Null Algebra resolutions of Complex Exponentiation
Null Algebra Extension III

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Abstract:

This paper explores the Null Algebra and traditional Algebra resolutions for the complex number i^i. It explains the apparent differences between the Null Algebra resolutions, and those of traditional Algebra, which uses the substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$. The methods shown herein explore the full set of subspace equations implied by a given equation whose output results in i^i for a specific input. It is shown that the values obtained from the various possible resolutions of i^i are found on some aspect of the given equation, or its expanded subspace equation sets. It shows i^i equals -1 and +0.20788.

It is assumed the reader has read and understood Null Algebra, as well as Null Algebra Extensions I and II in addition to standard Algebra and Trigonometry. The Null Algebra texts are available for download at (https://vixra.org/abs/2103.0131), (https://vixra.org/abs/2206.0135) and (https://vixra.org/abs/2304.0205). If you have not yet read these texts and attempted the examples contained therein for yourself it is highly suggested you do so before reading further as some concepts explained in detail there, are given only cursory review here. Without reading these prerequisites you may not fully understand the reasoning behind logic used in the equations of this text.

This version of the paper is submitted as a correction to original submission on 9 August 2023 of the same title. The explanation of how various solutions were arrived at, and the graphs of those solutions plotted, did not include a necessary step addressing $\oplus$ numbers, raised to $\oplus$ exponents. Those errors have been corrected in this paper.

1.0—Review of Logs and Exponents:

Before exploring complex exponentiation and their Null Algebra resolutions it's important to understand the common rules for logarithms and exponents. Where exponents raise a base value to a given power, logarithms provide the power necessary to obtain a result from a given base. Logs are the inverse of exponentiation. It is hoped these will provide the reader with a refreshed understanding of exponentiation and logarithms, to better facilitate understanding during progression through the text.

1.1.a—Exponentiation and Logarithm definition:

Exponentiation:	Logarithm:
$a^n = m$	$\log_a m = n$

Note, if the base value $a = \mathbf{e} \approx 2.71828$ the nomenclature used is the natural log: **ln**

1.1.b—Rules of Logs, and Natural Logs:

Log Product Rule	$\ln(x \cdot y) = \ln(x) + \ln(y)$	$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$
Log Quotient Rule	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$	$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
Log Reciprocal Rule	$\ln\left(\frac{1}{x}\right) = -\ln(x)$	$\log_a\left(\frac{1}{x}\right) = -\log_a(x)$
Log Power Rule	$\ln(x^y) = y \cdot \ln(x)$	$\log_a(x^y) = y \cdot \log_a(x)$
Logs of negative values	$\ln(-x) = \emptyset$	$\log_a(-x) = \emptyset$
Log of 0	$\ln(0) = \emptyset$	$\log_a(0) = \emptyset$
	$\lim\limits_{x \to 0^+} \ln x = -\infty$	$\lim\limits_{x \to 0^+} \log_a x = -\infty$
Log of 1	$\ln(1) = 0$	$\log_a(1) = 0$
Log of ∞	$\ln(\infty) = \infty$	$\log_a(\infty) = \infty$
Argument = base value	$\ln(\boldsymbol{e}^x) = x$	$\log_a(a^x) = x$
If $x = 1$	$\ln(\mathbf{e}) = 1$	$\log_a(a) = 1$

Value raised to power of a log with same base	$e^{\ln(x)} = x$	$a^{(\log_a x)} = x$
Log Derivative	$\frac{d}{dx}\ln x = \frac{1}{x}$	$\frac{d}{dx}\log_a(x) = \frac{1}{x\log a}$
Log Integral	$\int \ln(x)\ dx = x \cdot \ln(x) - x + C$ $\int \log(x)\ dx = x \cdot \log(x) - x + C$	
Euler Identity	$\ln(-1) = i\pi$	$\mathbf{e}^{i\pi} = -1$
Change of base	$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$	

*** Miller's Identity:** $1^i = 1$

Reciprocal-*i* Identity: $i = -\frac{1}{i}$

Exponentiation of a Product: For: $a = b \cdot c$ Let: $a^n = d$

Then: $a^n = d \quad \equiv \quad (b \cdot c)^n = b^n \cdot c^n = d$

<u>Rules for Exponents:</u>

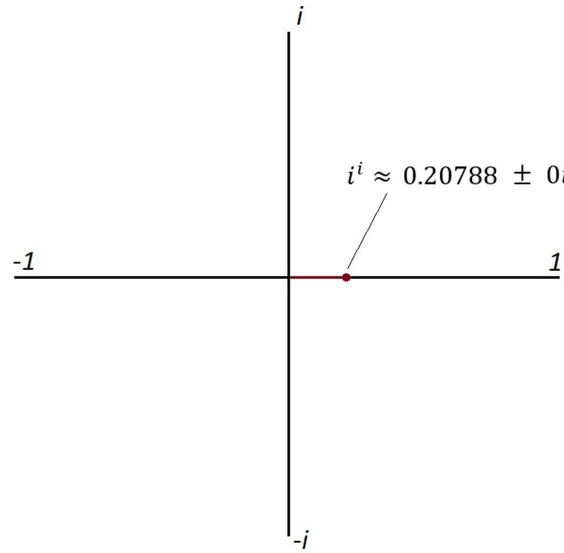
$n^0 = 1$ $\qquad n^1 = n$ $\qquad n^a \cdot n^b = n^{(a+b)}$

$\frac{n^a}{n^b} = n^{(a-b)}$ $\qquad n^{-a} = \frac{1}{n^a}$ $\qquad (n^a)^b = n^{ab}$

$(nm)^a = n^a m^a$ $\qquad \left(\frac{n}{m}\right)^a = \frac{n^a}{m^a}$

* I apologize for the small arrogance here. I could not find another reference to this identity beside that which I published at https://vixra.org/abs/2307.0124 on 24 July 2023. It seems an important identity. I have to call it something.

2.0—Aspects of i^i:



Null Algebra resolutions of i move values away from the complex plane into a real space, which includes one or more subspaces. The calculation of values using i as a base or as an exponent have more commonly been resolved using standard trigonometric properties to provide solutions. Even when the result of a trigonometric substitution is a real number it still can be thought of as possessing an imaginary part of $\pm 0i$.

The standard calculation of i^i will provide a real number solution. However, this is still on the complex plane. The trigonometric substitutions of *i* are still on the complex plane whereas the Null Algebra resolutions are removed to a plane composed of real and subspace axis.

This real value, $i^i \approx 0.20788$, on the complex plane is simply not displaced along the *i*-axis; it represents an output value, defined as a two-dimensional number of the form $a \pm bi$ for which the complex magnitude $b = 0$.

$$i^i \approx 0.20788\ldots \pm 0i$$

The Complex Plane is used as an output plane, showing single values of the output variable. For an equation of the form $y = f(x)$, the complex plane is composed of a real *x*-axis and the imaginary *i*-axis. We plot on it two-dimensional points for each *y*-axis output value in terms of *x* and *i,* as defined by a given $y = f(x)$ equation. So, though we plot points for equations of the form $y = f(x)$ on the Central Plane (*the XY-Plane*), we do not see the actual output *y*-axis of the central plane or any of the subspace axis associated with either the *x,* or the *y* axis. We see the Complex Plane, the entire expanse of which represents the two-dimensional *y*-axis output points.

More complicated is the trigonometric property used to obtain a value for i^i. It removes the *i* from the equation. This results in the standard value of ≈0.20788, obtained through Hyperbolic Trigonometry. The following sections will explore the calculation of i^i and its resolution from the complex plane to subspace hyperplanes.

2.1—Traditional Algebraic and Trigonometric calculation of i^i:

Usage of trigonometric identities and a Maclaurin series expansion on i^i yields the following:

$$i^i \approx 0.20788\ \ldots$$

This value is slightly larger than $^1/_5$, by not quite eight-thousandths. It is also a real number. Before we explore how this value is reached by traditional methods let's explore the immediately obvious Null Algebra resolution, which appears to stand in direct contradiction to the approximate 0.20788 value.

2.1.a:

Null Algebra resolves instances of i at their place of occurrence for a given equation on the central plane to its positive, up value, and to its negative, down value on the corresponding co-adjoining subspace. For an equation of the form $y = f(x)$, where elements of $f(x)$ produce i-multiples for inputs of x, we have

$$y = i = \oplus 1 = \begin{cases} xy - \text{Plane} & y = \hat{1} \;\; \rightarrow \;\; +1 \\ sy - \text{Plane} & y = \check{1} \;\; \rightarrow \;\; -1 \end{cases}$$

This result is the standard way of resolving a plus and minus number. One example of how this can arise would be an equation of the form $y = \sqrt{-|x|}$. However in the given example equation we will use in this paper, $y = \sqrt{x}^{\sqrt{x}}$ we have a $\oplus$ number being raised to a $\oplus$ power for sections of the domain which generate i-multiples. In this instance we must apply the $\oplus$ sign as an exponential power before resolving other features, which results in a squaring of the sign. It will also cause for this example $y = i^i$ to resolve to -1 on both the xy and sy planes. Failure to follow this augmentation to rules of operations will result in incorrect values. This will be shown in more detail momentarily.

2.1.b:

If we use the equation, $y = \sqrt{x}^{\sqrt{x}}$

- The point i^i, occurs when $x = -1$. It is but one of an infinite number of points on this graph.

Thus for $x = -1$

If we ignore the need to raise the base plus-and-minus number to its plus-and-minus exponent we receive the following erroneous result.

2.1.b.i:

$$y = \sqrt{-1}^{\sqrt{-1}} \doteq \oplus\, 1^{\oplus 1} \;\doteq\; \hat{1}^{\hat{1}} \;\doteq 1$$

If we apply the exponential value first it results in the correct answer.

2.1.b.ii:

$$y = \sqrt{-1}^{\sqrt{-1}} \doteq \oplus\, 1^{\oplus 1} \doteq\; -1^{1} = -1$$

The exact process for this and reasoning for these steps are covered later.

2.1.b.iv—The Apparent Contradiction between Algebraic, Trigonometric and Null Algebra:

Algebraic: i^i Trigonometric: $i^i = 0.20788$ Null Algebra: $i^i = -1$

Which of these is correct? They all are. As we explore the *xy, sy*, *xu* and *su* planes. We'll find on these graphs both $y = -1$ and $u = 0.20788$.

2.1.c—Calculating $\boldsymbol{i^i}$ using Euler's Identity:

Euler's Identity is derived from the Maclaurin expansion of $\mathbf{e}^{ix}$ for the value $x = \pi$. This can be used to assist in calculating values on the complex plane.

Euler's Identity: $\mathbf{e}^{i\pi} = -1$

This is a 180 degree turn on the complex plane. So to reach i we need only a 90° rotation of half Pi.

$$\mathbf{e}^{i\frac{\pi}{2}} = i$$

Thus the expression i^i may be rewritten as:

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}}$$

All that remains then is to raise the base value of **e** to the power of $-\frac{\pi}{2}$.

$$\mathbf{e}^{-\frac{\pi}{2}} \approx 2.71828^{-1.570796} \approx \frac{1}{2.71828^{1.570796}} \approx 0.20788$$

2.1.d—The Hyperbolic Calculation of $\mathbf{e}^{-\frac{\pi}{2}}$:

Recall that the Maclaurin Series expansion for approximating a function centered at the point $x = 0$, is given by:

$$\sum_{n=1}^{\infty} \frac{f^n(0) \cdot x^n}{n!}$$

2.1.d.i:

When performing the Maclaurin Series expansion on $\mathbf{e}^x$ and $\mathbf{e}^{-x}$ (*with no* ***i*** *in the exponent*) the resulting series components are equated to $\mathbf{e}^x = \cos x + \sin x$, and $\mathbf{e}^{-x} = \cos x - \sin x$. This results in the Hyperbolic Pythagorean identity for which the nomenclature of *cosh* and *sinh* are used to ensure understanding that hyperbolic trigonometry is being used.

$\mathbf{e}^x = \cosh x + \sinh x$ $\mathbf{e}^{-x} = \cosh x - \sinh x$

$\mathbf{e}^x \cdot \mathbf{e}^{-x} = 1 = \cosh^2 x - \sinh^2 x$

From this perspective you can easily verify that $\mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$

$\mathbf{e}^x = \cosh x + \sinh x$ for $x = -\frac{\pi}{2} = -1.5707963$

$\mathbf{e}^{-\frac{\pi}{2}} = \cosh(-1.5707963) + \sinh(-1.5707963) \quad \rightarrow \quad 2.509178478658 - 2.3012989$

$\mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 \ldots$

3.0—Rationalizing Null-Algebraic and Traditional Algebraic Solutions of i^i:

Thus 2.1.c provides a confirmed algebraic solution for i^i. 2.1.d provides a confirmed trigonometric solution for the same. The two methods of approach are in agreement that

$$i^i \approx 0.20788 \ldots$$

- *Remember this is still a real-only component of a number $z = a + bi \;=\; a + 0i$. It is still on the complex plane. Null Algebra resolutions translate values onto planes composed of real and subspace axis. Due to these differences there is no reason to believe the value obtained from the trigonometric exchange, which is really a different equation than the given equation will appear on the XY-Plane, or even at the same input..*

3.1—Is $i^i \approx 0.20788$ on the *y*-axis?

The trigonometric substitutions used to derive the ≈ 0.20788 solution involves squaring an *i*.

3.1.a:

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{i \cdot i \frac{\pi}{2}} = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$$

Null algebra and traditional algebra are in agreement that a i^2 equals -1

3.1.b:

$$i^2 = i \cdot i = -1$$

The Null Algebra resolutions indicate this value could be on one of two given axis. $i^2 = -1$ represents a 180 degree rotation on the complex plane. Null Algebra Extension II indicates, given an equation of the form $y = f(x)$, when squaring i we are referring to a boundary between the Central Plane and the Co-adjoining Subspace. The arguments provided in Null Algebra Extension II focus on the input values. If instead we are talking about the output *y*-axis, that same boundary could be referring to the Central Plane, the Posterior Subspace or the Transverse Plane. Once we determine which of these axis that value pertains to, we would also expect to find its opposite sign value (*its conjugate*) on the paired subspace axis. See Null Algebra Extension II for more detailed analysis of this topic.

So because we have squared *i* the value we get according to Null Algebra Extension II the value obtained could occur on a real axis or its parried subspace. The trigonometric substitution is in fact made against the *y*-axis output value. The equation which generates i^i is dependent upon the *x*-axis input, $y = \sqrt{x}^{\sqrt{x}}$ and occurs when $x = -1$. But the value $y = i^i$ is an unresolved, two-dimensional number value, plotted for this example, on only the *y*-axis.

3.1.c:

$$\boldsymbol{y = i^i} = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} = \mathbf{e}^{i \cdot i\frac{\pi}{2}} = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 \pm 0i$$

So this value 0.20788 could be on either the *y* or *u* axis. Another reason for this that the resolution, $\left(\mathrm{e}^{i\frac{\pi}{2}}\right)^{i}$, though equal to i^i, the general equation which produces that substitution is not identical to our example equation producing i^i. It just happens that for $x = -1$ on the $y = \sqrt{x}^{\sqrt{x}}$, we have the trigonometric identity $i = \mathrm{e}^{i\frac{\pi}{2}}$ which can be used to swap out for the base *i*. As a general equation it the source of the trigonometric substitution has the form

3.1.d:

For $-\infty < x \leq 0$ in $y = \sqrt{x}^{\sqrt{x}} = ri^{ri}$ where $r = \sqrt{|x|}$

Then, $$ri^{ri} = \left(r\mathrm{e}^{i\frac{\pi}{2}}\right)^{ri} = r^{ri} \cdot \mathrm{e}^{-r\frac{\pi}{2}}$$

It is only when $x = -1$ that $r = 1$ and $r^{ri} \cdot \mathrm{e}^{-r\frac{\pi}{2}} = \mathrm{e}^{-\frac{\pi}{2}}$.

This substitution provides a value for i^i but comes from a different equation than our given example equation of the $y = \sqrt{x}^{\sqrt{x}}$. So although it shares one value with our given equation, which we can use to obtain a resolution for i^i, there is no guarantee that value will actually be on the *y*-axis, nor is there any guarantee that value occurs when $x = -1$. Afterall the two-dimensional *y*-axis complex points, are composed of un-resolved values containing the number *i*. Resolving these values expands us into a series of subspace equations. The same situation occurs with resolving $r^{ri} \cdot \mathrm{e}^{-r\frac{\pi}{2}}$. So, $i^i = 0.20788$ may occur on the *u*-axis at an input value as yet unknown in terms of either *x* or *s*. The resolved instances of these numbers shows where values exist on several planes. We will find that the above listed solutions of -1 and 0.20788 are both present.

4.0—Resolving ⊕ number raised to ⊕ powers:
The following steps show the methods or resolving *plus-and-minus* numbers raised to *plus-and-minus* powers.

<table>
<tr><td colspan="2">Direct Resolution of i</td></tr>
<tr><td>$y = f(x) = \sqrt{x}^{\sqrt{x}}$ for $x = -1$</td><td>Given equation and a specified value.
Results in i^i for $x = -1$</td></tr>
<tr><td>$y = i^i = \oplus 1^{\oplus 1}$</td><td>Null Algebra resolution of $i = \oplus 1$. Applied to both instances of i.</td></tr>
<tr><td>$y = i^i = (\{\oplus\}^2 \cdot 1)^1$</td><td>In the standard trigonometric substitution $i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}}$ results when i in outside position is multiplied by the i inside the parentheses. This is consistent with rules for exponents on page 3, that $(n^a)^b = n^{ab}$. It results in a squared ⊕ sign for Null Algebra.

** See note below</td></tr>
<tr><td colspan="2">**

Due to the Null Algebra resolution of $i = \oplus 1$, we are now dealing with a sign rather than a number: $\left(\mathbf{e}^{\oplus\frac{\pi}{2}}\right)^{\oplus 1} = \left(\mathbf{e}^{(\oplus)^2 \cdot \frac{\pi}{2}}\right)^1 = \mathbf{e}^{-\frac{\pi}{2}}$. The ⊕ sign in the $y = i^i = (\{\oplus\}^2 \cdot 1)^1$ is simply a resolved i . In this resolved state, we distribute the outer ⊕ sign, leaving the expression raised to a given positive value, and now multiplies the base value by a squared sign.

The only way we can apply the sign, is as follows:
The situation of $(\oplus)^{\oplus}$ tells us the plus-and-minus sign in the parenthesis is both $(\oplus)^{+1}$ and $(\oplus)^{-1}$. Meaning $(\oplus)^{\oplus} = \frac{\oplus}{\oplus} \equiv (\oplus \cdot \oplus) = (\oplus)^2 = -1$.

To review the result of squaring a ⊕ sign see text on Null Algebra: The Math of Division by Zero and The Negative Radical (https://vixra.org/abs/2103.0131).</td></tr>
<tr><td>$y = (\{\oplus\}^2 \cdot 1)^1 = (-1 \cdot 1)^1 = -1$</td><td>All values are resolved.

For $x = -1$ $y = \sqrt{x}^{\sqrt{x}} = -1$</td></tr>
</table>

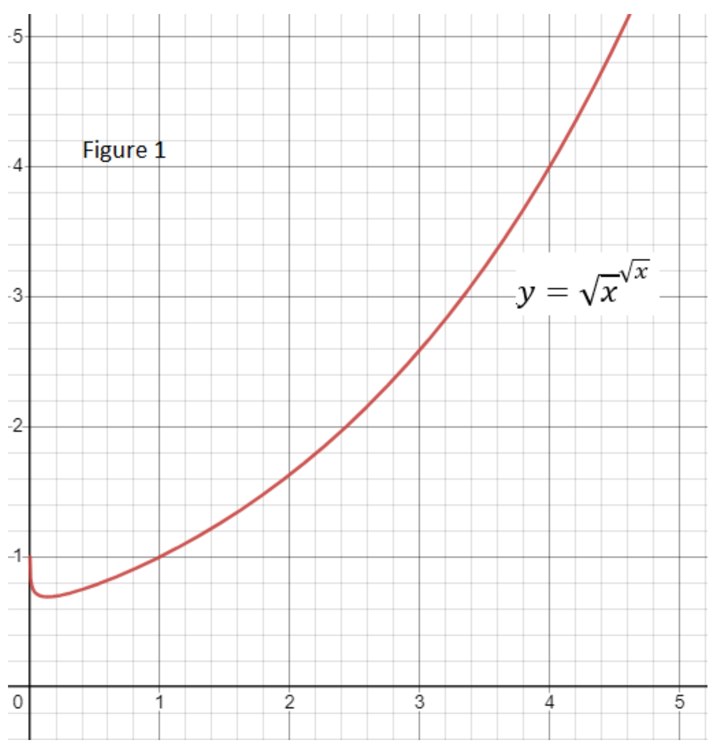
Substitution, Then Resolution of i	
$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i \doteq \left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1}$	i values resolved to $\oplus$ 1
$\left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1} = \left(\mathbf{e}^{(\oplus)^2\frac{\pi}{2}}\right)^1$	i exponent distributed as $\oplus$ into base value
$\left(\mathbf{e}^{(\oplus)^2\frac{\pi}{2}}\right)^1 = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$	Fully resolved value of 0.20788.

5.0—Analyzing graphs and values:

The graphs of the various equations are shown below, exploring the nature of the full subspace expansion of our given equation, $y = \sqrt{x}^{\sqrt{x}}$. Both -1 and 0.20788 are valid solutions, occurring on some section of the overall subspace hyper-plane sets.

5.1.a—$y = \sqrt{x}^{\sqrt{x}}$ For $0 \le x < \infty$

The graph of this function is shown here in Figure 1 over its traditionally valid domain. Its straight forward traditional algebra. For simplicity the graphs shown display only the positive output values for the roots.

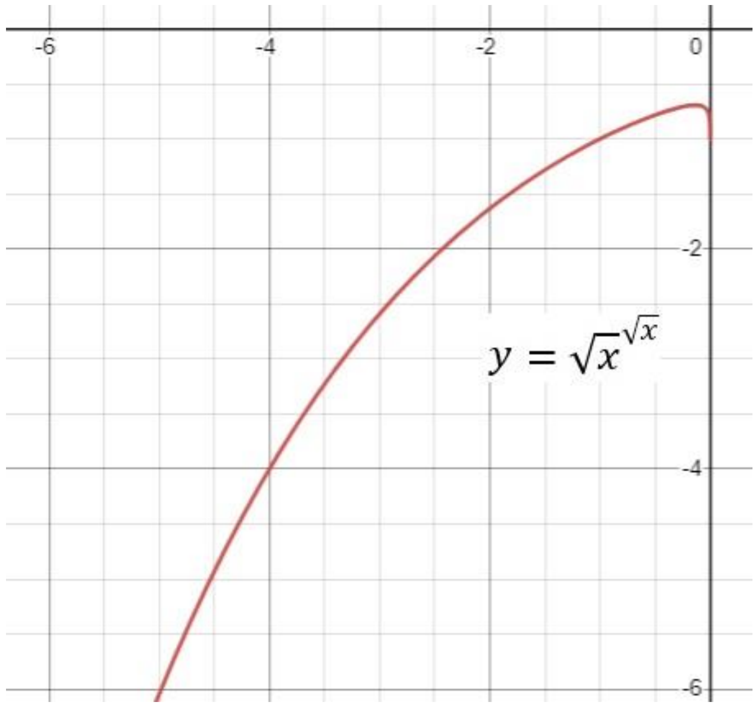


5.1.b— $y = \sqrt{x}^{\sqrt{x}}$ For $-\infty < x \leq 0$:
Figure 1a shows the output values for the roots, resolved from their i-multiple, to real number values using the methods discussed above in section 4.0.

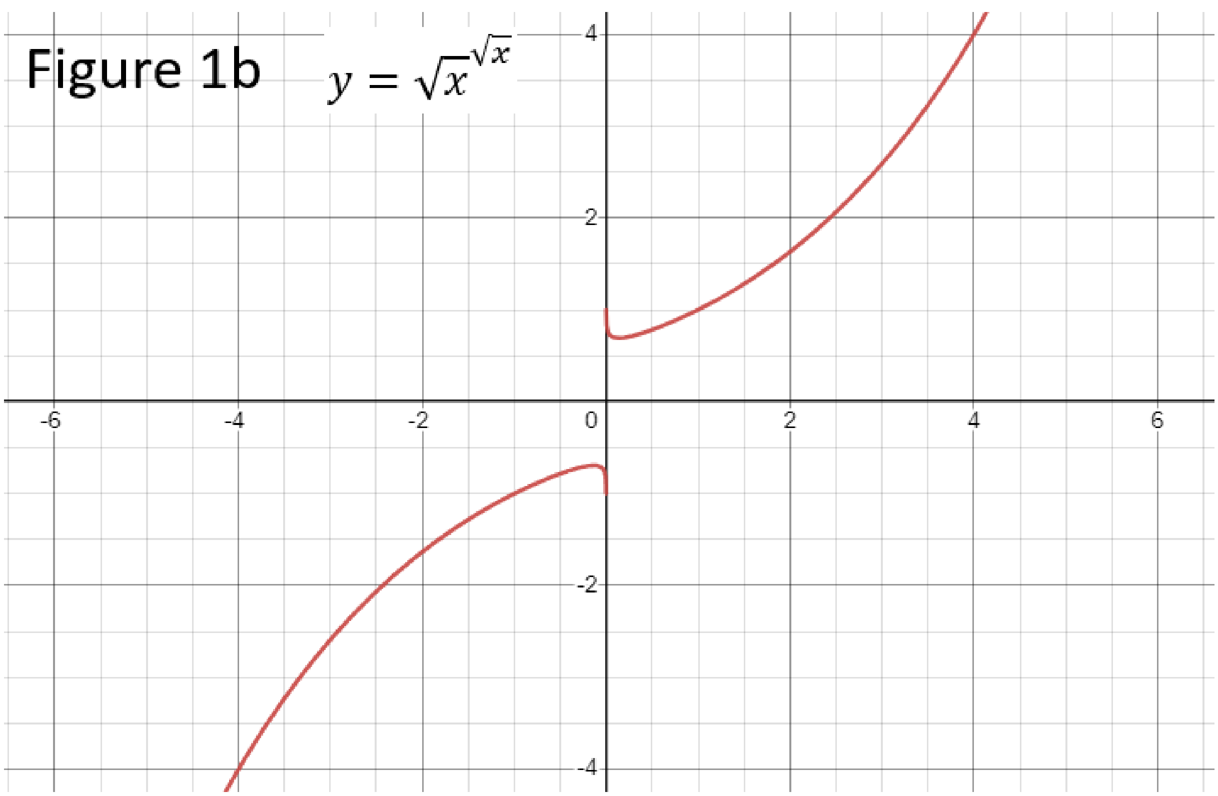
You can force a graphing utility to graph the function by the following steps.

$y = f(x) = \sqrt{x}^{\sqrt{x}}$ for $x = -1$	Given equation and a specified value. Results in i^i for $x = -1$
$y = \sqrt{-x}^{\sqrt{-x}}$	The inputs are switched with their negative value to force a standard graphing utility to graph the negative inputs. But we are still really using $y = \sqrt{x}^{\sqrt{x}}$ which generates i-multiples for each input over the domain $-\infty < x \leq 0$.
$y = \oplus\sqrt{-x}^{\oplus\sqrt{-x}}$	The recognize that the roots of the equation $y = \sqrt{x}^{\sqrt{x}}$ will be $\oplus$ number, resolved from an i-multiple state. We keep track of that by inserting the $\oplus$ sign in front of each radical.
$y = \{\oplus\}^2\sqrt{-x}^{\sqrt{-x}}$ $y = -\sqrt{-x}^{\sqrt{-x}}$	The $\oplus$ signs are squared in accordance with section 4.0 above. The equation is then simplified to its final state. Note this version of the equation of the one needed to force a graphing utility graph this function. The actual equation we are graphing is $y = \sqrt{x}^{\sqrt{x}}$ over the domain $-\infty < x \leq 0$. It is seen in Figure 1a.

In Figure 1aThe graph of $y = \sqrt{x}^{\sqrt{x}}$ over the domain $-\infty < x \leq 0$ shows clearly the domain $y = -1$ when the domain $x = -1$.



5.1.c:
We can put these two graphs directly together. Using the Null Algebra resolutions we see the full graph of $y = \sqrt{x}^{\sqrt{x}}$. See Figure 1b.



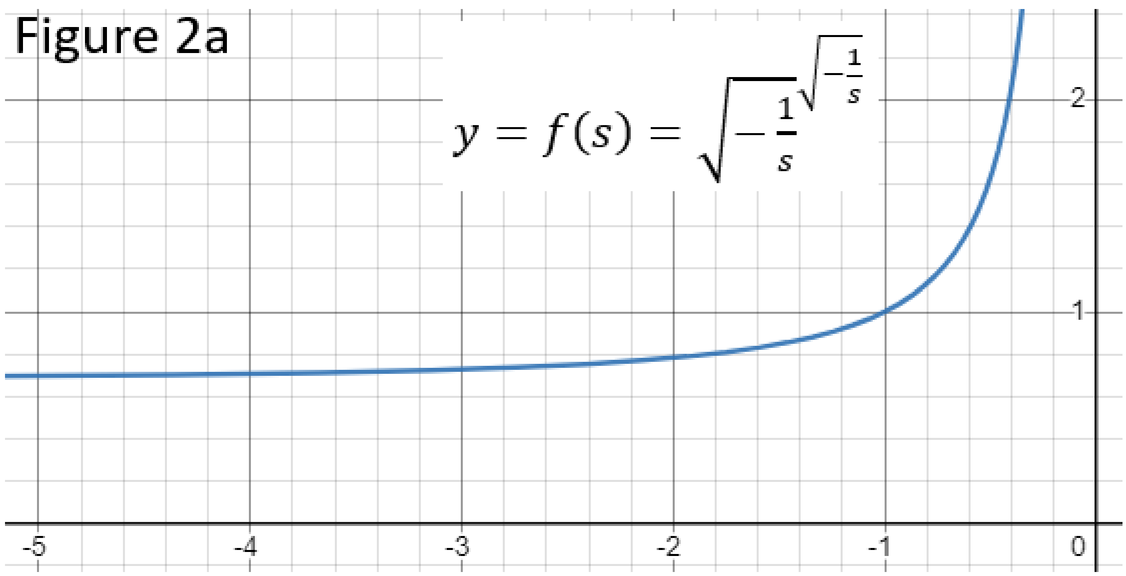
5.2.a—The Co-Adjoining Subspace $y = f(s)$, For $-\infty < x \leq 0$

Using the equation $y = \sqrt{x}^{\sqrt{x}}$ we use a subspace transform on the x input to obtain the s-axis subspace values which comprise the co-adjoining subspace equation.

5.2.a.i:

$x = -\frac{1}{s}$ $\qquad\qquad$ $y = f(s) = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$

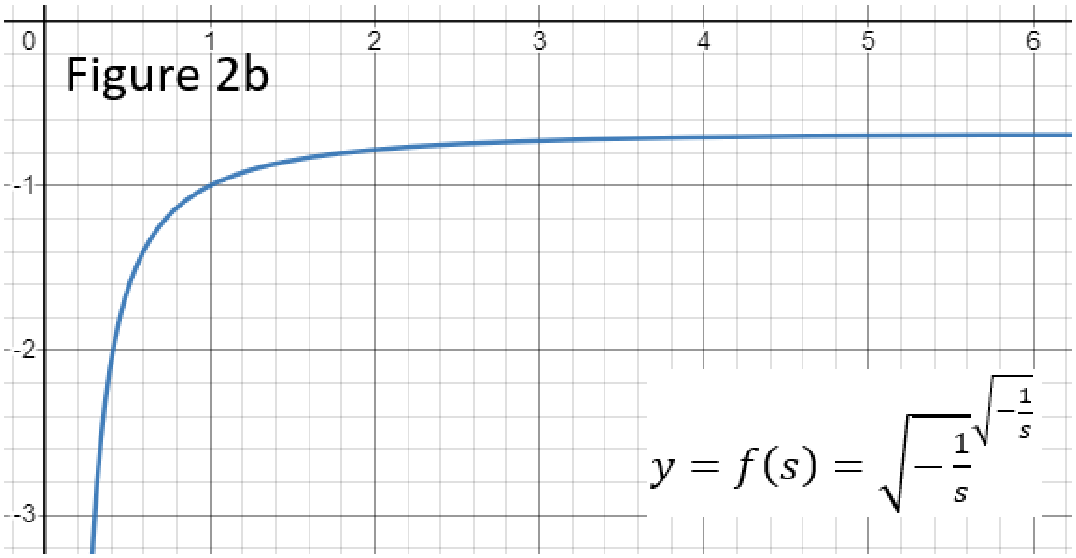
5.2.a.ii:
$y = f(s)$ is defined under the traditional Algebra domain of $-\infty < s \leq 0$. This is shown in Figure 2a.



5.2.b:
The extended domain of $0 \leq s < \infty$ will result in i-multiples. These *plus-and-minus* numbers we resolve to their down values on the s-axis of the co-adjoining subspace.

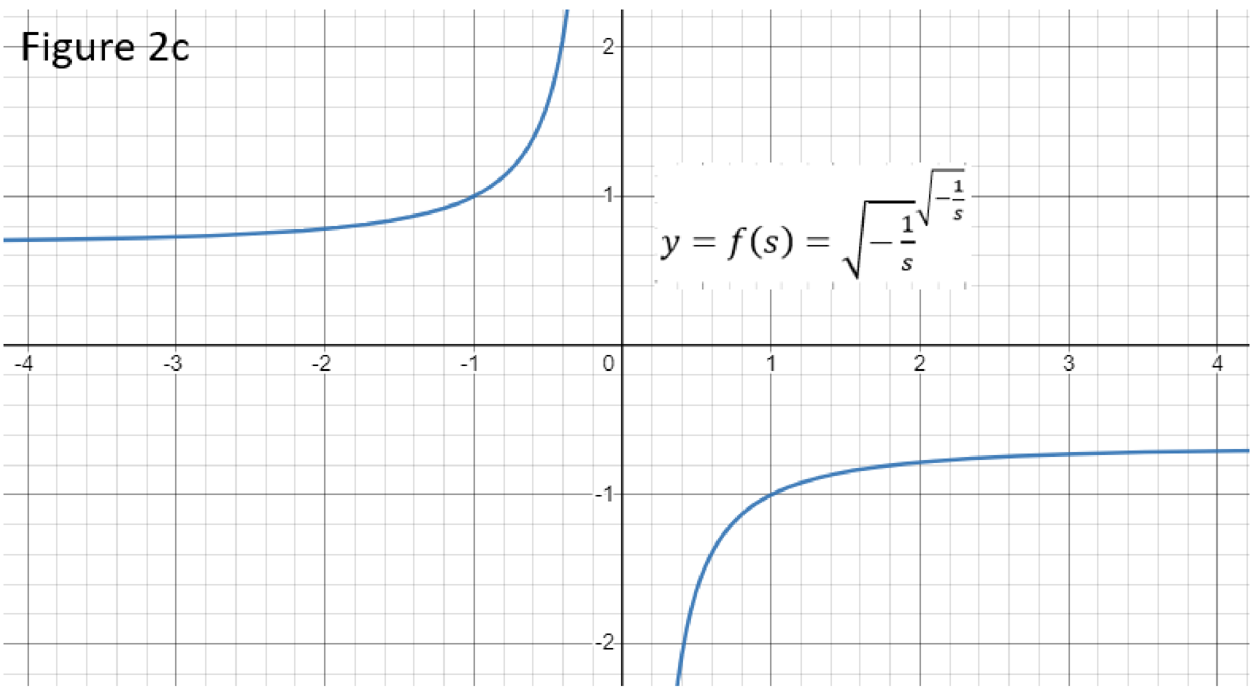
Using the same reasoning shown in section 5.1.b we can force a graphing utility to graph the equation as follows:

$y = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$ for $0 \leq s < \infty$	Given equation and a specified value. Results in i^i for $s = 1$ which corresponds to the value $y = i^i$ when $x = -1$. The $s = 1$ is the corresponding subspace value for $x = -1$. ‡ We should expect to find the output value $y = -1$ when $s = 1$. This is the squared value discussed in section 2.1.a above.
$y = \sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}$	The inputs are switched with their positive value, removing the negative sign. This will force a standard graphing utility to graph the positive inputs. But we are still really using $y = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$. It generates i-multiples for each input over the domain $0 \leq s < \infty$.
$y = \oplus\sqrt{\frac{1}{s}}^{\oplus\sqrt{\frac{1}{s}}}$	The recognize that the roots of the equation $y = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$ will be $\oplus$ number, resolved from an i-multiple state. We keep track of that by inserting the $\oplus$ sign in front of each radical.
$y = -\sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}$	The $\oplus$ signs are squared in accordance with section 4.0 above. The equation is then simplified to its final state. Note this version of the equation is the one needed to force a graphing utility graph this function. The actual equation we are graphing is $y = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$ over the domain $0 \leq s < \infty$. It is seen in Figure 2b.



‡ Indeed when $s = 1$, we find $y = -1$.

5.2.c:
The combined graphs showing the full domain of the $f(s)$ function is shown in Figure 2c.

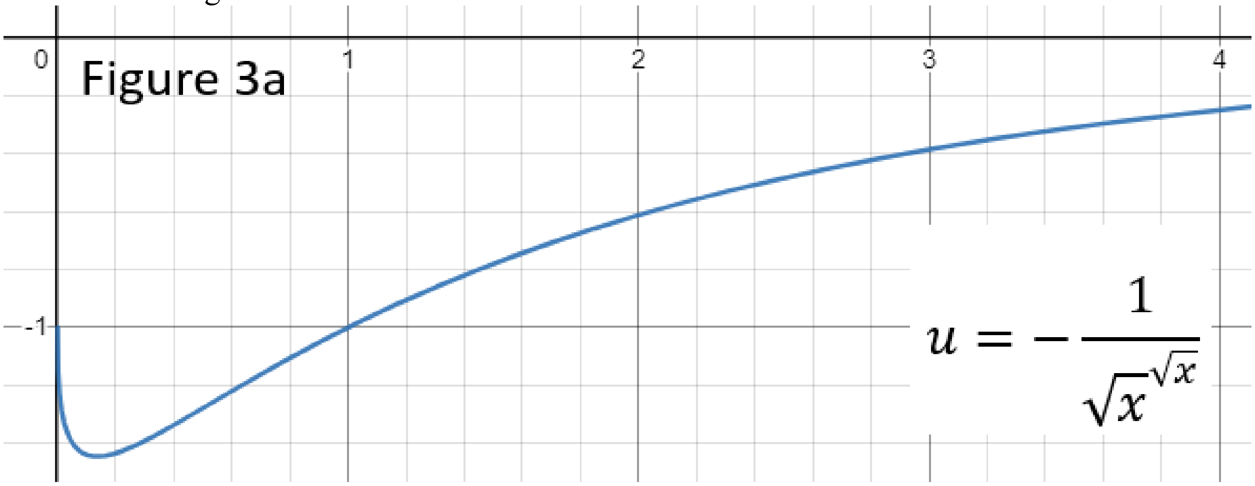


6.0—The Posterior Subspace $u = f(x)$:
Using the definition of section 6.0 we have the following.

$y = \sqrt{x}^{\sqrt{x}}$ $\qquad$ $u = -\frac{1}{y}$ $\qquad$ $u = f(x) = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$

6.1:

The function is traditionally defined over the domain of $0 \leq x < \infty$ and may be graphed directly. It is shown in figure 3a.

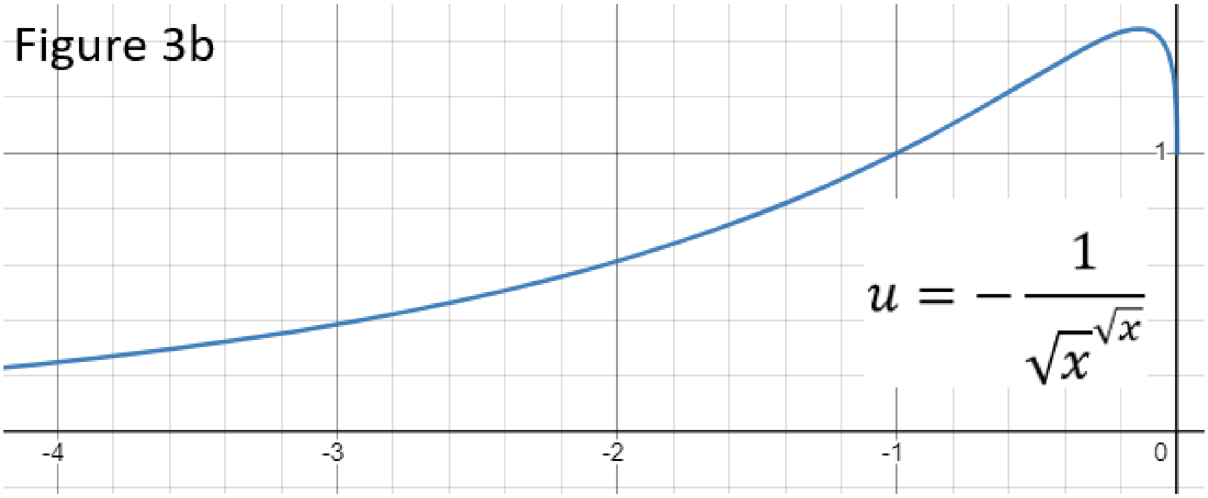


6.2:

The extended domain will result in i-multiples for $-\infty < x \leq 0$. We can force a graphing utility to graph the equation in accordance with section 4.0.

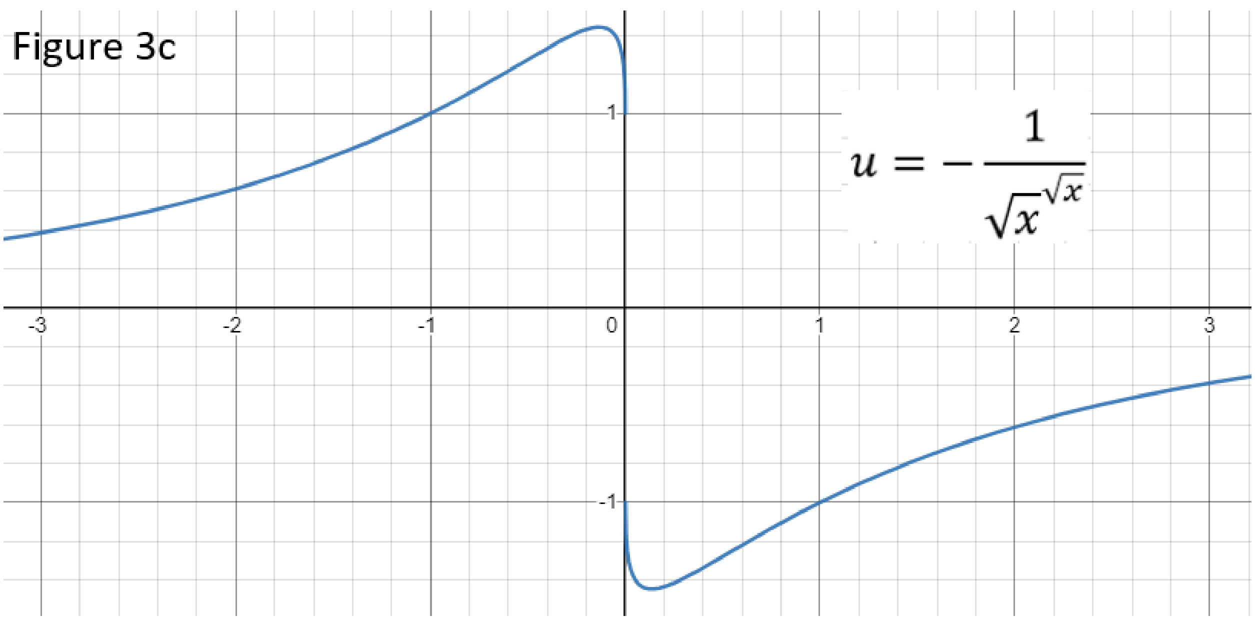
$u = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$	Given equation and a specified value. Results in i^i for $x = -1$.
$u = -\frac{1}{\sqrt{-x}^{\sqrt{-x}}}$	The inputs are switched with their negative value. This will force a standard graphing utility to graph the negative inputs. But we are still really using $u = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$. It generates i-multiples for each input over the domain $-\infty < x \leq 0$.
$u = -\frac{1}{\oplus\sqrt{-x}^{\oplus\sqrt{-x}}}$	We recognize that the roots of the equation $u = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$ will be $\oplus$ number, resolved from an i-multiple state. We keep track of that by inserting the $\oplus$ sign in front of each radical.
$u = -\frac{1}{-\sqrt{-x}^{\sqrt{-x}}} = \frac{1}{-\sqrt{-x}^{\sqrt{-x}}}$	The $\oplus$ signs are squared in accordance with section 4.0 above. The equation is then simplified to its final state. Note this version of the equation is the one needed to force a graphing utility graph this function. The actual equation we are graphing is $u = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$ over the domain $-\infty < x \leq 0$.

	It is seen in Figure 3b.



6.3:

The full graph of $u = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$ is shown in Figure 3.c.



7.0—The Transverse Plane $u = f(s)$:

$u = f(s)$ is obtained by the following subspace transformations

$y = \sqrt{x}^{\sqrt{x}}$ $\quad u = -\frac{1}{y}$ $\quad s = -\frac{1}{x}$ $\quad x = -\frac{1}{s}$ $\quad u = f(s) = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$

7.1:
The function is traditionally defined over the domain of $-\infty < x < 0$ and may be graphed directly. If is shown in Figure 4a.

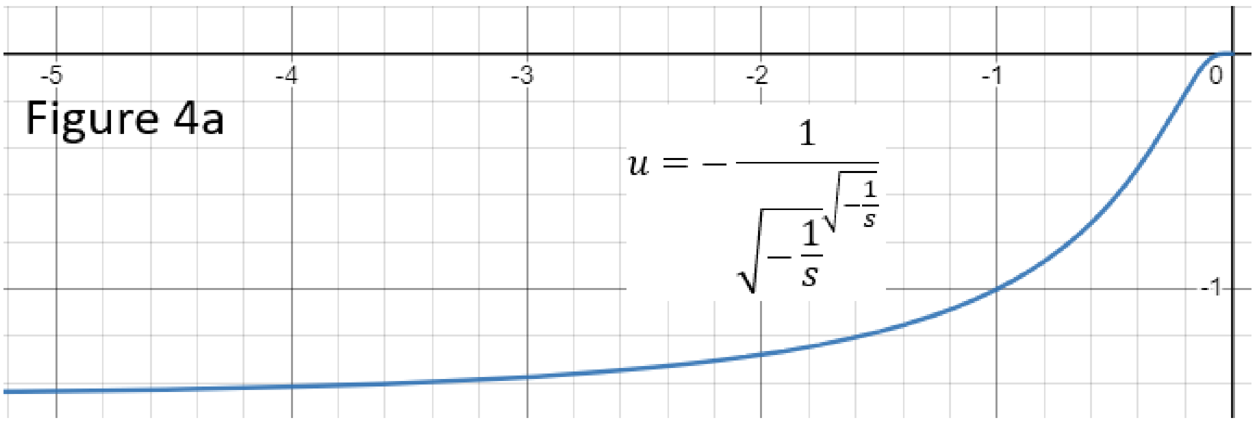
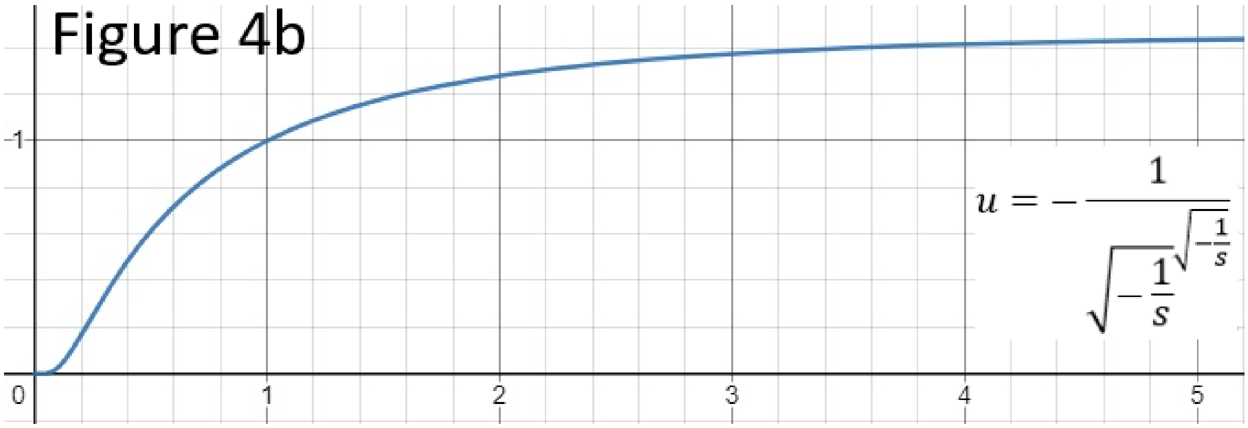


Figure 4a

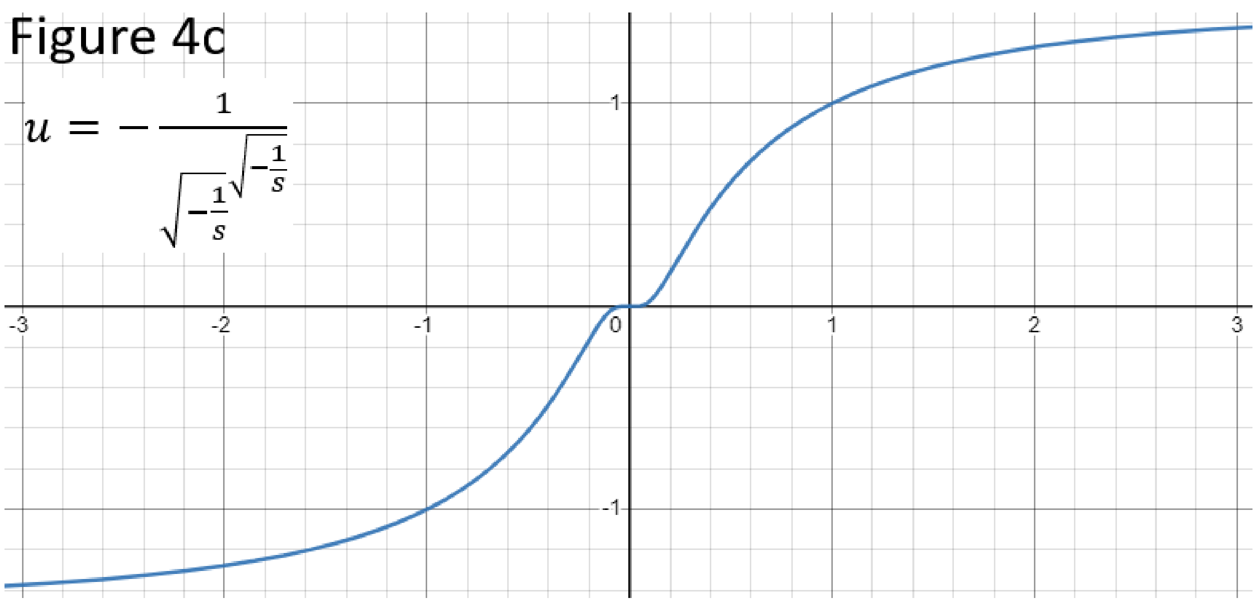
7.2
The extended domain will result in i-multiples for $0 \leq x < \infty$. We can force a graphing utility to graph the function over this domain in the following way, according to section 4.0.

$u = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$	Given equation and a specified value. Results in $-\frac{1}{i^i}$ for $s = 1$.
$u = -\frac{1}{\sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}}$	The inputs are switched with their positive value by removing the negative signs. This will force a standard graphing utility to graph the positive inputs. But we are still really using $u = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$ It generates i-multiples for each input over the domain $0 \leq x < \infty$.
$u = -\frac{1}{\oplus\sqrt{\frac{1}{s}}^{\oplus\sqrt{\frac{1}{s}}}}$	We recognize that the roots of the equation $u = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$ will be $\oplus$ number, resolved from an i-multiple state. We keep track of that by inserting the $\oplus$ sign in front of each radical.
	The $\oplus$ signs are squared in accordance with section 4.0 above. The equation is then simplified to its final state. Note this version of the equation is the one

$u = -\frac{1}{(\oplus^2)\sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}} = \frac{1}{\sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}}$	needed to force a graphing utility graph this function. The actual equation we are graphing is $u = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$ over the domain $0 \le x < \infty$ It is seen in Figure 4b.



<u>7.3:</u>
The full graph of $u = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$ is shown in Figure 4c.



7.3.a:

If we zoom in we can see the value obtained for $i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$ is on this graph, as was suggested in section 3.1.d above. The function $u = f(s) = 0.20788$ when $s = 0.22519$. See Figure 4d.

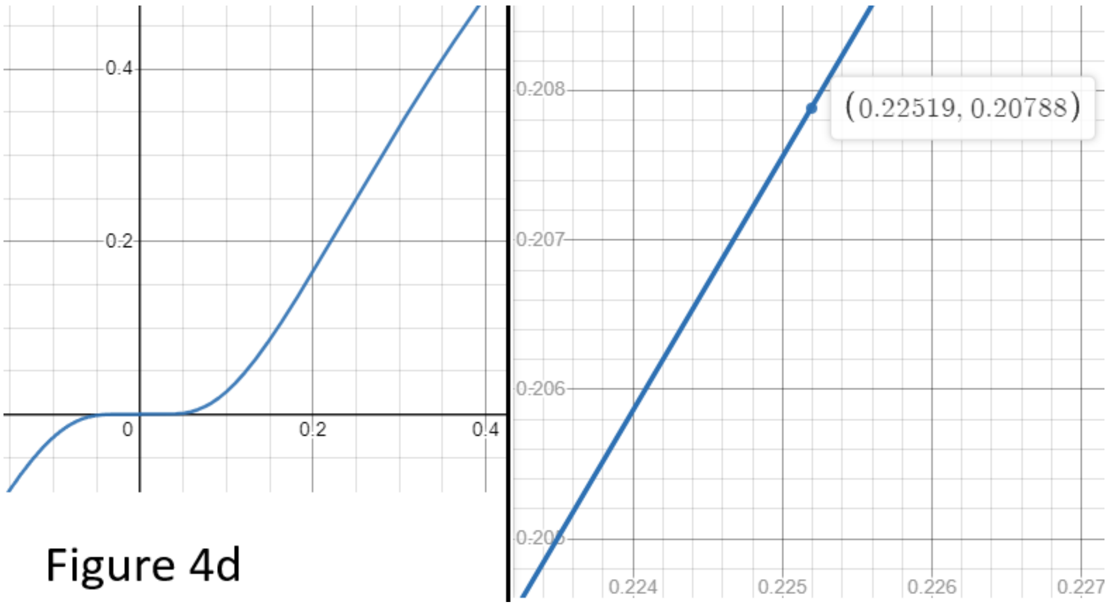


Figure 4d

7.4—Additional Graphs:

The following graphs are three-directional hypervolumes. These are graphed parametrically as specified below.

Figure 5a:

Graph of xsy-hypervolume: $x = t$ $s = -\frac{1}{t}$ $y = \sqrt{t}^{\sqrt{t}}$

For $-\infty < t \leq 0$ in blue. Graphing y as $y = -\sqrt{-t}^{\sqrt{-t}}$ in accordance with section 4.0.

For $0 \leq t < \infty$ in gold. Graphing y as $y = \sqrt{t}^{\sqrt{t}}$.

Figure 5a

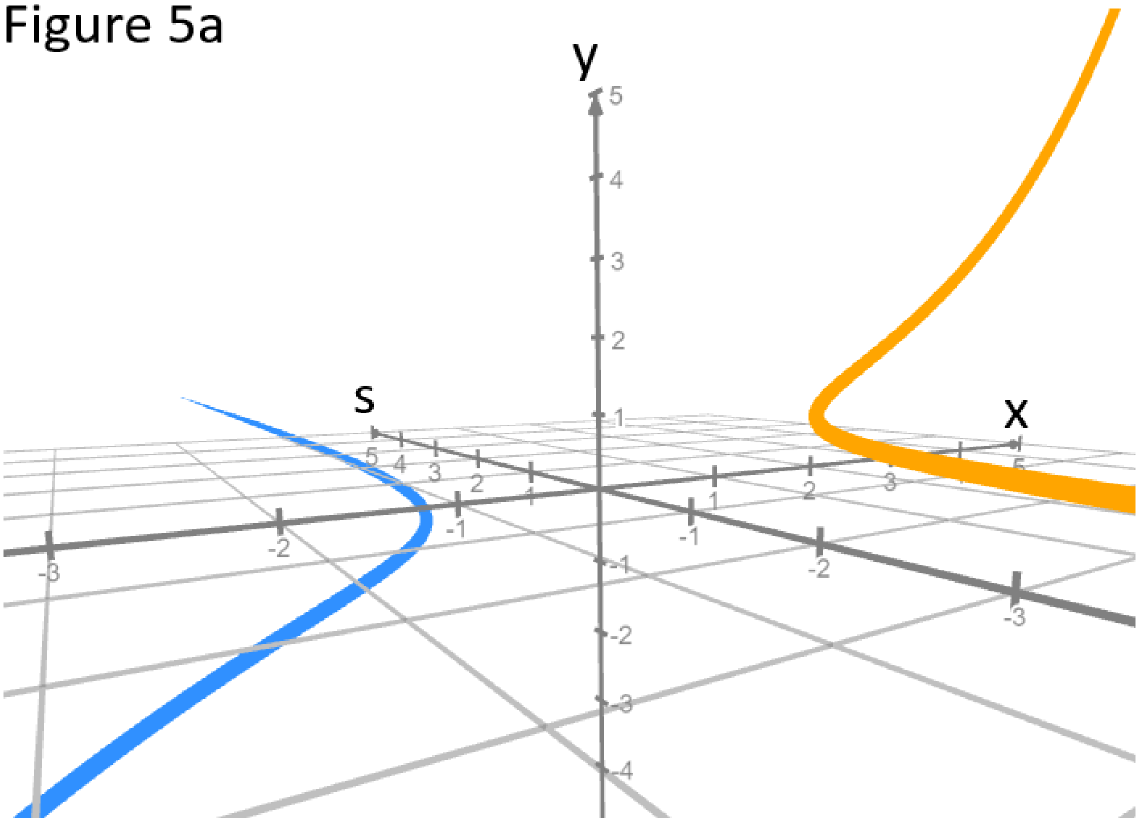


Figure 5b:

Graph of xsu-hypervolume: $x = t$ $s = -\frac{1}{t}$ $u = -\frac{1}{\sqrt{t}^{\sqrt{t}}}$

For $-\infty < t \leq 0$ in blue. Graphing u as $u = \frac{1}{\sqrt{-t}^{\sqrt{-t}}}$ in accordance with section 4.0.

For $0 \leq t < \infty$ in gold. Graphing u as $u = -\frac{1}{\sqrt{t}^{\sqrt{t}}}$.

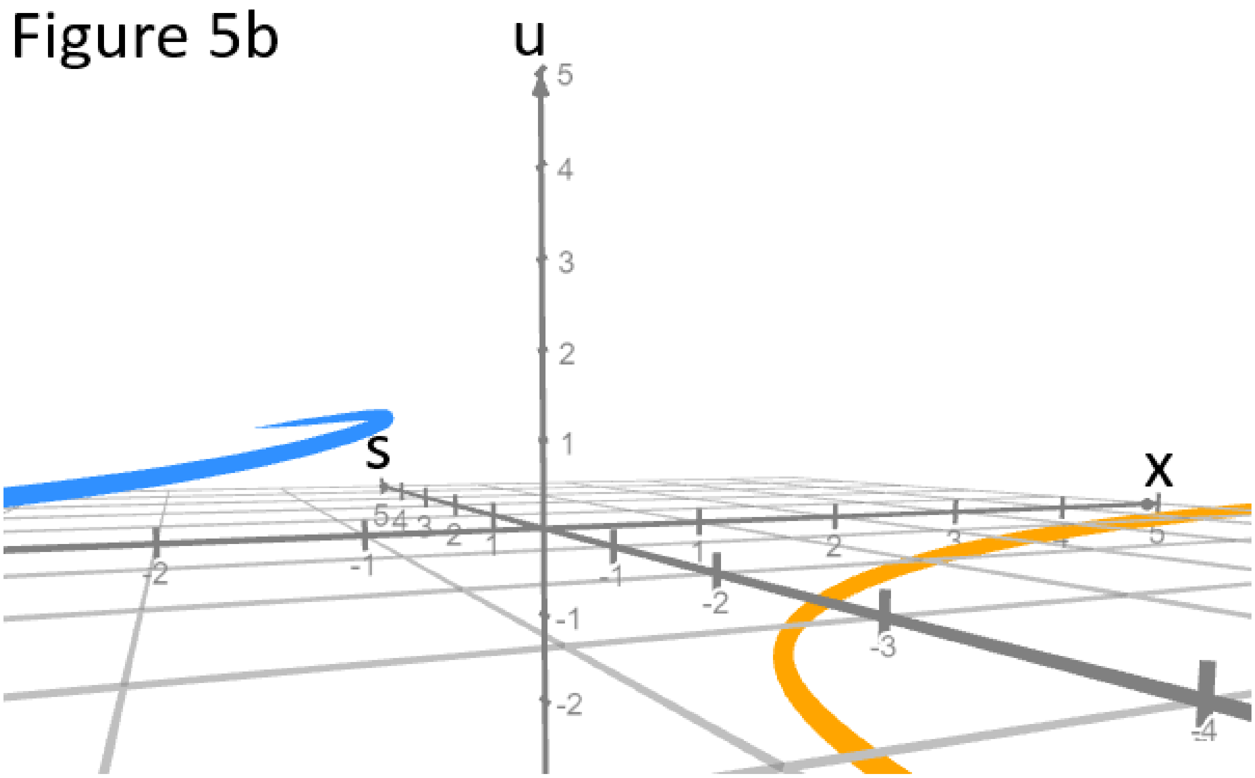


Figure 5c:

Graph of xyu-hypervolume: $x = t$ $y = \sqrt{t}^{\sqrt{t}}$ $u = -\frac{1}{\sqrt{t}^{\sqrt{t}}}$

For $-\infty < t \leq 0$ in blue. Graphing u as $u = \frac{1}{\sqrt{-t}^{\sqrt{-t}}}$

Graphing y as $y = -\sqrt{-t}^{\sqrt{-t}}$

Both in accordance with section 4.0.

For $0 \leq t < \infty$ in gold. Graphing u as $u = -\frac{1}{\sqrt{t}^{\sqrt{t}}}$

Graphing y as $y = \sqrt{t}^{\sqrt{t}}$

Figure 5c