Null Algebra resolutions of Complex Exponentiation
Null Algebra Extension III

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Abstract:

This paper explores the Null Algebra and traditional Algebra resolutions for the complex number i^i. It explains the apparent differences between the Null Algebra resolutions, and those of traditional Algebra, which uses the substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$. The methods shown herein explore the full set of subspace equations implied by the given equation, as well as why powers of i resulting from the trigonometric substitution must be considered in deriving those equations. It is shown that the values obtained from the various possible resolutions to i^i are found on some aspect of the given equation, or its expanded subspace equation sets. It shows i^i equals +1, -1, +4.81047738 and +0.20788.

It is assumed the reader has read and understood Null Algebra, as well as Null Algebra Extensions I and II in addition to standard Algebra and Trigonometry. The Null Algebra texts are available for download at (https://vixra.org/abs/2103.0131), (https://vixra.org/abs/2206.0135) and (https://vixra.org/abs/2304.0205). If you have not yet read these texts and attempted the examples contained therein for yourself it is highly suggested you do so before reading further as some concepts explained in detail there, are given only cursory review here. Without reading these prerequisites you may not fully understand the reasoning behind logic used in the equations of this text.

1.0—Review of Logs and Exponents:

Before exploring complex exponentiation and their Null Algebra resolutions it's important to understand the common rules for logarithms and exponents. Where exponents raise a base value to a given power, logarithms provide the power necessary to obtain a result from a given base. Logs are the inverse of exponentiation. It is hoped these will provide the reader with a refreshed understanding of exponentiation and logarithms, to better facilitate understanding during progression through the text.

1.1.a—Exponentiation and Logarithm definition:

Exponentiation:	Logarithm:
$a^n = m$	$\log_a m = n$

Note, if the base value $a = \mathbf{e} \approx 2.71828 \ldots$ the nomenclature used is the natural log: **ln**

1.1.b—Rules of Logs, and Natural Logs:

Log Product Rule	$\ln(x \cdot y) = \ln(x) + \ln(y)$	$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$
Log Quotient Rule	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$	$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
Log Reciprocal Rule	$\ln\left(\frac{1}{x}\right) = -\ln(x)$	$\log_a\left(\frac{1}{x}\right) = -\log_a(x)$
Log Power Rule	$\ln(x^y) = y \cdot \ln(x)$	$\log_a(x^y) = y \cdot \log_a(x)$
Logs of negative values	$\ln(-x) = \emptyset$	$\log_a(-x) = \emptyset$
Log of 0	$\ln(0) = \emptyset$	$\log_a(0) = \emptyset$
	$\lim_{x \to 0^+} \ln x = -\infty$	$\lim_{x \to 0^+} \log_a x = -\infty$
Log of 1	$\ln(1) = 0$	$\log_a(1) = 0$
Log of ∞	$\ln(\infty) = \infty$	$\log_a(\infty) = \infty$
Argument = base value	$\ln(\boldsymbol{e}^x) = x$	$\log_a(a^x) = x$
If $x = 1$	$\ln(\mathbf{e}) = 1$	$\log_a(a) = 1$

Value raised to power of a log with same base	$\boldsymbol{e}^{\ln(x)} = x$	$a^{(\log_a x)} = x$
Log Derivative	$\frac{d}{dx} \ln x = \frac{1}{x}$	$\frac{d}{dx} \log_a(x) = \frac{1}{x \log a}$
Log Integral	$\int \ln(x)\ dx = x \cdot \ln(x) - x + C$	
	$\int \log(x)\ dx = x \cdot \log(x) - x + C$	
Euler Identity	$\ln(-1) = i\pi$	$\mathbf{e}^{i\pi} = -1$
Change of base	$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$	
* Miller's Identity	$1^i = 1$	

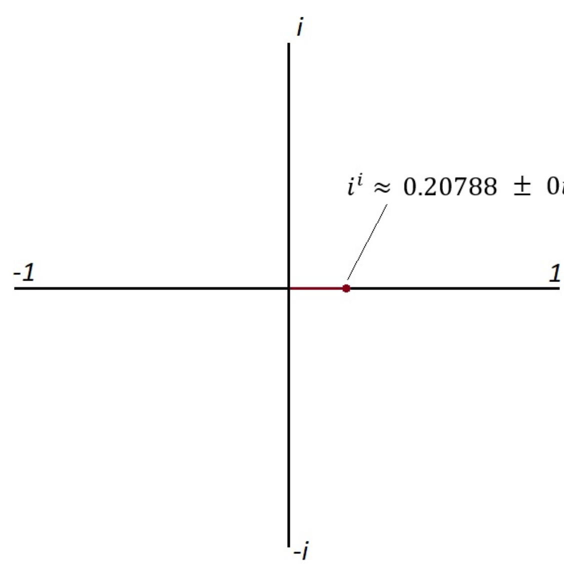
Exponentiation of a Product: For: $a = b \cdot c$ Let: $a^n = d$

Then: $a^n = d \quad \equiv \quad (b \cdot c)^n = b^n \cdot c^n = d$

2.0—Aspects of i^i:

Null Algebra resolutions of i move values away from the complex plane into a real space, which includes one or more subspaces. The calculation of values using i as a base or as an exponent have more commonly used standard trigonometric properties to provide solutions. Even when the result of a trigonometric substitution is a real number it still can be thought of as possessing an imaginary part of $\pm 0i$.

The standard calculation of i^i will provide a real number solution. However, this is still on the complex plane. The trigonometric substitutions of i are still on the complex plane whereas the Null Algebra resolutions are removed to a plane composed or real and subspace axis.



* I apologize for the small arrogance here. I could not find another reference to this identity beside that which I published at https://vixra.org/abs/2307.0124 on 7/24/2023. It seems an important identity. I have to call it something.

This real value, $i^i \approx 0.20788 \ldots$, on the complex plane is simply not displaced along the i-axis; it represents an output value, defined as a two dimensional number of the form $a \pm bi$ for which the complex magnitude $b = 0$.

$$i^i \approx 0.20788 \ldots \pm 0i$$

The Complex Plane is used as an output plane, showing single values of the output variable. For an equation of the form $y = f(x)$, the complex plane is composed of a real x-axis and the imaginary i-axis. We plot on it two-dimensional points for each y-axis output value in terms of x and i, as defined by a given $y = f(x)$ equation. So, though we plot points for equations of the form $y = f(x)$ on the Central Plane (*the XY-Plane*), we do not see the actual output y-axis of the central plane or any of the subspace axis associated with either the x, or the y axis on the Complex Plane. Instead the entire plane represents the individual y-axis output points.

More complicated is the trigonometric properties used to obtain a value for i^i. These methods remove the i from the equation. This results in the standard value of 0.20788… and equates the solution with Hyperbolic Trigonometry. The following sections will explore the calculation of i^i and its resolution from the complex place to a subspace hyperplane.

2.1—Traditional Algebraic and Trigonometric calculation of i^i:

Usage of trigonometric identities and a Maclaurin series expansion on i^i yields the following:

$$i^i \approx 0.20788 \ldots$$

This value is slightly larger than $^1/_5$. Its larger by not quite eight-thousandths. It is also a real number. Before we explore how this value is reached by traditional methods let's explore the immediately obvious Null Algebra resolution, which appears to stand in direct contradiction to the approximate 0.20788 value.

2.1.a:

Null Algebra resolves instances of i at their place of occurrence for a given equation on the central plane to its positive, up value, and to its negative, down value on the corresponding co-adjoining subspace. For an equation of the form $y = f(x)$, where elements of $f(x)$ produce i-multiples for inputs of x, we have

$$i = \oplus 1 = \begin{cases} x - axis & \hat{1} \ \rightarrow \ +1 \\ \\ s - axis & \check{1} \ \rightarrow \ -1 \end{cases}$$

2.1.b:

If we use the equation, $y = \sqrt{x}^{\sqrt{x}}$

- The point i^i is but one of an infinite number of points on this graph occurring when $x = -1$.

Thus for $x = -1$

$$y = \sqrt{-1}^{\sqrt{-1}} \doteq \oplus 1^{\oplus 1} \doteq \hat{1}^{\hat{1}} \doteq 1$$

2.1.b.i—The Apparent Contradiction:

$$+1 \neq 0.20788 \neq \hat{1} \rightarrow +1$$

Later we'll more deeply explore the *sy*, *xu* and *su* planes. You'll find as we explore those graphs the approximate 0.20788 by way of the Null Algebra resolutions is on the transverse plane. We will explore the explanation for why the values appear where they do.

2.1.c—Calculating i^i using Euler's Identity:

Euler's Identity is derived from the Maclaurin expansion of $\mathbf{e}^{ix}$ for the value $x = \pi$. This can be used assist in calculating values on the complex plane.

Euler's Identity: $\mathbf{e}^{i\pi} = -1$

This is a 180 degree turn on the complex plane. So to reach i we need only a 90° rotation or half Pi.

$$\mathbf{e}^{i\frac{\pi}{2}} = i$$

Thus the expression i^i may be rewritten as:

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}}$$

All that remains then is to raise the base value of **e** to the power of $-\frac{\pi}{2}$.

$$\mathbf{e}^{-\frac{\pi}{2}} \approx 2.71828^{-1.570796} \approx \frac{1}{2.71828^{1.570796}} \approx 0.20788$$

2.1.d—The Hyperbolic Calculation of $\mathbf{e}^{-\frac{\pi}{2}}$:

Recall that the Maclaurin Series expansion for approximating a function centered at the point $x = 0$, is given by:

$$\sum_{n=1}^{\infty} \frac{f^n(0) \cdot x^n}{n!}$$

2.1.d.i:
When performing the expansion on $\mathbf{e}^{x}$ and $\mathbf{e}^{-x}$ (*with no i in the exponent*) the resulting series is equated to $\cos x + \sin x$, and $\cos x - \sin x$ respectively. This results in the Hyperbolic Pythagorean identity for which the nomenclature of *cosh* and *sinh* are used to ensure understanding of hyperbolic trigonometry being used.

$\mathbf{e}^{x} = \cosh x + \sinh x \qquad\qquad \mathbf{e}^{-x} = \cosh x - \sinh x$

$\mathbf{e}^{x} \cdot \mathbf{e}^{-x} = 1 = \cosh^2 x - \sinh^2 x$

From this perspective you can easily verify that $\mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$

$\mathbf{e}^{-\frac{\pi}{2}} = \cosh(-1.5707963) + \sinh(-1.5707963) \quad \rightarrow \quad 2.509178478658 + \ -2.3012989$

$\mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 \ldots$

3.0—Rationalizing Null-Algebraic and Traditional Algebraic Solutions of $\boldsymbol{i^i}$:
Thus 2.1.c provides a confirmed algebraic solution for i^i, and 2.1.d provides a confirmed trigonometric solution for the same. The two methods of approach are in agreement that

$$i^i \approx 0.20788 \ldots$$

- *Remember this is still a real-only component of a number $z = a + bi \ = \ a + 0i$. It is still on the complex plane. Null Algebra resolutions translate values onto planes composed of real and subspace axis. Due to these differences there is no reason to believe the value obtained from the trigonometric exchange, which is really a different equation than the given equation in this example will appear on the XY-Plane.*

3.1.a:
There is another difference in values which results depending on when you decide to apply Null Algebra resolutions to the i multiples. Consider the two methods shown here below:

Direct Resolution of i	Substitution, Then Resolution of i
For $y = f(x)$ equation: x-axis: $i^i \doteq \oplus 1^{\oplus 1} \doteq \hat{1}^{\hat{1}} \doteq \dot{1}^{\dot{1}} = 1$ s-axis: $\doteq \check{1}^{\check{1}} \doteq -1^{-1} = -1$	$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} \doteq \left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1} \doteq \begin{cases} \left(\mathbf{e}^{\hat{1}\frac{\pi}{2}}\right)^{\hat{1}} \doteq \mathbf{e}^{\frac{\pi}{2}} \\ \mathbf{e}^{(\oplus 1)^2\frac{\pi}{2}} \doteq \mathbf{e}^{-\frac{\pi}{2}} \end{cases}$

3.1.a.i:

These steps show we have four possible solutions to the expression i^i depending on when and how we select to resolve the i-multiple values. The expression can be said to be

$$i^i = \{1\,, -1\,, \mathbf{e}^{\frac{\pi}{2}}\,, \mathbf{e}^{-\frac{\pi}{2}}$$

- *All of these are valid resolutions, and they all occur somewhere on the full expansion of subspace equations implied by the given equation in this example.*

The given equation, for the negative domain on $y = \sqrt{x}^{\sqrt{x}}$ results in expressions that indicate the *y*-axis values are represented in the form of $y = 0 + bi$ before being resolved by either Null Algebra or Traditional Algebra methods. The Null Algebra resolutions will involve *y* but also its subspace of *u*. We should anticipate to find these four solution values on either the *y*-axis or *u*-axis output. The direct resolution of i using Null Algebra in this example produces +1 and -1 values on the *xy*-plane and *sy*-plane respectively.

The substitution of $i = \mathbf{e}^{i\frac{\pi}{2}}$ then permits again either direct substitution of $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} = \left(\mathbf{e}^{\hat{1}\frac{\pi}{2}}\right)^{\hat{1}} \doteq \mathbf{e}^{\frac{\pi}{2}}$ or the squaring of plus-and-minus 1 via

Null Algebra	$\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} \quad \doteq \quad \left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1} \quad \doteq \quad \mathbf{e}^{-\frac{\pi}{2}}$
Traditional Algebra	$\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} \quad \longrightarrow \quad \mathbf{e}^{-\frac{\pi}{2}}$

3.1.a.ii:

Note $\left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1} \doteq \mathbf{e}^{-\frac{\pi}{2}}$ is not the only method of arriving at the $\mathbf{e}^{-\frac{\pi}{2}}$

From the section above on *Substitution, Then Resolution of* i, the top row of the resolution process, $\left(\mathbf{e}^{\hat{1}\frac{\pi}{2}}\right)^{\hat{1}}$ will resolve to $\left(\mathbf{e}^{\frac{\pi}{2}}\right)$.

However if we chose to specify the presence of the +1 coefficient in both exponents as $\left(\mathbf{e}^{1\frac{\pi}{2}}\right)^{1}$ we may then further choose to keep track of these as resolved *i*-multiples by including their dotted value accent marks:

$$\left(\mathbf{e}^{\dot{1}\frac{\pi}{2}}\right)^{\dot{1}}$$

These dotted values, indicate these are resolved components of *plus-and-minus* numbers and as such, when multiplied together it is identical to squaring the plus and minus sign, which will result in a negative sign.

$$\left(\mathbf{e}^{\dot{\mathrm{i}}\frac{\pi}{2}}\right)^{\dot{\mathrm{i}}} = \mathbf{e}^{(\dot{\mathrm{i}})^2\frac{\pi}{2}} = \mathbf{e}^{-\frac{\pi}{2}}$$

<u>4.0—Analyzing graphs and values:</u>
The graphs of the various equations are shown below, exploring the nature of all four solution values. All four are in fact valid, occurring on some section of the overall subspace hyper-plane set.

<u>4.1.a:</u>
The values of +1 and -1 require no special explanation; they follow known Null Algebra resolutions. We have likewise explored the trigonometric resolution value of $\mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$. This leaves us to evaluate $\mathbf{e}^{\frac{\pi}{2}}$.

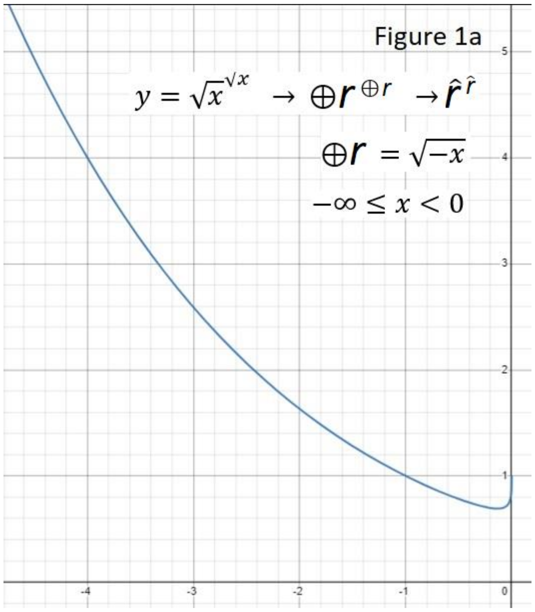
$$\mathbf{e}^{\frac{\pi}{2}} \approx (2.71828\,\ldots)^{(1.57079632\ldots)} \approx 4.81047738$$

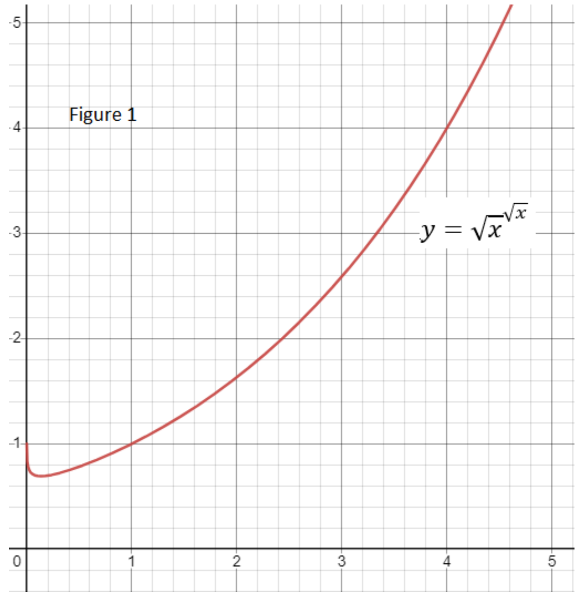
$$\mathbf{e}^{\frac{\pi}{2}} \approx \cosh\left(\frac{\pi}{2}\right) + \sinh\left(\frac{\pi}{2}\right) \approx 2.509178478658 + 2.30129890230 \approx 4.81047738$$

All four of these values, 1, -1, 0.20788 and 4.81047738 will appear somewhere on the subspace expansions of the given equation $y = \sqrt{x}^{\sqrt{x}}$.

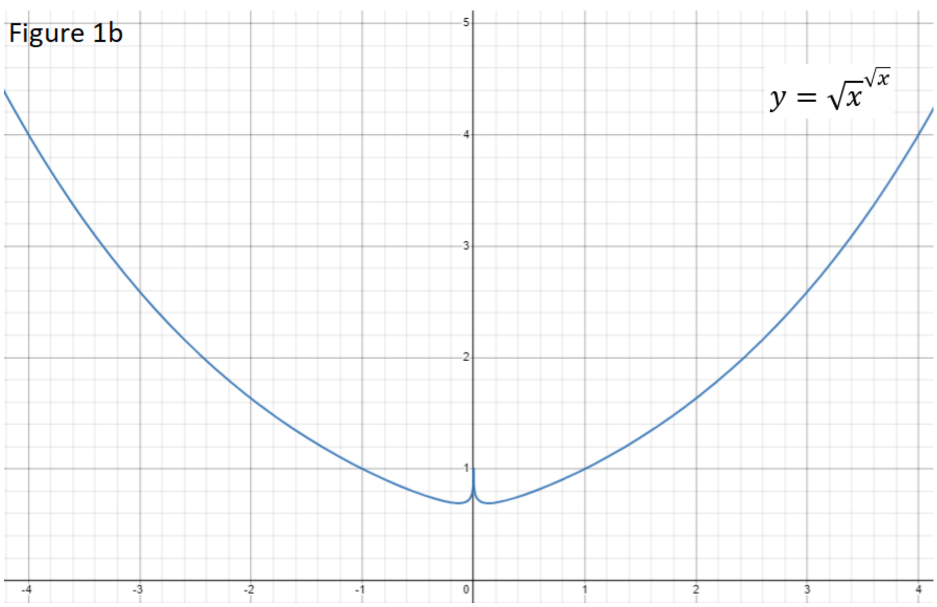
<u>4.2.a</u>
<u>$y = \sqrt{x}^{\sqrt{x}}$ For $0 \leq x < \infty$</u>

The graph of this function is shown here in Figure 1 over its traditionally valid domain. Its straight forward traditional algebra. For simplicity the graphs shown display only the positive output values for the roots. Figure 1a shows the output values for the roots, resolved from their *i*-multiple, to their positive, *up* value; the value positive-and-negative numbers take when resolved on the central plane over the extended domain of $-\infty < x \leq 0$.

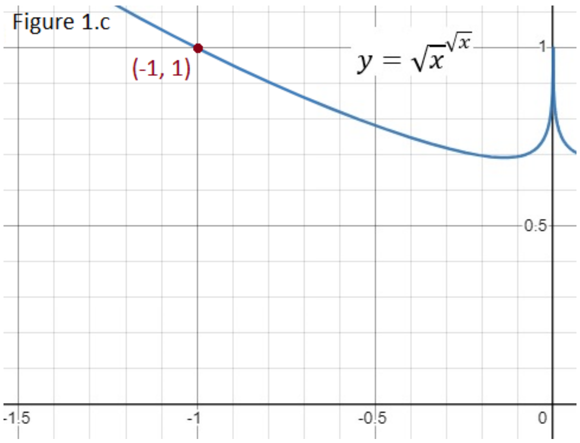


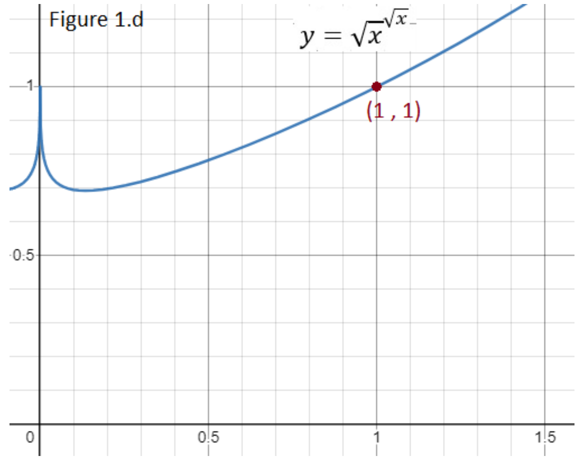


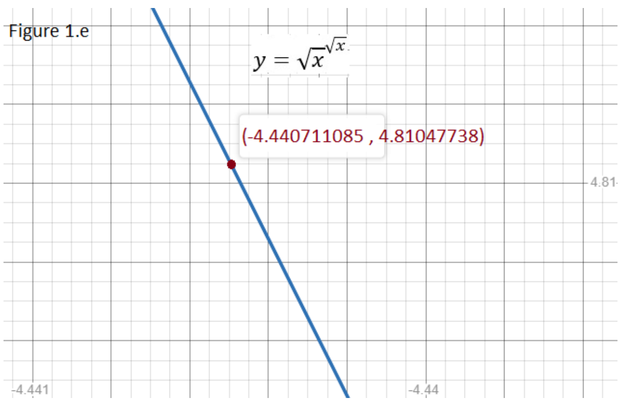
4.2.a.i:
We can put these two graphs directly together. Using the Null Algebra resolutions we see the full domain of the graph of $y = \sqrt{x}^{\sqrt{x}}$. See Figure 1b.

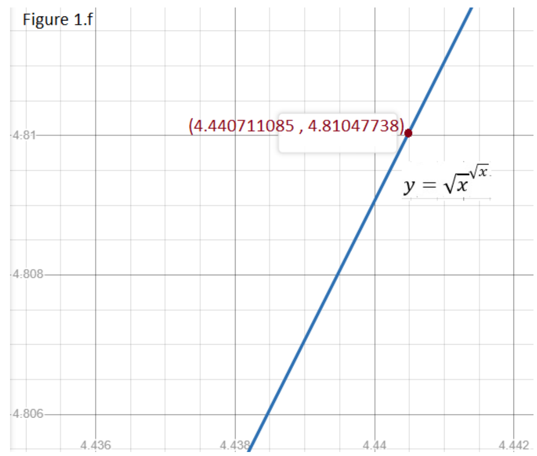


4.2.a.ii:
We can see by zooming in that the output value of $y = 1$ is present for the input value, $x = -1$. In fact, it also equals +1 when $x = +1$. Additionally we can see this graph also contains the output values of $y = 4.81047738$ when $x = 4.440711085$ and -4.440711085.









4.2.b:

$y = \sqrt{x}^{\sqrt{x}}$ For $-\infty < x \leq 0$ using $\mathbf{e}^x$ resolution:

We have graphed functions generating i-multiples before in Null Algebra Extension II. In those graphs we showed the traditional output *xi*-plane used to plot two-dimensional points representing y, and later the same functions as three-directional *xiy*-volumes with both inputs and outputs showing all three axis together. Thereafter we resolved the i-multiples and showed the final *xsy*-hyperplane graph.

The issue here is every input in the domain of $-\infty < x \leq 0$ produces an output of the form ri^{ri}. Without resolving the i values, what does it mean to raise an imaginary number (*or for that matter any number*) to an imaginary exponential power? Even on the output plot, complex *xi*-plane it's not defined what it means to raise a number ri to a power, ri.

- I am using *r* within the text to represent the magnitude of the root of a negative argument such that $\sqrt{-n} = ri$.

In this instance, without resolving the *i*-multiples we use the expression $i = \mathbf{e}^{i\frac{\pi}{2}}$. Because each *x* input for the domain $-\infty < x \leq 0$ will produce *i*-multiples for both the base value and the exponent in the equation $y = \sqrt{x}^{\sqrt{x}}$ we may make the following substitutions.

Let: $r = \sqrt{|x|}$ $ri = \sqrt{-x}$ $i = \mathbf{e}^{i\frac{\pi}{2}}$

Then for: $-\infty < x \leq 0$ $y = \sqrt{x}^{\sqrt{x}} \rightarrow ri^{ri} = \left(r\mathbf{e}^{i\frac{\pi}{2}}\right)^{ri} = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$

<u>4.2.b.i:</u>

Without resolving the i value in the exponent on the first term, we cannot actually graph $r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ on a Cartesian style plane. We would be limited to using the complex plane. It is only when $x = -1$, that $y = \sqrt{x}^{\sqrt{x}} = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}} = 0.20788$, a completely real number. All other values for the negative domain, will produce some complex number with an imaginary only part when using the trigonometric substitution. For all values $x \neq -1$ in the negative domain, the outputs will have no real part and are plotted directly on the *i*-axis at varying heights, confined to the complex plane.

0.20788 is one point on negative domain of the complex equation $\sqrt{|x|}^{\sqrt{|x|}i} \cdot \mathbf{e}^{-\frac{\sqrt{|x|}\pi}{2}}$. This equation is only equal to $\sqrt{x}^{\sqrt{x}}$ when $x = -1$. Though the output value resulting from the trigonometric substitution is valid and may be used to substitute and solve for i^i when not using Null Algebra substitutions, the two equations are not identical. So we should not be surprised the general form of the equation derived from the trigonometric substitution is not only different from the $y = f(x)$ equation, but also results in complex numbers for all but one input value over the negative domain.

<u>4.3:</u>

So we have two separate resolution methods for i^i. One which solves for the general equation $y = \sqrt{x}^{\sqrt{x}}$ using Null Algebra resolutions over the negative domain, and another which uses trigonometric substitutions for the complex number i^i but provides a value for only one point, which is not guaranteed to be on the XY-Plane. The trigonometric substitution can be used to derive a completely different general equation that happens to provide a real number output, to the same expression i^i, for the same input value shared on both non-identical general equations.

4.3.a
The differences between some key values can be summarized thus:

4.3.a.i
For: $y = \sqrt{x}^{\sqrt{x}}$ Over Domain: $-\infty < x \leq 0,$ At $x = -1$

Null Algebra Resolution	$+1$
Trigonometric Resolution	$\approx +\, 0.20788$

4.3.a.ii
To understand these apparent differences we'll need to examine the graphs involving the *x*, *y*, *s* and *u* axis.

4.3.a.iii:
Remember that the substitution of $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i} \approx 0.20788$ though a real number, does not remove us from the complex plane. Its generalized equation over the negative domain is of the form:

$$\sqrt{x}^{\sqrt{x}} = \sqrt{|x|}i^{\sqrt{|x|}i} = \left(\sqrt{|x|} \cdot \mathbf{e}^{i\frac{\pi}{2}}\right)^{\sqrt{|x|}i} = \sqrt{|x|}^{\sqrt{|x|}i} \cdot \mathbf{e}^{-\frac{\sqrt{|x|}\pi}{2}} = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$$

4.3.a.iv:
The outputs for this equation, over the negative domain will result in complex numbers of the form *a* + *bi* as defined below:

For: $x \neq -1$ and $-\infty < x \leq 0$ $z = 0 + bi$

For: $x = -1$ and $-\infty < x \leq 0$ $z = a + 0i$

Thus the value obtained from the trigonometric substitution is still on the complex plain. It just simply has no imaginary part. $z = a + 0i$. In other words a complex number with a zero magnitude imaginary component.

4.3.a.v:
Another factor to consider with $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^{i}$, is that it results in squaring i. Recall from Null Algebra Extension II this will result in a 180 degree rotation on the complex plane, something that places us on the border between values that apply to the given real space axis in the upper quadrants of the complex plane and the corresponding subspace values in the lower quadrants of the complex plane. This will become important in showing the resolution to the value 0.20788 as well as the axis it lays on.

4.3.a.vi:

The ≈ 0.20788 value will be found on the expanded graphs. However we should not assume this output value will occur at the value of $x = -1$. This is because the i^i at $x = -1$ is generated by $y = \sqrt{x}^{\sqrt{x}}$. The substitution of $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i$ provides a solution which is equivalent to i^i but is itself generated by a different equation, $r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$, which remains on the complex plane and shares only one point with $y = \sqrt{x}^{\sqrt{x}}$.

4.3.a.vii:

When $x = -1$, the equation $y = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ equals $\mathbf{e}^{-\frac{\pi}{2}}$, the value equivalent to the i^i at $x = -1$ on the equation $y = \sqrt{x}^{\sqrt{x}}$. Because the result 0.20788 comes from $r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$, as a result of a substitution, defining a different general equation than the given equation, the result of 0.20788 occurring at $x = -1$ on $r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$, may not occur at the same input value of $x = -1$ on any part of the expanded subspace equations of $y = \sqrt{x}^{\sqrt{x}}$. And yet it will occur on some portion of the expanded set of subspace equations of the given $y = \sqrt{x}^{\sqrt{x}}$ equation.

4.3.a.viii:

In other words:

	$y = \sqrt{x}^{\sqrt{x}}$	$\neq$	$y = \sqrt{\lvert x\rvert}^{\sqrt{\lvert x\rvert}i} \cdot \mathbf{e}^{-\frac{\sqrt{\lvert x\rvert}\pi}{2}} = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$
For $x = -1$	$y = \sqrt{x}^{\sqrt{x}} = i^i$	$=$	$y = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 \ldots$
	$y = \sqrt{x}^{\sqrt{x}} = i^i$	$\not\equiv$	$y = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 \ldots$

*** $i^i = \mathbf{e}^{-\frac{\pi}{2}}$, but $i^i \not\equiv \mathbf{e}^{-\frac{\pi}{2}}$.

4.4—The Co-Adjoining Subspace $y = f(s)$:

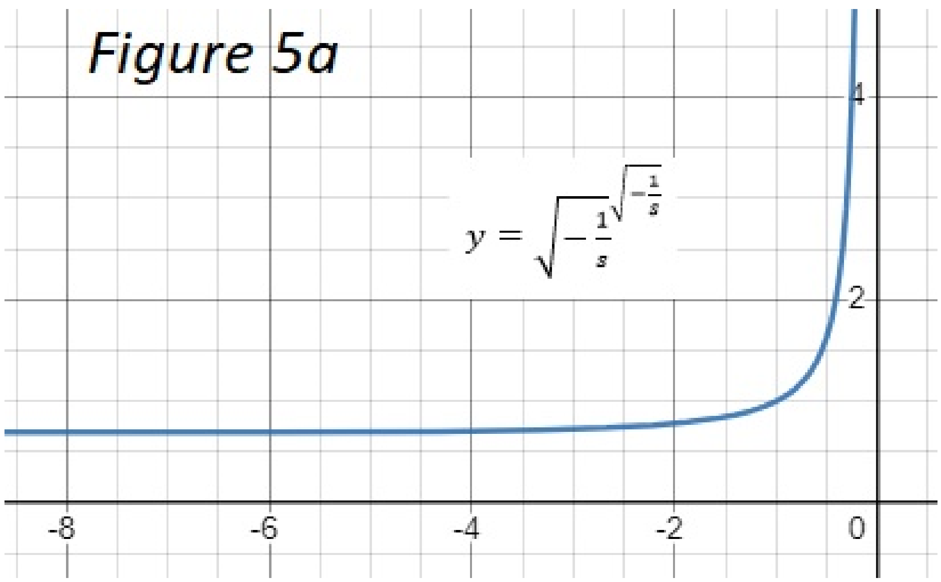
Using the equation $y = \sqrt{x}^{\sqrt{x}}$ we use a subspace transform on the *x* input to obtain the *s*-axis subspace values which comprise the co-adjoining subspace equation.

4.4.a.i:

$x = -\frac{1}{s}$ $\qquad\qquad$ $y = f(s) = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$

4.4.b:

$y = f(s)$ is defined under the traditional Algebra domain of $-\infty < s \leq 0$. This is shown in Figure 5a.



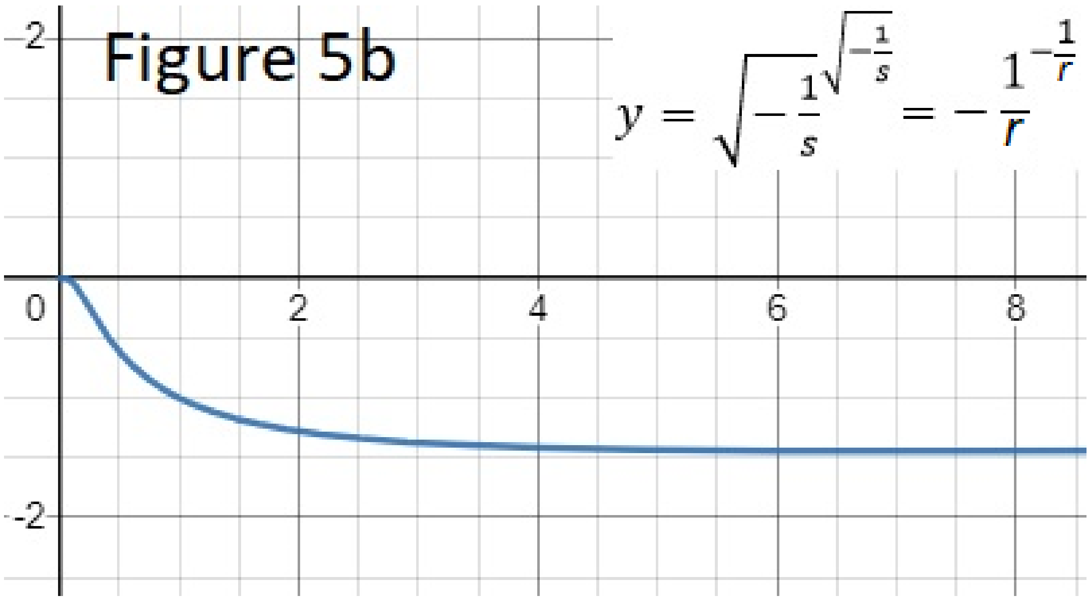
4.4.c:

The extended domain of $0 \leq s < \infty$ will result in *i*-multiples. These *plus-and-minus* numbers we resolve to their down values on the *s*-axis of the co-adjoining subspace.

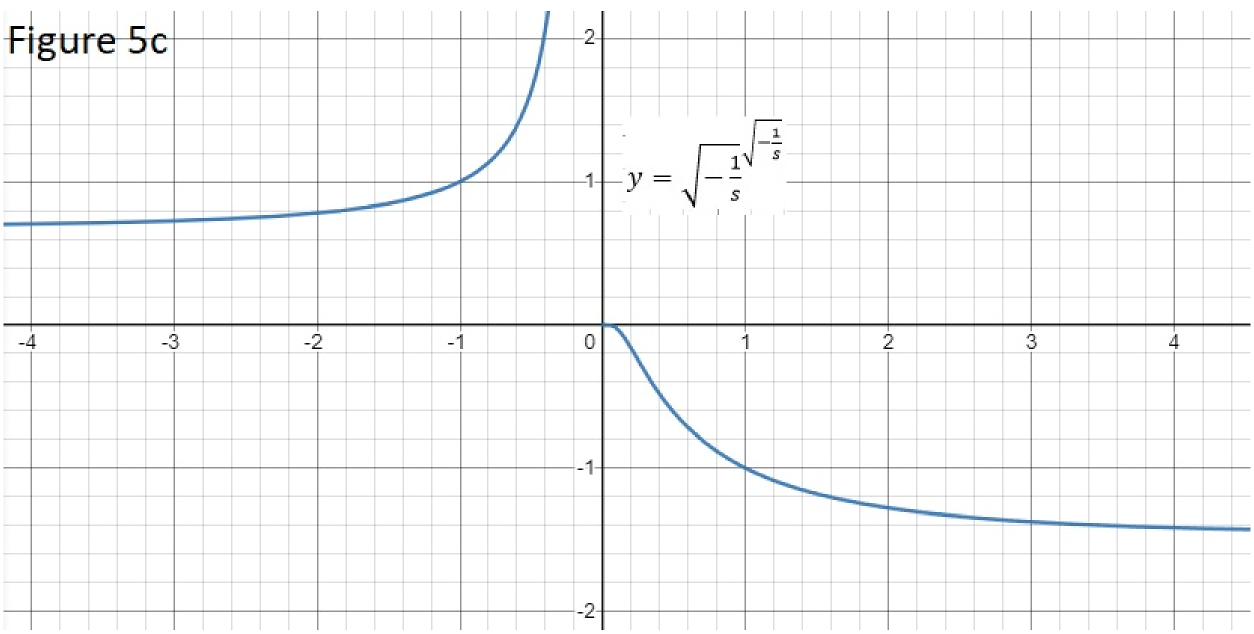
$y = \sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}$ $\qquad$ for $\qquad 0 \leq s < \infty$ $\qquad\qquad$ with $\qquad \sqrt{-s} = \oplus\, r \doteq \check{r} \doteq -r$

$$y = -\frac{1}{r}^{-\frac{1}{r}} = -r^{1/r}$$

This is shown on the graph of Figure 5b.



4.4.d:
The combined graphs showing the full domain of the $f(s)$ function is shown in Figure 5c.



5.0—Review of the Resolutions of i^i:
Thee resolutions of i^ihave already been shown to take several forms depending on the approach we take. We will take a short review of these resolutions before moving on to ensure proper understanding of the next steps in examining the Posterior Subspace and eventually the Transverse Plane.

Resolutions of i^i require this expression be understood as a real number.

5.0.a:
One of these are the Null Algebra resolutions. They declare that *i*-multiples occurring on the complex plane as a result of the given $y = f(x)$ equation, will take their positive, *up*, resolved values.

$$i^i \quad \longrightarrow \quad \hat{1}^{\hat{1}} \quad \longrightarrow \quad 1$$

This is but one single value found on the example equation $y = \sqrt{x}^{\sqrt{x}}$ when $x = -1$. Obviously the point $(-1\,, 1)$ generated when $x = -1$ is on this graph.

$$y \;=\; f(-1) \;=\; \sqrt{-1}^{\sqrt{-1}} \;=\; \bigoplus 1^{\oplus 1} \;=\; \hat{1}^{\hat{1}} \;=\; 1$$

5.0.b:
The trigonometric Resolution derives from

$$\mathbf{e}^{ix} = \cos x + i \sin x$$

$$\text{for} \qquad x = \frac{\pi}{2} \qquad\qquad \mathbf{e}^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \;=\; 0 + i \;=\; i$$

The base value of *i* in i^i is replaced vis this substitution

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788\,...$$

5.0.c:
It does not require explanation that $1 \neq 0.20788$

5.0.c.i:
Yet there is no discrepancy here. i^i is one point of an infinite set of points on the graph of $y = \sqrt{x}^{\sqrt{x}}$ over its defined and extended domain. It is the Null Algebra resolutions which in 5.0.a define $i^i = 1$ when i^i occurs on the central plane, generated by $y = \sqrt{x}^{\sqrt{x}}$ when $x = -1$.

5.0.c.ii:
Likewise, the point i^i is but one point of an infinite number of points on the graph

$y = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ where $r = \sqrt{|x|}$

And generated from the substitutions on *i* in the equation

$y = \sqrt{x}^{\sqrt{x}}$ Over the domain $-\infty < x \leq 0$

such that: $y = ri^{ri} = \left(r\mathbf{e}^{i\frac{\pi}{2}}\right)^{ri} = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$

When $x = -1$ $y = (\mathbf{e})^{-\frac{\pi}{2}} \approx$ 0.2.0788 ...

5.1:
These two resolutions for i^i are ***Equivalencies*** but they are not ***Identities***.

Equivalency	$y_1 = f_1(x) = \sqrt{x}^{\sqrt{x}}$ $y_2 = f_2(x) = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ For $-\infty < x \leq 0$ and Using Trigonometric Resolutions $y_1 = f_1(x) = i^i$ $y_2 = f_2(x) = \mathbf{e}^{-\frac{\pi}{2}} = 0.20788$ $y_1 = y_2$
Non-Identity	$y_1 = f_1(x) = \sqrt{x}^{\sqrt{x}}$ $y_2 = f_2(x) = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ For $-\infty < x \leq 0$ and Using Null Algebra Resolutions And, $x \neq -1$ $y_1 = f_1(x) = \sqrt{x}^{\sqrt{x}} = r^r$ $y_2 = f_2(x) = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$ $y_1 \not\equiv y_2$ $\sqrt{x}^{\sqrt{x}} \not\equiv r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$
For $x = -1$	$y_1 = y_2$

5.1.a:

There are likely infinite equations one could create which will contain the output value of i^i for some input value.

On all of these equations there will be Null Algebra and Trigonometric resolutions which produce the values already discussed. The Null Algebra resolutions show we receive four valid values of +1, -1, +0.20788, and 4.81047738. The trigonometric resolution provides only +0.20788.

Yet the equations which produce these values are not identical and produce different values from each other. However, because the Null Algebra precepts have been applied to resolve i^i, arising from an equation of the form $y = f(x)$ on the central plane, then there also exists equations of the form $y = f(s)$, $u = f(x)$ and $u = f(s)$ all present and related to the originally resolved $y = f(x)$ equation.

The various values located for the trigonometric and null algebra resolutions of i^i shall occur within the collection of these four equations.

5.2:

Because of the equivalence between $y = i^i$ and the substitution $\mathbf{e}^{-\frac{\pi}{2}} = 0.20788$ we should expect to find the value 0.20788 on some portion of expanded subspace equations set for $y = \sqrt{x}^{\sqrt{x}}$.

5.2.a:

The equation $y = f(x)$ uses an extended domain resulting from Null Algebra resolutions when $-\infty < x \leq 0$.

5.2.b:

The presence and resolution of *i*'s in the equation $y = \sqrt{x}^{\sqrt{x}}$ requires us explore the remaining three subspace equations to comprehend the full extent of the given equation.

$$y = f(s) \qquad u = f(x) \qquad u = f(s)$$

5.2.c:

The Trigonometric Resolution of $i^i = \mathbf{e}^{-\frac{\pi}{2}} = 0.20788$

- Will appear on the equations representing the subspace expansions of $y = \sqrt{x}^{\sqrt{x}}$
- It is not guaranteed to occur on the Central Plane. Can see from the graphs of $y = f(x)$ it is not on the XY-plane.
- Because it is a different equation which produces a merely numeric equivalent of $i^i = r^{ri} \cdot \mathbf{e}^{-\frac{r\pi}{2}}$, it is also not guaranteed to occur at the same input value of $x = -1$. We will explore

why this is so after making our first attempt to produce the remaining subspace expansion equations implied by $y = \sqrt{x}^{\sqrt{x}}$.

6.0—Definition of the *u*-axis subspace:

The Null Algebra resolutions dealing with the *u*-axis define it as:

6.0.a:

$$u = -\frac{1}{y}$$

We will proceed with this process using this definition provided in 6.0.a. to illustrate a point about trigonometric substitution which will also help show why the value 0.20788 exists within the subspace expansion equations of $y = \sqrt{x}^{\sqrt{x}}$.

6.1—Reversal of Up and Down values.:

The central plane, holding equations of the form $y = f(x)$ and the co-adjoining subspace $y = f(s)$ have been fairly well explored in earlier Null Algebra texts. Something we have not explored until this Extension to Null Algebra is that the *up* and *down* resolutions to i-multiple values are dependent on whether the input and output values match as real-space axis, subspace-axis or a mixed set between the two. The output axis determines which axis will take the *up* and *down* value resolutions to i-multiples. The *up* values will apply to the input axis which matches the nature of the output axis. The input axis which is in opposition will take the *down* value.

6.1.a:
The Central Plane

$y = f(x)$ — i-multiples result in positive *up* value resolutions

6.1.b:
The Co-Adjoining Subspace

$y = f(s)$ — i-multiples result in negative *down* value resolutions

6.1.c:
The Posterior Subspace

$u = f(x)$ — i-multiples result in negative *down* value resolutions

6.1.d:
The Transverse Plane

$u = f(s)$ — i-multiples result in positive *up* value resolutions

This will affect the way we produce the $u = f(x)$ and $u = f(s)$ equations. We will discover this process is still not yet complete. Because the unique example value of i^i, it will ultimately require both u-axis equations to be multiplied by -1. This is actually occurring in the process of obtaining the trigonometric substitution for i^i. After exploring why obtaining the u-equations without additionally multiplying them by a -1 is not accurate, we will show exactly where this additional magnitude of -1 comes from and why it is necessary.

7.0—The Posterior Subspace $u = f(x)$:

Using the definition of section 6.0 we have the following.

$y = \sqrt{x}^{\sqrt{x}}$ $\qquad u = -\frac{1}{y}$ $\qquad u = f(x) = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$

7.1:

The function is traditionally defined over the domain of $0 \le x < \infty$ and may be graphed directly.

7.2:

The extended domain will result in i-multiples for $-\infty < x \le 0$. The requirement that the input axis which is in opposition to the output axis' type take the negative *down* value resolutions will be applied here for the i-multiples on the x-axis components of this equation. A standard graphing utility can be forced to graph it appropriately by using

$$u = f(x) = -\frac{1}{-\left(\sqrt{-x}\right)^{-\left(\sqrt{-x}\right)}} = \left(\sqrt{-x}\right)^{\left(\sqrt{-x}\right)}$$

Figure 6a, 6b and 6c respectively show the positive domain, negative extended domain and combined full graph of $u = f(x) = -\frac{1}{\sqrt{x}^{\sqrt{x}}}$. They ignore the special requirement for this example that we multiply the equation by -1 due to the trigonometric substitution.

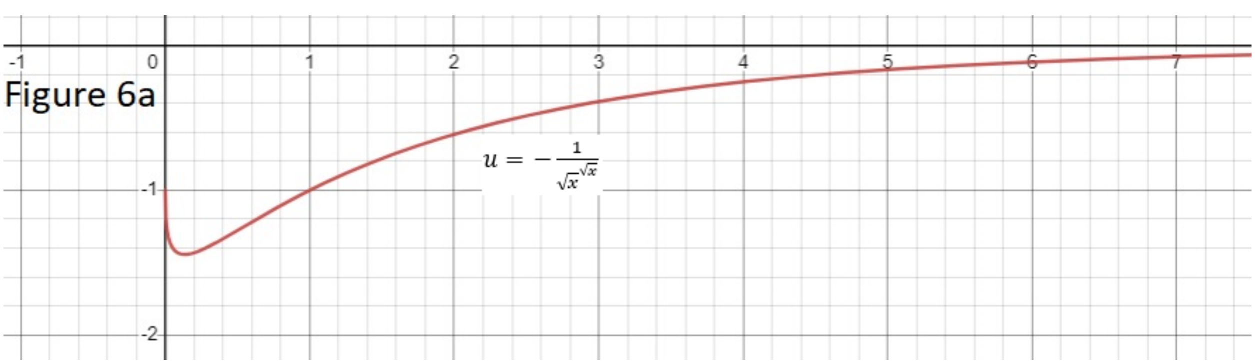
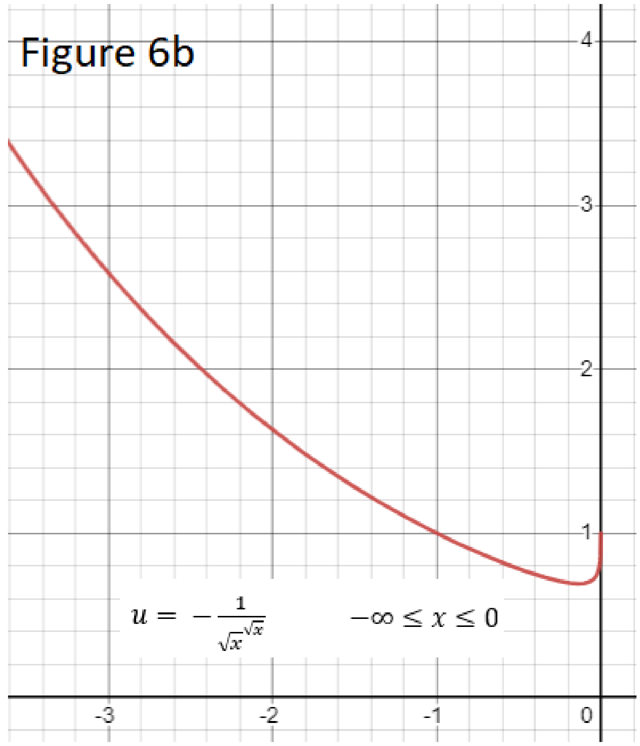


Figure 6a shows the positive domain of the graph of $u = f(x)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the traditionally defined domain for this example $0 \le x < \infty$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.

Figure 6b shows the negative domain of the graph of $u = f(x)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the extended domain for this example $-\infty < x \leq 0$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.



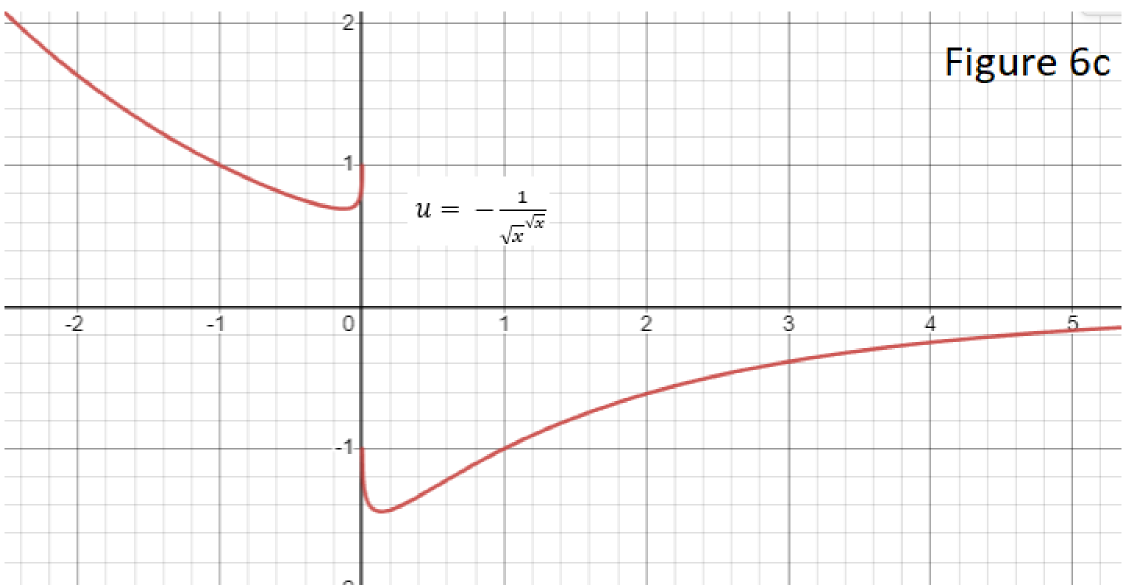


Figure 6c shows the full domain and range of the graph of $u = f(x)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the full domain, both the extended and traditional domain, $-\infty < x < \infty$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.

8.0—The Transverse Plane $u = f(s)$:

Using the definition of section 6.0 we have the following.

$$y = \sqrt{x}^{\sqrt{x}} \qquad u = -\frac{1}{y} \qquad s = -\frac{1}{x} \qquad x = -\frac{1}{s} \qquad u = f(s) = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$$

8.1:

The function is traditionally defined over the domain of $-\infty < x < 0$ and may be graphed directly.

8.2:

The extended domain will result in *i*-multiples for $0 \leq x < \infty$. The requirement that the input axis which matches the output axis' type will take the positive *up* value resolutions will provide we use positive *up* resolutions for the *i*-multiples on the *s*-axis components of the equation. *A* standard graphing utility can be forced to graph it appropriately by using

$$u = f(s) = -\frac{1}{\left(\sqrt{\frac{1}{s}}\right)^{\left(\sqrt{\frac{1}{s}}\right)}}$$

Figure 7a, 7b and 7c respectively show the negative domain, positive extended domain and combined full graph of $u = f(s) = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$.

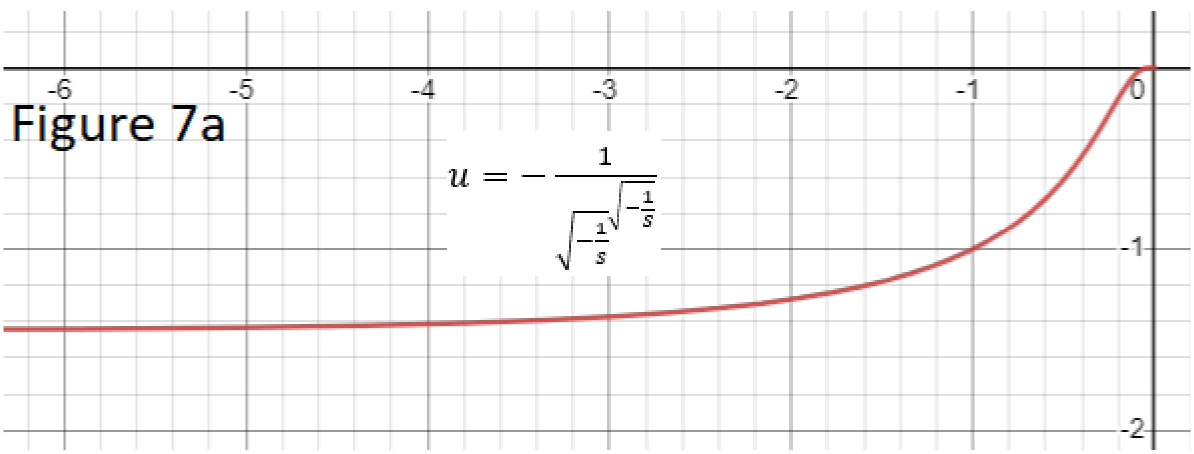


Figure 7a shows the negative domain of the graph of $u = f(s)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the traditionally defined domain for this example $-\infty < x \leq 0$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.

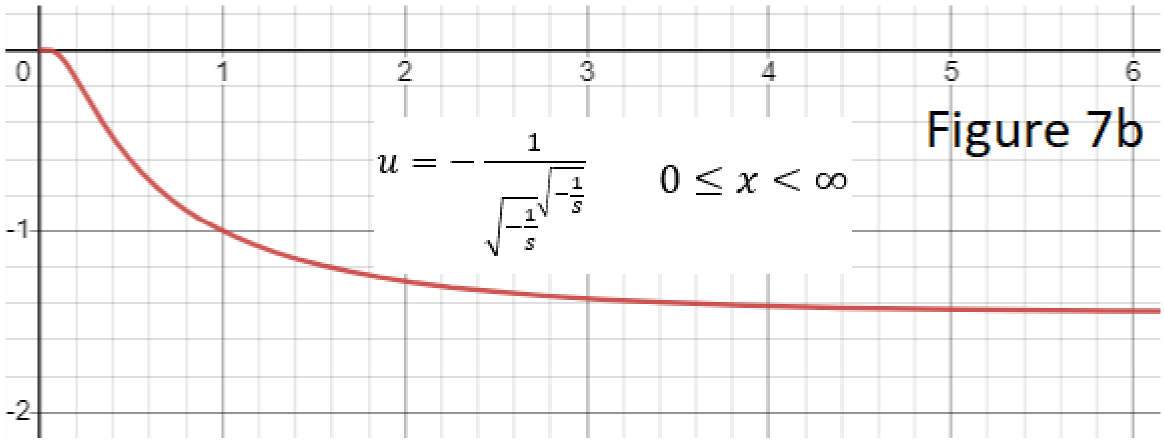


Figure 7b shows the positive domain of the graph of $u = f(s)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the extended domain for this example $0 \leq x < \infty$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.

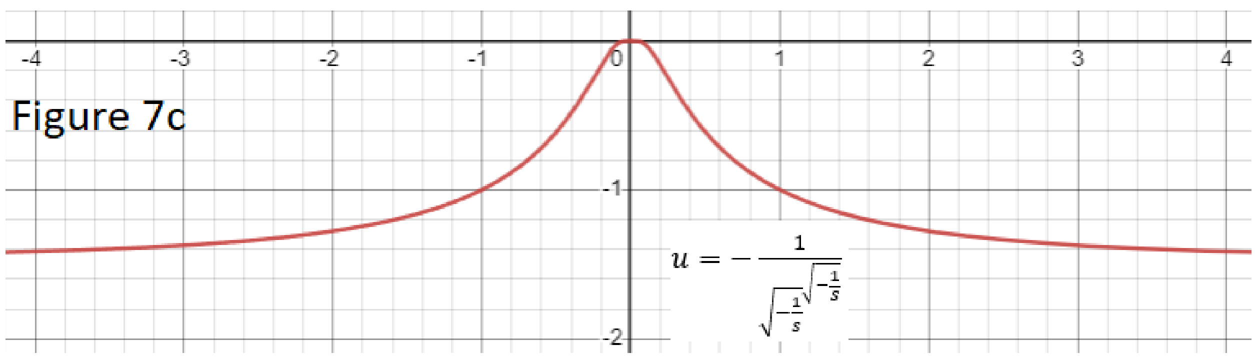
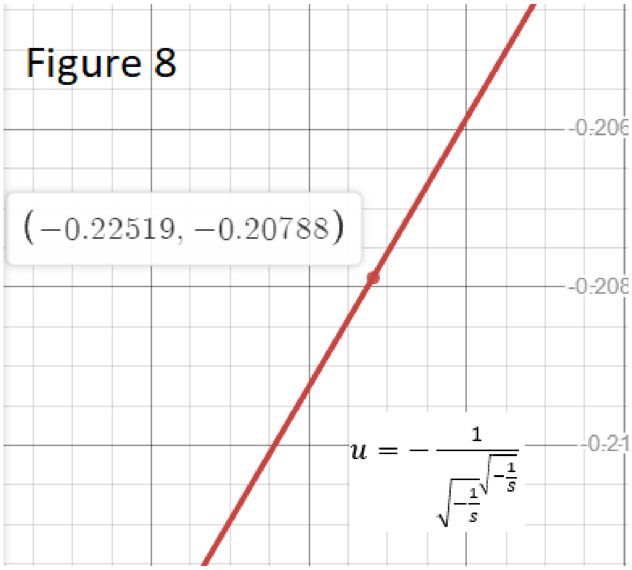
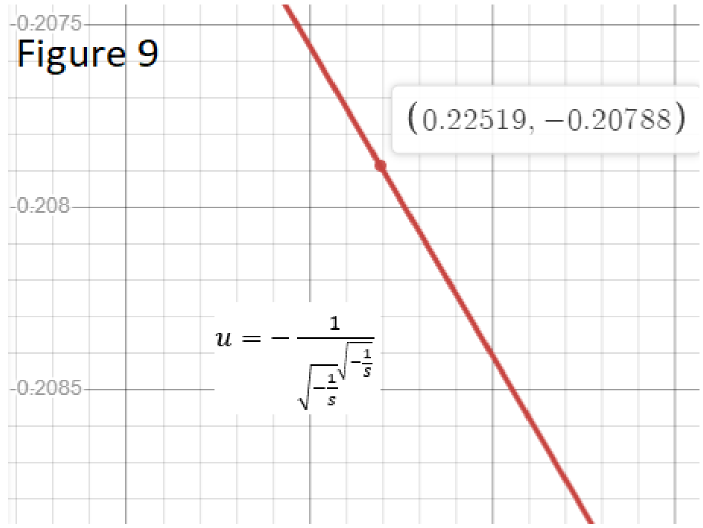


Figure 7c shows the full domain and range of the graph of $u = f(s)$. This graph is presented to show the standard approach to deriving this equation from the given equation on the Central Plane. It is graphed over the full domain, both the extended and traditional domain, $-\infty < s < \infty$. It ignores the need to multiply the equation by -1, a step obtained from the nature of the trigonometric substitution of i^i. This will be expounded upon later.

9.0—Locating $i^i \approx 0.20788$:

The $u = f(s)$ equation is the first instance in which output values lie in the range of an approximately $^1/_5$ magnitude. We can examine this graph in both the positive and negative domain and see where this value exists. See figures 8 and 9 below.





9.1:

We can clearly see that an input value of approximately ± 0.22519 will produce an output value of approximately -0.20788. This is the correct magnitude, but the wrong sign. The trigonometric substitution which provided a solution specified a positive value.

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$$

We need to go back and re-examine the process by which we arrive at this value from the substitution process and see how it relates to the *u*-axis.

9.2—Squaring i:

It is known that squaring i will produce -1.

$$i^2 = -1$$

The trigonometric resolution of i^i involves the squaring of i

$$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{i \cdot i\frac{\pi}{2}} = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788$$

9.2.a:

Null Algebra Extension II provides a list of resolutions for apparent discrepancies that exist between Traditional Algebra and Null Algebra instances for powers of i^n for $n \geq 1$. These were provided because Null Algebra resolutions for each higher power of i not only rotate the value around the complex plane, but change the axis to which it references, in terms of the originally provided variable.

For example the Null Algebra resolution of $i^3 = +1$. This is the value obtained on the *s*-axis subspace, seen in term of the *x*-axis. Yet on the *x*-axis its value will resolve to -1. Using the subspace transformations we can easily see this.

$s = -\frac{1}{x}$ For: $i^3 = +1 = s$ Then: $i^3 = \frac{\Xi}{\varsigma x}(+1) = -1 = x$

These resolutions for multiples of *i* provide meanings for which axis is implied at a given power of *i*. They do not require usage of that value alone as it has a simultaneous conjugate pair. Just as this resolution of i^3 provides the output value in terms of $s = +1$, if using the *x*-axis one may simply convert this to -1.

9.2.b:
The Null Algebra resolution for squaring *i* is especially important in this regard.

$$i^2 = i \cdot i = -1$$

9.2.b.i:
This represents a 180 degree rotation on the complex plane. Null Algebra Extension II indicates, given an equation of the form $y = f(x)$, when squaring i we are referring to one of two boundaries between the Central Plane and the Co-adjoining Subspace. The value obtained, $i^2 = -1$, is the value on the Central Plane in terms of the *xy*-plane. Though this is representative of the value as it appears on the *xy*-central plane, it simultaneously represents the value of $+1$ on the *sy* Co-Adjoining subspace plane.

9.2.b.ii:
All *i*-multiples have an *up* and a *down* conjugate value. All values assigned to a given axis, will have a corresponding values on subspace axis. Powers of *i* of the form i^n may result in a real value of the form $a \pm 0i$. These values through completely real still lay on the complex plane with conjugate values that are identical. They will also have corresponding values on subspace axis.

$y = 0 + bi$	For $x = a$	
$y = a + 0i$	has conjugate	$y = a - 0i$
$y = a$	No difference	

This is the situation we have with the expression: $y = i^i = 0.20788 \pm 0i$

We are examining a single value for the *y* variable. It just happens to be a complex number of the form $z = 0.20788 + 0i$, which results from the trigonometric substitution. For such single values of *y* we have the following relations:

But there are corresponding subspaces

XY-Plane	$y = a + 0i \ = \ 0.20788$		
SY-Plane	$y = -\frac{1}{a} + 0i$	$y = -\frac{1}{a} - 0i$	$y = -\frac{1}{a}$
XU-Plane	$u = -\frac{1}{a+0i}$	$u = -\frac{1}{a-0i}$	$u = -\frac{1}{a}$
SU-Plane	$u = -\frac{1}{-\frac{1}{a}+0i}$	$u = -\frac{1}{-\frac{1}{a}-0i}$	$u = a$

The number 0.20788 is on the complex plane and results from trigonometric substitutions on $y = i^i$. Here we have $y = i^i = a$.

$$0.20788 \ \pm 0i \ = \ 0.20788 = a$$

From section 9.2.b.ii we may conclude that 0.20788 may equal either *y* or *u*.

$$y \,||\, u = 0.20788$$

<u>9.3.a:</u>

The trigonometric substitution $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788 ...$ is a hyperbolic trigonometric expression because the value of *i* has been removed from the expression. It remains a resolution of i^i. Because the substitution $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i \approx 0.20788$ is applied to $y = i^i$ we are addressing the output y axis. Because that same substitution squares i we are not only addressing y but by extension its subspace u. The answer is a completely real value of the form $y = a \pm 0i$. It is not found on the y-axis, so we can expect to find the value of $u = a$ on the *su* transverse plane. This is seen in 9.2.b.ii. This is due to the Null Algebra resolutions for Powers of *i* discussed in Null Algebra Extension II.

9.3.b.i:

The given equation we began with, $y = \sqrt{x}^{\sqrt{x}}$

For $x = -1$ $\qquad\qquad$ $y = i^i$

The trigonometric substitution provides

$$i = \mathbf{e}^{i\frac{\pi}{2}} \qquad\qquad i^i \rightarrow \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}} \approx 0.20788\ldots$$

9.3.b.ii:

When i is squared in $\left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i = \mathbf{e}^{-\frac{\pi}{2}}$ occurs, it still refers to $i^2 = -1$. That -1 applies, as detailed in Null Algebra Extension II, to the output axis value on the plane of occurrence but also still lies on the border between the real-space axis and its corresponding. Thus we may in fact be dealing with $u = f(s)$ equation. Finding the magnitude of -0.20788 on the $u = f(s)$ equation, we know we are referring to the u axis when applying the trigonometric substitution.

9.3.b.ii:

Thus the equations obtained when converting to $u = f(x)$ and $u = f(s)$ must account for the squaring of *i* that occurred with trigonometric expression substitution. Seeing the output of 0.20788 does not occur on *y* and does occur on *u* confirms this. Further failing to account for the squaring of *i* will produce the wrong value of the *u*-axis outputs; the value of -0.20788 is present but should be positive. When we account for this and multiply the *u*-axis equations by -1 we get the correct form of the *u*-axis equations.

10.0—Re-defining the *u*-axis equations:

Now we may re-explore the *u*-axis equations. The subspace $u = f(s)$ equation will take positive *up* values for instances of *s* which generate *i*-multiples. Likewise the subspace equation $u = f(x)$ will take the negative *down* values for instances of *x* which generate *i*-multiples.

10.1:

If we do not account for the squaring of *i* observed in the trigonometric substitution we would assume to use the equations of the form:

$$u = f(x) = -\frac{1}{\sqrt{x}^{\sqrt{x}}} \qquad\qquad u = f(s) = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$$

With caveat that we must multiply the *u* equations by -1 we obtain their actual form.

$$u = f(x) = -\frac{1}{\sqrt{x}^{\sqrt{x}}} \cdot (-1) \;=\; \frac{1}{\sqrt{x}^{\sqrt{x}}}$$

Can force a graphing utility to graph the full domain as:

$0 \leq x < \infty \qquad u = f(x) = \frac{1}{\sqrt{x}^{\sqrt{x}}}$

$-\infty < x \leq 0 \qquad u = f(x) = \frac{1}{-\sqrt{-x}^{-\sqrt{-x}}} = -\sqrt{-x}^{\sqrt{-x}}$

$$u = f(s) = -\frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}} \cdot (-1) \quad = \frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$$

Can force a graphing utility to graph the full domain as:

$0 \leq s < \infty \qquad u = f(s) = \frac{1}{\sqrt{\frac{1}{s}}^{\sqrt{\frac{1}{s}}}}$

$-\infty < s \leq 0 \qquad u = f(s) = \frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$

The following figures show the various graphs of $u = f(x)$ and $u = f(s)$.

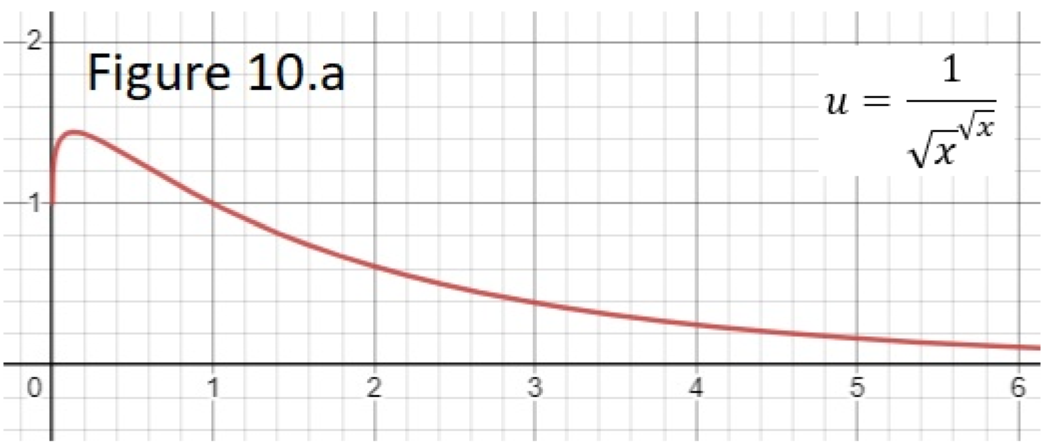


Figure 10.a shows the positive domain of the graph of $u = f(x)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is otherwise obtained using the standard approach to deriving the $u = f(x)$ equation from the given equation on the Central Plane. It is graphed over the traditionally defined domain for this example $0 \leq x < \infty$.

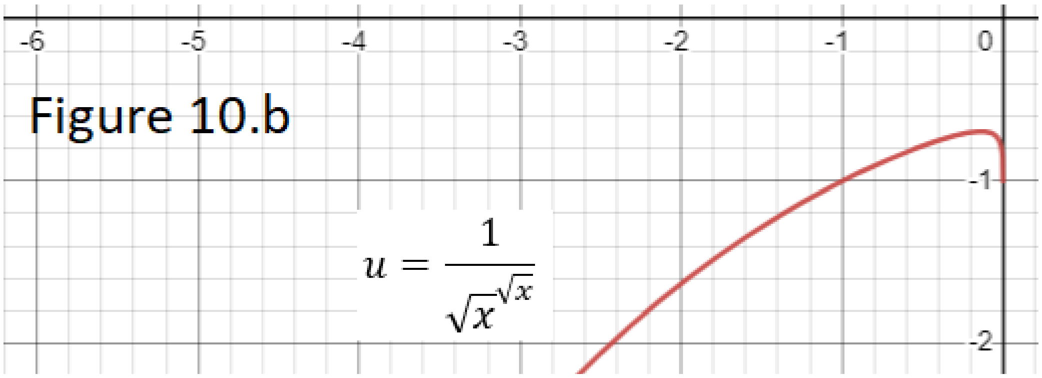


Figure 10.b shows the negative extended domain of the graph of $u = f(x)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is otherwise obtained using the standard approach to deriving the $u = f(x)$ equation from the given equation on the Central Plane. It is graphed over the extended domain for this example $-\infty < x \leq 0$.

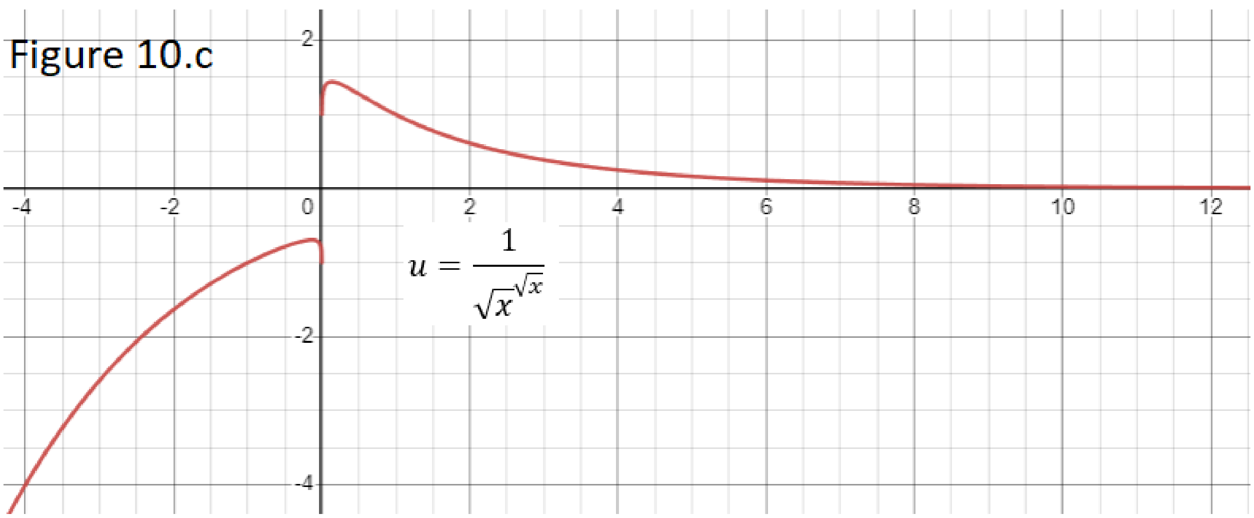


Figure 10c shows the full domain and range of the graph of $u = f(x)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is graphed over the full, both the extended and traditional domain, $-\infty < x < \infty$.

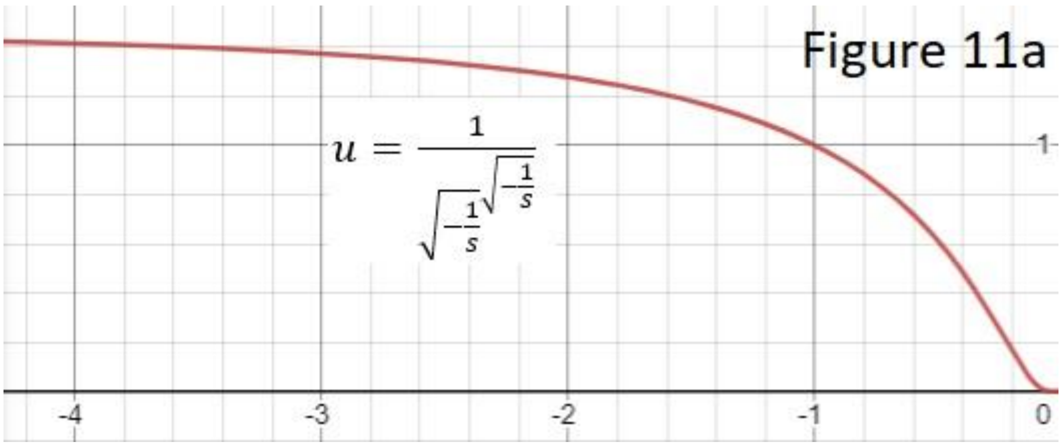


Figure 11.a shows the negative domain of the graph of $u = f(s)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is otherwise obtained using the standard approach to deriving the $u = f(s)$ equation from the given equation on the Central Plane. It is graphed over the traditionally defined domain for this example $-\infty < s \leq 0$.

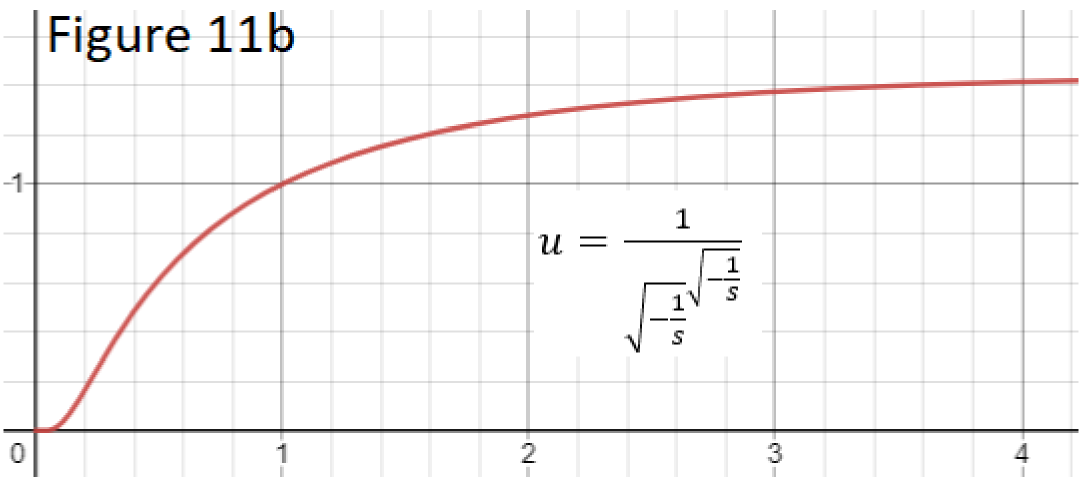


Figure 11.b shows the positive extended domain of the graph of $u = f(s)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is otherwise obtained using the standard approach to deriving the $u = f(s)$ equation from the given equation on the Central Plane. It is graphed over the extended domain for this example $0 \leq s < \infty$.

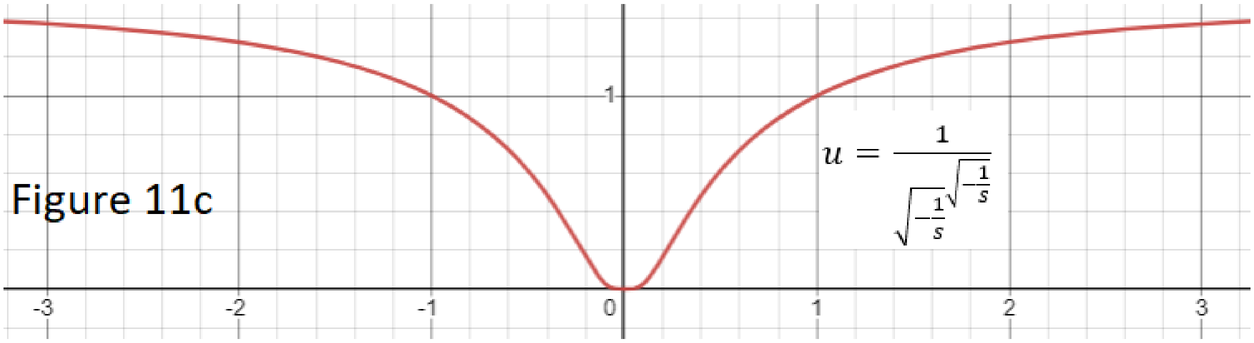
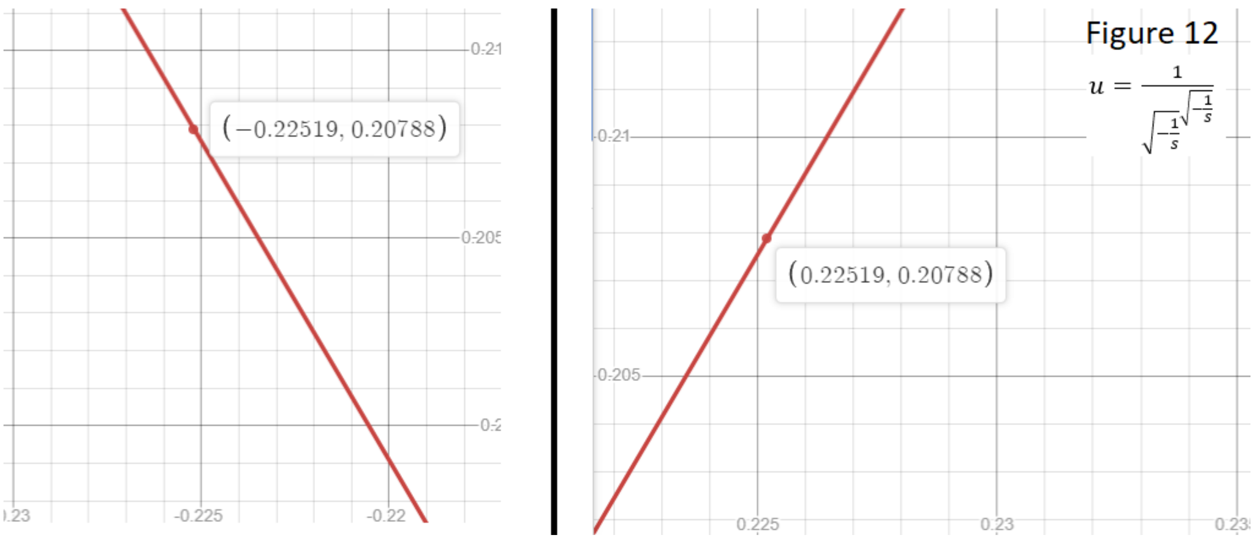


Figure 11.c shows the full domain and range of the graph of $u = f(s)$. This graph includes the need to multiply the function by an additional -1 due to the consideration of the trigonometric substitution $i = \mathbf{e}^{i\frac{\pi}{2}}$ in the expression i^i. It is graphed over the full domain, both the extended and traditional domain, $-\infty < s < \infty$.

$$u = \frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$$

<u>10.2:</u>

If we closely examine the graph of Figure 11.c we can see the values for which this subspace extension of the original equation does equal 0.20788. If is shown in Figure 12.



$$u = \frac{1}{\sqrt{-\frac{1}{s}}^{\sqrt{-\frac{1}{s}}}}$$

<u>11.0—A list of all pertinent values:</u>

When we began the exploration of the expression i^i as one output value in a function of the form $y = \sqrt{x}^{\sqrt{x}}$ by considering the various output values we obtain, depending on the method for resolving the instances of *i*.

The Null Algebra resolutions provide four possible values for i^i as denoted in section 3.1.a:

Direct Resolution of i	Substitution, Then Resolution of i
For $y = f(x)$ equation: x-axis: $i^i \doteq \oplus 1^{\oplus 1} \doteq \hat{1}^{\hat{1}} \doteq \dot{1}^{\dot{1}} = 1$ s-axis: $\doteq \check{1}^{\check{1}} \doteq -1^{-1} = -1$	$i^i = \left(\mathbf{e}^{i\frac{\pi}{2}}\right)^i \doteq \left(\mathbf{e}^{\oplus 1\frac{\pi}{2}}\right)^{\oplus 1} \doteq \begin{cases} \left(\mathbf{e}^{\hat{1}\frac{\pi}{2}}\right)^{\hat{1}} \doteq \mathbf{e}^{\frac{\pi}{2}} \\ \mathbf{e}^{(\oplus 1)^2\frac{\pi}{2}} \doteq \mathbf{e}^{-\frac{\pi}{2}} \end{cases}$

The trigonometric substitution only provides for $i^i = \mathbf{e}^{-\frac{\pi}{2}} = 0.20788$.

<u>11.1.a:</u>

Thus the possible values we should see represented in the full extended subspace graphs of the given equation $y = \sqrt{x}^{\sqrt{x}}$ must include:

$$+1 \quad , \qquad -1 \quad , \quad \mathbf{e}^{\frac{\pi}{2}} = 4.81047738 \quad , \quad \mathbf{e}^{-\frac{\pi}{2}} = 0.20788$$

On some section of the full extended series of graphs originating from $y = \sqrt{x}^{\sqrt{x}}$ include all four of these values on some section of those collective graphs.

Thus: $$i^i = \begin{cases} +1 \\ -1 \\ 4.81047738 \\ 0.20788 \end{cases}$$