Instead, we proposed the Ultimate Unification (UU) [1,27,108,153]. We see the result here as also consistent with UU, and our prediction of a fundamental particle desert above the energy scales of the gravity electroweak symmetry breaking energy scale [28,72,73].

### 6.4.1 Below gravity electroweak symmetry breaking

At energy scales below the gravity electroweak symmetry breaking energy scale, fermions have masses.

Numerous mechanisms are identified as contributing to the chiral symmetry breaking for QCD [11], in addition to what we discussed above, like spacetime orientation and gravity induced chirality flips. Also, as we will see later, when confinement exists, chiral symmetry is typically completely broken, with exceptions (all axial current contributions). It will be rediscussed later.

In all these cases, because the chiral symmetry is broken, the Nielsen–Ninomiya theorem does not apply. Yet Fermion doubling is a numerical simulation problem.

[11,64] detail several plausible ways to address the challenge. In particular, we note the combination of the domain wall fermions (from a 5D spacetime)<sup>18</sup>, with a 5D Wilson fermion term along with a Chern Simons term [11]. The result removes Dirac fermion doubling.

For the rest, no inconsistency is expected.

At high quark density, a phase of quark gluon plasma appears with deconfinement (due to the asymptotic freedom property), and (partial) chiral symmetry restoration, because of the link between confinement and (partial) symmetry breaking, then also delocalization with percolation. As quark density increase [12,64,74,75], we can recover the type of phase diagrams from [12,75].

### 6.4.2 Above gravity electroweak symmetry breaking

Above the energy of the gravity electroweak symmetry breaking energy scale, spacetime is globally not oriented. Fermions (quark) are massless, except for fluctuations. Higgs does not necessarily break the chiral symmetry breaking any more, as already discussed in terms of the impact on the Yukawa potentials. Many of the contributions to chiral symmetry breaking disappear progressively and so confinement disappears even at low(er) quark densities. Finally everything becomes colorless within hadron matter: the strong interaction effect disappear completely screened away just as for the chiral effects of the electroweak interaction.

The Nielsen–Ninomiya theorem applies, and we for sure need to address fermion doubling, which we can do as we already proposed in section 6.4.1.

At that point, and above, only gravity and weak hypercharge-based electromagnetism matter. We are ready for particle desert [1,28], and UU discussion mentioned earlier. We will revisit in an upcoming section.

## 7. Yang Mills Asymptotic Freedom

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<sup>18</sup> We will discuss the link to 5D space time matter induction and scattering [5,13,19,22,ii5] in section 13 on the multi-fold theory. As the reader will see, this approach is very consistent when it comes to the multi-fold theory, including the handling of the chirality.

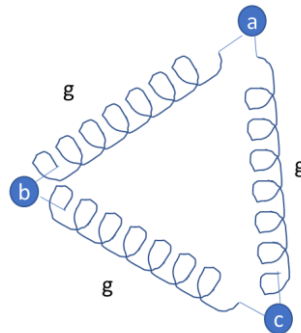
[65,68] assert that only (renormalizable)<sup>19</sup> non-abelian Yang Mills fields can have asymptotic freedom, abelian field don't. [65] also asserts that Fermions reduce the possibility of asymptotic freedom, as do scalars, and certainly scalars involved in Higgs mechanisms. In section 6.3.3, with steps (i) and (ii), we invalidated the absolute nature of the scalar and Higgs statement simply by explicitly eliminating the Yukawa couplings above the Higgs driven (gravity) electroweak symmetry breaking. It is a new mechanism that we haven't encountered suggested anywhere else so far, and a new approach to model physics above the electroweak scale. Yet it modifies things little beyond what we discussed.

The proofs of these rules for asymptotic freedom rely on exhaustive analyses of all the possible cases, with complicated renormalization computations. These can only tell you as much as the running of the coupling constants models, and for the groups  $G$  under considerations and associated actions. As we are not aiming at providing a lecture on this topics, something a priori well known for 40 to 50 years by now, we would rather explain the physics behind asymptotic freedom. In our view, although some mathematics addicts may find it not rigorous enough, it is qualitatively providing the right insight on what happens, and allowing much more generic confirmation of the rule assertions mentioned in the previous paragraph, and in section 6.3.3.

Let is first understand asymptotic freedom.

## 7.1 Abelian (Yang Mills) fields have no asymptotic freedom

The (Yang Mills) abelian field case can be understood as follows. See figure 1.



*Figure 1 – The blue circles indicate charges involved in the exchange. They may be carried by any colored particle. This is a 4D spacetime scenario.*

In figure 1, we see that multiples fermions can exchange bosons. Because the group, and therefore the interactions, are abelian, all these interactions can take place uninhibited by the order of arrivals of the bosons, as the commutator is always null. With the usual  $r^{-2}$  properties of the propagators (in 4D; it is key), effects are amplified at smaller scales and high energies. There is no evolution towards a reduction of the interactions. There is no asymptotic freedom. Sources are moving, at  $c$  when massless, so the drawing must be understood, with a grain of salt, as symbolic.

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<sup>19</sup> That is a given to be physical.

The vacuum polarization will create many configurations as in figure 2, where virtual pairs are created. The attracted ones being closer while the other ones being further away. The effects screens outwards, not inwards. A good example is QED, with non-colored, i.e., carrying no charge, photons.

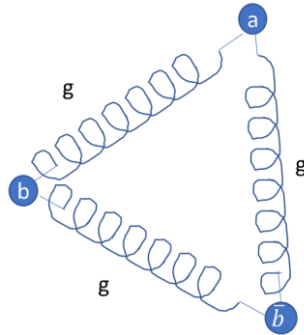


Figure 2 – Particular 4D case involving vacuum created pairs and anti-pairs of fermions.

As we will see this is quite different from the non-abelian case. However this can lead to confinement, as discussed below, under certain conditions. For example, in lower dimensions, QED can confine. *Note added on July 23, 2023: see for example [179] and references therein.*

Sources are moving, at  $c$  when massless, so the drawing must be understood, with a grain of salt, as symbolic.

## 7.2 Non-abelian yang Mill fields always start with asymptotic freedom

With no, or few, fermion types, we always have asymptotic freedom in the case of non-abelian fields.

It results from figure 3.

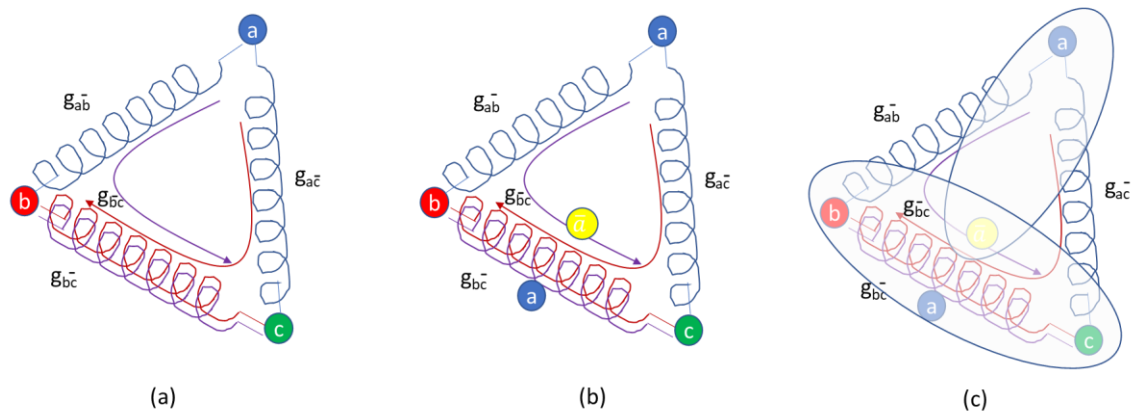


Figure 3 – (a) is the same setup as figure 1. (b) However now the non-abelian properties systematically produces the charge “a” when coming from both paths (that do not commute), and it means creating a and  $\bar{a}$  charges. Drawing of a located lower than  $\bar{a}$  is just for the convenience of the explanation. (c) Within a small enough regions, the region is colorless (white) and the interactions mutually neutralized. Particles behave as if they were free (in terms of the Yang Mills colored interactions). Figure 1, has no equivalent because it is abelian. The reasoning can be repeated with others combinations of colors and anti-colors. It always results into the pattern shown in (c). The example here is for  $SU(3)$ .

Figure 3, illustrates the asymptotic freedom within small regions for a non-abelian Yang Mills field. No equivalent exists in figure 1, hence no systematic asymptotic freedom in abelian cases, only the screening effects due to vacuum polarization perceived further away. In figure 3, the behavior comes from the non-commutativity of the interactions a-to-b to-c vs a-to-c to b that generates a and  $\bar{a}$ . Sources are moving, at c when massless, so the drawing must be understood, with a grain of salt, as symbolic.

As the distances increase, the effect disappear, and interaction appears and increase. It is a first step towards confinement as discussed later on.

Section 8.2 discusses the effects of scalars and fermions, but let us see if we can find a simple intuitive explanation so far for their impact on asymptotic freedom.

*Note added on July 25, 2023: In the presence of many (colored) Fermions, at scales where they are felt, see figure 1 in [92] to understand what we mean in the multi-fold case, the fermions can't overlap per the Pauli exclusion principle. It may limit the coexistence of a color in figure 3 I, and therefore limit asymptotic freedom, if too many species are present. The multi-fold explanation address this challenges: everything ends up rather patterns of random walks of massless bosons: in a multi-fold universe, asymptotic freedom should ultimately prevail no matter what.*

*In terms of scalars, as discussed in [92], they can pile over each other, and with gravity, create other attraction centers and obstacles that limit asymptotic freedom, especially if too many species are involved.. Again the multi-fold explanation address this challenges: everything ends up rather patterns of random walks of one species of massless bosons: in a multi-fold universe, asymptotic freedom should ultimately prevail no matter what.*

### 7.3 Other properties

The rules compiled at the beginning of the section will become obvious after we understand confinement. This is discussed in the next section.

## 8. Confinement

### 8.1 Physical explanation

Confinement was famously proven using the running of the coupling constants in renormalization of QCD [11,70], which led to a Nobel prize. However, that was just in the context of SU(3). On lattice, confinement has also been perturbatively proven [11,70], showing the area law for the Wilson loop criteria [11,70,74], at least for strong coupling. It also works for abelian fields, also implying possible confinement for them, as already mentioned in section 7.1. So, at strong enough coupling, electromagnetism can become confining [11,70], as is electroweak. We also know that the Higgs mechanism is compatible with confinement [11]. However, for the electroweak interaction, the energy scales of the gravity electroweak symmetry breaking are way larger than the confinement scales. As energy lowers, bosons become massive and we never encounter confinement. *Note added on July 23, 2023: This is confirmed by 2D/3D QED as discussed in [179] and references therein.*

Let us explain confinement graphically starting from a source that can be a colored fermion, or one of the color carried on by a gluon.

Figure 4 shows how both radiated gluons and created pairs of (virtual) particles and anti-particles amplify the color of a source. It is called anti-screening. The larger the distance considered (and far away to not be within the asymptotic freedom regime). The energy grows at least linearly with the distance, until new bound states are cheaper than continuing to try to get further from the source. It is illustrated for SU(3), SU(2) or other groups can be similarly analyzed.

Again the effect is a repeat of the effects of non-commutativity across the interaction paths, as in figure 3, but now having to account all possible paths, as in path integrals, and therefore the different combinations of pairs of colors as shown in figure 4 (a) to (c). The effects in figure 4(c) reflect the properties of SU(3) and its impact on the gluons that can be encountered, namely as 2 combinations of the previous ones. *For a simple tutorial see [188].* The amplitudes should not matter here. [143] shows how non-commutativity is key to confinement as a key property of the non-abelian symmetry.

Sources are moving, at c when massless, so the drawing must be understood, with a grain of salt, as symbolic.

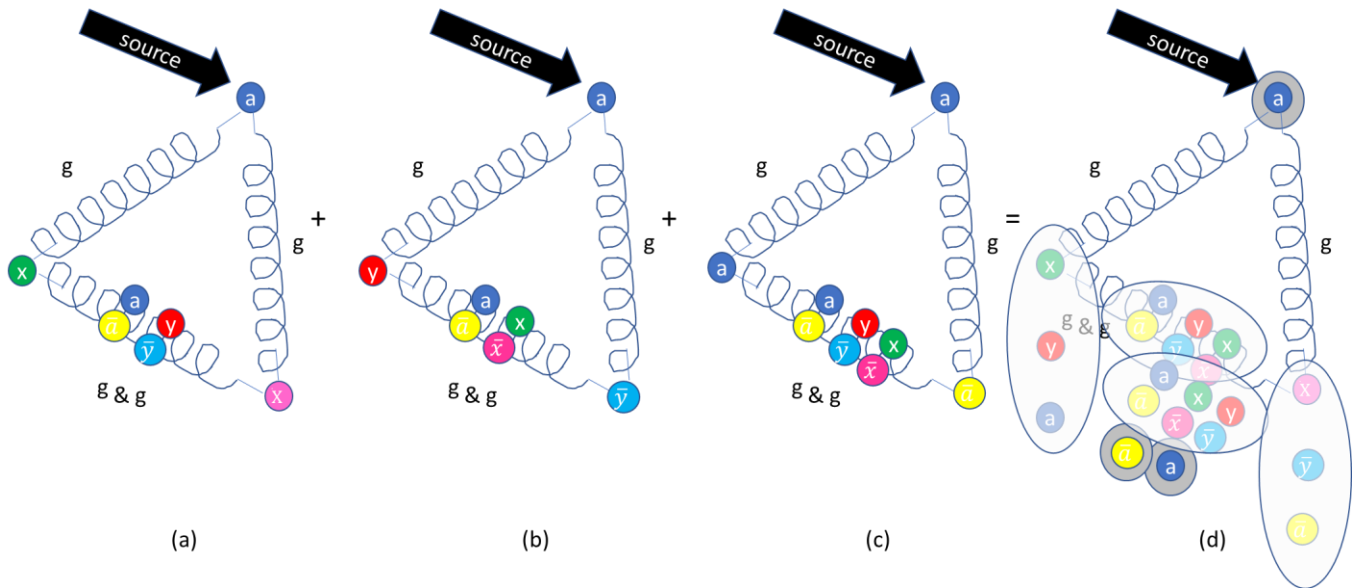


Figure 4: Confinement around a source  $a$  due to the color of emitted gluons and surrounding creates pairs of fermions. Blue holds for the color of the source. The example is illustrated for  $\sim SU(3)$ . (a) (b) and (c) show three cases of particle and anti-particle pairs created in the vacuum. We can see at the end in (d) the anti-screening effect resulting from these different options: 2  $a$  charges have now appeared. The anti color also appears emphasizing also the ant behavior. Repeating this often explodes the effect. The further away from the source, the more often this repeat, as more pairs can be created. From further away, the color charge increases with distance (think of a variation on the Birkhoff theorem [183]). (d) results from the 2 (instead of 3) last two gluons expressed as combinations of color/anti color pairs, e.g., [188]. Per figures 1 and 2 versus figure 3, this explains only the case of non-abelian Yang Mills fields.

The result works for non-abelian fields. It does not depend on spacetime being continuous or discrete as long that Yang Mills is a viable and consistent theory.

For abelian fields, figure 4 does not help. When confinement of an abelian field occurs, it is rather a reflection of the changes of the interaction potential evolution with the distance. For example, with 2D QED, the potential evolves in  $r$ , instead of  $1/r$  in 4D spacetime, and therefore, fulfill automatically the criteria of rowing with the distance: we have confinement and can expect chiral symmetry breaking as discussed later on. See [92,189] and references therein. In 3D spacetime, QED is also confined (and as a result with chiral symmetry breaking) because the Coulomb potential is now in  $\ln(r)$ , which implies a increasing effect, just as in 2D, it is in  $r$ , an even stronger effect. These imply anti-screening and confinement in the presence of (charged) fermions. It is consistent with conventional views [88-90]. The massive gauge vector boson in 2D QED (Schwinger model) can be thought as the result of the chiral anomaly and hence chiral symmetry breaking and confinement, and a massless boson cancelled by the massless (confined) fermions, ensuring that confinement dominates (and massless fermions are what is encountered above the energy scales of the electroweak symmetry breaking energy) [92]. In 4D because the potential decreases with the distance, we have no confinements for 4D QED, and no chiral symmetry breaking; it can however appear at string coupling, where an effect growing with the distance can again take place, e.g., as fermion pairs appear in between, now possibly with anti-screening effects.

Again the conventional model matches the approach we take in the multi-fold analysis. This is why in UU regime, even the electromagnetic effect disappears both at ranges larger than the fermion scales (confinement), and very small scales (within the random walk patterns of solitons). Within the patterns, only collisions and entanglement remain. Note added on April 16, 2023: this is further expanded in [153] – see figure 1 in [153].

Note that Polyakov showed that instantons, i.e. tunneling effects contributions to path integrals, as quasi particles tunneling, may also be a way to characterize, and regulate (compact) abelian gauge fields (QED) confinement in 3D [11,83]. In 4D, the same computations show no confinement at weak coupling, but as already mentioned it exists at strong coupling. Physically, the contribution of instantons can be understood as follows. When confinement occurs, a potential barrier appears. Instantons, which attempt to tunnel through, characterize that event. They also happen to keep the correlation length finite. Note also that abelian gauge fields do not dynamically change the axial charge: they do not break the chiral symmetry. But instantons do, for example in the presence of massless fermions [11], and produce a chiral condensate.

However, it does not apply the same way for non-abelian Yang Mills in 4D as in such case, even without confinement, instanton break the axial chiral symmetries [11], independently of confinement.

Confinement occurs inside Hadrons. Examples include baryons and mesons, nuclei, charmed and strange ones and hypothetical, but not likely, strangelets [191,192].

## 8.2 Effects of fermions and scalars on confinement and asymptotic freedom

Note that the more fermion flavors exists in the theory for the interaction, the larger the effect of figure 4: adding Fermions increases the anti-screening effect shown in figure 4, or they makes the renormalization  $\beta$  function<sup>20</sup> more positive for each new flavor (valid if coupling is weak enough<sup>21</sup>) [11,70]. This is why adding fermions flavors will always reduce the chances of encountering asymptotic freedom.

Similarly, adding scalars, breaks chiral symmetry [11], a way to encourage confinement as we will discussed in an upcoming section. A way to understand that is to consider that scalars increase gravity effects by piling up, and therefore they amplify gravity flips that create chiral symmetry breaking and hence favors confinement which is anti-screening. In addition, without spin, they present obstacles, and gravity fluctuations that prevent a clean figure 3, limiting asymptotic freedom.

Scalar involved in Higgs mechanisms further support confinement [11], if/when the Yukawa potentials do not disappear, e.g., post condensation i.e., at energies below the electroweak symmetry breaking: the Yukawa interactions interfere with the scenario of figure 3, and increasing the figure 4 effects.

In both cases one can see that if scalar interact colorlessly with the quarks and gluons, they will create openings in figure 3 that weaken the whitening<sup>22</sup>. Yukawa interactions associated to Higgs add such systematic effects close to colored fermions, absolutely preventing asymptotic freedom, unless if tamed above spontaneous symmetry breaking by the additional mechanisms proposed in section 6.3.3.

With this we have recovered an explanation for all the additional exception rules discussed earlier in section 6, and ways to invalidate aspects of them (abelian factor, Higgs scalars above Higgs mechanism etc.).

The implications of confinement is that every colored particle is limited to a certain box or quantum well, with its instanton. Just as photons acquire mass in a (reflective) box, all colored particles reaching the confinement boundaries will be massive [141]. Breaking confinement results from creating new particle pairs, or percolating in a dense quark gluon plasma or as superconductor pairs. Colored particles are therefore also remaining massive, even if massless, just as photons in matter. As we saw this also applies to QED at lower dimensions, and therefore can be encountered always for non-abelian Yang Mills field, and sometimes for abelian fields. These considerations

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<sup>20</sup> [11] provides a computation result for all the relevant group G worth considering.

<sup>21</sup> Figures 1-4, also work at strong coupling and in confinement.

<sup>22</sup> Think of an ice breaker creating lanes in the iced sea.



are very important when discussing the mass gap problem, as in upcoming sections. Note in particular the caveat though: these conclusions apply only if the particles are aware of the boundary, created by the confinement. Or densities are high enough (and/or dimensionalities low enough / couplings high enough for example).

*Note added on July 23, 2023: In the multi-fold case, as already explained, figure 1 of [153] explains why we eventually will always have asymptotic freedom, and UU.*

## 9. Chiral symmetry breaking

It is known that the relationship between chiral symmetry breaking and confinement is not always clear: they usually appear together but sometimes not as alleged by different lattice QCD results [11,46,64].

It should be clear that there are multiple sources of chiral symmetry breaking and different levels, depending on what groups are affected [11]. And in [11], we can see proofs that baryons (and mesons which are handled as pseudo Goldstone bosons, massive because of chiral symmetry breaking), at low energies where we have only 2 or 3 quarks, can't be with confinement, and leave the chiral symmetry breaking unbroken, and that it also implies that baryons are always massive, even if the (light) quarks were massless. This does not change our analysis earlier: massive quarks exist only below the energy scale of the electroweak symmetry breaking; but baryons can still be massive at energy scales above. And the mass of the baryons results from such effects rather than the Higgs mechanism. Only higher density may "restore" the chiral symmetry by deconfining.

There are multiple sources of chiral symmetry breaking:

- Spacetime orientation from local to global around and below the energy scale of gravity electroweak symmetry breaking. This is generic, for non-multi-fold and multi-fold universes.
- Gravity chirality flips, albeit that is more a local effect, and really identified in the context of the  $SM_G$  and multi-folds. But it could also apply. The rest is derived directly without any multi-fold; considerations.
- Chiral anomaly [11] whenever fermions are presents due to the fermion triangles [11]
- Chiral symmetry breaking due to electroweak symmetry breaking, Higgs mechanisms and presence of scalars, Higgs scalar and massive fermions [11].
- Chirality symmetry breaking due to confinement<sup>23</sup>, within the quark  $q\bar{q}$  condensate, in order to accommodate 't Hooft gauge anomalies via anomaly matching [11,77,78].

So far, the proof for the last bullet, from 't Hooft, must be done case by case, showing contradictions that imply that the chiral symmetry is fully broken, but not really explaining why. We want an easier proof, because we want to be generic, and not limited to SU(3) or SU(2), or having to guess (or compute) for every single other possible case.

### 9.1 Always broken, except if no triangles

Here is our argument. In a confined situation, many virtual fermions and gluons are created, per the mechanisms of confinement sketched in figure 4. There is place for all possible fermion triangles for any type of symmetry of the underlying Yang Mills field, which create all sorts of axial currents. This implies breaking of all chiral symmetries

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<sup>23</sup> This last full chiral symmetry breaking is really the one that is associated to confinement and baryon masses [11]. The masses of nucleons are indeed much larger than what would be endowed just by the Higgs mechanisms.

that would still be unbroken. Nothing can compensate for that. More (massless or massive) fermions complicate the task. Heavy fermions, and therefore, additional flavors, further ensure confinement and break of the symmetry breaking as discussed above. Any running away baryon<sup>24</sup> or meson (or higher) involves similar material.

The only case, where this full chiral symmetry breaking can be prevented, is when triangles can be prevented. It means that the density of fermions (particles not the type of flavors) must be large enough to overlap so that asymptotic freedom kicks in. Deconfinement takes place and Yang Mills interactions fade away: all anomalies are fading away as does therefore the chiral symmetry breaking. As overlap takes place with percolation, delocalization can also follow as discussed in [12,75]. These are states of quark gluon plasmas as in [12,75]. Then at high density but low enough energy (i.e., as measured by the temperature), one can also expect that quarks can interact with each others as entangled fermions and create a combination of flavor specific superconductors that may also contribute to trigger the electroweak symmetry breaking [12,75,79].

Mass of hadron, nuclei and other exotic composites of quarks and gluons comes from the chiral symmetry breaking by raising the potential of the soup of gluon quarks [1,11,42,193,197,198].

We have therefore shown why and how full chirality and symmetry breaking is associated to confinement and the confined  $q\bar{q}$  condensate [11].

It is worth noting again that gravity breaks chiral symmetry, as is needed to handle anomalies. This is encountered with the  $SM_G$  and in multi-fold universe, and conventionally known (see section 3.2.4 in [11]). *Note added on July 23, 2023: [184,185] hints that stability and consistency of the  $SM_G$  always implies a multi-fold universe.*

## 9.2 Another explanation

There is another way to link confinement and chiral symmetry breaking by referring to figure 4. Consider just 2 light quarks. When the separation between the source and another particle is too large compared to the confinement effects, two new particles are created. The created pair of new particles and anti-particles can be a priori following the  $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$  [140]. After being created, it must then match the chirality imposed by the existing source and other particle and their orientation of spacetime [160]. They were left or right. Therefore, the symmetry of the new pair is broken from  $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$  to  $SU(2)_V \times U(1)_V \times U(1)_A$ , when combining with the existing particles.  $U(1)_A$  is anomalous and also broken. It explains why confinement in general comes hand in hand with chiral symmetry breaking. However note that there may be cases when it's not the case as already mentioned, if the original particles do not preserve their chirality and spacetime orientation in the process, which could happen, for example if the density, pressure or temperature allows for it. When gravity effects are strong enough, below energy scales of the gravity electroweak symmetry breaking, chiral symmetry is always broken by the gravity contributions. Interaction with the new pairs of old and new quarks is to be seen as chiral currents, as carried by the pseudo-scalars Goldstone bosons: virtual mesons [11].

For  $N_f$  quarks, the chiral symmetry breaking of the theory is:

$$U(1)_V \times SU(N_f)_L \times SU(N_f)_R \rightarrow U(1)_V \times SU(N_f)_V \quad (1)$$

Interestingly, exceptions qualitatively match between the explanations of section 9.1 and 9.2.

It is consistent with [153]: deconfinement occurs for non-abelian fields when the symmetry from non-commutativity is broken; which requires enough density to much with the non-commutativity. Otherwise, in the

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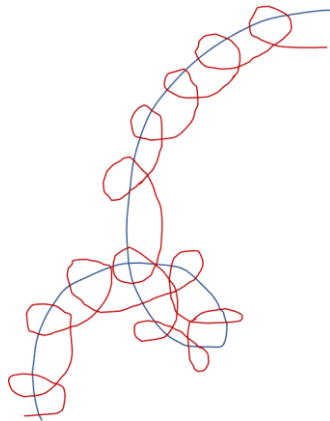
<sup>24</sup> E.g., If the quark escapes by creating new mesons (or baryons).

presence of confinement, when trying to escape confinement, new particle / anti-particle pairs are created which explains chiral symmetry breaking on one hand, and results also from the same non-commutativity.

### 9.3 About instantons

As a side note, in section 8.1, we discussed how instantons seem to play a role in characterizing compact abelian gauge field confinement. With massless fermions instantons can no more (easily) tunnel, and instead, they break the chiral symmetry, and creates chiral condensates. This is when confinement occurs [11]. Again chirality symmetry breaking is directly linked to confinement. Compact non-abelian gauge fields have not been characterized this way, as far as we know, because the first axial chiral symmetry is already broken, and somehow prevent us to easily see such effects, assuming they also are present.

## 10. Conventional mass gap in continuous spacetime



*Figure 5: Possible trajectories between two gluons forming a glue ball as used in the reasoning in the text. In practice for the configuration under consideration the blue trajectory curve is also similarly spiraling around the red trajectory, and the two spirals are tight, with radius  $r \rightarrow 0$  and overall staying in a narrow region within the "confinement region" domain, i.e. not encountering these boundaries.*

### 10.1 Generic proof and invalidation of the mass gap

Section 8 provides a generic proof of the mass gap for non-abelian yang Mills Field: colored particles, including massless ones, which are aware of the confinement boundaries are confined and therefore massive [141].

When deconfinement takes place, some particle might become massless, but we argue that they should remain massive due to the high density needed to encounter deconfinement, so that any deconfinement and delocalization is burdened by the associated percolation that can still confer mass, but not necessarily always. So that means that, when deconfinement takes place, the gap may not be assured, especially if the Quark Gluon

plasma is spatially infinite. When finite, we have a gap not matter what, unless if these are the boundaries of a finite and expanding universe, as discussed later. This is consideration that invalidates the Millenium prize formulation of the mass gap.

The proof presented here is valid for any non-abelian group  $G$ , and works, with an updated terminology/scope, on continuous or discrete spacetime. This “proof” results from the equivalence between gapped systems and exponential decorrelation of QFTs correlation functions as discussed in [142,144-146].

For abelian Yang Mills fields, 4D QED is a counter example to the mass gap, at low coupling, even if we have seen that abelian Yang Mills field can sometimes confine and have a mass gap lower dimensions, or possible at higher couplings. These counter examples invalidate the generic Millenium prize mass gap formulation for any Yang Mills field: it is not generically true for abelian fields. Of course, it could be argued that the Millenium prize mass gap challenge is meant only for non-abelian field, if Yang Mills field were accordingly limited. We will not try to argue what is the correct taxonomy, just noting that everybody well knows the QED counter example.

But can we therefore conclude that the mass gap is generically satisfied for non-abelian Yang Mills fields?

## 10.2 Gluon balls

Let us consider a continuous spacetime, the potential of one colored charge can be approximated by equation (2) in [80], between two gluons. If we treat it like set of spiral in an axial direction, or on a torus or other trajectory (See sketch in figure 5), with the other gluon similarly circling the trajectory of the first one. We can see that this problem is not that different from a complex variation of the hydrogen atom [81], where we model each gluon particle as massive with, as mass, its energy content in the direction orthogonal at any time to a planar orbit projection. Solutions will have specific quantum numbers. Solutions exist as they have been found say in that paper, or in lattice QCD [2-4]. By changing continuously the orthogonal trajectory, we can change the energy contained in the system. The approach only works for colored/charged gluons. It is not encountered with abelian fields without charged bosons.

Rigorously the model needs to be computed in the center of mass framework of the system, leading to equations as in [42,194], with also spin states to consider. However as we are looking here only into the lowest possible state above or other than the ground state, we can limit the model to 0 spins: the spin couplings and interactions will only further split the energy level, but not change a discussion of a massless particle modeled by Yang Mills / QCD, if any.

The  $\sim$  orbital radius,  $\sim$  because we look at a local projection on an orbital plane instead of the actually twisted/curved spiral in 3D, can expand then contract around a certain position, possibly oscillating between  $\sim 0$  and the confinement range. As a result the binding potential energy is maximized (as does the axial kinetic energy). In all cases, it is in addition to the energy of the other particle that can also be dictated by the wavelength or go to zero, if also in orbit with the same or another particle. This demonstrates the existence of glue balls: the resulting bound states are glueballs or gluon balls as discussed for example in [42,194,196,199].

If the speed in the orbital plane goes towards zero (and the axial speed goes to the speed of the gluon ( $c$  or a massive value)), then for each value of the radius, that becomes large, but bounded by confinement ranges, if there is confinement, a set of quantized energy level could exist. On the other hand, if the speed in the orbital plane goes towards  $c$ , the radius goes to zero, with the potential  $\propto 1/r$  [82] increasing to infinity. The feasibility of the latter, i.e., a limit of the radius to zero, depends on:

- (i) The presence or absence of asymptotic freedom around the axis. One would expect that it may occur with massive gluons/bosons. With massless ones, each gluon radiates within a conical wake behind it, as in [1] and reference therein about gravity effects and radiated boson by photons / massless particles. The orbiting gluon sees the forward potential and may not be able to reproduce the scenario of figure 3, to generate whitening of the colors near the axis: asymptotic freedom does not dominate and may not play a role. It rather all depends on density, fermions and scalars and past history. If the gluons are massive, because they see / saw the confinement boundary, then asymptotic freedom will kick in ensuring a low cutoff for  $r$ .
- (ii) The actual details of the potential à la [82] dictates if the relativistic “Bohr atom orbital” can be contracted to zero, or is limited to a low cut off by relativistic effects, where the orbital speed would reach  $c$  before the limit.
- (iii) Other low cutoff for  $r$  like (a) non-point-like particles, or (b) discrete spacetime minimum length.

If this is satisfied, when  $r \rightarrow 0$ , (5) allow the speed of the glueball is  $c$ . Therefore that particle would be massless, and the bound gluons would have had to been massless. Note that this is not contradictory with section 11.1: in such a gluon ball, the gluons circle each other with orbits where  $r \rightarrow 0$ : they never have to reach the confinement boundary, they are not aware of it and indeed are massless.

The existence of a potential massless gluon ball would invalidate the mass gap, despite confinement: the boundaries do not constraint the gluon ball and its orbiting gluons to be massive. The result is generic. It may not be encountered in the case of the SM Yang Mills fields.

In a continuous spacetime with point particles, as for conventional QFT, QCD and SM, we cannot guarantee in general that we have a mass gap, for Yang Mills potentials that would allow  $r \rightarrow 0$ . They assume it was the ground state, which it is, with a mass gap. But it really shows that the lowest bound state is the same as the vacuum state. (and if  $r$  can't go to zero, we do no reach  $V(0)$  and have certainty to have a gap). This model also handles spins and spin couplings.

With this analysis, we see that in a continuous spacetime, a gluon ball could be encountered that moves slower than  $c$ , meaning that its mass is non-zero, while the energy is smaller than the energy of two free gluons (and converging to such energy): the lowest excitation is therefore lower than the energy of two free gluon and massive but converging to a massless particle. There is no mass gap.

The reader may be concerned by the approximations. Note that if you follow a 2 particles, relativistic, in their center of mass reference frame, with path integrals as in [194], one obtains the same result. Just that [194] assumes the case where masses go to zero to be the vacuum. But it is in fact a bound state of  $M_0 = 0$ , with  $m_1 = 0$ ,  $m_2 = 0$ ,  $\mu_1 = 0$  and  $\mu_2 = 0$  with  $r \rightarrow 0$ ... The model is actually more rigorous that it may look.

For abelian fields, we already noted that the issue does not exists if the bosons carrying are colorless, like in electromagnetism. A mass gap can exist at lower dimensions as already discussed. No gluon ball risk to imperil the result.

With this, we have invalidated the main clause of conventional formulation of the mass gap conjecture formulated by the Millennium Prize that was to be proven: it is not always guaranteed to hold in a continuous spacetime for any symmetry group  $G$ , and the associated Yang Mills potential. That is even if the Yang Mills field of the SM may be mass gapped for the Yang Mills  $G$  and Yang Mills potentials involved.

It is a rather good reason why the mass gap was never proven so far, even if other clauses are also to be proven in the context of the millennium prize formulation [34], and they are also not necessarily trivial to prove. We treat these extra considerations<sup>25</sup> in sections 8 and 9, equally valid for continuous or discrete spacetime.

At the scales involved when  $r \rightarrow 0$ , the presence of quarks, massless or massive, (or other fermions or scalars) may help prevent gaps, in the continuous case, by squeezing in between the gluons, even if they also hurt asymptotic freedom.

The reasoning of [119] does not answer the question of existence or not of a mass gap, in the terminology of that paper, the surfaces to consider for integration are now within the three-form  $P$ . there is no contradiction.

Quark bindings, are widely captured with QCD models, Hadron, and Meson physics, nuclei physics, and exotic tetra quarks, tetra quarks, etc. For these, we have argued, in section 9, referring to [11], that these composite particles are all massive, when formed. Therefore, they are not candidates to break the mass gap and don't seem to exist alone without the soup of quarks and gluons embedding these valence quarks. One can argue that this also include the models of bound quark and gluons that again do not exist on their own with same kind of soup of quarks and gluons and interaction energies sufficient to always create the surrounding soup. Only gluon / gluon bindings (with 2 or 3 gluons) are relevant candidates. They are what we discussed. Note that we use the term soup instead of the correct term of Quark Gluon Plasma (QGP), that also covers the deconfined phases.

Finally, note that the absence of algorithm to decide the existence of a spectral gap in systems in 1 or more dimensions (spatial), other than by experimentally looking at its non-stop evolution, as discussed in [142,147-150] and references therein, is interesting in the context of the mass gap problem and its Millenium prize formulation. Ut explains why or undecided result is not surprising<sup>1</sup>, but instead aligned with these results.

In the upcoming sections, we manage to provide proofs by finding loopholes for this results: discrete or multi-fold spacetime as well as a 2D (1+1) proof that aligns all the pieces.

*Note added on July 23, 2023: [184-185] hint that if gravity is present, spacetime must be discrete, and multi-fold, and particles non-point-like, or the Yang Mills would be unstable. We will see in upcoming sections that this ensures mass gaps.*

The conclusions of this section is that the Millenium prize formulation of the mass gap challenge is invalidated in a 4D continuous spacetime, on technicalities/scope challenges (e.g., abelian counter example in 4D of 4D QED, cases of deconfinement in an expanding universe), and on the undecided possibility of a tight massless gluon ball, not propagating and unaware of the confinement boundaries. If its terminology/scope is reworked, the mass gap could be valid for specific fields and groups/potential/couplings.

## 11. Mass gap in a discrete spacetime

Here, let us explain why a mass gap exists in general for Yang Mills on a discrete spacetime (for abelian and non-abelian fields). In a discrete spacetime, the minimum distance guarantees that under no circumstances can the particles collapse on each other: indeed we now have discontinuity of the possible radius values and therefore energies. The spectrum is possibly discrete and, in any case, minimally bound. This proves the mass gap for non-abelian Yang Mills field in a discrete spacetime, no matter what  $G$  and Yang Mills potential are involved.

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<sup>25</sup> Except the compactness consideration that we handle in section 12.1.

For abelian fields we already know that they have no mass gap in the case of 4D QED and gapped at lower dimensions. As already discussed, this again invalidates the more generic version of the mass gap problem if it were to include abelian fields.

The presence or absence of massive or massless quarks, other fermions, or scalars, does not change the conclusions in a discrete spacetime.

So we have now encountered a few new mass gap theorems:

- On a discrete spacetime, the Yang Mills mass gap is guaranteed.
- On a GR-based spacetime, that is therefore discrete per [6], the Yang Mills mass gap is guaranteed.
- *Note added on July 23, 2023: Per [184-185], in the presence of gravity, GR-based or not, the universe is multi-fold, and the Yang Mills mass gap is guaranteed (and particles can't be point particles). This reinforces the previous two bullets.*

It looks like for our real universe; the mass gap is guaranteed if we accept that it is based on GR, or if we consider the SM<sub>G</sub>, the standard model (SM) with gravity non-negligible at its scales [1] (*Note added on July 23, 2023: And yes it means that the universe is multi-fold per [184,185]*).

About our past results from [1,2]. We already know, from section 2, that the mass gap is satisfied in lattice QCD [2-4]. With lattice QCD, lattices are at the QCD scale, not the Planck scales. The glueball mass encountered in such simulation are large enough to be large and compatible with the energy scale modeled by lattice QCD: from a few hundreds of MeV to a few GeV [12,13,87,199]. Yet if scales were to be way smaller, which has not been tested as far as we know, one does not know of these estimates evolve, and if another region would be met with continuous spectrum leaning towards zero. But for a while it will appear stable. Even for SU(3), this may happen and based on figure 5, we are concerned that it happens if the spacetime evolves to become continuous and no minimal length exists. Indeed, in such simulations, one does not study well enough the region around a gluon, which is key to the invalidation presented in section 10. So lattice QCD estimates risk to appear with a gap and it will appear to be around the QCD scale (i.e. between 210 MeV and 1.275 GeV [86]). This is aligned with results like [12,13,87]. It explains the potential contradiction.

We must again stress that we do not claim that it happens with SU(2) or SU(3) Yang Mills and potentials as in SM. But we do know already that with gravity these must exist in a discrete spacetime with non-point like particles [184,185].

The conclusions of this section is that the Millenium prize formulation of the mass gap challenge is validated in general in a 4D discrete spacetime, where it is only invalidated, on technicalities/scope challenges (e.g., abelian counter example in 4D of 4D QED, cases of deconfinement in an expanding universe).

## 12. Millennium prize related results

Additional clauses are associated with the mass gap problem formulated for the Millenium prize [32-34].

### 12. 1 Group Compactness

As explained in [11], picking compact (which one among the quotient groups) or non-compact groups leads to different theories, and we do not always know which one to pick for our real world.

We like the reasoning of [82] for picking compact group, based on many cases, where there are no other options, e.g. SU(2) has no non-compact version, and the considerations of the charge quantization<sup>26</sup> [82], that we also invoked when we forbade the existence of magnetic monopoles [1,36]. This is what we will assume for the rest of the discussion, and as meant in the mass gap problem.

## 12.2 What we have proven

We have proven that for Yang Mills associated on any compact G on a discrete spacetime, and therefore on a GR-based spacetime per [6], (or on a discrete spacetime with gravity) that:

- For non-abelian Yang Mills field associated to any compact group
- (\*) A mass gap is guaranteed.
- Confinement can be met at low enough energies to have massive fermions, and it is associated to chirality symmetry breaking. This statement may be invalidated when the density of fermions is too high, when we encounter quark gluon plasma, or a superconducting phase.
- Yang Mills is consistent and viable in a discrete spacetime. *Note added on July 23, 2023: the QFT / quantum cellular automata invoked in [1,49,153,179] and references therein, provides the necessary reconstruction of QFTs in 2D to 4D.*
- Abelian fields are not generically gapped<sup>27</sup>, but they can be.
- Deconfined phases may be gapless.

We have also proven that while the other statements remain valid for a continuous spacetime, there is no way to guarantee a mass gap, unless when at least one of the (i) thru (iii) conditions of section 10.2 is satisfied.

We have therefore invalidated, in general, the mass gap Millennium prize conjecture as formulated in [34]:  $\mathbb{R}^4$  must be replaced by something like  $\sim\mathbb{Z}^4$ , and the formulation must not include abelian fields.

## 13. A multi-fold take

### 13.1 Multi-fold universes at 4D scales

In [1,5,6,153,163], multi-fold<sup>28</sup> spacetime reconstruction shows as discrete, fractal, 2D process driven by random walk spacetime which is also non-commutative and Lorentz invariant. Doing so we addressed some of the strongest objections to concerns with Lorentz invariance preservation on discrete spacetime and paradox like Weyl

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<sup>26</sup> After all, crushing the existence of magnetic monopoles [1,36] in a multi-fold universe, because of gravity, already forced us to make similar choices to continue to explain electric charge quantization [1].

<sup>27</sup> Abelian groups do not have the results of non-abelian fields, and depend on further handling like the WGC (Weak gravity conjecture) [1,31], or the Ultimate unification [1,27]. As we already mentioned, there is no much room for other GUTs or TOEs: strong and weak chiral contributions fade away, leaving only electromagnetism, albeit with hypercharge for the SM, and gravity for the SM<sub>G</sub>.

<sup>28</sup> Tracking the work in progress on the multi-fold theory can be done at [84]. Overviews are available there [1,72,73].



tile arguments [1,5,187]. [6] expanded on it and [1] showed that the spacetime reconstruction algorithm from [1], is encountered at Planck scales in GR.

A 2D (1+1) spacetime is central to high energy quantum gravity and ensuring that it is asymptotically safe [1,32,49,85,153]. We have also proven that conventionally one can derive asymptotic safety of gravity from the yang Mills double copy duality for gravity [41,49,153].

The multi-fold gravity electroweak symmetry breaking justifies spacetime orientation, and gravity based chiral symmetry breaking [13]. Combined with models of particles as solitons of Higgs condensate Qballs matching the geometry of over extremal microscopic black holes [1,5,13,52], we can provide microscopic models for the 2D spacetime as random walk, inflation and gravity electroweak symmetry breaking, as well as massless particles models soliton random walk patterns [1,5,39,66,67,153]. All these models have conventional non-multi-fold universe equivalent, except maybe for the random walk patterns, which are further discussed in references [1,49,153,179].

The generation of the solitons relies on a 7D, but essentially 5D, space time matter induction and scattering from an embedded 7D flat or Einsteinian space [18,22,73,143]. These models are also inspired from non-multi-fold theories [19-21,23-25]. Handling of chiral fermions, absent in odd spacetime of 5 or larger dimensions, matches the proposal for 5D wall domain Wilson fermions [11,64].

Multi-fold universes and the  $SM_G$ , the SM with gravity effects non-negligible at its scales, allow us to introduce non-interacting right-handed neutrinos that give additional physical explanations for multi-folds as traversable wormholes [45], and, along with the  $SM_G$ , can qualitatively explain many of the challenges still encountered with the SM. They are compiled and can be tracked at [72,73,84,154], and later on in the appendix.

For the rest, the paper has shown consistency of the SM, or rather  $SM_G$ , on a discrete spacetime or lattice, with the non-interacting right-handed neutrinos<sup>29</sup> [11].

In a multi-fold universe, the mass gap for discrete spacetime holds, as formulated by the millennium prize formulation (i.e. compact group and with clauses on confinement and chiral symmetry breaking), because a multi-fold universe always has a discrete spacetime. We have shown that SM and  $SM_G$  is consistent on such a discrete spacetime. *Note added on July 23, 2023: [184,185] further elaborates of consistency of QFT, Yang Mills and SM (as SMG) in the presence of gravity and seems to indicate that the universe must be multi-fold with a discrete spacetime and particles modeled as 7D space time matter induced or scattered QBall solitons or random walk patterns [18-25,119,153,165].*

It is also aligned with the Ultimate Unification proposed in a multi-fold theory [1,27,119,153,165].

The conclusions of this section is that the Millenium prize formulation of the mass gap challenge is in general validated in a multi-fold universe at 4D scales, and only invalidated on technicalities/scope challenges (e.g., abelian counter example in 4D of 4D QED, case of deconfined space).

*Note added on August 6, 2023:*

### *13.2 2D Physics (in Multi-fold spacetime)*

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<sup>29</sup> For conventional physics, i.e., a priori non-multi-fold, we saw that right-handed neutrinos are also needed, for consistency of SM and physics in general on a lattice, and to handle associated anomalies [11].

*Per [194], all our multi-fold theory results indicate that Physics in a multi-fold universe and in the real universe is fundamentally built on 2D Physics*

*In a multi-fold universe, spacetime and gravity is a 2D (1+1) process from random walk, fractal yet non-commutative and Lorentz symmetric [1,5,6.39-41,49,67,85,153,129,156,163,179]. It affects SM or rather  $SM_G$ , including QED, non-abelian Yang Mills, and gravity/spacetime. All of our Physics is fundamentally 2D, and it's all what matters. 4D effects of spacetime are macroscopic approximations. Furthermore, [184,185] showed that in the presence of gravity, stability of QFT, Yang Mills and SM, implies that particles are non-point-like, but rather stable black hole Qballs, and spacetime is multi-fold.*

*This 2D Physics is frustration free as it only involves UU interactions and massless bosons random walks [151]. [152] shows that the 2D physics has therefore a positive probability equal to 1 to be gapped on an translation invariant 1D spatial lattice.*

*Therefore, 2D QED is gapped, we have seen that it is the case, and 2D Yang Mills is gapped, as we also established. It is related also to the confinement that ensures that each are bound by confinement domains. It does not matter that the universe is expanding and finite, the latter also helping. When deconfinement takes place at high enough density, the Pauli exclusion principle for fermions is expected to ensure also a mass gap as colored quarks can't cross the 1D spatial barrier other than by slow tunneling. Gravity, by adding attraction, could ensure the same for colored scalars and colored bosons. They all appear gapped.*

*2D gravity and spacetime on the other hand is not gapped as we know 1) that there are no gravity shields [1,200] and 2) that the massless Higgs bosons are dilatons, that implements 2D gravity, and concretize spacetime [1,41,153,165,172,179], as well as play key roles in the multi-fold theory like the gravity electroweak symmetry breaking [13]. See all the papers compiled at [84]. Therefore, one could incorrectly expect that the massless Higgs boson i.e., the (Higgs field) would also be gapped. Indeed, it isn't gapped, because the multi-fold spacetime expands [1,38,66], and so, we re-encounter our earlier boundary considerations: the massless Higgs bosons never encounters the spacetime boundary. Without a mass gap, bosons can be freely created anytime from the ground / vacuum, which hints how it can create and concretize at will spacetime locations [1,32,41,66,153,165,172,179].*

*In other words, we also show that in truth, when considering a 2D random walk model, the system is mass gapped, except for gravity and the massless Higgs boson dilaton. For the rest, we can validated the mass gap formulation, including abelian fields, in multi-fold universes. As already stated, much of the results of the multi-fold theory like [1,6], as well as how it contributes to potentially address open issues with the SM and the standard cosmological model ( $\Lambda$ CDM) [1,2,5,6,13,18,22,27,28,35-45,47-52,66,67,72,73,84,85,92-105,108-139,153-186,190-193,200].*

*The conclusions of this section is that the Millenium prize formulation of the mass gap challenge for Yang Mills non-abelian, and abelian fields is always validated for 2D physics, except for gravity / space time and associated Higgs field modeled with massless Higgs bosons.*

## 14. Ultimate Unification (UU), or Weak Gravity Conjecture (WGC)

The asymptotic freedom mechanisms that describe how the electroweak chiral interaction and strong interaction fade away, match the expectation of the UU described in [1,27,119,153,165]. Only hypercharge-based electromagnetism and gravity remain in play. The reasonings can then be repeated as in [1,27,119,153,165].

These fading away effects certainly apply also to more conventional, a priori non-multi-fold universe<sup>30</sup>, to ensure that GUTs / TOEs have no places anymore, something we also became convinced based on proton decay, magnetic monopole, superstring and super symmetry considerations [1,28,36,38,40,41,47,72,73]. In such a universe, one may also pursue the WCG [31], if one wants to. But we do not agree with the strict inequality [1], and we understand that it also does not lead to GUTs, nor superstrings.

In either cases, it indicates a particle desert above the electroweak symmetry breaking energy scales, unless if there are new interactions (non-supersymmetry based), as we already discussed in [28,72,73]. Others have also argued this [29,30]. As a result we conjecture that no new fundamental particle will be discovered at the next run of the LHC at CERN, or elsewhere in the future [28].

## 15. Conclusions

This paper provides some key contributions, granted that some will not like the approach, and certainly that our result may be a surprise.

We have proven the millennium price mass gap for a discrete spacetime, especially if GR-based, or just includes gravity. It does not have to be multi-fold, although there are many indications that in the presence of gravity, such spacetime is multifold, as is therefore our real universe.

We have shown that on a continuous spacetime the mass gap may sometimes be invalidated: it does not always hold for non-abelian Yang Mills fields, and it does not hold for abelian fields in 4D. It is an important results<sup>31</sup>. That result does not mean that there is no mass gap for the SM Yang Mills fields; it is a generic result for any Yang Mills field, in 4D, even if just considering non-abelian fields. indeed, we noted that asymptotic safety, and the detailed structure of the Yang Mills potential may allow for a mass gap, say in the context of the SM. This generic undecidability is aligned with the undecidability of the mass gap problem that was already established in the literature. In particular, it could but may not hold for SU(2) and SU(3). And we explained why a discrepancy with lattice QCD analyses makes sense. This result is not tied to multi-fold universes.

We also proved the mass gap for multi-fold 2D Physics, including abelian fields, but excluding Higgs boson based dilaton 2D gravity, and argue compelling hints that this holds in our real universe.

The paper also reemphasized a few key aspects:

- How to make SM ( $SM_c$ ) consistent on a discrete spacetime. *Note added on July 23, 2023: [184-185] seems to indicate that coexistence of the SM and gravity implies taking place in a multi-fold universe to be consistent, where particles are microscopic black hole Qballs, or random walks, instead of point particles, and spacetime is discrete.*
- The likelihood that we will encounter a fundamental particle desert at energies above the electroweak symmetry breaking energy scales [28], except for the massless Higgs boson.

The paper also presented some new ways to explain some properties of asymptotic safety and confinement, and chiral symmetry breaking, as well as counter examples of deconfinements and abelian fields. We also explained

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<sup>30</sup> Hard to say with a straight face considering [6]. But then again one may dispute that our real universe is fully based on GR. *Note added on July 23, 2023: Yet based on [184,185], SM and gravity coexistence seems to imply a multi-fold universe, no matter the (reasonable) nature of gravity.*

<sup>31</sup> Unfortunately, proving something different and invalidating the original conjecture may not be what the Millennium Institute intended, when it set up the prize for resolution of its mass gap problem.

the link between confinement, mass gap and chiral symmetry breaking. Much of this analysis is not limited to multi-fold or discrete spacetime. The explanations provided are different from what is usually described in the literature where symmetry/group theory [11], or lattice computations are usually invoked, or even derivations à la Simonov when assuming that the ground state is the same as the 0 mass case [42,193,197,198].

We further discuss suitability of the SM, or rather  $SM_G$ , on a lattice, including the Nielsen–Ninomiya theorem, anomalies and fermion doubling. The latter is resolved in a way directly related to the 7D multi-fold space time matter induction and scattering. The analysis is done above and below the energy scales of the multi-fold energy electroweak symmetry breaking; ensuring full consistency of the model. Above, the mechanisms to handle Yukawa potentials impact on asymptotic freedom, recovers consistent hints of the UU.

We proposed new scenario above the energy level of the electroweak symmetry breaking, where asymptotic freedom kicks in and the strong interaction as well as the chiral part of the weak interaction fade away, opening the door to a ultimate unification approach instead of GUTs. The proposal does not change the conclusions that the mass of hadrons, nuclei and other quark/gluon composite particles results essentially from the chiral symmetry breaking and confinement, rather than from the mass conveyed to the constituents via the Higgs mechanisms.

Interestingly the 2D result is all what matters: there, when modeled bottom up as 2D random walks, except for gravity, everything's physics is indeed gapped. The 4D cases are more effective theories that may still appear and remain not gapped. It is what we may have for 4D Yang Mills non-abelian fields and what we have for 4D QED, which are abelian.

While maybe not what we expected or demanded by the Millenium prize map gap problem formulation, we argue that we have settle the problem, in 2D, multi-fold, discrete or continuous spacetime, to the best of what can be generically said. The conclusions on the strict formulation of the Millenium prize:

- Invalidated in general in a 4D continuous spacetime unless the formulation of the problem is reworked. It can then be valid for specific fields and groups/potential/couplings.
- In general valid in a 4D discrete spacetime if the formulation of the problem is reworked.
- In general valid in a multi-fold universe (4D) if the formulation of the problem is reworked.
- Valid for the 2D Yang Mills Physics that underlines bottom up all 4D physics in a multi-fold universe, and possible or real universe if it is multi-fold.

## Appendix: Review of the multi-fold theory

Before revisiting the alleged cracks in the context of a multi-fold universe, with otherwise the same Physics and observations as in our real universe, we probably need to provide some points to the multi-fold theory. It was introduced in [1]. Tutorials and overviews can be found at [72,73,84,154] while the latest developments, updates and discussions can be found at [84].

In a multi-fold universe [72,73,84,154], gravity emerges from entanglement through the multi-fold mechanisms. As a result, gravity-like effects appear in between entangled particles [1,96,97], whether they be real or virtual. Long range, massless gravity results from entanglement of massless virtual particles [1,96]. Entanglement of massive virtual particles leads to massive gravity contributions at very small scales [1,114]. It is at the base of the E/G Conjecture [96], and the main characteristics of the multi-fold theory [154]. Multi-folds mechanisms also result in a spacetime that is discrete, with a random walk fractal structure and non-commutative geometry that is Lorentz invariant and where spacetime nodes and particles can be modeled with microscopic black holes [1,5,13,39,52,66,67,156]. All these recover General Relativity (GR) at large scales, and semi-classical model remain

valid till smaller scale than usually expected. Gravity can therefore be added to the Standard Model (SM) resulting into what we define as  $SM_G$ : the SM with gravity effects non-negligible at its scales. This can contribute to resolving several open issues with the Standard Model without new Physics other than gravity. These considerations hint at an even stronger relationship between gravity and the Standard Model, as finally shown in [155].

Among the multi-fold  $SM_G$  discoveries, the apparition of an always-in-flight, and hence non-interacting, right-handed neutrinos, coupled to the Higgs boson is quite notable. It is supposedly always around right-handed neutrinos, due to chirality flips by gravity of the massless Weyl fermions, induced by 7D space time matter induction and scattering models, and hidden behind the Higgs boson or field at the entry points and exit points of the multi-folds. Massless Higgs bosons modeled as minimal microscopic black holes mark concretized spacetime locations. They can condensate into Dirac Kerr-Newman soliton Qballs to produce massive and charged particles [1,52], thereby providing a microscopic explanation for a Higgs driven inflation, the electroweak symmetry breaking, the Higgs mechanism, the mass acquisition and the chirality of fermions and spacetime; all resulting from the multi-fold gravity electroweak symmetry breaking. Massless particle on the other hand result from patterns of the random walks. The multi-fold theory has also concrete implications on New Physics like supersymmetry, superstrings, M-theory and Loop Quantum Gravity (LQG) [1,72,73,84,38-41,47,52-57].

The multi-fold paper [1] proposes contributions to several open problems in physics, like the reconciliation of General Relativity (GR) with Quantum Physics, explaining the origin of gravity proposed as emerging from quantum (EPR- Einstein Podolsky Rosen) entanglement between particles, detailing contributions to dark matter and dark energy, and explaining other Standard Model mysteries without requiring New Physics beyond the Standard Model other than the addition of gravity to the Standard Model Lagrangian [1,2,5,6,13,18,22,27,28,35-45,47-52,66,67,72,73,84,85,92-105,108-139,153-186,190-193,200]. All this is achieved in a multi-fold universe that may well model our real universe, which remains to be validated.

With the proposed model of [1], spacetime and Physics are modeled from Planck scales to quantum and macroscopic scales, and semi-classical approaches appear valid till very small scales. In [1], it is argued that spacetime is discrete, with a random walk-based fractal structure, fractional and noncommutative at, and above Planck scales (with a 2-D behavior and Lorentz invariance preserved by random walks till the early moments of the universe). Spacetime results from past random walks of particles. Spacetime locations and particles can be modeled as microscopic black holes (Schwarzschild for photons and concretized spacetime coordinates, and metrics between Reissner Nordström [50], and Kerr Newman [51] for massive, and possibly charged, particles – the latter being possibly extremal). Although possibly surprising, this recovers results consistent with others (see [52], and its references), while also being able to justify the initial assumptions of black holes from the models of gravity or entanglement in a multi-fold universe. The resulting gravity model recovers General Relativity at larger scale, as a 4D process, with massless gravity, but also with massive gravity components at very small scales, which make gravity non-negligible at these scales. Semi-classical models also turn out to work well till way smaller scales than usually expected.

Multi-folds are encountered in GR at Planck scales [6,50] and in Quantum Mechanics (QM) if different suitable quantum reference frames (QRFs) are to be equivalent relatively to entangled, coherent or correlated systems [51]. This shows that GR and QM are different facets of something that they cannot well model: multi-folds.

Considering results as in [6,41,48-51,118,155,167,176,179], and our answers to so many open issues with the SM and the  $\Lambda$ CDM can be qualitatively explained with the  $SM_G$  and multi-fold mechanisms, as discussed for example in [1,2,5,6,13,18,22,27,28,35-45,47-52,66,67,72,73,84,85,92-105,108-139,153-186,190-193,200], we can then argue that these conclusions can probably apply to our real universe, especially considering how the multi-fold mechanisms recover GR [1,50], and can be encountered in GR at Planck scales, with the spacetime reconstruction [1,156], and with the top-down-up-and-upper derivation of the multi-fold theory [6]. At the risk of repeating ourselves, as a result spacetime is, at very scales sales, discrete, generated by random (Levy) walks, and therefore (multi-fractal), non-commutative and yet Lorentz symmetric [1,5,6,41,49,153,156,161,176,179].

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