

Notes of Black Holes

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Abstract

The primary purpose of this article is to fundamentally invoke non-standard analysis throughout any interaction to thereby holographically “de-centralize” the hidden conformal modes (large central tower OPEs). Ultimately, measurements, continuum, and singularity mechanics are dualized against Wilson partitions, chaos representation, and renormalization flow, to produce a universal topological field theory. The Primary results established are: sub-harmonic chaos in $U(1)$ gauge theory is identified and quantized of the the flat space celestial hologram, a background model of virial loop-information in Einstein’s gravity is $U(1)$ probed at a critical { sub-topology, pressure } measurement-phase and found to be a dual basis of $d=4$ $2x2$ Gaussian Unitary Ensemble gauge theory, in uniformity, establishing it as a candidate of non-perturbative, loop QCD in gravity; the state preparation simultaneously produces a post-selected supersymmetry algebra (over the canonical log-partition) which survives all possible no-go tests. Finally, the uniform net weight of the canonical partition state identifies the $2x2$ GUE critical vacuum as a $\frac{1}{8}$ -BPS topological phase measurement prepared as a quasi-continuous $SO(4)_{KK} \times \mathbb{R}_{BPS}$ state of information decay; indeed, the partially conformal background is identified as a $\frac{1}{4}$ -BPS shadow and given a mechanism of spin-entanglement.

Notably also, the Cosmological Hierarchy problem is resolved, the fine structure constant is derived (up to 5 orders of magnitude) using analytic black hole decay, and a new, $21 - pt$ emergent universal holographic constraint bound between celestial gravitons, quantum information stability is shown at loop level, which further resolves the naturalness of $d = 4$ emergent spacetime and the directedness of time.

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Dedicated to my brother, Andrew B. Chanson, the only person who understands. I love you.

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1 Introduction

Rather than proper contextualization of modern physics[1], this paper will instead focus on the direct mathematical embedding of physics from a modern perspective. To that end, it is only important to remember what shadows analytic techniques presently cast on the past. Particularly, the following crude observations:

- 1) Newton didn't know about electric/magnetic duality;
 - 2) Maxwell didn't know about space/time duality;
 - 3) Einstein didn't know about thermodynamic/gravitational dualities;
 - 4) Hawking did know about AdS/CFT type dualities.
- (1)

To this end, the remarkable state of physics currently may be painted as an extension of the increasing scope of the sense of duality generally, with the AdS/CFT correspondence [2] representing the breakneck development of a categorical closure mechanism of physical principles themselves[3]. To that extent, the difference between 21st century physics and 20th century physics is, crudely, that this recent series of revolutions have revolved around dualities themselves, not any specific a priori set of dualities. The full utilization of functional principles of symmetry has yielded continual, and remarkable, mathematical architecture in a relatively short period of time across nearly every subfield [4][5] [6] [7] [8] [9] [10] [11] [12]. Perhaps unsurprising from the strong partnership between maths and physics, this has all occurred in the background of a similarly incredible set of unifications in pure mathematics [13].

This paper represents a sophomoric attempt to categorize modern features of dualities from a functionally *out-to-in* perspective, generally reasoning that finding well defined (and minimal) coordinates over phase space is the hard part of physics; this may be read as a series of extensions towards my previous collaborations [14] [15] [16] (summarized in my PhD thesis, to be published shortly [17]). Less (or more) concisely, a central thesis is that measureability is, generically, a function of measures while, inversely, measures are not necessarily a function of measureability: they are, by construction, the decider functions of the minimal embedding topology. The number of functional measures available vastly exceeds the number of measures; still, heuristically it is always possible to “measure measures of measureability”[7], known as the (quasi)uniformity embedding, and, accordingly, particular work is made to avoid analytically standard asymptotic/perturbation constructions.¹

Less (or more) coherently, hidden symmetries may themselves produce symmetries *in* mea-

¹Put cheekily , all apples fall at the same accelerative rate on (the same area of) Earth...unless the apple is actually hiding a hidden system (such as a rocket/explosive device), in which case there may be a sudden, less *outside* observable explanation for the trajectory. Dually, and beyond more cheekily, if it is expected some subsets of apples may contain explosives it is less appropriate to drop it on the head of an expectant, $\sqrt{\frac{2\Delta[h]}{g}} = t$ certain, Sir Isaac Newton.

sure, although this may not always be measurable for every set of measures; this is exactly the idea of “emergence”. To this, black holes provide an interesting model of this effect by producing strong effects on *out*-measurability without, necessarily, strongly changing any *in*-moments (event horizons of active galactic nebula often have planet-scale surface gravity); indeed hawking radiation [18] and black hole jets [19] are both strongly induced by boundary effects of cosmic censorship [10], or a strict condition of stable black hole embeddings [20]. In-fact, because of subtleties to be covered (concerning the existence of a pullback cover), the information paradox cannot be resolved with classic hawking analysis for the same reason that Newton could not unify light/matter, why Maxwell couldn’t unify electricity and gravity, and why Einstein couldn’t unify quantum mechanics and relativity: all considered the notion of duality to be itself rigid [21] (in the sense of being always, everywhere thermodynamically actionable, or of Noether class). This paper will set out to show, across a series of basic examples, black holes present a different class of symmetries (in-interaction) that have interesting applications to the meaning of duality itself simply because they exactly embody the classical notion of “action at a distance” (essentially because of their starkly censored interactions with the observable universe).

To that extent, this section ends with one final observation: everything observable in the universe is unavoidably measured across time and under (weak) gravity [22] [23]. Still, Banach compactification is always available in physics [7]; accordingly, it is computationally optimistic to hypothesize that emergent dualities exist in the physics of measurement as an interaction itself. Reflectively, measurement emergence can also be thought of as absolutely axiomatic² under classical thermodynamics [24], causality [25], and quantum stangeness [26] conditions.

1.1 The Kerr Geometry

The primary system of interest throughout this paper will be the pure gravitational interaction pole found in the metric given by the Kerr geometry. In Boyer-Lindquist[27], or thermal, type coordinates,

$$ds_K^2 = 2\omega_\phi d\phi(dt - d\phi) + \Sigma \left[\Delta^{-1} dr^2 + \frac{\omega_\phi}{a} d\phi^2 + d\theta^2 \right] + \frac{\Delta - a\omega_\phi}{\Sigma} (dt - \omega_\phi d\phi)^2 \quad (2)$$

with $\omega_\phi = a \sin^2 \theta$, $\Sigma = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 - r_s r + a^2$.³

In $D = 4$, the no hair theorems guarantee that classical Kerr black holes are uniquely identified by two asymptotic invariants: the apparent event horizon area (which is time-like tangent, a.k.a. the geodesic mass) and the angular momentum (constructed as a near-horizon, null co-tangent e.g., a geodesic sheet index). Quickly, these invariants can be

²And, classically, the trivially relative dual of emergent measurements, a.k.a., experimental results/proof construction, known as “error correction”.

³Note that Δ is polynomial, and can be readily inverted to find the coordinate singularities $\frac{r_\pm}{r_s} = \frac{1}{2}(1 \pm \sqrt{1 - \alpha^2})$, where $\alpha = \frac{2a}{r_s}$. Under the no hair theorems, the geometry of this region determines the black hole’s interaction space, lending to the idea of α as a dynamic parameter and r_s and a canonical scale.

extracted by considering their Newtonian limits in the far-field regimes; rigorously[28], they can be found by propogating the near horizon Killing-frame throughout the spacetime (and noting it's peeling invariants[29] at \mathcal{I}^+ , under at matching condition at \mathcal{I}^0). Physically, these invariants can be found by coupling a conformal, dynamic probe field to use as a spectral couple[10], [30].

Critically the event horizon, despite being a completely contact shadowed surface, is interactive: in fact, the second thermodynamic invariant is explicitly related to this classical extended region of interaction, known as the ergosphere⁴.

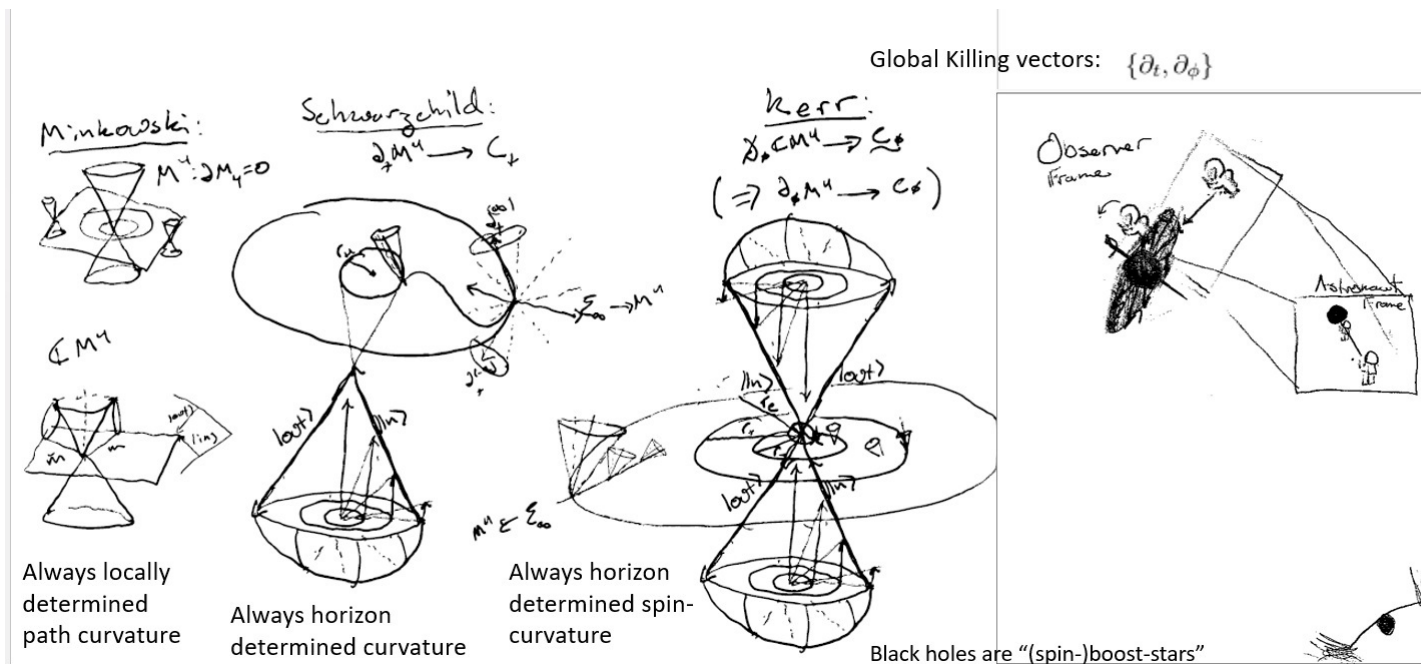


Figure 1: Some Stable Spacetime Interactions in d=4

Indeed, black holes with angular momentum change the geodesic states of spacetime (embeddings) local to neighborhoods of the event horizon: timelike orbits are rotationally excited near Kerr black holes and, if the orbits are also massive⁵, they may exchange boost radiation. ⁶ Indeed, there are a number of well known mechanical orbital procdures, known as Penrose processes, which can be used to extract this boost energy from a Kerr black

⁴In fact, the Kerr black hole was originally built by looking for boosted symmetries of the Schwarzschild solution in Eddington-Finkelstein (advanced and retarded) coordinates, which generatively led to the Kerr-Schild metric (from which Boyer-Lindquist coordinates are derived)[31]. Thus, in fact, the available thermodynamic energy comes from a bound boost state in the Kerr-Schild metric: hence, Kerr black holes can roughly be thought of as "boost-frame radiators". This line of thought is pushed further in what's known as the Inverse Scattering Method, a constructive technique in general relativity which is the jumping off point to constructing higher dimensional black holes (or strings, or saturns, or otherwise generalized horizon structures)[32]

⁵Have timelike legs at the conformal boundary, e.g., have timelike, near-asymptotic quasi-invariants which may be deformed (on-sheet)⁽⁴⁾

⁶The edges of this region is known as the ergosphere, and are given by the points where the sign of g_{tt} changes: $r_+^e = \frac{r_s}{2} (1 \pm \sqrt{1 - 4a^2 \cos^2 \theta})$. This is a Penrose Process.

hole. More generally, it is interesting to consider how this boost radiation may affect other types of fields throughout physics⁽⁵⁾; this paper will specifically concern how vector gauge fields respond.

1.2 The Cosmological Constant

Starting from real-algebraic topology in $d = 4$ dimensions, $\mathbb{R}^{1,3}[t, \vec{x}] \rightarrow \mathbb{R}[w] \times \mathbb{R}^3[y]$ s. t. $\mathbb{A}[M] \rightarrow \mathbb{A}^\lambda[U(1)] \times \mathfrak{A}_\lambda[\mathbb{R}^{(3)}]$, two solutions to the low-dimensional geodesic parameterization can be found immediately. Firstly, it can be immediately shown that the local Poincare group can be globally extended to a global solution manifold $\mathbb{R}^{1,3} \equiv M^4$ known as Minkowski space. Killing-groups in this geometry are constructed to satisfy the Lorentz algebra globally; explicitly, the Minkowski metric is given by:

$$ds_{\text{Minkowski}}^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \sum_{i=1}^{D-1} (dx^i)^2 \quad \Rightarrow \quad g^{\text{Min}}[\cdot, \cdot] = (-\partial_t^2 \otimes \mathbb{I})[\cdot, \cdot] \quad (3)$$

It is useful to note that, in this metric, the canonical differential form is pushed to within a sign of the universal partial wave $\nabla_\mu[\cdot] \rightarrow (-\delta_t^\mu + \delta_i^\mu) \partial_\mu[\cdot]$. Iconically, the effect is just to change the sign-weight of the sub-dimensional partial wave, which can immediately be understood as an exact algebraic decomposition of the null topology⁷. This immediately motivates the secondary canonical solution form, the Euclidean metric⁸:

$$ds_{\text{Euclidean}}^2 = \sum_{i=0}^D (dx^i)^2 \quad \Rightarrow \quad g^{\text{Euc}}[\cdot, \cdot] = \mathbb{I}[\cdot, \cdot] \quad (4)$$

Then, it is immediate that a $d = D - 1$ Euclidean sub-space canonically extends throughout Minkowski spacetime,

$$g^{\text{Min},(D)}[\cdot, \cdot] = \left(-\partial_t^2 \otimes g^{\text{Euc},(d)}\right)[\cdot, \cdot] \quad (5)$$

Still, it is important to note that the converse is NOT universally true: $d = D$ Minkowski space must be canonically embedded in a $D+1$ dimensional Euclidean spacetime (and thus only up to isomorphism of choice [33])⁹. Two "naive" manipulations may be used to isolate the relevant subspaces:

$$ds_{EM}^2 := ds_E^2 + ds_M^2 = 2dx_i dx^i \equiv ||ds_E + id s_M||^2 \quad (6)$$

$$ds_{E/M}^2 := ds_E^2 - ds_M^2 = 2dt^2 \equiv (ds_E + ds_M)(ds_E - ds_M) \quad (7)$$

⁷ $ds_M^2 = 0 \Rightarrow dt^2 = dx_i dx^i$; this can also be seen by the homogeneous wave equation in this geometry, or the (flat) d'Lambertian, $\square[*] = 0 \Rightarrow \omega^2 = k^2$

⁸which, of course, was historically discovered first through it's universal applications in partial-wave (Fourier) decomposition

⁹Existence, and the converse, immediately follows from Tychonoff's theorem and the axiom of choice: the $D+1$ Euclidean compactification exists iff \mathbb{R} is compact over $\mathbb{R} \otimes \mathbb{R}$. It is exactly this subtlety that allows the compactification scheme of the AdS-CFT correspondence to produce actual bulk dispersion relations.

Accordingly, the geometry of flat space may be considered as a "Minkowski polarization" of the Euclidean embedding; further, the internal topology of flat time may be considered as a "Minkowski factorization" of the Euclidean embedding.

Starting from the Einstein-Hilbert action:

$$S_{\text{EH}} = \int R \quad \Leftrightarrow \quad S[x^\mu] = \int d^n x \sqrt{|g|} R \quad (8)$$

Here, it is important to note that the curvature/metric coupling comes about through the Jacobian form, a canonical result from analysis which pulls the curved manifold back to its domain coordinate form.¹⁰ Importantly, the Jacobian is a universal result from real analysis which relies on sub-domain measures¹¹ Thus, identically, the Einstein-Hilbert action minimizes the local curvature measure by imposing a universal, though coordinate specific, measure envelope: the Jacobian. In particular, this can be interpreted as an equation relating the curvature weighted metric (the canonical coordinate chart) and the curvature tensor (the coordinate chart of the curvature¹², manifold) together, known as Einstein's Equations (here presented with no matter sources):

$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 0 \quad \Rightarrow \quad g_{\mu\nu} = 2 \frac{R_{\mu\nu}}{R} \quad (11)$$

The final, cavalier manipulation attempts to interpret the metric as a trace-normalization of the curvature form. Ignoring the fact that the curvature is a function of the metric, the RHS representation characterizes the metric as a "form-unit of curvature" a.k.a., the local geodesic deviation-form or the acceleration of a test particle relative to a flat-Minkowski coordinate embedding. Moreover, on face this naive interpretation has a difficult time explaining $R = 0$ -curvature manifolds.

In fact, supposing a solution $\{g[x^\mu], R[x^\mu]\}$ exists under some spacetime-symmetric δ -diffeomorphism, it is immediately clear that another solution can be generated under a

¹⁰In physics there are two classical pullback spaces: the Euclidean domain, \mathbb{R}^D , and the Minkowski domain, $\mathbb{R}^{(1,D-1)}$. Importantly, classical General Relativity assumes a local Minkowski symmetry and thus domain coordinates will almost always be Minkowski (type unless otherwise specified or projected).

¹¹In particular, it combines diffeomorphic symmetries to parameterize a local coordinate basis in (parameterized) units of the D -2 relative dual(-basis) velocities:

$$\int d^n x [\lambda^\mu] f[x^\mu[\lambda^\mu]] = \int d\lambda^\nu \int d^n x [\theta^\mu] \lambda_\nu f[x^\mu[\lambda^\mu]] = \int d\lambda^\nu \lambda_{\dot{\mu}} \int d^n x [\theta^\mu] \theta_\nu^\mu f[x^\mu[\lambda^\mu]] \quad (9)$$

¹²or "tangent of the tangent": $R^\alpha_{\mu\beta\nu} V^\mu = ([\nabla_\beta, \nabla_\nu] + T^\lambda_{\beta\nu} \nabla_\lambda) V^\alpha$ where $T^\lambda_{\mu\nu} = 2\Gamma^\lambda_{[\mu\nu]}$ is the torsion tensor. Also, $R_{\mu\nu} = \delta^\beta_\alpha R^\alpha_{\mu\beta\nu}$. Then, Einstein's equations read:

$$\left([\nabla_\mu, \nabla_\nu] + T^\lambda_{\mu\nu} \nabla_\lambda - \left(G_{\mu\nu} + \frac{R g_{\mu\nu}}{2} \right) \right) V^\mu = 0 \quad (10)$$

This can be interpreted as saying that local area gaps in flat embeddings of geodesic (bundles) is sourced by the torsion operator (as an affine velocity field) and/or by the Einstein tensor and the metric weighed by the curvature scalar (which behave as mass terms). In this sense, energy-density and the curvature of spacetime may be thought to warp the test path of vector probes. Note that a fine balance can, in principle, develop between $T^\lambda_{\mu\nu} \nabla_\lambda$ and $G_{\mu\nu}$ such that $g_{\mu\nu} \sim \frac{2}{R} [\nabla_\mu, \nabla_\nu]$.

constant shift in the curvature action,

$$S_\Gamma = \int d^d x \sqrt{|-g|} (R + 4\Lambda) \quad \forall \Gamma \in \mathbb{R} \quad (12)$$

simply by adding a Λ -scaled Jacobi-density variation term to Einstein's equations:

$$G_{\mu\nu}^{(\Lambda)} := G_{\mu\nu} - \Lambda g_{\mu\nu} = 0 \quad \Rightarrow \quad g_{\mu\nu} = 2 \frac{R_{\mu\nu}}{R + 2\Lambda} \quad (13)$$

Again ignoring the functional dependence of the curvature of the metric, the RHS attempts to interpret the metric as a shifted-trace-normalization of the curvature form. Then, $\forall \Lambda \neq 0$ it is more plausible to interpret the low curvature limit directly by considering its trace comparison to the "cosmological constant":

$$\Lambda \gg R \quad \Rightarrow \quad g_{\mu\nu} \approx \frac{1}{\Lambda} R_{\mu\nu} \sum_{n=0}^{\infty} \left(\frac{R}{2\Lambda} \right)^n := f\left[\Lambda, \frac{R}{2}\right] R_{\mu\nu} \quad (14)$$

$$\Lambda \ll R \quad \Rightarrow \quad g_{\mu\nu} \approx \frac{2}{R} R_{\mu\nu} \sum_{n=0}^{\infty} \left(\frac{2\Lambda}{R} \right)^n := f\left[\frac{R}{2}, \Lambda\right] R_{\mu\nu} \quad (15)$$

Each expansion has a slightly different interpretation. To zeroth order, the relatively large cosmological constant (the first formulation) treats $g_{\mu\nu}$ as a (globally) re-scaled $R_{\mu\nu}$, not a local trace density; as such, this represents a global curvature density limit. At the same order the second formulation recasts the metric as a tower of scalar interactions off of the flat-space limit $\Lambda \rightarrow 0$ ¹³.

¹³whence the cosmological constant behaves as a Lagrangian multiplier between the relevant (curvature perturbation) splines: $\sum_{n=i}^k \left(\frac{2\Lambda}{R}\right)^n \rightarrow 0 \Rightarrow \sum_{n=0}^k (2\Lambda)^n R^{k-n} \rightarrow \sum_{n=0}^i b_n (2\Lambda)^n R^{k-n}$. Note that, in the relatively small cosmological constant limit $g^{\mu\nu}[*] \approx \frac{1}{R} \frac{\delta^{\mu\nu} Tr[*]}{f[\frac{R}{2}, \Lambda]}$, where $(\cdot)[*]$ represents the continuous functional embedding of the tensor algebra. Immediately, this may be interpreted as a Weinberg pole in the cosmological phase space of the trace class measures under the propagation of a cosmological braid between the curvature poles and the background cosmological pressure. Perhaps surprisingly, this naive deconstruction will prove deep. For example, considering the KG equation on the massive (massless probe[14]) $\varphi[\cdot]$:

$$\left(\partial^\nu + \frac{d-1}{2} \partial^\nu \ln [Rf] \right) Tr [\partial_\nu \varphi] = (Rf)m^2 \varphi \quad \Rightarrow \quad \begin{cases} \partial^\nu Tr [\partial_\nu \varphi] = (Rf)m^2 \varphi & 0 \ll |R| \gg \sup\{|\partial_\nu \varphi|\} \\ [\partial^\nu + \frac{d-1}{2} \partial^\nu \ln [Rf]] Tr [\partial_\nu \varphi] = 0 & R \rightarrow 0 < |\partial^\nu [Rf]| \end{cases} \quad (16)$$

Accordingly, neighborhoods of large curvature are governed by a Gauss-type Law over the curvature (coupling to the cosmological constant) weighed mass $\tilde{m} = Rf m^2$, which can be seen by taking the trace over both sides (RHS is $d = 0$) and defining $D^\mu = Tr \partial^\mu$ (or also by defining an adjoint representation, $\varphi = Tr[\tilde{\varphi}]$); this immediately implies a weak conservation of some cosmological form factor over coordinate curvature poles (of scalar, or thermal, probes). In the relatively small, stable curvature phase note that the gauge-trace classification $\partial^\mu Tr[\] \rightarrow -\frac{d-1}{2} \partial^\nu \ln [Rf] Tr[\]$ gives a collection of neighborhoods with continuous Trace-momentum eigenstates.

Note that the low curvature case reduces to the massless high-curvature case if ($d=1$ or) $f \sim \frac{1}{R}$; then, both relative scales of the cosmological constant produce $\frac{1}{6} \sum_{n=1}^{\infty} e^{n\chi} = -\frac{1}{12}$ where $\chi|_{\lambda > R} = \ln \frac{R}{2\Lambda} = -\chi|_{\lambda > R}$.

Finally, this matching may be directly compared to Ramanujan's series $-\frac{1}{12} = \sum_{n=1}^{\infty} n$ if $\chi = \frac{\ln 6n}{n}$, or $\frac{R}{2\Lambda} = e^{\pm \frac{1}{2} \frac{d}{dn} (\ln 6n)^2}$; in tune, this final expression may be naively inferred to resemble some (renormalized) number (state-)operator over uniformly stable thermal strings in (perturbatively) curved spacetimes. This

Considering how readily the initial difficulty of the low-curvature, flat-space limit fled upon considering $\Lambda \neq 0$ leads immediately to the consideration of some common sub-index expansion of the global and trace curvatures: $\{R, \Lambda\} \rightarrow \{R[l], \text{sign}[\Lambda]l^2\}$ (which amounts to an effective lexicographic parameterization of the functional forms). Assuming the trace density has a saddle-stable sub-index zero-point, $l \rightarrow 0$, limit ¹⁴ gives $R[l] = \sum_{n=0} R[l]_n l^n$ and:

$$g_{\mu\nu} = \frac{R_{\mu\nu}}{2l^2} \frac{1}{1 + \text{sign}[\Lambda] \sum_{n=0} R_n[l] l^{n-2}} \approx \frac{R_{\mu\nu}}{2l^2} \sum_{k=0} \left(\text{sign}[\Lambda] \sum_{n=0} R_n[l] l^{n-2} \right)^k \quad (17)$$

Although the RHS looks messy, it has a clean interpretation in terms of a topological circle sub-basis (of the sub-scale order) $S^1[\sim l]$ by remembering the intrinsic curvature of a circle of radius l is $R_{S^1[l]} = 1/l$, giving:

$$g_{\mu\nu} \approx \sum_{k=0} \frac{R_{\mu\nu}}{2R_{S^1[l]}^{2(1-k)}} \left(\text{sign}[\Lambda] \sum_{n=0} \frac{R_n[l]}{R_{S^1[l]}^n} \right)^k \quad (18)$$

Then, interpreting the square function as the canonical $\mathbb{R}^{\pm/\{\infty\}}$ left/right (L/R) sub-measure gives $\frac{R_{\mu\nu}}{R_{S^1[l]}^{2(1-k)}}$ an interpretation as an L/R sub-density $\mu[\mathbb{R}_{S^1[l]}^{1-k}]$. This can be seen immediately by recognizing that if the curvature tensor could be expanded (or split) in only even (or odd) powers of the circle curvature then the RHS would simplify greatly. ¹⁵

Under what's known as the minimal coupling model, it is assumed that Einstein-Hilbert action may be minimized simultaneously with some action represented, coupled fields:

$$S_{\text{min-model}} = S_{\text{EH}}[g, R; x^\mu] + \hat{S}[\phi^{(i)}, \partial_\mu \phi^{(i)}, \dots; g, R; x^\mu] \quad (19)$$

Notice the above separates a model action into a purely geometric form and a set of interpretation is directed (over the Ricci-tensor), which can be seen by the number sign change between the $\Lambda \rightarrow \Lambda_0 \in \{0, \infty\}$ limiting metric forms (under this "thermal uniformity condition"); further, note that including (high curvature) asymptotic (anti-)mass strings implicitly present another emergent, \mathbb{Z}_2 -symmetric thermal membrane. A complete approach is to minimally express known \tilde{m} algebras (from the standard model) as trace-adjoint, asymptotically flat gauge membrane-charges[34] [35] [33].

¹⁴and that $\mu[\mathbb{R}^{+/\{\infty\}}] = -\mu[\mathbb{R}^{-/\{\infty\}}]$ (where $\mu[\cdot] = |[\cdot]|$) is a global measure on the covering set of the canonical product topology $[-\Lambda, \Lambda] \subsetneq \mathbb{R}^{/\{\infty\}} \otimes \mathbb{R}^{/\{\infty\}}$, while $\mu[\cdot] = \text{sign}[\cdot]|[\cdot]|^2$ gives a measure on each signed domain $[\mathbb{R}^{-/\{\infty\}}, 0] \oplus [0, \mathbb{R}^{+/\{\infty\}}]$, but not on both. As such, $\Lambda[l] = \text{sign}[\Lambda]l^2$ represents a topological splitting into AdS and dS sub-topologies

¹⁵Furthermore, this immediately motivates higher derivative corrections to GR: minimal coupling models canonically connect at the source level to Einstein's equations. Still, it is plausible that these contact expansions may also have higher derivative prescriptions at the level of the action. These higher derivative terms are known as Gauss-Bonnet contributions and will be largely ignored throughout the rest of this work.

gravitationally interacting subfields. Then, Einstein's equations may be written as:

$$R_{\mu\nu} - \left(\frac{1}{2}R + \Lambda\right)g_{\mu\nu} = T_{\mu\nu} \quad \Rightarrow \quad g_{\mu\nu} = \frac{R_{\mu\nu} - T_{\mu\nu}}{\frac{1}{2}R + \Lambda} \quad (20)$$

Therein, the metric may be interpreted, in units of local curvature $\frac{1}{2}R + \Lambda$, as a direct difference between the spacetime curvature tensor and the stress-energy tensor. In fact, by assuming the curvature form to be much larger than the stress-energy form in the discrete measure, $|R_{\mu\nu}| \gg |T_{\mu\nu}|$, the metric then may be interpreted as a canonically re-scaled

curvature, a.k.a. the embedded acceleration¹⁶:

$$|R_{\mu\nu}| \gg |T_{\mu\nu}| \quad \Rightarrow \quad g_{\mu\nu} \sim \frac{R_{\mu\nu}}{\frac{1}{2}Tr[R_{\mu\nu} + \delta_{\mu\nu} \frac{2\Lambda}{D}]} \quad (21)$$

It is also naively useful to consider the large auxillary field limit; still considering the

¹⁶Starting from the standard, scalar geodesic equation, $t^a \partial_a v^\alpha + t^a \Gamma_{a\beta}^\alpha v^\beta = 0$, consider some other (non-linear choice of affine representation) parameterization(s), λ_I , s.t. $t^a \nabla_a v^\alpha = \nabla_\lambda [x_{(\alpha)}^\mu [\lambda_I] x_{\mu}^{(\alpha)} [\lambda_I]]$. Then, if the hidden index, $x_{(\alpha)}^\mu [\lambda_I] := C_{(\alpha)}^I \tilde{\lambda}_I x^\mu \equiv \lambda_{(\alpha)} x^\mu$ is covered by gradient operators (which commute with the base metric), $0 = \nabla_\sigma [x_{(\alpha)}^\mu [\lambda_I] x_{\mu}^{(\alpha)} [\lambda_I]] \equiv \nabla_\sigma (g_{\mu\nu}^{(\alpha)} \langle x^\mu [\lambda_I], x^\nu [\lambda_I] \rangle_{(\alpha)}) = g_{\mu\nu} [\nabla_\sigma [g^{(\alpha)(\beta)} x_{(\alpha)}^\mu x_{(\beta)}^\nu] + \langle x^\mu | \Gamma_\sigma | x^\nu \rangle]$ OR: $\langle x^\mu, x^\nu \rangle_{(\alpha)} = -\oint_I d\lambda_I (\langle x^\mu | \Gamma_\lambda | x^\nu \rangle - g^{\mu\nu} [0])$, and assuming the metric is not singular lets the final term be dropped (otherwise, the index will need to strongly converge on the asymptotic algebra to regulate this term). Immediately, the product coordinate basis can be interpreted as a (globally phased) tangent space co-form residue over itself (the so-called second canonical tangent push up the trivial cohomology chain) weighed over the affine space densities (indexed/measured by $\{I, d\lambda_I\}$). When the affine parameter directly represents a spacetime coordinate the geodesic is massive; when the affine parameter directly represents a tangent space coordinate the geodesic is massless. The latter symmetry is exactly responsible for the enhanced little group symmetry of the collinear graviton envelope in (2, 2).[35]

Indeed, picking $\langle *_{(i)}, \cdot_{(j)} \rangle \sim \delta_{(i)(j)}$ to be sub-orthogonal and minimal gives a natural (ordered) inner index expansion with real weights: $g^{(\alpha)(\beta)} |* \rangle \langle \cdot | \sim |* \rangle \langle * | g^{(\alpha)(\beta)}$. Then, imposing that the index expansion be r -sub additive: $(v^a v_a)^r = \|\partial_\lambda x^\mu\|_{\lambda_I}$, gives: $\delta_{(i)(j)} \sim -\oint_I d\lambda_I \langle *_{(i)} | \Gamma_\lambda | \cdot_{(j)} \rangle$ and $v^a v_a = \left(\langle *_{(i)}, \cdot_{(j)} \rangle_{(\alpha)} \right)^{\frac{1}{r}} = (i)^{\frac{2}{r}} \sqrt[r]{\oint_I d\lambda_I \langle * | \Gamma_\lambda | \cdot \rangle}$. Critically, this shows that geodesic involutions may be constructed such that the extended index connection is flat (within the inner automorphisms) if the number field $\mathbb{F}_0 \sim \mathbb{R}[x^\mu]$ is algebraically exact over the geometric connection (or, equivalently, iff every sub-measure can be analytically continued, to a convergent extension, in a complete convex extension domain). It's important to note the $r = 2$ case, $v^a v_a \sim i \sqrt{\oint_I d\lambda_I \langle * | \Gamma_\lambda | \cdot \rangle}$, which (correspond to single-spin operator product expansions, OPEs, known as Penrose-spinners and) show that the canonical Field extension, $\mathbb{R} \rightarrow \mathbb{C}$, is resultant from finding a "square-root" (or self dual) basis; this construction, in fact, exists (and will be discussed in more detail later) even when black holes are present, and is known as the Kerr-Taub-Nut double cover [36].

Finally, the above co-dimension-2 split – which doesn't necessarily corresponding to base topology vector projections – is always possible (over real, finite symmetric topologies) and corresponds to the canonical Jordan normal index-form of the "inner" λ_I -morphisms under the Riemann-Roch theorem, and universally follows from $x^{-2n} = x \Rightarrow x \in \{\pm 1, \pm i^{\frac{1}{n}}\}$ having a universal \mathbb{R} -linearly dependent pair of solutions ($\forall n < \infty$), ± 1 , and a set of \mathbb{R} -independent field extensions, $i^{\frac{1}{n}}$, dependent on the size of the \mathbb{R} -represented eigenspace; then, it is always possible to find an operator covering basis which pairs the top, nilpotent eigen-operators with a long set of sub-space eigen-shift operators which are also nilpotent. This can also be considered a practical algorithmic proof of the Uchida theorem for finite dimensional, (topologically) real vector spaces over finite inner product algebras. Considering proper OPEs (with affinely infinite dimensional diffeomorphisms over GL_n vector spaces), this has been shown to compactly cover measurable functions under the functorial Langlands classification[13] and the converse classification of the analytic extension-dual transfer cohomologies by Cogdell and Piatetski-Shapiro [37], which construct strongly measurable inner OPE (conversly, a co-kernel outer R-support stabilizer) functor covers by tracking the local Galois density extensions and showing that they can be arranged to uniformly converge within a principle convergence index. Of particular importance later is to keep in mind the Ramanujan's weight 12 modular form[38]: $\Delta(z) = q \prod_{n=1}^{\infty} (1 - q^n)^{24} = \sum_{n=1}^{\infty} \tau(nq^n)$, where $q[z] = e^{2\pi iz}$ and, letting τ be multiplicatively homogeneous, $\tau(mn) = \tau(m)\tau(n)$ gives the following descendancy amongst prime (radical) towers: $\tau(p^{n+1}) = \tau(p)\tau(p^n) - p^{11}\tau(p^{n-1})$, and $|\tau(p)| \leq 2p^{\frac{11}{2}}$

minimal coupling limit:

$$|1|_R \ll |R_{\mu\nu}| \ll |T_{\mu\nu}| \quad \Rightarrow \quad g_{\mu\nu} \sim \frac{T_{\mu\nu}}{\frac{1}{2}Tr[R_{\mu\nu} + \delta_{\mu\nu} \frac{2\Lambda}{D}]} \quad (22)$$

Here, the interpretation is emergent, hence the care in subindexing the scale. In practice this will be an IR flowed (tree-level re-summed) sub(-limit-)sequence; this interpretation grants that the large field limit is self-dual descendant.

More naively yet, the RHS may be expanded in two senses: firstly, the auxillary radiation modes may be canonically connected to the stress tensor through some set of induced (adjoint) Ward identities over the momentum modes. By pulling these adjoint identities back to the base space $T_{\mu\nu}[\lambda_I] \rightarrow \sum_{k_I} \delta f_{k_I}[\lambda_I] \tilde{T}_{\mu\nu}^{k_I}[x^\mu]$ the stress tensor can be decomposed into sub-shelled mixing modes within the canonical algebras. Typically, the basis is chosen to be the canonical Jacobian (resulting in the classical spin representations), although string theory/ $w_{1+\infty}$ [33] suggest that richer spin-multiplet representations should be included. Either way, their exact forms are irrelevant at this level; substituting in:

$$g_{\mu\nu} \sim \frac{\sum_{k_I} \delta[f_{k_I}[\lambda_I]] \tilde{T}_{\mu\nu}^{k_I}[x^\mu]}{\frac{1}{2}Tr[R_{\mu\nu} + \delta_{\mu\nu} \frac{2\Lambda}{D}]} = \sum_{k_I} \delta[f_{k_I}[\lambda_I]] \frac{\tilde{T}_{\mu\nu}^{k_I}[x^\mu]}{\Lambda} \left[\frac{1}{Tr[\frac{\delta_{\mu\nu}}{D} + \frac{R_{\mu\nu}}{2\Lambda}]} \right] \quad (23)$$

Secondly, consider the divisor as some induced $Tr[\cdot]$ -level propagator¹⁷. Then¹⁸ :

$$\begin{aligned} Tr\left[\frac{\delta_{\mu\nu}}{D} + \frac{R_{\mu\nu}}{2\Lambda}\right] &= \ln \left[\det\left[e^{\frac{\delta_{\mu\nu}}{D} + \frac{DR_{\mu\nu}}{2\Lambda}}\right] \right] = 1 + \ln[\det[e^{\frac{DR_{\mu\nu}}{2\Lambda}}]] \\ \Rightarrow \frac{D}{Tr[\delta_{\mu\nu} + \frac{DR_{\mu\nu}}{2\Lambda}]} &= \frac{D}{D + \ln \left[\det\left[e^{\frac{DR_{\mu\nu}}{2\Lambda}}\right] \right]} \sim \sum_n \frac{(-1)^n \left(\frac{R}{\Lambda} \partial \ln \frac{D}{\Lambda} + Tr\left[e^{-\frac{DR_{\mu\nu}}{2\Lambda}} \partial R_{\mu\nu} e^{\frac{DR_{\mu\nu}}{2\Lambda}}\right] \right)^n}{2^n} \\ &\sim \sum_{n=0}^{\infty} \sum_{k=0}^n \binom{n}{k} \left(\frac{R}{2\Lambda} \partial \ln \frac{\Lambda}{D} \right)^k \left(\frac{-1}{2} Tr\left[e^{-\frac{DR_{\mu\nu}}{2\Lambda}} \partial R_{\mu\nu} e^{\frac{DR_{\mu\nu}}{2\Lambda}}\right] \right)^{n-k} \end{aligned}$$

Here, only the first term in the \ln Taylor series was kept¹⁹.

In this approximation, there is a clear ordering among the fixed points: dimensions D are the coarsest degree of freedom, followed by Λ , and then the curvature tensor. Accordingly: the dimension functional does not depend on the cosmological constant or the (*in*-)curvature, the cosmological constant may be a functor of the dimension functional (but not the curvature tensor), and the curvature tensor may be a function of both: $\{D, \Lambda, R_{\mu\nu}\} \sim \{D[\cdot], \Lambda[D[*]; [\cdot]], R_{\mu\nu}[D[\cdot], \Lambda[D[*]; [\cdot]]; [\cdot], [*]]\}$. Then, combining ev-

¹⁷For rigor, the subindexing set $k_n \in I$ has been included to allow for divergence functions/transition representations. As a rule of thumb, when $[\cdot]^n < 1$ $k_n \equiv n$; $[\cdot]^n \subset \{1 \geq |z| \in \mathbb{C}\} \Rightarrow k_n \in [-1, 1]$; $[\cdot]^n \in \{1 \leq |z| \in \mathbb{C}\} \Rightarrow k_n \in \mathbb{C}^{-n}[z]$. Then, the Lorenz terms represent the representation's peeling of the sum's divergence; because the LHS clearly converges it must be that these terms balance. Generally, if the function is $\mathbb{C}^{m+1}[z]$ represented then $\mathbb{C}^{-n}[z]$ functions may be uniformly connected to a $\mathbb{C}^{m-n}[\bar{w}]$ smooth function (the harmonic dual coordinate) through the Cauchy-Riemann (a.k.a., topological transition) equations on some contour(s) envelope $\Gamma_{\bar{z}z} \subset (\bar{w}, w)$ within the dual annulus of convergence. Generically there will be poles within the annulus of convergence but outside of the contour path: they present the representation's momentum modes/boundary compactification states and are incorporated in a path-specific residues on the inverse harmonic functions (a.k.a., as Green's functions). Conceptualizing in the casework: $[[\cdot]] < 1$ gives dual coordinates which cover the annulus of convergence; the complex convergent case has a(n) (ir)regular puncture at the origin (when $[[\cdot]] = 1$) which represent the holomorphic momentum modes (compactified along the support domain, a.k.a. the unit circle, develops fixed points on its real branch representing fixed sub-domain parameterization, a.k.a., bound Goldstone modes). Here, as $[[\cdot]]$ gets larger the transition equations become over-constrained at (near) the origin of \bar{w} (because the oscillating, growing terms dominate without bound) the origin resulting in point (regional) discontinuities within the annulus of convergence of the harmonic dual \bar{w} . When $m-n \geq 0$ the \bar{w} representation is smooth and the interpretation is as a (pure) series in the antiholomorphic (transition) coordinate

¹⁸ $\det[e^{[\cdot]}] = e^{Tr[\cdot]} \Rightarrow Tr[\cdot] = \ln[\det[e^{[\cdot]}]]$ and, $\partial \det B = \det B (B^{-1} \partial B)$

¹⁹And the ∂ operator is understood as an exact differential (linear form) operator under each Maxwell relation. Temporarily granting the the un-indexed ∂ operator a new definition, and letting every indexed ∂_l form be a regular partial differentiation, leads to:

$$\partial D = \delta[\cdot] \partial \cdot D, \quad \partial \Lambda = \partial \cdot \Lambda \delta[\cdot] + (\partial_* D[*])^{-1} \partial_* \Lambda \partial D[*] \equiv \partial \cdot \Lambda \delta[\cdot] + \partial_* \Lambda \delta[*] \quad (27)$$

$$\text{and} \quad \partial R_{\mu\nu} = \delta[\cdot] \left(\partial \cdot R_{\mu\nu} \Big|_{\partial \cdot \Lambda=0} + \partial \Lambda R_{\mu\nu} \Big|_{\partial \cdot R_{\mu\nu}=0=\partial D[*]} \right) + \delta[*] \left(\partial_* R_{\mu\nu} \Big|_{\partial D[*]=0} + \left(\frac{\partial \Lambda}{\partial D} \right)^{-1} \partial D R_{\mu\nu} \Big|_{\partial_* R_{\mu\nu}=0=\partial D[\cdot]} \right) \quad (28)$$

Then, considering $[\cdot]$ to be the induced Λ (sub-coordinatization) basis, the second term shows that dimensionally reduced sub-domains (such as membranes between trapped surfaces) have a natural contact form with the cosmological constant

everything²⁰, and defining²¹ $\Theta[D, \Lambda, R_{\mu\nu}] := \pi + i\frac{DR_{\mu\nu}}{2\Lambda}$:

$$g_{\mu\nu} \sim \sum_{k_I} \delta[f_{k_I}[\lambda_I]] \tilde{T}_{\mu\nu}^{k_I}[x^\mu] \left[\sum_{n=0}^{\infty} \sum_{k=0}^n \binom{n}{k} \left(\frac{R}{2\Lambda} \partial \ln \frac{\Lambda}{D} \right)^k \Lambda^{-1} \left(\frac{1}{2} \text{Tr}[e^{i\Theta} \partial R_{\mu\nu} e^{-i\Theta}] \right)^{n-k} \right] \quad (29)$$

$$\Rightarrow T_\nu^\nu \sim \sum_{k_I} \delta[f_{k_I}[\lambda_I]] T^{\mu\nu} \tilde{T}_{\mu\nu}^{k_I}[x^\mu] \left[\sum_{n=0}^{\infty} \sum_{k=0}^n \binom{n}{k} \left(\frac{R}{2\Lambda} \partial \ln \frac{\Lambda}{D} \right)^k \Lambda^{-1} \left(\frac{1}{2} \text{Tr}[e^{i\Theta} \partial R_{\mu\nu} e^{-i\Theta}] \right)^{n-k} \right] \quad (30)$$

Then, the weak energy condition can be interpreted as saying that both $T_\nu^\nu \geq 0$ and $T^{\mu\nu} \tilde{T}_{\mu\nu}^{k_I}[x^\mu] \geq 0$, which in turn put strong constraints on the curvature portion of the sum on the RHS.²²

Critically, the above shows the metric descending to a peeling product between continuous (formal field extension) parameters Λ, R, D and the quantized degrees of freedom $R_{\mu\nu}$ (adjointly represented by the out of order trace operator) over Λ ; immediately, Λ gains a clear interpretation as a vacuum propagator under a continuous boundary frame and a partially saddled charge algebra(sub-quantization) [40]. This motivates the higher order derivative terms of Lovelock gravity as a tree-level boundary connection to a conformal fluid. In particular, as derivatives are peeled off of the continuous field the matter currents (top symmetries) descend into $\{[\cdot], [\ast], D[\cdot], \Lambda[D[\cdot]; [\cdot]]\}$ constraint forms (bottom symmetries). Then, often the ‘‘middle’’ symmetries exhibit unique (or reduced) algebraic geometries (e.g., have simple holographic duals) which have well behaved field extensions (or, equivalently, have well defined boundary kernels almost everywhere).

Interestingly, it turns out that:

$$21 [kg^{-1}m^{-4}] = \frac{\Lambda_0 m_e c^2}{h^2} \equiv \frac{8\pi G \rho_\Lambda m_e}{(hc)^2} \quad (31)$$

$$\Rightarrow F_N[M = \rho_\Lambda, m = m_e; r = hc] = \frac{1}{4} \frac{21 [kg^{-1}m^{-4}]}{2\pi} \quad (32)$$

where Λ_0 is the measured cosmological constant, m_e is the mass of the electron, ρ_Λ is the cold dark matter density, G is Newton’s constant, h is plank’s constant, c is the speed of light, and $F_N[\ast; \cdot]$ is Newton’s law of gravitational attraction. Note the units are flux mass-spacetime that reduce to units of space-flux in geometrized units $[kg^{-1}m^{-4}] \sim_{[hc] \sim 1} [m^{-3}]$; then, although this expression seems unitful (and thus not fundamental), note that the LHS is actually a force density (from $M = \rho_\Lambda$), meaning this presents as an invariant ratio of unitful measures²³.

²⁰And assuming $D[\cdot]$ is at a fixed point (so that Λ^{-1} may be commuted with the fluid derivative terms); Notably, the continuous terms are symmetric in indices, and further only contains contact terms involving sub-thermalized fields (e.g. $\frac{R[D, \Lambda, \cdot]}{\Lambda}, -\ln \frac{\Lambda[D, \cdot]}{D}$ contacts expand Maxwell relations between sub-commutative equivariant OPE representations.)

²¹Note: $i\partial R_{\mu\nu} = \partial[(\theta - \pi) \frac{2\Lambda}{D}]$

²²In particular, this may be compared to superpositions of spacetimes, such as the BTZ spacetime explored in [39]

²³then note that 2π is the circumference of a unit circle, and that Hawking’s area law for the entropy runs as $S_H \sim \frac{A}{4}$, so that the RHS appears as a (geometrized) volume-flux invariant over circular (β -type) units; this may be considered a virial force representation of some cosmological connection. Perhaps surprisingly, it will indeed be shown (in the final section of this paper) that $n = 21$ represents a unique number density

This exactly leads to an incredibly important example of bottom(-strong) symmetry emergence: black hole jets.

1.3 Astrophysical Black Hole Environments

Emergent symmetries (mean-field fixed point degeneracies) are wide phenomena across physics which, as mentioned above, may occur when complex boundary conditions contact large degrees of (*internal*) freedom across deep, narrow channels [41]: then (interactively) critical phase transitions can be interpreted as quasi-static tower (boundary) channel propagators. In fact, this phenomena has a clear place in black holes physics when electromagnetism is constructed as an indexing field in strong gravitating systems.

Similar to above, this model involves an order of coarseness; here, considering the gravitational symmetries as top, let the metric topology be the coarsest fixed functor in the system. Further, let the electromagnetic field be minimally (charge) perturbative and maximally (spacetime, or *in-metric*) extended: the first condition requires that the field be force free, while the second demands that current and field tensor exactly decompose the spacetime²⁴. This can also be interpreted as the fluid (charge quantization \rightarrow zero) limit of the electromagnetic field [43]

So, consider a magnetically dominated, charge free electromagnetic field in the presence of a dominate, fixed spinning black hole. Then, it is natural to enforce the top symmetries on the bottom form by only considering functional derivatives which keep the metric fixed.

$$S_{EM} = \int \sqrt{g} (R + F^2) \quad (33)$$

Let the axisymmetric and stationary (a.k.a., the global symmetry, or toroidal) coordinates be denoted by Greek indices, α, β , and poloidal (or path) coordinates by Latin indices, i, j . Further, preserve $\{\mu, \nu\}$ to be free (any sector) coordinates. Accordingly, this leads to the (curved spacetime) covariant formulation of electromagnetism which, under the force free condition, then descends to a specific matching form:

$$\begin{aligned} S_{bh \times EM} &:= S_{BH} \oplus S_{EM} \quad \wedge \quad \delta^* S_{BH} \sim 0 \\ &\Rightarrow \delta S \sim \delta^* S_{EM} \\ S_{EM} &:= F^{\mu\nu} F_{\mu\nu} - A_\sigma J^\sigma & \text{then, } 0 = F_{\nu\mu} j^\nu = F_{\nu\mu} \nabla_\sigma F^{\sigma\nu} \\ \text{s.t. } F_{\mu\nu} &:= A_{[\nu;\mu]} \quad \wedge \quad \nabla_\mu F^{\nu\mu} = J^\nu & \Rightarrow \nabla_\sigma [F_{\nu\mu} F^{\sigma\nu}] = F^{\sigma\nu} \partial_\sigma F_{\nu\mu} \end{aligned} \quad (34)$$

Critically, because the black hole remains functionally fixed (as a fixed spinning geometry), the top symmetrization (/“fixed background”) hypothesis supports vector fields which can be gauged in the top (functor-form) symmetries; therein, let the spacetime, and therefore the gauge field A_μ , be axisymmetric and stationary. Then, this immediately leads to the

in quantum gravity.

²⁴This condition can always be met by a slightly stronger requirement of degeneracy, $F_{\mu[\nu} F_{\alpha]\beta} = 0 \Rightarrow F_{ab} = [a, b]$, which guarantees that $\{J, a, b, B\}$ are orthogonal and complete [42].

conclusion that $\sigma \in \{i, j\}$, and that the toroidal magnetic field lines act as a strongly gauged partition: $F_{\alpha\beta} = 0 \Rightarrow A_\phi[r, \theta] := \varphi$.²⁵ Indeed, considering the toroidal field form (defined as everything not completely in the Path sector)²⁶:

$$F_T := F_{\{\mu\nu\} \neq \{i,j\}} dx^\mu \wedge dx^\nu = F_{\alpha i} dx^\alpha \wedge dx^i = -(A_{t,i} dt + A_{\phi,i} d\phi) \wedge dx^i \quad (35)$$

$$\Rightarrow F = \varphi_{,i} dx^i \wedge \left(\frac{A_{t,i}}{\varphi_{,i}} dt + d\phi \right) + F_{ij} dx^i \wedge dx^j \quad (36)$$

$$\text{and } 0 = F \wedge F = F_T \wedge F_T = [A_{t,j} \varphi_{,i} - A_{t,i} \varphi_{,j}] dx^i \wedge d\phi \wedge dx^j \wedge dt \quad (37)$$

$$\Rightarrow \frac{A_{t,j}}{A_{t,i}} = \frac{\varphi_{,j}}{\varphi_{,i}} \Rightarrow \tilde{A}_t[\cdot] \equiv A_t[\varphi[\cdot]] \quad (38)$$

It's critical to note that the final equality of the top line is induced because the global symmetries exactly "chop" the antisymmetric field index *into* an exact (partial gauge) field index in the non-zero, non-poloidal (path) sector; then,²⁷ this allows a well defined (toroidal sector complete) functor to be defined between the global gauge forms:

$$F_{\alpha j} = A_{[j,\alpha]} \equiv - \left(\delta_\alpha^t A_{t,\varphi} + \delta_\alpha^\phi \right) \varphi_{,j} := \left(\delta_\alpha^t \Omega[\varphi] - \delta_\alpha^\phi \right) \varphi_{,j} \quad (39)$$

$$\Rightarrow F = d\varphi \wedge (\Omega[\varphi] dt - d\phi) + F_{ij} dx^i \wedge dx^j \quad (40)$$

This can be cleanly interpreted as saying the poloidal sector has an everywhere well defined electric potential, or that the toroidal sector has a well defined magnetic potential²⁸. This

²⁵Quickly, from: $[1] \sim \frac{A_{\phi,t}}{A_{t,\phi}} \sim \frac{\partial \phi}{\partial t} \frac{\partial A_\phi}{\partial A_t}$, where $\frac{\partial \hat{\phi}}{\partial \hat{t}}$ is interpreted as a coordinate (4-space) rotation of the $\hat{\phi}$ under the unit \hat{t} time translation (which, by the global symmetries and the $d = 4$ saturation, implies it must be exactly the poloidal angular velocity, and " \sim " is given under Ω^{-1} iso-sets, and " \rightarrow " runs from Cartan's magic formula, $\mathcal{L}_{\hat{x}} = d\hat{x} \cdot dX + d(\hat{x} \cdot X)$)[42][43]. This follows exactly from the force free condition, which runs to $\left[\frac{\mathcal{L}[A_\phi]}{\mathcal{L}[A_t]} \right] \sim \frac{\partial_\phi \partial A_\phi[\cdot]}{\partial_t \partial A_t[\cdot]}$ and, when the spacetime has constant (local) angular velocity, can be envisioned as Archimedian spirals (of iso-gauged field line flux) in time. Most concretely, $\partial_t \int_{\gamma[\phi]} \phi = \int_{\gamma[\phi]} \partial_\phi A_t \equiv A_t[r, \theta] + C[r, \theta] \Rightarrow A_t = \partial_t \int_{\gamma[\phi]} \varphi - C[r, \theta]$; by choosing a Gaussian (iso-)gauge (or, equivalently, choosing a path γ s.t. the (constant weight) A_t measure (residue) is an exact $A_\phi[r, \theta]$ multiple. Because the toroidal symmetries are global (constant gauged), and the $T \oplus P$ decomposition is exact, P -pointwise compactification is always possible (and topologically smooth) because the field is close under the Uchida theorem [44]: $\Delta t_\gamma A_t = \int_{\gamma[\varphi]} \varphi$, $C[r, \theta] \Delta t_\gamma \equiv C[\varphi]$. Under the pointwise rescaling of the time-gauged (iso-)sets (equivalent to modding out the spacelike magnetic charge accumulation) leads to: $A_{t,i}[r, \theta] \sim A_{t,\varphi}[\varphi] := -\Omega_F[\varphi]$

²⁶Note: $dF_T = [A_{t,ij} dx^i + \varphi_{,ij} dx^\phi] \wedge dx^j \wedge dx^i = 0 = -dF_P = 0$; further, noting the uniqueness of the determinate function $F_{[\mu\nu]F[\lambda\rho]} \sim \epsilon_{\mu\nu\lambda\rho}$, the force free condition implies $0 = f[\cdot] \epsilon_{\mu\nu\sigma\rho} j^\sigma \Rightarrow f \equiv \hat{0}$, or that every force free field is "degenerate" $F_{[\mu\nu]F[\sigma\rho]} = 0 = F \wedge F$, although the converse may not be true.

²⁷generically, the low dimensions require the antisymmetric field algebra to interact with the canonical representation algebra effectively widening the cover space of (antisymmetric field) kernel operators as the domain descends onto canonically commuting pairs, $\partial_{[i} \partial_{j]} \equiv 0$ [45]

²⁸that lies exclusively across gradients of the φ field; this is known as the Alfvén's theorem [42], and shows that equipotentials are exactly those which conserve the (toroidal) magnetic flux. See Appendix A for more details.

is seen exactly from the Jacobi identity, $d \star F = J$, which ensures the conservation form ²⁹:

$$\oint_{\gamma_\beta} \oint_{\gamma_i} [d \star F - J] = 0 = \oint_{\gamma_\beta} \left[(\epsilon_{\alpha\beta}{}^{mn} F_{mn} dx^\alpha + \epsilon_{j\beta}{}^{\alpha i} F_{\alpha i} dx^j) \wedge dx^\beta - \oint_{\gamma_i} J \right] \quad (41)$$

$$\text{and} \quad 0 = \oint_{\gamma_\beta} \oint_{\gamma_i} \left[\begin{aligned} & \left(\nabla_i (\epsilon_{\alpha\beta}{}^{mn} F_{mn}) + \epsilon_{i\alpha\beta}{}^k j_k \right) dx^\alpha \wedge dx^i \\ & + \left(\nabla_i (\epsilon_{j\beta}{}^{\alpha i} F_{\alpha i}) + \epsilon_{ij\beta}{}^\alpha j_\alpha \right) dx^j \wedge dx^i \end{aligned} \right] \wedge dx^\beta \quad (42)$$

Particularly, the second line implies that j_α must be a function of φ (and the toroidal determinate), which (under current closure³⁰) then implies j_k , and therefore, F_{ij} must be a function of φ (and the toroidal determinate)³¹. So, in totality,

$$F = d\varphi \wedge (\Omega[\varphi]dt - d\phi) + F_{ij} dx^i \wedge dx^j := d\varphi \wedge \eta + \sqrt{-\frac{g_P}{g_T}} I[\varphi] dx^i \wedge dx^j \quad (43)$$

OR:

$$\begin{aligned} F_{\nu\mu} \nabla_i F^{i\nu} = 0 & \quad \text{AND} \\ F_{\alpha i} = \eta_{\alpha} \varphi_{,i} & \quad \text{give:} \\ F_{ij} = -\sqrt{-\frac{g_P}{g_T}} I[\varphi] \epsilon_{ij} & \quad \frac{\eta_\alpha}{\sqrt{-g}} \nabla_i [\eta^\alpha \sqrt{-g} g^{ij} \varphi_{,j}] = \frac{1}{-g_T} \frac{dI^2}{d\varphi} \end{aligned} \quad (44)$$

This is the fundamental equation of (degenerate) force free electrodynamics (in any geometry), and is known as the stream equation. Note the similarity to the Klein-Gordon equation. $\nabla_\nu [\sqrt{-g} g^{\mu\nu} \nabla_\mu \varphi] = -\sqrt{-g} m^2 \varphi$, becomes immediate when it's rewritten as: $\frac{1}{\sqrt{-g}} (\eta_\alpha \eta^\alpha \nabla_i [\sqrt{-g} g^{ij} \varphi_{,j}] + \nabla_i [\eta_\alpha \eta^\alpha \sqrt{-g} g^{ij} \varphi_{,j}]) = \frac{1}{-g_T} \frac{dI^2}{d\varphi} + g^{ij} \varphi_{,j} \eta^\alpha \nabla_i \eta_\alpha$.³²

²⁹Using $\star J = \epsilon_\sigma{}^{\mu\nu\rho} J_{\mu\nu\rho} dx^\sigma := j = j_\sigma dx^\sigma \Rightarrow J = \epsilon_{\mu\nu\lambda}{}^\sigma j_\sigma \wedge dx^\mu \wedge dx^\nu \wedge dx^\lambda$

³⁰ $\nabla_\varphi (\epsilon_{ijkt} \varphi^{j^k}) = \nabla_k (\epsilon_{i\varphi\beta j} j^k)$

³¹and can be seen explicitly by $dJ = 0 \Rightarrow \nabla_j \nabla_i \epsilon_{\varphi\beta}{}^{mn} F_{mn} = \nabla_\varphi \nabla_i \epsilon_{j\beta}{}^{\varphi i} F_{\varphi i}$. Thus, under Stoke's theorem, $\sqrt{g} F^{\varphi i} = \oint_\varphi \nabla_j \epsilon_{\varphi\beta}{}^{mn} F_{mn}$ is an exact function of φ , and dually $\sqrt{g} F^{ij} = \oint_j \nabla_\varphi \epsilon_{j\beta}{}^{\varphi i} F_{\varphi i} := -I[\varphi]$; this is also known as Faraday's Law, $\sqrt{|-g_T|} F_{ij} dx^i \wedge dx^j = I[\varphi] \epsilon_P$

³²Defining $\nabla_i [\eta_\alpha \eta^\alpha \sqrt{-g} g^{ij} \varphi_{,j}] := \sqrt{-g} \tilde{m}^2 \tilde{\varphi}$, this becomes: $\frac{\eta_\alpha}{\sqrt{-g}} \nabla_i [\eta_\alpha g^{ij} \varphi_{,j}] + \sqrt{\frac{1}{-g_T}} \frac{dI^2}{d\varphi} = (||\eta||^2 m^2 \varphi + \tilde{m}^2 \tilde{\varphi})$, from which the limiting result follows when $\sqrt{-g} \gg m^2$. More generally, it may be expected that algebraic lifts split the measure states over the affine length of the corotation vector or, equivalently, that the radical basis elements in the extended manifold: $\sqrt{\tilde{m}^2 \tilde{\varphi}} = \sqrt{m^2 \varphi \tilde{\varphi}} \wedge ||\eta|| m \varphi = i\sqrt{\lambda} \varphi \tilde{\varphi}$. Accordingly, it is instructive to try and split the global wedge units, $a\vec{c} \wedge (b\vec{d}) \equiv ab (\vec{c} \wedge \vec{d})$ as a Lie series over linear and sub-linear ("square-root") representations $\sim \oplus_{(k)} \odot_k \wedge^{(k)} *_k$. Considering the units between the center and rightmost extensions, the natural guess is to push the scaling factor against the massive scalar and some dual unit vector, \hat{e} , against the "light dressed" massive scalar field: $0 \sim ||\eta|| m (m^2 + i\sqrt{\lambda}) \varphi \tilde{\varphi} \wedge (\varphi - \frac{\hat{e}}{||\eta|| m})$. Then, this approximate condition may be met in several ways:

$$\lambda \sim -m^4 \quad \vee \quad ||\eta|| \sim 0 \quad \vee \quad m \sim 0 \quad \vee \quad \varphi \sim \frac{\hat{e}}{||\eta|| m} \quad \vee \quad \varphi \tilde{\varphi} \sim_{(k)} \varphi - \frac{\hat{e}_{(k)}}{||\eta|| m} \quad (45)$$

While the top row of algebraic conditionals represent fixed point expansions (of the functional basis elements), the second line represents a projective, finite difference conditional on the extended phase space. Suppose that (the not necessarily regulated basis) obeys $||m\eta||^2 \ll ||\hat{e}||^2$ and is chosen in the (freely extension) dual product expansion, $\hat{e}_{(k)} \sim_{(k)} \varphi \tilde{\varphi}$, then this final condition reduces to (sheets of exact closure

Indeed³³, $\|\eta\|^2 \rightarrow 0$ corresponds to light surfaces, $F^2 = 0$, iff the poloidal cap current conditions):

$$\varphi\tilde{\varphi} \sim_{(k)} \varphi \quad (46)$$

$$\Rightarrow -m^2\varphi\tilde{\varphi} + \lambda(\varphi\tilde{\varphi})^2 + \frac{1}{-g_T} \frac{dI^2}{d\varphi} = -(\varphi\tilde{\varphi})^\alpha \left(m^2\varphi\delta_\alpha^\beta + g^{ij}\varphi_{,j}\nabla_i \right) (\varphi\tilde{\varphi})_\beta \quad (47)$$

$$\text{OR:} \quad \left(\left(\frac{2\lambda}{m^2}\varphi\tilde{\varphi} - e\hat{e} \right) + \sqrt{1 - \frac{4\lambda}{-g_T m^4} \frac{dI^2}{d\varphi}} \right) \left(\left(\frac{2\lambda}{m^2}\varphi\tilde{\varphi} - e\hat{e} \right) - \sqrt{1 - \frac{4\lambda}{-g_T m^4} \frac{dI^2}{d\varphi}} \right) \\ = -(\varphi\tilde{\varphi})^\alpha \delta_\alpha^\beta \left(m^2\varphi + g^{ij}\varphi_{,j}\nabla_i \right) (\varphi\tilde{\varphi})_\beta \quad (48)$$

which makes contact with the Higgs potential (note the (k) indexes were suppressed under the $\sim_{(k)}$ equivariance). Note that the RHS is zero iff $\lambda\varphi\tilde{\varphi} = \frac{m^2}{2}e\hat{e}$ this implies an integrability constraint (emergent soft pole) running between the four-point moment and the current(-squared) flux: $\frac{dI^2}{d\varphi} = \frac{-g_T m^4}{4\lambda}$, or $\lambda \sim e \frac{4}{-g_T m^4} \int \frac{dI^2}{d\varphi} d\lambda$.

Finally, the adjoint gauge condition $\nabla[\tilde{m}^2\tilde{\varphi}] = 0$ induces $\nabla \ln(\varphi\tilde{\varphi})^2 = -\nabla \ln \lambda$; supposing the bare mass moment m is constant over each integration domain and pulling $d\lambda$ over the gradient dependent sub-forms f^α (representing Maxwell relations) yields: $\nabla \ln(\varphi\tilde{\varphi})^2 \sim \frac{4}{m^4} \int \nabla \left[\frac{f^\alpha[\cdot]}{-g_T} \frac{dI^2}{d\varphi} \right] d\lambda_\alpha$ or $(\varphi\tilde{\varphi}) \sim e^{\frac{1}{2} \int d[\cdot] \frac{4}{m^4} \int \nabla \left[\frac{f^\alpha[\cdot]}{-g_T} \frac{dI^2}{d\varphi} \right] d\lambda_\alpha}$. In particular, this paints the poloidal ‘‘cap’’ current as an iterative derivative in the (induced) mass moduli (η dual push), $\varphi \rightarrow (\varphi\tilde{\varphi})^\alpha \varphi_\alpha$ and is critical to understanding the Menon-Dermer solutions (and how to generically analytically continue them in the $\mathfrak{su}(2)$ ‘‘Fermi-gauge’’).

³³This implies both $m \rightarrow 0$ as well as $\tilde{\varphi} \rightarrow 0$; the latter condition amounts to, under some well defined perturbative scheme, a possibly closed ladder residue OPE [43] that may require proper field extensions over some perturbation state regulators $\mathbb{F}[x^\mu] \rightarrow \mathbb{F}[x^\mu; r_i[a_i]]$ exactly when the perturbative scheme has a hidden (large series) shell closure among series-residue algebras (see previous footnote).

For example, approximating the real function $\sin[\cdot]$ by exclusively cubic monomials will never produce a zero difference form although it will produce a universally stable (stabilizer OPE) residue: $|\mathbb{T}_{|I}[\sin[x]] - \sum_{n^i}^I b_n x^{3n}|_{\text{deg}} - |\mathbb{T}_{|I}[\sin[x]] - \sum_{n^i}^I b_n x^{3n}|_{\text{deg}} = [I\frac{2}{3}]_{\text{deg}}$, and naming the difference in residue interpolation monomials between $I \rightarrow I+1$ truncations gives an exact, quadratic closure tower $\Delta_{\mathbb{T}_{|x^3}}^n[\sin[x]] = 0$; repeating for cosine shows the same effect, $\Delta_{\mathbb{T}_{|x^3}}^n[\cos[x]] = 0$, which stems from the choice of the low prime 3, as well as a sign of $\sin \cos$ duality. Indeed, extending the ring by a derivative of the cubic monomials gives a zero residue in both cases, and is directly descendant from the $(\sin x)'' = (\cos x)' = -\sin x$ (S^1 cohomology) identities.

Still, the practical ‘‘perturbation fidelity’’ depends not just on the functional (*in*) convergence, but also on the iterative smoothness of the *out* stability conditions. Suppose the missed truncation basis elements may be (stepwise, log-convergence) weighed as $\sim \frac{1}{3}$ and the truncated overlap basis measure as $\sim -\frac{1}{3}$; then, this gives an difference alternating form, $\sim \pm \frac{2}{3}$, representing the ‘‘left/right waving’’ ends of the $3I$ -interpolant outside the convergence domain.

Indeed, this measure may be justified using the Fourier identity $\mathcal{F}[x^n] \sim i^n \delta^{(n)}[x]$ and considering the residual basis elements as higher geometry boundary terms with induced crossing forms; in fact, using a generalized field extension $i \rightarrow \hat{a}_i$, this shows exactly that regulator extensions (such as $f[\log a]$) should be expected to emerge at the $\min\{\text{lcm}\{|I_{\text{Scheme}}|, |I_{\text{Field}}|\}\}$ perturbative order. So, as will be seen shortly, there exists a successful model of black hole jets generated from family of EM perturbation solutions to Kerr-geometrized models of black holes (with an exact analytic horizon constraint form), known as the Blandford-Znajek, model built from force free electrodynamics onto an $O(a^2)$ -towered stabilizer form; then, it may be expected that power-convergence corrections, such as $\{i^2, \log a\}$, should occur at $\text{lcm}\{2, 3\} + 1 = 7$ seventh order, as has been shown in recent high series expansions showing a sign degeneracy at α -seventh order and a $\log|\alpha|$ non-analytic regulator at eighth order [43].

Interestingly, this also points to the small iterative seed generation’s confluence with the global \mathbb{Z}_2 as the

is zero³⁴. Indeed, within light-surface solution covers, the stream equation reads as an integrability class (P-current to Tb-field crossing) OPE condition,

$$| -g_T |g^{ij} \varphi_{,j} \eta^\alpha \nabla_i \eta_\alpha = -\frac{dI^2}{d\varphi} \quad (51)$$

operative emergent point: finding un-even pushes of the split-monopole BZ solution may produce relatively hyperfine solutions or new perturbation regimes. For example, considering the example from [43], which appears to have a \mathbb{Z}_2 alpha symmetry being broken at fifth order

$$\frac{1}{\alpha^2 + 1} + \sqrt{(\alpha^2)^{\alpha^4} - \frac{1}{\alpha^2 + 1}} \sim 1 - \alpha^2 + |\alpha|^3 + \alpha^4 + |\alpha|^5 (\log |\alpha| - \frac{1}{2}) + \mathcal{O}(\alpha^6) \quad (49)$$

(from the appearance of the $|*|^{2k+1}$ despite the evenness of the LHS). Considering smooth, infinite dimensional extension $\alpha = i \sin \theta$ shows that the LHS is harmonically incomplete over a single copy of the reals $R[\alpha]$,

$$\sec \theta^2 \left[1 + i \sqrt{\cos \theta - (i \sin \theta)^2 \sin^4 \theta} \right] \sim 1 - \alpha^2 + |\alpha|^3 + \alpha^4 + |\alpha|^5 (\log |\alpha| - \frac{1}{2}) + \mathcal{O}(\alpha^6) \quad (50)$$

because a single complex extension $\mathbb{C}(z, \bar{z})$ cannot cover the entire LHS entirely; instead, the extension is topological: $\mathbb{C}[K, \bar{K}]$ s.t. $K \subset \mathbb{C}[z, \bar{z}]$. This is directly manifest in the RHS, where the log emergence at fifth order can be directly attached to a winding ambiguity in the ln/sin functionals' pullbacks; further, the constant index at fifth order, $-\frac{1}{2}$, is exactly related to the nilpotency of the (global) sub-cover symmetry $(\pm i)^2 \rightarrow (iI)^2 = -I$. The lesson that perturbation representations may require properly (scheme) independent scalar field extensions when every \mathbb{C} embedding has a pushed (winding) degeneracy.

³⁴Notably, $F^2 = B^2 - E^2$ represents the local field partition weights. $F^2 := F^{\mu\nu} F_{\mu\nu} = \frac{I^2}{2\pi^2 | -g_T |} + |\tilde{\mu}(\varphi)|^2 \eta^\alpha \eta_\alpha$ shows that real cap currents have an exact minimal bound under the field scalar index: $F^2|_{\eta^2 \geq K^2} \geq K^2$. Still, pushing all the transcendental Field extensions together gives a "light-scalar index": $\tilde{\eta}^2 := [\pi \sqrt{| -g_T |} \eta]^2 = \frac{2[\pi \sqrt{| -g_T |} F]^2 - I^2}{2|\tilde{\mu}(\varphi)|^2} := \frac{\tilde{F}^2 - I^2}{2|\tilde{\mu}(\varphi)|^2}$. This light-surface orientated framework shows some interesting things.

Firstly, the co-rotation form may be considered an (magnetic flux) operationalized measure density of the second order field extension (closures) $\Delta_{\text{tilde}\mu, \varphi}[\tilde{F}^2 - I^2] \sim ([F - I], [F + I])_{\tilde{\Delta}}$; therein, some mileage is gained in understanding it as a magnetically graded deflection between the local P(ath) current density and the Faraday-tensor (scalar) density. Then, the sign of the Faraday scalar may be understood as the composite tolerance of P-currents and spacelike co-rotation vectors: > 0 induces some dual support, $= 0$ induces Null support, and < 0 induces only timelike co-rotation support. It is important to note the crucial role the lower boundedness of the $I^2[\varphi]$ variety plays in the well definedness of this construction.

Secondly, $F^2 \rightarrow 0$ immediately understands light surface co-vector (g_T -scaled)magnitudes as projective varietal surfaces of the magnetic flux(-measure) over the P-current (real-measure); further, η^2 is maximized by the (reality) lower bound on the P-current algebra: covectors must be null (iff $I = 0$) or timelike at light surfaces. Finally, and perhaps most excitingly, this reasoning can be inverted. Large, forced currents at light surfaces must either (quasi-statically) induce: a nearly fixed field density with large, negative changes in the co-rotation form (measure), or a nearly fixed co-rotation form (light-surface topology) with large, (positively) balanced change in F^2 , OR a huge positive field scalar induction (a large magnetic flux induction) and a relatively small change in the co-rotation form. Note the first case asymptotically preserves the interior of the light-region, while the second case asymptotically preserves the exterior of the light-region; then, the final state represents an interactive membrane state.

Although the $\mu[y, x] = |y^2 - x^2|$ is well ordered under the (algebraically axiomatic) global \mathcal{Z}_2 Field extension of the canonical commutative "+" ring, it is important to note that this equation is necessarily field action degenerate: extensions of the field *in*-parameterizations (transported gauge relaxations on the field shelling dynamics) certainly trickle directly through (the absolute value). For example, $(n-)$ complex extensions of $(F, \eta) \rightarrow (\mathcal{Z}, \chi)$ may be used to push off a quasi-regionalized $|\tilde{F}^2 - |\mathcal{Z}|^2| \sim 0$ profile and, using a real (quasi-literally Coulomb) P-current density, into a properly flowed (measure inverted) magnetic flux partition, $[\tilde{\varphi}] = \frac{1}{2} \int \mu^{-1}[\frac{\mathcal{Z}^2 - I^2}{\chi^2}]$. In fact, the equivariant $[\tilde{\varphi}]$ algebra may even be well represented by a non-trivial sector of the P-current Field variable algebra (iff the extended difference $([\mathcal{Z} - I, \mathcal{Z} + I]$ can be exactly completed, or closed, over the flux measure functorization). Indeed, this is exactly how field regulation and classical off-shell propagation functionally works, but these schemes (almost always) require

which universally support the field's wave guiding kernels as (light-surface) asymptotic boundary charges can be uniformly matched to any disjointly smooth harmonic OPE and vice-versa; this is the Neukirch-Uchida theorem applied to the topological extension cover of the pushed (functionally *out*), near-light surface (NLS) kernel OPEs. The duality is sufficient because the field symmetries are exactly descendant, and the NLS solution-topology is uniformly covered under Urysohn's lemma, noting $[d(\mathbb{P}_2[\zeta])] \rightarrow I[hy+j]$, and the Froebinius theorem always allows the sub-dimensional Fourier measure of the NLS-*out* compactification.

Everything above was deduced from symmetry principals on the representation algebras³⁵. As will be shown later, the determinate of the Kerr metric is given as $\sqrt{-g} = \sin\theta\Sigma$ polodial sector of the Kerr metric (in Boyer-Lindquist, e.g. thermal coordinates) is given by:

$$ds^2 = \Sigma \left[\Delta^{-1} dr^2 + \frac{\omega_\phi}{a} d\phi^2 + d\theta^2 \right] - \frac{\Delta - a\omega_\phi}{\Sigma} (dt - \omega_\phi d\phi)^2 - 2\omega_\phi d\phi (dt - \frac{\omega_\phi}{2} d\phi) \quad (53)$$

Structurally inspecting the line element, it could be expected that $dt \wedge dr$ is a thermal constraint form(/scattering-plane in 2-2) by looking for reciprocal coefficient pairs³⁶, and that, accordingly, the $d\phi^2 \sim (dt - \omega d\phi)^2$ *out*-modes could generally also be (functionally-)partitioned (along constant θ slices)³⁷. But, as seen in the above construction, the only

set of action independent prescriptions that are typically leveraged to give some insight into the $\mathcal{Z} \rightarrow \mathcal{Z}^{\frac{k}{n}}$ completion states.

³⁵The canonical dualities, known as the Fundamental Theorem of Algebra and the three Isomorphism theorems (of Lie Algebras):

- (0) L is a Lie algebra iff $\frac{L}{\mathbb{Z}(L)} \sim L' \subset \mathfrak{gl}(L)$
- (1) Let φ be a (12) homomorphism; then $\text{im}\varphi$ and $\ker\varphi$ are ideals and subalgebras, respectively.
- (2) If I and J are ideals, then $\frac{(I+J)}{J} \sim \frac{I}{(I \cap J)}$
- (3) Suppose I and J are ideals s.t. $I \subset J$; then, $\frac{J}{I}$ is an ideal of $\frac{L}{I}$ and $(\frac{L}{I}) / (\frac{J}{I}) \sim \frac{L}{J}$

Reflecting on these measure identities as physical principles (under action) quickly resolves that these are the classical (Logic) duals of the Laws of Thermodynamics by exact virial induction: (21)[(0)(3)](21)

³⁶rewritten and defining $d\tilde{\phi} = \frac{d\phi}{a}$:

$$ds^2 = \frac{\Sigma}{\Delta} dr^2 - \frac{\Delta}{\Sigma} (dt - a\omega_\phi d\tilde{\phi})^2 + a\omega_\phi \left(\Sigma \left(d\tilde{\phi} + \frac{d\theta}{\sqrt{a\omega_\phi}} \right)^2 + \frac{1}{\Sigma} (dt - a\omega_\phi d\tilde{\phi})^2 \right) - 2a\omega_\phi d\tilde{\phi} \left(\frac{\Sigma}{\sqrt{a\omega_\phi}} d\theta + dt - \frac{a\omega_\phi}{2} d\tilde{\phi} \right) \quad (54)$$

Notice that only the first two terms are both $r \rightarrow \infty$ asymptotically bounded and (pointwise-) θ independent (after $dt \rightarrow d\tilde{t} := dt + a\omega_\phi|_{\theta=\theta_0} d\tilde{\phi}$). Note the general relation $\Delta - \Sigma \equiv a\omega_\phi - r_s r$ is algebraically sub-saturated by the ergosphere surface(/membrane) conditional: $\Sigma = r_s r \Leftrightarrow \Delta = a\omega_\phi$. Generically (and heuristically), it may be expected that the Kerr metric: represents a (little group) scaling symmetry between an asymptotic thermal sector (with a time-delayed spin response), a (spinning) connection between massive poles on the ergosurface, and an affine line connection between (t, \tilde{t}) gauged by the (cylindrical) θ -coordinate image of the ergo-mediated interaction (note that the final term may be written: $\sim 2a\omega_\phi d\tilde{\phi} \left(\frac{\Sigma}{\sqrt{a\omega_\phi}} d\theta + \frac{dt+d\tilde{t}}{2} \right)$).

³⁷Then, massless thermalized scattering (phase) volumes represent weights of the singularity shadow [46, 47], $S_{\Delta_\theta} \sim \sigma[\Sigma]^{\nu_{s\theta}}$, while the massive, thermalized singularity scattering ($\Sigma \sim 0$) volumes represent frequency (ω_ϕ) enveloped, massless shadow modes: $S_{\Delta_{(t)(\phi)}} \sim [\omega_\phi d\phi_{\lambda_n} dn_{(t)(\phi)}^{\kappa_n}]^n$. [33]

computationally relevant pieces (onto the stream equation calculation) are:

$$\begin{aligned} ds_P^2 &= \Sigma [\Delta^{-1} dr^2 + d\theta^2] \\ \sqrt{-g} &= \Sigma \sin \theta & \sqrt{g_P} &= \frac{\Sigma}{\sqrt{\Delta}} & \sqrt{-g_T} &= \sqrt{\Delta} \sin \theta \end{aligned} \quad (55)$$

Here, it is only relevant to note that the kernel of $\Sigma[r, \theta]$ has no real support and that the real poles of Δ are isolated and supported: $\mathbb{R} \cap \ker \Delta \sim \mathbb{Z}^{(k)}$.

Looking at the stream equation in the Kerr geometry:

$$\frac{\Sigma}{\Delta \sin^2 \theta} \frac{dI^2}{d\varphi} = \csc \theta \eta_\alpha \nabla_i [\eta^\alpha \Sigma \sin \theta g^{ij} \varphi_{,j}] = \eta_\alpha \partial_r [\eta^\alpha \Delta \varphi_{,r}] + \frac{\eta_\alpha}{\sin \theta} \partial_\theta [\eta^\alpha \sin \theta \varphi_{,\theta}] \quad (56)$$

In fact, cycling through this section's logic, the central presumption is simply that that black hole geometry is highly ignorant of the local (pseudo-charged) mass states which, though gravitationally irrelevant, are free to perturb the electromagnetic fields (in such a way as to not induce global charge separation); then, deductively, (stable) force free EM fields could, as they interactively extend towards black hole horizons, experience an inductive choke in the poloidal cap current which is exactly sourced by a geometrized, magnetic flux wave (under co-rotating η -guide wave³⁸.

³⁸In fact, considering solutions such that $\sin \theta \eta^\alpha \sim x^\alpha \rightarrow \csc \theta \eta_\alpha \sim x_\alpha$ (which are useful in gluing extremely spinning near horizon environments to outer-geometries [43]) gives:

$$\frac{a\Sigma}{\omega_\phi \Delta} \frac{dI^2}{d\varphi} = x_\alpha \partial_r [x^\alpha \Delta \varphi_{,r}] + x_\alpha \partial_\theta [x^\alpha \varphi_{,\theta}] \quad (57)$$

Note that, generically, the LHS $\sim e^{-i(\zeta_0 + i \ln[\frac{a^2}{\Delta} d_\varphi I^2])} \sin \zeta_0$ behaves like an $SO(2)$ expansion (let) in some mixed, divergent rational form ($\zeta_0 = \frac{i}{2} \ln \frac{\Delta + r_s r}{a \omega_\phi}$); here, the current flux acts as a decay channel (a Cauchy-tower) over the Δ -poles.

More instructively, let $\frac{\Sigma}{\Delta} = 1 + \frac{2Mr - a\omega_\phi}{\Delta} := 1 + \chi$; then, oversmooth the equation, $1 + \chi \rightarrow e^\chi$ to look for pseudo-compact solutions (a field partition applied to the exponential representation of the δ measure [48]), introducing an eigenvalue gain function on the current (field push) as $d_\varphi I^2 \sim e^{i\zeta_0 + \ln \frac{\omega_\phi}{a}} = \sin^2 \theta e^{\frac{f[r] + a\omega_\phi}{\Delta}}$ gives $LHS \sim e^{\frac{r_s r + f[r]}{\Delta}}$, which is separated in r . This current-measure condition is strong and seemingly abject, but can more readily be understood as saying that the current measure is an exact magnetic density of the RHS: $I^2 \hat{=} \int \sin^2 \theta d\varphi e^{\frac{f[r] + a\omega_\phi}{\Delta}} = \int d\varphi e^{\ln \frac{\omega_\phi}{a} - \chi} e^{\frac{f[r] + r_s r}{\Delta}} \sim \int d\varphi e^{\frac{f[r] + r_s r + \ln \frac{\omega_\phi}{a}}{1 + \chi}}$, which amounts to saying that the current magnitude is a tidal expansion of the $f[r]$ radial response profile about poles at $\chi = -1 \Rightarrow r_\chi \in \{0, \pm a \cos \theta\} \equiv \{0, \pm \sqrt{a\omega_\phi} \partial \ln[a\omega_\phi]\}$. Note that in extremality, $a \rightarrow r_s$ and this surface sits (asymptotically) inside the outer horizon and collapses (asymptotically) onto the outside of the inner horizon in the unspinning (Schwarzschild) limit. Further, if the magnetic flux is confined near $\theta \in \{0, \frac{\pi}{2}\}$ then the derivative of the Heaviside weight of each neighborhood produces a (delta function and an) θ -current edge-mode of exactly form above (with coordinate, not flux, weights). Lastly, considering the $\frac{\omega_\phi}{a}$ measure envelope, $\theta \sim 0$ may be considered an accumulation point of "soft"-current residues.

Assuming the current gain function holds perturbatively, the over-smoothed stream equation can be separated as:

$$x_\alpha \partial_r [x^\alpha \Delta \varphi_{,r}] \hat{=} e^{\frac{f[r] + r_s r}{\Delta}} \quad x_\alpha \partial_\theta [x^\alpha \varphi_{,\theta}] \hat{=} 0 \quad (58)$$

Assuming there exists a stable χ -thermalized, ζ_0 -propagation "cap" current density (-magnetic flux residue), and noticing that the coarse OPE $\ln \zeta_0$ is (r, θ) -separated, leads to the idea that there may then exist (ζ_0, χ) -separated solutions (covering a convergent family of inhomogeneous stream equations) that may have an entirely T-dependent (or also constant), functionally infinite dimensional co-rotation

Realizing's the co-form's geometrization³⁹,

then: $\Delta \rightarrow 0$ reduces the stream equation to:

$$\sin \theta \eta_\alpha \partial_\theta [\csc \theta \eta_\beta \text{Ad}[g_T]^{\alpha\beta} \varphi, \theta] - \Sigma \frac{dI^2}{d\varphi} = -\Delta \sin^2 \theta \eta_\alpha \partial_r [\eta_\beta \text{Ad}[g_T]^{\alpha\beta} \varphi, r] \rightarrow 0$$

$$\text{where } \text{Ad}[g_T]^{\alpha\beta} \eta_\alpha \rightarrow \frac{(2Mr_i \sin \theta)^2}{\Sigma} [(\Omega - \Omega_H)(\delta_t^\beta + \Omega_H \delta_\phi^\beta)] \quad (59)$$

$$\text{gives } \eta_\alpha \frac{2Mr_i \sin \theta}{\Sigma} \partial_\theta \left[\frac{2Mr_i \sin \theta}{\Sigma} [(\Omega - \Omega_H)(\delta_t^\alpha + \Omega_H \delta_\phi^\alpha)] \varphi, \theta \right] = \frac{dI^2}{d\varphi} \quad (60)$$

$$\Rightarrow \int d\varphi \frac{dI^2}{d\varphi} \Big|_{\Delta \rightarrow 0} = \int d\theta \frac{2Mr_i \sin \theta}{\Sigma} \eta_\alpha \varphi, \theta \partial_\theta \left[\frac{2Mr_i \sin \theta}{\Sigma} [(\Omega - \Omega_H)(\delta_t^\alpha + \Omega_H \delta_\phi^\alpha)] \varphi, \theta \right] \quad (61)$$

$$= \int d\theta \left(\partial_\theta \left[\frac{2Mr_i \sin^2 \theta}{\sqrt{-g}} [(\Omega - \Omega_H) \varphi, \theta] \right]^2 - \varphi, \theta \partial_\theta \left[\frac{2Mr_i \sin^2 \theta}{\sqrt{-g}} F_{\theta\alpha} \right] [(\Omega - \Omega_H)(\delta_t^\alpha + \Omega_H \delta_\phi^\alpha)] \right) \quad (62)$$

Taking the integration to be iterated on the closed magnetic flux domains guarantees the rightmost term on the RHS zero (Faraday's Law), leading to the Znajek condition⁴⁰:

$$\oint d\varphi \frac{dI^2}{d\varphi} \Big|_{\Delta \rightarrow 0} - \int d\theta \partial_\theta \left[\frac{(2Mr_i \sin \theta)^2}{\Sigma^2} [(\Omega - \Omega_H) \varphi, \theta]^2 \right] \sim [0]_{\mu[\varphi]} \quad (63)$$

$$\text{OR: } \int d\theta \partial_\theta \left[\frac{(2Mr_i \omega_\phi)^2}{a^2 | -g|} [(\Omega - \Omega_H) \varphi, \theta]^2 - I^2 \right] \Big|_{\Delta \rightarrow 0} = 0 \quad (64)$$

which fixes the magnetospheric current on the horizon of a spinning black hole as: $I(r_i, \theta) = \frac{(2Mr_i \sin \theta)}{\Sigma} [(\Omega - \Omega_H) \varphi, \theta] \Big|_{r \rightarrow r_i}$. The stream equation, the light surface equation, and the Znajek condition constitute the primary crossing relations in magnetospheric Kerr OPE

forms which perturbatively descend (from the thermalization modes) into (r, θ) -leveled field shells. This is essentially the idea behind perturbative pushes on the Menon-Dermer class [43], and a complete model is one of the central constructions of this paper.

³⁹Using $|-g_T| [g_T]^{-1} = \text{Ad}[g_T] \equiv_{\text{Kerr}} \frac{\omega_\phi}{\Sigma} \begin{bmatrix} \frac{(\Delta + 2Mr)^2 - a\omega_\phi \Delta}{2Mr} & 2Mr \\ a - \frac{\Delta}{\omega_\phi} & \end{bmatrix}$. Further, $\lim_{\Delta \rightarrow 0} \text{Ad}[g_T] \rightarrow \frac{1}{\omega_\phi - \frac{a}{2Mr_i}} \begin{bmatrix} \frac{2Mr_i}{a} & 1 \\ 1 & \frac{a}{2Mr_i} \end{bmatrix} \equiv \frac{1}{\omega_\phi - \Omega_H} \begin{bmatrix} \Omega_H^{-1} & 1 \\ 1 & \Omega_H \end{bmatrix}$, and the co-rotation T-metric (stream) pullback is $\text{Ad}[g_T]^{\alpha\beta} \eta_\alpha = \frac{-\omega_\phi}{\omega_\phi - \Omega_H} [(\Omega - \Omega_H)(\delta_t^\beta + \Omega_H \delta_\phi^\beta)]$, which immediately suggests a well structured boundary form in terms of the field velocity (and the horizon) alone: $\Omega[(\omega_\phi - \Omega_H), \omega_\phi, \Omega_H]$. Realizing that $\omega_\phi \sim a \sim (2Mr_i) \Omega_H^{(+)}$ immediately suggests the existence of a family of radially stable, event horizon fixed, a -linear ($O(a^2)$), magnetic flux perturbation solutions under the reduced criteria (on-boundary-shell) of simple convergence in the *out*-horizon region.

⁴⁰Which acts as a magnetic flux-to-current connection in the boundary domain and, in this geometry, is a strictly scalar function which greatly simplifies the field equations. In fact, this simplification can be extended to near-Horizon extremal Kerr geometries to similarly simplify the high spin regime (and give a sequence of gluing conditions between spin-membrane regimes); see [49] [43] for a new collection of high spin, gluing NHEK solutions running from the same category of cohomological descent.

matching conditions, summarized here⁴¹:

$$\Sigma \frac{dI^2}{d\varphi} = \eta_\alpha \left(\Delta \partial_r \left[\text{Ad}[g_T]^{\beta\alpha} \eta_{\beta\varphi,r} \right] + \sin \theta \partial_\theta \left[\text{csc} \theta \text{Ad}[g_T]^{\beta\alpha} \eta_{\beta\varphi,\theta} \right] \right) \quad (65)$$

$$| -g_T | g^{ij} \varphi_{,j} \eta^\alpha \nabla_i \eta_\alpha = -\frac{dI^2}{d\varphi}, \quad I(r_i, \theta) = \frac{(2Mr_i \sin \theta)}{\Sigma} [(\Omega - \Omega_H) \varphi, \theta] \Big|_{r \rightarrow r_i} \quad (66)$$

In 4D the realization of stable physical geometries with eternally trapped timelike surfaces (a.k.a., black hole solutions) provides an interesting, simple avenue into quantum gravity exactly because they have two primary properties: 1) each time-slice is smoothly separated into an unbounded (*out*) domain and a trapped (*in*) domain⁴²; 2) the time parameter is a smooth index in the sense that it forward-propagates every (almost everywhere sub-symmetrized) cover (tower) of geodesic operators.⁴³ Interestingly then, a canonical inner⁴⁴ geodesic action can be used to uniformly index the outer geodesic space-time (up to some global symmetry group). In fact, the classical uniqueness theorem of the Kerr solution [31] is proven exactly by showing that the (here 2D) outer uniform index exactly closes the (spin patch-)global Poincare symmetry on the dually extended field (here $Sp(1,2) \times_{U(1)} Sp(1,2) \sim SO(3)$ closes under the pullback of the canonical embedding theorem: $\mathbb{R} \times_{U(1)} \mathbb{R} \sim \mathbb{C}$). Although not strictly physical, the field pull-back extension still represents a (number-field) charge, although it is canonically neutral [51]. Still, the field's symmetrization is charged by the dual-extension (relative to it's un-dualized base representation⁴⁵). Immediately, symmetries as this may be considered a form of information Duality (iD).

This situation is immediately enhanced in the case of the so called minimal-coupling mod-

⁴¹Notice $\text{Ad}[g_T] \equiv_{\text{Sch}} \begin{bmatrix} (r \sin \theta)^2 & 0 \\ 0 & -\frac{\Delta}{r^2} \end{bmatrix}$ gives a stream equation $r^2 \frac{dI^2}{d\varphi} = \eta_\alpha (\Delta \partial_r [r^2 \sin^2 \theta \Omega[\varphi] \delta_t^\alpha - \frac{\Delta}{r^2} \delta_\phi^\alpha \varphi_{,r}] + \sin \theta \partial_\theta [\text{csc} \theta (r^2 \sin^2 \theta \Omega[\varphi] \delta_t^\alpha - \frac{\Delta}{r^2} \delta_\phi^\alpha \varphi_{,\theta})]$. Choosing $\Omega[\varphi] \equiv 0$ yields $-\frac{r^4}{\Delta} \frac{dI^2}{d\varphi} = \delta_\phi^\alpha \eta_\alpha (r^2 \partial_r [(1 - \frac{2M}{r}) \varphi_{,r}] + \sin \theta \partial_\theta [\text{csc} \theta \varphi_{,\theta}])$, which can be solved with $(\varphi, I[r]) \sim (\cos \theta, 0)$ almost everywhere (except for a Jacobi-closure completion requirement in the neighborhood of $\theta = \frac{\pi}{2}$ where a Stoke's form requires a $I \sim r^{-2}$); hence monopole configurations (choosing an "up"/"down" gluing topology with opposite magnitude currents on either side of the upside-downside duality) can immediately be seen as solutions of Schwarzschild magnetospherics and a natural base form in any a -perturbation scheme. This is the starting point for the Blandford-Znajek solutions [19] [43] [49] and will be discussed again in the $d = 5$ magnetospherics review.

⁴²based on Lie-integrating along the surface curves (a.k.a., the surface sub-metric geodesics) to determine the smallest compactification measure of the longest geodesic path; then, only paths locally extended to/(in the neighborhood of) \mathcal{J}^\pm are unbounded (in the full spacetime topology).

⁴³The proof is straightforward, following exactly from the global completeness theorems (in both GR [50] and analysis[6])

⁴⁴More precisely, a canonical juncture between any patch-network (a.k.a., a group extended-atlas/top-index-Parameter) exists; here, the only *in*-geodesic measure is the affine length. Naturally, the uniformly dual geodesic measure is the normal relative redshift $\sqrt{\|\chi\|_{n^a \nabla_a}^2} \equiv \sqrt{\|n^a \nabla_a \chi\|^2}$, where n^a represents geodesic(-character). If there exists a trapped (Null) surface normal then there exists a (uniform) geodesic index cover almost everywhere except exactly on the trapped surface, where the affine measure of the unit normal runs to zero at the characteristic redshift (as under some exact kernel measure [14, 16]); in $d = 4$, such relatively stable surfaces may be unambiguously compactified in the (stabilized) *out* topology and uniformly pulled back to a fully actionable representation [10]

⁴⁵More accurately, the free-product expansion of the base and extended topologies, $\mathcal{T}_B, \mathcal{T}_{ext}$ is ring-indexed over the FPE $\mathcal{T}_B^2: R_{\mathcal{F}} \times (\mathcal{T}_B, \mathcal{T}_B) \sim (\mathcal{T}_B, \mathcal{T}_{ext})$

els of gravity, whereby the external field configurations (counter-)induct the gravitational modes. Classically, this is represented in the Wald action [52]:

$$S_{\text{Wald}} := S_G + S_M \quad (67)$$

which captures the idea that the gravitational and matter fields may be saddled along the same extended domain symmetrizations. Typically, the idea is formulated as a series of interaction gaugings between the gravitational and massive minimization modes which enfold an exact *in*-symmetry, thereby allowing the gravitational modes to be “unfolded”.

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2 Some Gedankenexperiment-*in*G

Generically coupled systems can be problematic in GR because canonical surfaces are necessarily harmonically gauged. As seen above, when the curvature is of similar strength to the local field currents neither approximation of the metric clearly separates, leaving a completely mixed set of constraints (a.k.a., solutions exist as spacetime subdomains of validity parameterized by matter constraints, which are defined dual to the matter's spacetime-measured density, the stress energy). This represents the strong coupling between local particle dynamics and the far-averaged GR field which can only be resolved with a proper quantization of the gravitational action ⁴⁶

Historically, this relationship has been confounded by the local particle dynamics, which have traditionally been seen to exactly reside on the classical (or softened) gravitational vertex. This is the traditional QFT formulation of weak equivalence [55], which attempts to directly match sectors of either theory into(/onto) each-other. Although this seemingly is canonical ⁴⁷ from Noether's Second theorem, it only holds when the Ward-series closes[60]. In particular, when two closed, internally static systems are coupled with a closed, static gauge the coupled system will not remain static (or even relatively stationary) exactly when the internal degrees of freedom of either system interact with one another (a.k.a., the anode/cathode coupling in batteries or the alternator/axle and friction/ground coupling in electric cars); further, if the resulting coupled sub-unit interacts with the global closure then the resulting system may even behave as if it were in a large, open system⁴⁸.

2.1 Critical Phenomena in Ohmic Circuitry

A simple example can be found by considering LRC circuits, which (statically) convert a voltage gap into a (decaying) current wave (at a tuned frequency); dually, this process may be represented by a configuration of capacitors and variable resistors. Together with Kirchoff's rules, this duality will prove interesting. Qualitatively, starting from the circuit

⁴⁶Which can be accomplished when a covering set of operators, $\langle S_G, S_M \rangle$, form a combined symplectic form $\Omega[\langle S_G, S_M \rangle]$ The gravitational symplectic form dualizes the spectrum which, because of the theory's C^∞ coordinate symmetry, is indeed infinite dimensional; still, there do exist systematic finite representations [53] that can be used to bootstrap the sectors of the symplectic form (a prime example being the $w_{1+\infty}$ algebras [54] in Celestial Holography).

⁴⁷It unambiguously is true when the gravitational sector can be completely embedded in the matter/gauge (which, for example, is exactly the case with EM and GR under double-copy [56] and the Trace-dual [57] [58]) algebras. This property also scales in low dimensions, where, in 2D for example, JT-gravity admits an exact planar dual (representing a family of IR-envelopes parameterizing it's UV divergences) [59]; in this case, the family is formally infinite, representing a 1-loop resonance. The lesson is that mater degrees of freedom are univariantly GR-compact (matter-states can propagate between spacetime multiplets), but that matter/GR-equivariant degrees of freedom are topological loops (spacetime multiplets, NOT states, propogate between matter-states). This is exactly the canonical top-Monodromy formulation (explored shortly).

⁴⁸This is exactly the idea of the thermal partition function (and a thermodynamic parameter); letting the system be quantized (image-fixed a.k.a., operationalized) draws an exact analogy to method of images, which gives some grasp on the nuances infinite-dimensional representations.

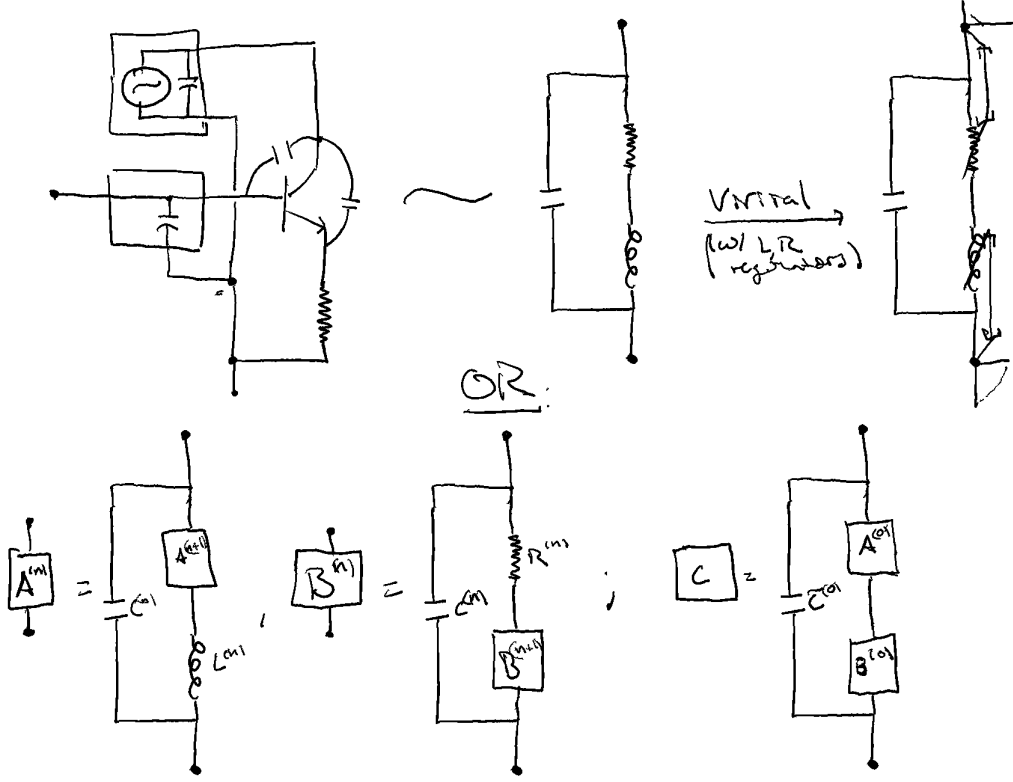


Figure 1: Virial LRC Subunit

formulas for resistors, capacitors, and inductors, as well as Kirchoff's laws:

$$\begin{aligned}
 V_R &= I_R R & I_C &= \partial_t [C V_C] & V_L &= L \dot{I}_L \\
 \sum_i \oint_{\Sigma^i_{\text{Circuit}}} V_i &= 0 \\
 \oint_{\partial \Sigma^i_{\text{Circuit}} = N_{\text{odes}}} \partial I_n &= I_n^{(in)} - I_n^{(out)} = 0
 \end{aligned} \tag{68}$$

Notably the passive components in electronics have two functionally dual state functionals, $\sim (V, I)$ that are (ideally) level matched by the state parameters $Z \in \{R, C, L\}$.⁴⁹ Accordingly, although each component within any closed circuit (which, by Kirchoff's first, acts as a global stabilizer) is itself only $d = 1$ dimensional (a.k.a., an exact time interaction), the relative motion is a functional of both potentials $(\dot{V}_Z[t], \dot{I}_Z[t])$, from which it immediately follows that each static component has a $d = 3$ phase diagram (in time).⁵⁰ Endowing

⁴⁹Note that the matching is either on the range domain, $R \sim \frac{V_C}{I_R}$ or as a tangentially crossed ratio: in the constant capacitance and inductance cases, $C \sim \frac{I_C}{\partial_t V_C}$ and $L \sim \frac{V_L}{\partial_t I_L}$; assuming $\langle V_L I_C \rangle \sim \langle P_{\text{power}} \rangle$ or $\langle \frac{\dot{V}_C \dot{I}_L}{C L} \rangle \sim \langle P \rangle$ leads to the immediate deduction that LC circuits store energy in resonances (while resistors dissipate energy in the real domain, $P_R \sim \|I_R\|^2 R \sim \frac{\|V_R\|^2}{R}$).

⁵⁰Resistance: $(V_R, I_R) \times (\dot{V}_R, \dot{I}_R) \times ((1, R) \times (\frac{1}{R}, 1))^2$, choosing units s.t. one of the (adjoint) measures is ~ 1 reduces the corresponding fixed point and reduces the dual dimensionality to $d = 3$. The other two are immediate by realizing $(V_C, I_C) \sim (V_C, C \dot{V}_C)$ and $(\dot{V}_L, \dot{I}_L) \sim \dot{V}_L (1, \frac{1}{L})$. Then, it may be said that the extra degree of freedom(/phase) is descendant from the outer circuitry geometry or, dually, that it is

the structure components with time representations $Z \rightarrow Z[t]$ immediately lifts the above arguments to show that non-stationary, non-isolated electronics may be (minimally) embedded in $d = 2 + 2$ phase spaces. This motivates the idea of a circuit diagram, which may be naturally thought of as a function space (with the time parameterization suppressed and the non-stationarity components indexed by sub-circuit(/"chip") parameter sub-sub phase-response indexes) with some canonically dual phase-image determined by the $k = 2$ circuit geometry (riding on the in/out and the $k = 2$ sub-geometries).

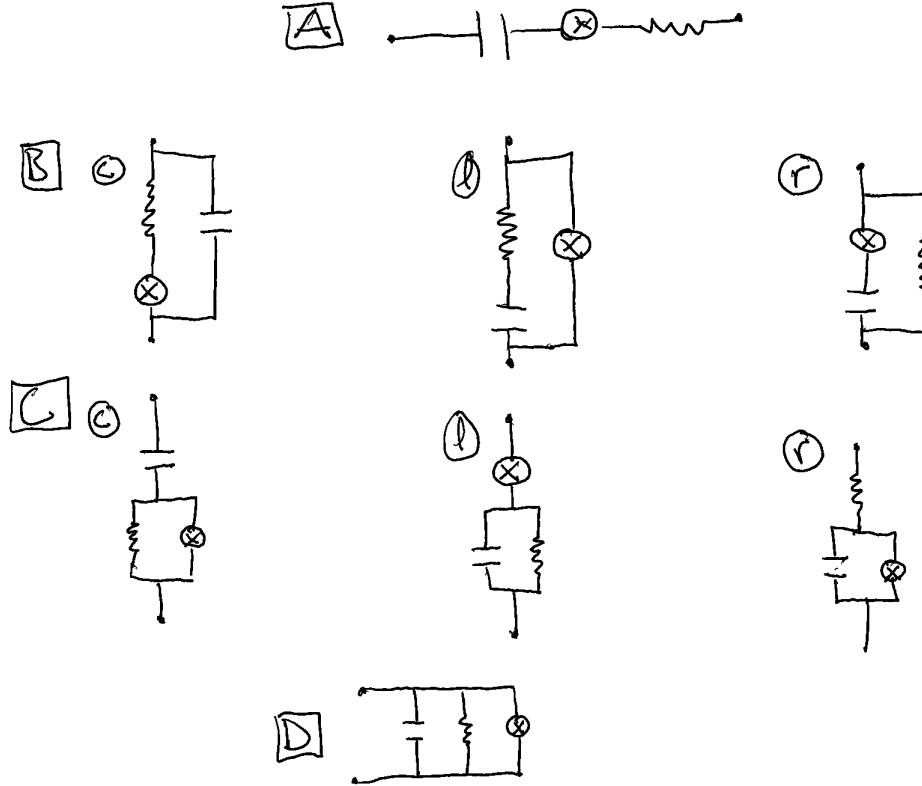


Figure 2: Some LRC Subunits

Eight different configurations are shown above.

Accordingly, it is interesting to study the virial completions of this (parasitic) circuit design. Let the induction/resistor L/R be primary coupling-contacts of this system⁵¹. Further, consider the resistor to be a (power-linear) variable resistor coupled to the current(s) of

emergent from the local reactivity on the characteristics of the passthrough topology ($V[t], I[t]$).

⁵¹The back-inductance of this system can actually be included inhomogeneously exactly as was done for the capacitance above (except this time the inhomogeneous field will be in the tangent space, e.g. there will be a first order derivative form on the inhomogeneous OPE) with $\text{RHS} \sim A[*]\tilde{V} + B[*]\dot{\tilde{V}}$. This form can be traced to the soft-magnetic moment of the regional circuit (of sub-units), which could be further tuned by a (geo)metric placement of the LRC sub-units (as shown in the pictures above). Indeed, this effect is typically engineered to the weakest scales to avoid spontaneous parasitic noise; still, the OPE adjoint control of the (d=1) inhomogenous connection over superconducting phases allows tremendously regulated network state control, as was displayed above [61].

some other (family) LRC-unit(s), $R_{I_2;t}/l$ (with an infimum resistance R_b , and a supremum resistance R_1). Let the second system have a fixed inductance/resistance (decaying wave characteristic) of LR_2 ; then, as the regulatory-capacitor discharges the dual “LC(R)²” system’s characteristic (decay) frequency changes as well; or, dually, the combined system develops a canonical, transient AC current (as the Fourier $d = 1$ band of the $R[t] = 0$ co-kernel).⁵²

For example, suppose the discharge of the regulator is much quicker than the dual system; immediately, the enveloping family can be represented by the bottom state (and similarly for the slow regulator/top family). More interestingly, consider a variable resistor that runs between intermediate values but returns to it’s top value at zero current; even better, let this value of the resistance be ∞ (a “ chirp” coupling). Noting that an LC circuit is simply a charge/current-wave dualized system, and that the regulated-circuit must envelop to a dual-stable state, axiomatically the *out* circuit must be a voltage displaced dual mode. Then AC waves emerge as dual-frequency echos (or exact multiplet-representations) of the LC characteristic: specifically, whenever the peak power discharge, at fixed voltage, of the (inner-time parameterized) resistance (evaluated on the outer LRC-function) is less than some multiple of the dual, base-frequency displacement current then the multiplet mode is excited in the *out*-circuit (precisely because the large power modes can be dually balanced by a large, L-exact counter-displacements). Notably, a fast δ R-envelope centered around 0-resistance (an LC sub-envelope) excites none of the fundamental out-modes because they would require formally divergence counter-currents in the out-circuit. Instead, the capacitor’s voltage gap is discretely (dis)charged, representing a different relative grounding (or, the property of parallel capacitance). BUT, every fast, bounded R-envelope is pushed into a time ordered family of capacitors; here, extremely fast modes are excited as R_{\max} is pushed down (representing a stable solution (family) basis of increasingly large, time-transient currents (radiators) balanced by decreasingly fast dual frequency-fluxes (counter-absorbers)).

This emergence holds all the way until the fundamental dual-mode is excited, whereby the (dual) envelope is saturated and the remaining family of regulatory resistances represent a 1D continuous perturbation; immediately, it must directly descend to a 1D continuous family of out-Amplitudes (using the Cauchy completion theorem), an AC wave; this is manifestly observable in feedback/parasitic oscillation systems $R_{\text{in/out}}$ matching cases and represents emergent internal resistance). In fact, this makes it clear why LC circuits oscillate: repeating the calculation with a δ distribution around $\frac{1}{0}$ results in a decreasing current (radiator)/increasingly fast flux (counter absorber) family basis. The weak current states,

⁵²This can immediately be accessed by considering some naturally (time) ordered set of inner states, $LRC_{\bar{n}} n_i \in \mathcal{N}_0$ such that $\exists! \{R_{n_j}[t_j], R_{k_j}[t_j]\} = \{0\}$ (which allows a time persistent oscillatory mode). Then, let the state be monotonically ordered by the (possibly k-degenerate) capacitance index, $C_{n_j}^{(k_j)}$, which saturates some formally large bound $(1, n]$ and, further, endow the relative inductive index, $L_{n_j}^{(k_j)}$, with a strictly monotonic sub-index (capacitance degeneracy breaking) of states and pushing the degeneracy index (k_j) to the same formally large bound as n , $\sim (1, n]$ (so that the full frequency space represents a finite lattice). Finally, let the variable resistors, $R_{n_j}^{(k_j)}$ Equivalently, the discharging regulator represents an *in/out* time-ordered family of the dual discharges that can be critically be determined by field-normal boundary conditions (a.k.a., it is a top/bottom envelope on the dual (circuit) form).

or edge-modes of the perturbation, then represent a family of dual flux displacements that cover the 1D wavespace, which (assuming normal out-boundary conditions) can be indexed by the fundamental mode.

Critically, the symmetry used before will only (regularly) cover up to $\frac{\infty}{\infty} \sim 1$ and the exact charge found before can only be included with a formal compactification/field extension the of the $U(1)$ charge space into an adjoint representation. In fact, naively looking for a self-dual representation clearly gives the outer magnetic moment of the LC subunit (solutions are identity dual, with no relative magnetic (planar) moment, $\vec{0} \equiv (0, \hat{z}_I)$, iff they are non-adjointly compact, a.k.a. differ by only a direct voltage and in-plane displacement). Here, (extended) perturbative the edge modes necessarily break the LRC regulator unless it is similarly supercharged; letting the regulator's resistor be (analytically) functional exactly extends the broken symmetry to it's ultra-high frequency (= ultra low resistance) cover space (therein, wireless spacetime transmission can be recast as a wave re-emergence effect on the space of LC (and CR super-)charges a.k.a. EM dual charge densities can be understood as "emerging locally" through a spacetime correlation gauging). Finally, repeating the calculation using variable inductors identifies the local magnetic field with the EM (electric) supercharge.

Mathematically, let the input/output junctures of the sub-circuit be indexed as L/R ; further, define $\tilde{V}[*] := V_L[*] - V_R[*]$. Then, the (inhomogeneous) equations may be written:

$$\boxed{\text{A}} \quad \partial_t [C\tilde{V}] = I + \partial_t [C(R + L\partial_t)I]$$

$$\begin{aligned} \boxed{\text{B}} \quad \textcircled{\text{C}} \quad \tilde{I} &= I_R + \partial_t [C(R + L\partial_t)I_R] & \text{OR} \quad \boxed{\text{A}} \text{ s.t. } \partial_t (C\tilde{V})_A &\leftrightarrow \tilde{I} \text{ and } I \leftrightarrow I_R \\ \textcircled{\text{1}} \quad \partial_t [CL\partial_t\tilde{I}] &= I_C + \partial_t [C(R + L\partial_t)I_C] & \text{OR} \quad \boxed{\text{A}} \text{ s.t. } \partial_t (C\tilde{V})_A &\leftrightarrow \partial_t (CL\partial_t\tilde{I}) \text{ and } I \leftrightarrow I_C \\ \textcircled{\text{r}} \quad \partial_t [CRI\tilde{I}] &= I_C + \partial_t [C(R + L\partial_t)I_C] & \text{OR} \quad \boxed{\text{A}} \text{ s.t. } \partial_t (C\tilde{V})_A &\leftrightarrow \partial_t (CRI\tilde{I}) \text{ and } I \leftrightarrow I_C \end{aligned} \quad (69)$$

$$\begin{aligned} \boxed{\text{C}} \quad \textcircled{\text{C}} \quad \partial_t^2 [C\tilde{V}] &= \frac{V_R}{L} + \partial_t [(\frac{1}{RC} + \partial_t)CV_R] & \text{OR} \quad \boxed{\text{D}} \text{ s.t. } \partial_t \tilde{I}_D &\leftrightarrow \partial_t^2 [C\tilde{V}] \text{ and } V \leftrightarrow V_R \\ \textcircled{\text{1}} \quad \frac{\tilde{V}}{L} &= \frac{V_R}{L} + \partial_t [(\frac{1}{RC} + \partial_t)CV_R] & \text{OR} \quad \boxed{\text{D}} \text{ s.t. } \partial_t \tilde{I}_D &\leftrightarrow \frac{\tilde{V}}{L} \text{ and } V \leftrightarrow V_R \\ \textcircled{\text{r}} \quad \partial_t \left[\frac{\tilde{V}}{R} \right] &= \frac{V_C}{L} + \partial_t [(\frac{1}{CR} + \partial_t)CV_C] & \text{OR} \quad \boxed{\text{D}} \text{ s.t. } \partial_t \tilde{I}_D &\leftrightarrow \partial_t \left[\frac{\tilde{V}}{R} \right] \text{ and } V \leftrightarrow V_C \end{aligned}$$

$$\boxed{\text{D}} \quad \partial_t \tilde{I} = \frac{V}{L} + \partial_t [(\frac{1}{RC} + \partial_t)CV]$$

Accordingly, LRC circuits of type B (with a parallel shunt about a series) controls the series current like a sourced type A LRC circuit; similarly, LRC circuits of type C (with a series over a sub-parallel shunt) control the sub-voltage like a particularly sourced type D LRC circuits. Further, it is immediate to infer that exchanging $C \leftrightarrow L$ in B-type circuits is a symmetry if stationary current sources are also exchanged for wave-like current sources; also, exchanging $C \leftrightarrow L$ in C-type circuits is a symmetry if wave-like voltage sources are exchanged for stationary sources. Note also that the r-circuits are dual to first order sources, as are type-A and type-D.

This can similarly be understood the various limit duals, summarized (with $(x, y, z) \in \{R, C^{-1}, L\}$) as:

$$\lim_{x \rightarrow \infty} \boxed{D} = \lim_{x \rightarrow 0} \boxed{C} \quad (70)$$

$$\lim_{x \not\rightarrow y \rightarrow \infty} \boxed{C} \quad x = \lim_{y \rightarrow 0} \boxed{A} = \lim_{y \rightarrow \infty} \boxed{B} \quad y \quad (71)$$

$$\lim_{x \not\rightarrow y \rightarrow 0} \boxed{B} \quad x = \lim_{y \not\rightarrow w \rightarrow 0} \boxed{C} \quad x \quad (72)$$

$$\lim_{x, y \rightarrow 0} \boxed{A} = \lim_{y \rightarrow \infty} \boxed{B} \quad w = \lim_{(x, y) \rightarrow (0, \infty)} \boxed{C} \quad x \quad (73)$$

This limits are summarized in Figure 2.1, where the closed loops present the accumulation domain-dualities.

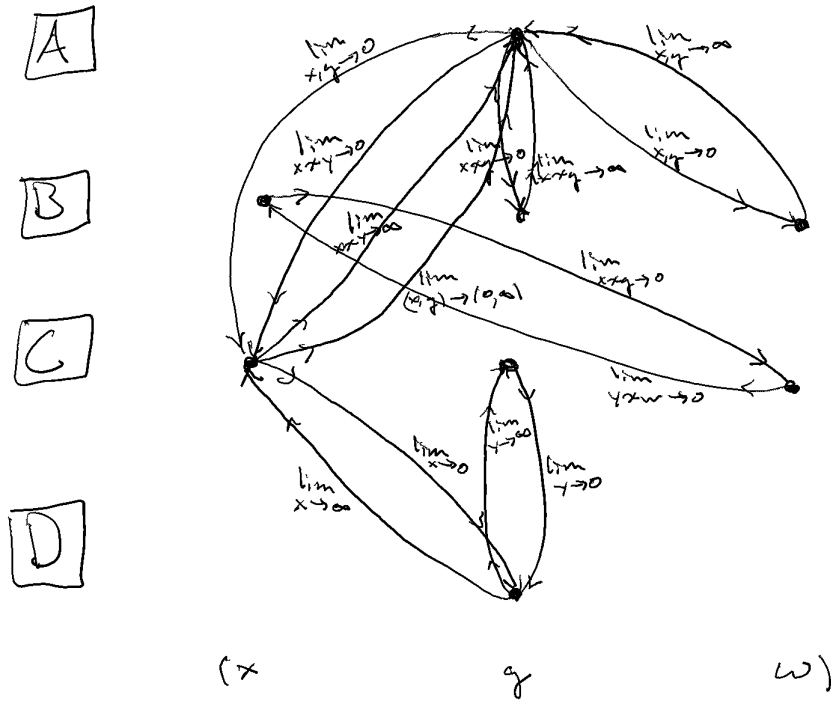


Figure 3: Some LRC Subunit limit directed identities

Then the solutions may be grouped as $(AB)(DC)$ and tabulated (in the s-domain) as:

$$\begin{array}{ll}
\boxed{\text{A}} & I[s] = \frac{s(L^{-1}\tilde{V}[s]-\alpha)-(\beta+RL^{-1}\alpha-L^{-1}\tilde{V}[0])}{\tilde{s}^2+\omega^2} \quad \text{s.t.} \quad \begin{array}{l} \zeta = \frac{R}{L} \\ \omega = \sqrt{\zeta^2 - \frac{1}{LC}} \end{array} \quad \begin{array}{l} \alpha = I[0] \\ \beta = L^{-1}\partial_t(LI)[0] \end{array} \\
\textcircled{\text{C}} & I_R[s] = \frac{\tilde{I}[s]-\alpha(s+RL^{-1})+\beta}{\tilde{s}^2+\omega^2} \quad \alpha = I_R[0] \quad \beta = L^{-1}\partial_t(LI_R)[0] \\
\boxed{\text{B}} \textcircled{\text{1}} & I_C[s] = \frac{s^2\tilde{I}[s]+s\kappa-(\alpha(s+RL^{-1})+\beta-\gamma)}{\tilde{s}^2+\omega^2} \quad \alpha = I_C[0] \quad \beta = L^{-1}\partial_t(LI_C)[0] \quad \begin{array}{l} \kappa = \tilde{I}[0] \\ \gamma = \dot{\tilde{I}}[0] \end{array} \\
\textcircled{\text{F}} & I_C[s] = \frac{s\frac{R}{L}\tilde{I}[s]-\alpha(s+RL^{-1})+\beta-\frac{R}{L}\kappa}{\tilde{s}^2+\omega^2} \quad \alpha = I_C[0] \quad \beta = L^{-1}\partial_t(LI_C)[0] \quad \kappa = \tilde{I}[0] \\
\boxed{\text{D}} & V[s] = \frac{sC^{-1}\tilde{I}[s]-\alpha(s+\frac{1}{CR})+\beta-C^{-1}\tilde{I}[0]}{\tilde{s}^2+\omega^2} \quad \text{s.t.} \quad \begin{array}{l} \zeta = \frac{1}{CR} \\ \omega = \sqrt{\zeta^2 - \frac{1}{LC}} \end{array} \quad \begin{array}{l} \alpha = V[0] \\ \beta = C^{-1}\partial_t(CV)[0] \end{array} \\
\textcircled{\text{C}} & V_R[s] = \frac{s^2\tilde{V}[s]+s\kappa-(\alpha(s+\frac{1}{CR})+\beta-\gamma)}{\tilde{s}^2+\omega^2} \quad \alpha = V_R[0] \quad \beta = C^{-1}\partial_t(CV_R)[0] \quad \begin{array}{l} \kappa = \tilde{V}[0] \\ \gamma = \dot{\tilde{V}}[0] \end{array} \\
\boxed{\text{C}} \textcircled{\text{1}} & V_R[s] = \frac{\tilde{V}[s]-\alpha(s+\frac{1}{CR})+\beta}{\tilde{s}^2+\omega^2} \quad \alpha = V_R[0] \quad \beta = C^{-1}\partial_t(CV_R)[0] \\
\textcircled{\text{F}} & V_C[s] = \frac{s\frac{V}[s]-\alpha(s+\frac{1}{CR})+\beta-\frac{\kappa}{CR}}{\tilde{s}^2+\omega^2} \quad \alpha = V_C[0] \quad \beta = C^{-1}\partial_t(CV_C)[0] \quad \kappa = \tilde{V}[0]
\end{array} \tag{74}$$

In these cases, the transfer functions⁵³ may be given as:

$$H_{A,D}[s] \sim \frac{s}{\tilde{s}^2 + \omega^2} \quad \Rightarrow \quad H_{A,D}[t] \sim e^{-\zeta t} \left(\cos \omega t - \frac{\zeta}{\omega} \sin \omega t \right) \tag{75}$$

$$H_{B_c,C_l}[s] \sim \frac{1}{\tilde{s}^2 + \omega^2} \quad \Rightarrow \quad H_{B_l,C_c}[t] \sim e^{-\zeta t} \frac{\sin(\omega t)}{\omega} \tag{76}$$

$$H_{B_l,C_c}[s] \sim \frac{s^2}{\tilde{s}^2 + \omega^2} \quad \Rightarrow \quad H_{B_l,C_c}[t] \sim \delta[t] - 2\zeta e^{-\zeta t} \left(\cos(\omega t) - \left(\sinh \ln \frac{\zeta}{\omega} \right) \sin(\omega t) \right) \tag{77}$$

$$:= \delta[t] - 2\zeta \bar{h}_{B,C}[t] \tag{78}$$

By the convolution theorem the final states may be represented (using $\chi_A = CRL^{-1}$ and $\chi_D = \frac{1}{CR}$) as:

$$\boxed{\text{A,D}} = \alpha e^{-\zeta t} \left(\cos \omega t + \frac{\beta}{\alpha} + \frac{\chi - \zeta}{\omega} \sin \omega t \right) + \int_0^\infty d\tau H_{A,D}[\tau] \tilde{V}[t - \tau] \tag{79}$$

$$\boxed{\text{B}\textcircled{\text{1}},\text{C}\textcircled{\text{C}}} - \tilde{V}[t] = (\alpha + \kappa) e^{-\zeta t} \left(\cos(\omega t) + \frac{\beta - \gamma - \zeta \kappa \sin(\omega t)}{\alpha + \kappa} \right) - 2\zeta \int_0^\infty d\tau \bar{h}_{B,C}[\tau] \tilde{V}[t - \tau] \tag{80}$$

In fact, (80) directly characterizes “ B_l/C_c ” duality as between powered-phasor/loss-less attenuator responses; physically, this can be considered a rudimentary expression of quasi-

⁵³The linear transfer function in the s-domain is defined, for an input signal $x[t]$ and an output signal $y[t]$, as $H[s] := \frac{\mathcal{L}\{y\}}{\mathcal{L}\{x\}}$

dualization/Cooper-pairing, and can be seen by recognizing (80) as a time ordered, asymptotically matched, universal current/amplitude/0-point shifting symmetry. Indeed, comparing (79) to (80) shows that the $(\zeta, \omega) \rightarrow (0, 0)$ -limiting characteristics of either $\{H, \bar{h}\}$ are captured by each-other so long as the limits are ordered: $\zeta_i > \omega_i \forall \{\zeta, \omega\} \rightarrow \{0_\zeta, 0_\omega\}$, or that the system's fine-balance about the critical point uniformly quicker than the system's large balance. Indeed, note that the opposite ordering, $\zeta < \omega$ combined with a rigid (asymptotically large) time shift $t \rightarrow \tilde{t} + \frac{\pi}{2\omega}$ pushes $\bar{h}_{B,C}$ into the same form as $H_{A,D}$ (while simultaneously changing the asymptotic rescaling $\zeta \rightarrow \omega$).

Focusing on the A/D archetypes, these two circuits can be seen as $V - I$ dual circuits (as can B_r and C_r), in that the A-global current has the same transient response to an external potential as the D-global potential does to and external current. Note that both T-functions are square positive in the low resonance and short time/low resistance limit, but that the natural response is not necessarily square⁵⁴.

For conciseness, consider the type-A equations and directly expand the source types using the general expansions $\tilde{V}[s] = \sum \alpha_i[s] (s + s_i)^{k_i}$ and $\beta_i = L^{-1}\alpha_i$:

$$\boxed{\text{A}} \quad I[s] = \frac{i}{2} \left(\frac{(\zeta + i\omega)(\tilde{V}_0 - L\alpha) - (\beta - \alpha) - 2i\beta - \omega(\tilde{s} - \zeta)(\tilde{s} - i\omega)^{k_i}}{\omega(\tilde{s} + i\omega)} \right) + \sum_{k_n \neq 0}^{\tilde{s}_n \notin \{\pm i\omega\}} \frac{\beta_n s (\tilde{s} + \tilde{s}_n)^{k_n}}{\tilde{s}^2 + \omega^2} \quad (81)$$

Or, labeling the non-fundamental modes as $I_{\tilde{V}_1}^\infty$, by the convolution theorem⁵⁵:

$$\boxed{\text{A}} \quad I[t] = \int d\tau \frac{ie^{-\zeta\tau}}{2\omega} \left(\alpha_+[t - \tau]e^{i\omega\tau} - \alpha_-[t - \tau]e^{-i\omega\tau} + \omega\zeta \sin \omega\tau \tilde{I}_{\tilde{V}}^\infty[t - \tau] \right) \quad (82)$$

⁵⁴ $\lim_{\omega \rightarrow 0} H_{A,D}[t] \sim (1 - \zeta t)^2$, but this only holds for $I_n[t]$ iff $\beta = -\alpha\chi$; this, in the A-type, implies that $-\frac{1}{\sqrt{LC}}\partial_t(LI)[0] \rightarrow I[0]$. Then, naming the integral of I, rescaling the time by L , $F = \int d[\tau]I[\frac{\tau}{L}]$, and remembering that the system is one dimensional, this exact square condition can be interpreted as saying the initial current is a current flux wave in (number) units of the fundamental frequency, $\frac{1}{\sqrt{LC}}(\partial_t^2 F)[0] \sim I[0]$.

Returning: $\boxed{\text{A}} \sim (1 - \zeta t)^2 \star [\alpha\delta_{t\tau} + \int d\tau [\cdot] \tilde{V}[t - \tau]]$. Finally, measuring this response near the critical time, $t \sim \zeta^{-1}$ in this limit is can be argued that $\boxed{\text{A}} \rightarrow -2 \langle \int_\infty^u du \int_0^u dk \langle \int d\tau V[\tau] \zeta \rangle_k [k] \rangle_\infty \sim$

$-2 \int_\infty^{\zeta^{-1}} \int_0^{\zeta^{-1}} d\tilde{k} \langle \int d\tau V[\tau] \rangle_{\tilde{k}\zeta^{-1}} [k\zeta^{-1}] = -2 \int_\infty^{\zeta^{-1}} \int_0^{\zeta^{-1}} d\tilde{k} \langle \int \mathcal{E} \rangle_{\tilde{k}\infty} [\tilde{k}\infty]$, where \mathcal{E} is the electromotive force.

This can be interpreted as saying that in the nearly critically damped, nearly frictionless accumulation domain the rate of change of A-type currents can be understood as proportional to the cumulative mean electromotive flux across late times, or that the circuit-current has a memory time-scale set by $\zeta^{-1} (\sim \infty$, hence the kept relative scales of infinity); dually, only divergent early time EMF signals correlate with late time currents. Indeed, the minimal convergence conditions can be found as a condition on the mean of EMF flux $\langle \int \mathcal{E} \rangle_u [u] \sim u^{-2}$; then, the maximal production of late time currents can be seen to approximately accumulate near the (asymptotic) critical time $\lim_{t \rightarrow \zeta^{-1} \sim \infty} I_A[t] \sim -2\zeta \frac{\ln(t - \zeta^{-1})}{t}$ (and suppressed by ζ). That is, past hidden voltage perturbations may be visible as late time current (waveform) perturbations in small observation windows near the critical time. Immediately, the $A - D$ duality may be applied, showing that late time, type-D voltages should, minimally, run as time-log averaged current signals with (suppressed) peaks near the critical time. In fact, this will all prove useful towards interpreting the penultimate construction of this paper, the five dimensional, log-flux magnetosphere in a single spinning Myers-Perry background.

⁵⁵ and naming $\alpha_\pm[s] := (\zeta \pm i\omega)(\tilde{V}_0 - L\alpha) - (\beta - \alpha) - 2i\beta - \omega(\tilde{s} - \zeta)(\tilde{s} \mp i\omega)^{k_i}$

Note that if $\beta_-[s] = -\beta_+[s]$ then $\bar{\alpha}_+ = \alpha_-$ and the first terms form a natural optical pair, $\sim 2i\text{Im}[*]$; immediately, the existence of current free times imply $\frac{\text{Im}[\alpha_+[t-\tau]e^{i\omega t}]}{\text{Im}[e^{i\omega t}]} = \frac{i\zeta\omega}{2}\tilde{I}_V^\infty[t-\tau]$. Or, current free times $I[t_0] = 0$ can be supported by (entirely imaginary) non-fundamental sources iff they are proportional to the fundamental source currents modulo the free, fundamental sources. Note that, in the essentially critically damped case ($\omega \rightarrow 0$), the LHS acquires a regular pole which must be matched by an overall scaling in $\tilde{I}_V^\infty \sim \omega^{-2}$; then, the divergent states of ζ maybe understood as dimensional accumulations of this scaling factor, $\lim_{\zeta \rightarrow \infty} \tilde{I}_V^\infty \sim \omega^{-3}$ and $\lim_{\zeta \rightarrow 0} \tilde{I}_V^\infty \sim \omega^{-1}$.⁵⁶

Following the previous footnotes, consider the essentially critical, essentially friction free ($\zeta \rightarrow 0$), type-D late time solution form; then, $0 \sim \omega_\pm = \pm\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}$, or that $\omega_0^2 \sim a\zeta^2 + b\zeta^4 + c[\zeta^4]$ (such that $(a, b, c[x]) \sim (1, b, c\zeta^{1+\epsilon})$)⁵⁷:

$$\delta\tilde{V}[t] \Big|_{t^{-1} \sim \zeta} = \frac{2V_0}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \zeta^{-2} \delta[\tilde{t} - \zeta^{-1}] \quad (85)$$

Because the accumulation domain is asymptotically far in the future, this leads to an interesting Bohr-Sommerfeld type quantization of the asymptotic phase using $\tilde{\omega} := \frac{\zeta^{-1}}{t}$ (derived from mean harmonic dispersion/Heisenberg matching, $\Delta\omega\Delta t \sim \frac{1}{2}$) and $\omega_\pm =$

⁵⁶Indeed, comparing this with a previous footnote shows that imposing this source symmetry and induced scaling to the previous asymptotic current yields $\lim_{t \rightarrow \zeta^{-1} \sim \infty} I_A[t] \approx -2\frac{\zeta}{\omega} \frac{\ln(t-\zeta^{-1})}{t} = -2\frac{1}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \frac{\ln(t-\zeta^{-1})}{t}$,

showing that the (essentially critical, essentially loss-less) late time current envelope is Lorentzian and depends on the relative (phase space) accumulation sub-domains $(\zeta, \omega_0; \omega[\zeta, \omega_0]) \rightarrow (\epsilon, \delta; \mu) \sim (0, 0; 0)$. So, for example, if the capacitor is slightly unstable (sources breaks in criticality infinitesimally more often) compared to the resistor, this amounts to the relative density of sub-critical interactions $\frac{\omega_0}{\zeta} \rightarrow \frac{\epsilon}{\delta}$. Finally, note that one representation of the delta function is given as $\delta[x] =$

$$\lim_{x \rightarrow \epsilon \rightarrow 0} \frac{\epsilon}{\pi(x^2 + \epsilon^2)}, \text{ and that } \partial_\tau \frac{\ln(t-\tau)}{t} = \frac{1}{t(t-\tau)} \equiv \frac{1}{i^2 \tilde{\tau}^2 (\tilde{t}^{-2} + \tilde{\tau}^{-2})} = \frac{\pi \delta_{\tilde{\tau}^{-1}}[\tilde{t}^{-1}]}{i^2 \tilde{\tau}} = \sum_{\tilde{\tau}_0 \sim \infty \sim \zeta^{-1}} \frac{\pi \tilde{\tau}_0^2 \delta[\tilde{t} - \tilde{\tau}_0]}{i^2 \tilde{\tau}}$$

(such that $\tilde{t} = t + \frac{\tau}{2}$ and $\tilde{\tau} = 2i\tau$); then $\lim_{t \rightarrow \zeta^{-1} \sim \infty} I_A[t] \approx \frac{i\pi}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \oint_{\tilde{\tau}_0 \sim \infty \sim \zeta^{-1}} d[\ln \tau] \frac{\tilde{\tau}_0^2 \delta[\tilde{t} - \tilde{\tau}_0]}{i^2} =$

$$\frac{i\pi}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \left(\frac{\tilde{\tau}_0^2 \delta[\tilde{t} - \tilde{\tau}_0]}{i^2} \ln \tau \Big|_{\tilde{\tau}_0 \sim \infty \sim \zeta^{-1}} - \oint_{\tilde{\tau}_0 \sim \infty \sim \zeta^{-1}} d\zeta \frac{\ln \zeta}{\zeta^2} \partial_\tau \left[\frac{\tilde{\tau}_0^2 \delta[\tilde{t} - \tilde{\tau}_0]}{i^2} \right] \right) \sim \frac{i\pi}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \left[\frac{\tilde{\tau}_0^2 \delta[\tilde{t} - \tilde{\tau}_0]}{i^2} \ln \zeta \right] \Big|_{\tilde{\tau}_0 \sim \infty \sim \zeta^{-1}} \approx$$

$$\frac{-i\pi \ln \zeta C_* \delta[t - \zeta^{-1}]}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} = \frac{-i\pi \zeta^{-2} \delta[t - \zeta^{-1}]}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}}, \text{ where } C_* \text{ is a dimensional factor running from the (delta functional)}$$

evaluation over the one dimensional subspace $\tilde{\tau}_0 \sim \infty$ (the boundary scale of the critical neighborhoods, or the change in circumference per unit neighborhood): $C \sim \frac{\partial_\zeta l[\zeta]}{\int d\zeta l[\zeta]} = \frac{\zeta^{-2}}{\ln \zeta}$, where $l[\zeta] = 2\pi\zeta^{-1}$.

⁵⁷Keeping in line with the previous spirit, let the late time parameter ζ serve as a perturbation index of an extended measure space $\mathbb{R}^* / \{\omega_0, \zeta; \omega_\pm[\omega_0, \zeta]\}$; then, expanding produces

$$\frac{\zeta^{-2}}{\sqrt{1-a}} \sim \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} \zeta^{-1} \left(\frac{b\zeta^2 + c\zeta^{4+\delta}}{1-a} \right)^n \quad (83)$$

$$\Rightarrow \frac{1}{2b_*} \left(\frac{\zeta^{-2}}{\omega_\pm}; b, c, c_*[x] \right) = \left(\frac{2b_*\zeta^{-2}}{\sqrt{\zeta^2 - 4b_*}}; \zeta^{-2}, \zeta^{-5}, x^0 \right) \quad \delta = 1 \quad c_*[x] = -2b_*[x] \quad (84)$$

Note that letting $b_* = -\frac{3}{4}\zeta^2$ leads to $(\omega_\pm; b, c) = (\frac{1}{6i}\zeta; \frac{3}{4}, \frac{3}{4}\zeta^{-3})$, and $c_*[x] = -\frac{3\sqrt{x}}{2} = -\partial x^{\frac{3}{2}}$

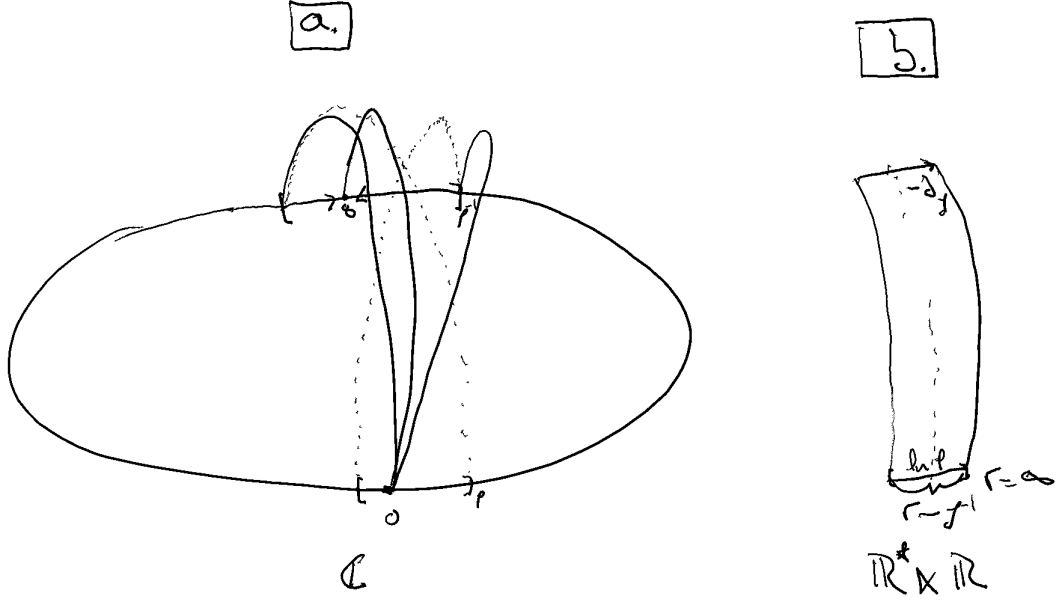


Figure 4: Asymptotic projective family densities, $\frac{\partial \zeta^{-1}[\zeta]}{\partial \zeta} = \frac{\zeta^{-2}}{\ln \zeta}$ embedded in a. \mathbb{C}_* and b. $\mathbb{R}^* \times \mathbb{R}$

$$\pm \sqrt{1 - \frac{\omega_0^2}{\zeta^2}}$$

$$\delta \tilde{V}[\omega] \Big|_{\sim \zeta^{-1} \infty} = \frac{2V_0}{\sqrt{1 - \frac{\omega_0^2}{\zeta^2}}} \zeta^{-2} \delta[\tilde{\omega} - \tilde{\omega}_{\pm}] \quad (86)$$

From the bulk perspective this may be interpreted as a formally divergent, uniformly measured source voltage at a specific asymptotic frequency. Note that the LHS is measure projected, indicating that this only properly shelled in the asymptotic geometry; deductively, this response-potential represents an $d = 0$ gauge (an index) of the asymptotic phase-topology over earliest (ω_-) and latest time (ω_+) sources.

Critically, the perturbative (voltage) amplitude of either pole is suppressed by the distance between the frequency poles.⁵⁸, which allows this representation to characterize the criti-

⁵⁸showing, for example, that $R \rightarrow 0$ implies a narrow response band $\frac{1}{V_0} \delta \tilde{V}[*] \sim iLC \delta[\tilde{\omega} \pm i]$ (with $\tilde{\omega} := \sqrt{LC} \tilde{\omega}$). Meanwhile $C \rightarrow 0$ implies $\delta \tilde{V} \rightarrow 0$ unless $\omega \rightarrow \infty$ (a.k.a., pure inductors uniformly annihilate time-ordered voltage deformations, effectively projecting into DC current phases). This perturbation limit may be explored by letting $\tilde{\omega} \sim \tilde{\omega}_C \sqrt{\frac{L}{C}}$, $C \sim \frac{\tilde{I}[\omega]}{\omega R \tilde{V}[\omega]}$, $\tilde{P}[\omega] := \frac{\tilde{V}^2[\omega]}{R}$ and $\tilde{P}_0[\omega] := \tilde{I}[\omega] V_0$ from where it can be shown that relative voltage perturbations run as $\omega R \frac{\delta \tilde{P}[\omega]}{\tilde{P}_0[\omega]} \sim \frac{-iL}{2} \delta[R\tilde{\omega}_C \pm 2i]$.

From here there are two possible ways to interpret the action of the system, which will be termed “in-to-out” or “out-to-in”, depending on whether the canonical 0–Voltage (measurement) surface is supported inside or outside the local circuit. Considering first the “in-to-out” case, let $\delta \tilde{P}$ represent the energy (interactively) stored by in some local fields and \tilde{P}_0 represent the energy dissipated/absorbed by the system; then it is immediate that:

$$Q_{i/o}[\omega] \sim \frac{iL}{R} \delta[R\tilde{\omega}_C \pm 2i] \quad (87)$$

cally damped case, $\frac{4L}{R^2C} = 1$, as a frequency agnostic step-functional (shown by integrating both sides by $\oint_{C_\alpha} d\alpha \int d\omega e^{i\alpha\omega}$ s.t. $\alpha = \sqrt{1 - \frac{4L}{R^2C}} \sim 0$)
It's possible to arrive at a similar equation of motion using a variable resistor, capacitor, and a current source; then $I - I_* = C\dot{V}$ and defining $\tilde{V} := V - V_*$ it can be shown that

$$\ddot{V} - \partial_t [\ln R] \left(\partial_t - \partial_t \left[\ln \dot{R} \right] \right) V = \frac{\dot{V}}{\dot{R}} \partial_t [\ln R] \quad (91)$$

Note that the zeros of R act as poles (sources) of this equation, representing the emergent LC circuit (purely oscillating modes).

The common envelope on the lower derivative pieces guarantees a uniform kernel OPE iff the source voltage is continuous⁵⁹. A number of immediate interesting limits may be produced⁶⁰, but an operator comparison may be made with the LRC circuit, producing the following classifiers of the instability point (by letting $L\partial_t = \partial_\tau$):

$$\partial_\tau R \sim R^2 \quad \wedge \quad \partial_\tau \left[\frac{1}{L} \partial_\tau R \right] = \frac{R}{C} \quad \Rightarrow_{\partial_\tau L=0} \quad R = \sqrt{\frac{2L}{C}} \quad (92)$$

For the "out-to-in" case let $\delta\tilde{P}$ represent the energy dissipated/absorbed by the system and (remembering V_0 is an arbitrary amplitude, this final $U(1)$ d.o.f. may be used to) define \tilde{P}_0 as the energy stored (in some local field); then, the LHS can be rewritten using the circuit's Q-factor, $Q = \omega \frac{\tilde{P}_0}{\delta\tilde{P}}$, the Q-factor of the bare inductor, $Q_L[\omega] := \frac{\omega L}{R}$, as well as the non-perturbative impedance, $|Z_{\max}| = Q_L^2 R$ giving:

$$\omega_{o/i}^3 \sim \frac{-iQ_L[\omega]Q[\omega]}{2} \delta[R\hat{\omega}_C \pm 2i] \quad \Rightarrow \quad Q[\omega] \Big|_{\omega_C = \mp \frac{2i}{R}} = \int d\hat{\omega}_C \frac{\omega_{o/i}^3}{\frac{|Z_{\max}|}{RQ_L}} \quad (88)$$

Finally, if the local Q-factor is assumed to be a thermodynamic partition of the global impedance(-functional), then it is reasonable to deduct the non-perturbative extension into the full frequency compactification under some Plankian-type measurement (gauge): $\int d\hat{\omega}_C \rightarrow \sigma \int d\omega_{o/i}[*]$ and $\frac{|Z_{\max}|}{RQ_L} \sim (e^{-\beta\omega_{o/i}} - 1)$, which then immediately grants an analogy between the non-perturbative Q-factor and the Stephan-Boltzmann Law:

$$Q[\omega] \Big|_{\omega_C = \mp \frac{2i}{R}} = \frac{\pi^4}{15} \sigma \beta^{-4} \sim_{i/o} \tilde{\sigma} \left(\frac{R}{L} \right)^4 \quad (89)$$

where the final identification was made by comparing the second "Plankian gauge" condition to (87) (at first order), a.k.a. $\beta \sim i \frac{L}{R}$. Lastly, this lets the Q -operator (87) be rewritten in an interactive picture as some boundary mediated interaction in the response (open) topology:

$$\mathcal{O}_Q \sim e^{iQ_{i/o}\beta} = e^{-\delta[R\hat{\omega}_C \pm 2i]} \sim e^{\int \Theta'[*]} \quad (90)$$

This is immediately recognizable as the photoelectric effect.

⁵⁹actually, a represented compact group, by the Peter-Weyl theorem. Note the written form: $\ddot{V} - \partial_t [\ln R] \left[\partial_t - \frac{\dot{R} - \dot{R}\dot{V}V^{-1}}{\dot{R}} \right] V = 0$

⁶⁰Firstly, $\dot{R} \rightarrow 0$ implies $\omega^2 \tilde{V}[\omega] = \mathcal{F} \left[\frac{\dot{V}}{\dot{R}} \right]$. More generally, $R \rightarrow 0$ points represent source poles; most generally, non-continuous resistance profiles, $\dot{R}, \ddot{R} \rightarrow \infty$, also present as phase-sources. Secondly, $\ddot{R} = -R$ gives $\ddot{V} - \partial_t \ln R \left[\partial_t - 1 - \frac{\dot{V}}{CV\dot{R}} \right] V = 0$, which shows the (outer) inhomogeneous current-flow as a fixed source over the (inner) voltaic conductor. Note that, generally, the poles will remain regular if $\dot{R} \rightarrow K$, where $\dot{R} = \int \ddot{R} \sim \in \frac{k[t]}{t-t_*} \sim 2\pi k'[*]$, which gives $k[t] \sim Kt + b$ s.t. $(K, b) \neq (1, -t_0)$; from here it can be deduced that $-\frac{b}{K} = t_0 - \frac{t_{20}t_{10}}{t_{12}}$ presents as the L -integrator's finite time domain. This should be read as a Diophantine-like construction of parasitic charge states.

Still, the (phase) stationary identification is not the only one. Notice that the leftmost identification is well solved by any τ -constant infinitesimal, ϵ s.t. $[\epsilon^2] \sim [0]$. Then, considering $[\tau - \tau_*]$ to be some canonical 0-form anti-derivative operation, the equations can be closed with $R \sim \tau - \tau_*$ and $L \sim (\tau - \tau_*)^{-1} \sim C$.⁶¹

This in turn lets the inhomogeneous component keep the degree of the operator the same if $\dot{V} \sim \frac{\tilde{C}}{(\tau - \tau_*)}$; OR

$$\dot{V} \sim \frac{\tilde{C}}{\tau - \tau_*} \sim \tilde{C}L \Rightarrow \partial_\tau \tilde{V} \sim \tilde{C}L^2 \Rightarrow \tilde{V}[\tau] \sim \int_\tau \tilde{C}L^2 \quad (93)$$

This is to say that the frequency states of the inductor (a.k.a., perturbation integrator) are dual to the inhomogeneous voltages; this is commonly known as a “phasor effect”. Note,

⁶¹this may seem strange, but indeed considering $R \sim e^\Delta$ s.t. $\partial\Delta \sim \tau - \tau_*$ gives the result. Reflecting, this is simply the skin-displacement dual formulation of the equations of motion, with the anti-derivative bounded in some (1-graded) local neighborhood representing the skin of the inductor’s restorative current.

letting q_e represent units of charge-to-mass, presents the units of the integrand as⁶²:

$$\left[\tilde{C}L^2 \right] \cdot [s] = [\text{kg } q_e^{-1}]^3 [\text{m}^4 \text{ s kg}^{-1}] = [s] \cdot \left[\frac{\rho_m q_e^3}{c} \right]^{-1} \quad (102)$$

⁶²Interestingly, this may be used to bootstrap into an analogy with the Friedmann-Robertson-Walker (FRW) type manifolds, which have a supporting matter equation of state determined by $p = w\rho$ $\rho \sim a^{-3(w+1)}$ (then, the bootstrap follows the $(w, a) \sim (0, q_e)$ phase curve). Examining the FRW Cosmological model:

$$ds^2 = dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \quad (94)$$

$$\Rightarrow \quad H^2[a; t] = \frac{\rho_{tot}}{3M_p^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad \ddot{a}[t] = a[t] \left(H^2 + \frac{k}{a^2} - \frac{4\rho_{tot} + 3p_{tot}}{6M_p^2} \right) \quad (95)$$

It is naively interesting to compare these equations of motion to (92), which produce the following model overlap constraints (defining $\rho_{vac} = M_p^2 \Lambda$):

$$\mathcal{Z}_k[a; t] := \sqrt{k} \cosh \frac{1}{2} \ln \frac{k}{a^4} \sim_H \frac{1}{6M_p^2} (\rho_{tot} + \rho_{vac}) \quad C \sim_K \frac{3M_p^2}{\rho_{vac}} \frac{1}{1 - \frac{\rho_{tot} + 3p_{tot}}{2\rho_{vac}}} \quad (96)$$

In fact, Fourier transforming $a[\cdot]$ against k , $\mathcal{F}[a[\cdot], k; *] \sim \int dk e^{ika[\cdot]}$ (being careful to define the twist conditions at $k=0$) the left equation can be shown equal to a radiation density like profile ($w \sim \frac{1}{3}$) under some (possibly trivial) group emdoding of the spectral action ($\mathcal{F}_{ak} \sim_H \mathcal{H}$):

$$\mathcal{F}_{ak} [\mathcal{Z}_k[a]] = -\rho_{rad} \Rightarrow 6M_p^2 \rho_{rad} + \mathcal{H} [\rho_{tot} + \rho_{vac}] \sim 0 \quad (97)$$

Then, comparing the next set of equations leads to the inference that $\partial_\tau \left[\frac{1}{L} \partial_\tau a \right] \sim \frac{a}{C} \sim_K \frac{\rho_{vac} - \frac{\rho_{tot} + 3p_{tot}}{2}}{3M_p^2} a[t] =_{w=\frac{1}{3}} \frac{\rho_{vac} - \rho_{tot}}{3M_p^2}$.

Or, generally using the Friedmann energy conservation equation, $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$, and rescaling $\tau \rightarrow \tilde{t} = M_p^{-1} \tau$:

$$3\partial_{\tilde{t}} \left[\frac{1}{L} \partial_{\tilde{t}} a[t] \right] \sim \left(\rho_{vac} - \rho_{tot} - \frac{\dot{\rho}_{tot} H^{-1}}{2} \right) a[t] \equiv \int dt \left[\left(\dot{\rho}_{vac} + \frac{3\dot{\rho}_{tot}}{2} \right) a + (\rho_{vac} + \rho_{tot}) \dot{a} \right] \quad (98)$$

$$\Rightarrow \quad 3LM_p^2 \ddot{a}[t] = (\rho_{tot} + \rho_{vac}) a[t] + \int dt a[t] \left(\frac{3w}{2} - 1 \right) \dot{\rho}_{tot} \quad (99)$$

Note that $w = \frac{2}{3}$ eliminates the smeared (mnemonic) charge uniformly, which corresponds to $\rho \sim a^{-5}$, which could deductively be thought of as a complex $k=1$ embedding invariant of $d=4$ (or, dually, a $D=5$ higher dimensional spacelike curvature invariant). In the case of the former, the RHS of (99) remains smeared (and the effective inductance is heuristically sensitive); in the case of the latter the solution basis of (99) is no longer distributed "at the cost" of finding the partition in (97) to actually be a (relatively prime) sub-modular partition, $\mathcal{F}_{ak} [\mathcal{Z}_k[a]] \sim_{w=\frac{2}{3}} \rho_{tot}^{\frac{4}{5}}$. Indeed, this construction will prove deeply consistent with the central results of this paper.

A more cavalier suggestion is to pick $w = \frac{1}{5}$, which produces:

$$\frac{2 \cdot 7 \cdot 3^4}{5^3} \frac{1}{a[t]} \int a - (\partial \ln \rho)^2 \left[\frac{18}{5} \frac{\partial \ln \dot{\rho}}{\partial \ln \rho} - 1 \right] = \left(3 + \frac{3}{5} \right)^2 (\rho_{tot} + \rho_{vac}) \quad (100)$$

Considering the partial derivative functional as over the tangent complete closure embedding, $\partial \in \mathcal{O}[*] \sim \mathcal{M} \oplus_I \mathcal{L} [U_I] \mathcal{U}$, a k -rescaling the tangent space independent of the coordinate base scaling n - amounts to a left-right (0)-dual rescaling of the measurement index, or $\{\partial \rightarrow \partial_t \partial_{\xi_I}, \rho \rightarrow \xi_i k^i \rho\}$. In the $w = \frac{1}{5}$ case, the symmetric, circuit dual configuration implies $(n, k) \sim (5, 13)$ and the reduced ansatz:

$$\frac{7!}{2^3} \frac{1}{a[t]} \int a = \left(\frac{5^2}{3} \right)^2 (\partial \ln \rho) (\partial \ln \dot{\rho}) + (5 \cdot 6)^2 (\rho_{tot} + \rho_{vac}) \quad (101)$$

This equation of state is very holographic: the measured self-similarity of the background acceleration (as a superconducting duality), weighed as $l = k - n$ eight independent degrees of freedom spherically constrained

Using geometrized units $[s] \sim_g [kg]$ presents as an equivalence-connection; then, integrating over some small proper time t interval, the t -measured integrand's units appear as a volume density of the \sim_g connection over the unit charge/mass ratio, mediated by the flat \sim_g connection over a Euclidean bundle.⁶³

Summarily, while the capacitor represents a first order integrator (a line-charge integrator), the inductor presents as an integrator (anti-differentiator) of the resistance form factor and, dually, the inhomogeneous current stabilizer forms accumulate as (outer) capacitance shells of the (positive definite) L-measure (viewed as a local, $d = 0$ dimensional topology governed by recent memory τ_*).

Importantly, the virial theorem may be used to qualify some large-N limit of subunits with globally thermalized boundary conditions; in particular, if the phases are almost everywhere quasi-stable (periodic) than the large-N limit will support quasi-stable (periodic) degrees of freedom almost everywhere⁶⁴. Critically, the converse is also (always) true (up to some equivalence sets): it is always possible to bootstrap using the (any isolated subsets of) quasi-stable (periodic) thermodynamic parameter(s)⁶⁵. Note that this says nothing about quasi-periodic circuit paths(/geodesics), which may lose stability (decay/feedback) in their Virial compactification.

$N[l]l^2 \sim ll$, is dual to the sum of a dimensional (density) contact fluctuation $n^2\sigma^2 \sim n^2 \left(\frac{n}{n-2}\right)^2$ and an (n -)index double trace weight $n^2(n+1)^2 = 4 \left(\sum_{k=1}^n k\right)^2$.

⁶³further, note that the units of $\left[\frac{\tilde{V}}{CL^2}\right] \sim [\text{kgm}^2\text{s}^{-2}\text{A}^{-2}] = \left[\frac{\sigma_q}{q_m/Q}\right]$ are in units of surface-charge density per mass-to-charge ratio, leading to the inference that the anti-differentiation is over a small, skin like topology (in the phase configuration domain) of fundamental interactions. This conceptually rounds back to the physical configuration of the inductor (as a volume boundary winding gauge). Note the analogy is well directed to (102) as a volume (or 3-)density (over a flat 4-measure mediated by the mass to proper time exchange).

⁶⁴For example, a static graph of non-interacting (magnetically shielded) LC circuits will exhibit only the fundamental mode; alternatively, a magnetically interacting graph (of LC circuits) also supports modes based on the (static) unit distribution.

Finally, a graph of LC-LCR subunits must also support a (boundary mode) degree of freedom descendant from the LC-LCR regulatory interaction itself.

⁶⁵This follows directly from Noether's Second Theorem, with the exact divergence form fixed by the nature of the quasi stability (or, the resulting fast-adiabatic response of the internal DoF); therein, this can also be seen as a manifestation of the Second Law of Thermodynamics. Lastly, the global divergence mode of the virial extension can be seen directly as a propogator between the sub-unit regulator to boundary domain modes by analytically extending the 2nd LoT (which represents time/spacial phase-distributions throughout the graph, a.k.a. global Fourier distributions, or top-down topology measure-distributions/uniformities. This is also a clean point of contact with the Second Isomorphism Theorem.

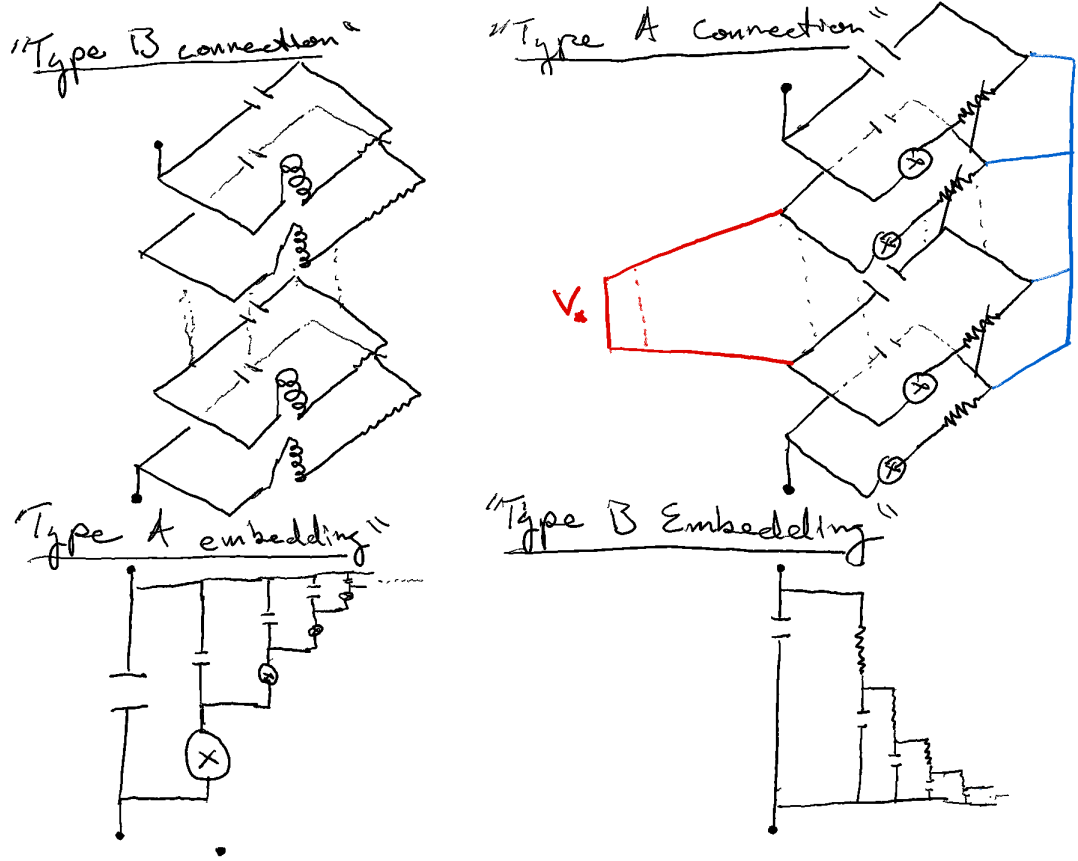


Figure 5: Some Examples of n -Uniform LRC Junctions. Note that the embedding schemes are exact iff $\{L^{(n)}\} \rightarrow \{0\} \leftarrow \{R^{(n)}\}$, which is a strong topological measure in the sense that $Q_L^{(n)} \rightarrow Q_R^{(n)}$. A slightly deeper understanding of this can be found by lifting the isolated, LRC sub-circuits into a neighborhood of interactions (depicted here as a $d = 1$ site ordering) mediated by either the L or R component. Because there are no magnetic monopoles, the discharge frequency of each (un-shielded) circuit leaks into the effective inductance of some neighboring circuit; contrarily, in the case of a type A connection Kirchoff's law of circuit completion requires that there exists a "bare" (arbitrary) inner DC voltage gap, V_* , to mediate the decaying perturbations induced by the variable resistors. Immediately, it can be concluded that effective monopoles can be understood as interactive modes of the internally grounded states of a type A connection.

2.2 Critical Stability Features in Euler-Lagrange Systems

In fact, this method of symmetry descent is a general feature of quasi-stable systems. This section (and paper) will explore this idea in two ways fundamental ways.

Starting from a field formulation constrained under some equations of motion \mathcal{E}_α , and degrees of freedom $\delta_{\{i\}}x^{(i)\alpha}$, under an action principle minimization $\delta_I S = 0 = \mathcal{E}_\alpha \delta_i x^{(i)\alpha} + \text{Div}_i[x^{(i)\alpha}, \delta_i x^{(i)\alpha}]$ and the identity $f^{,a} \nabla_a \delta(f(\vec{x})) = \sum_{f(\vec{x}_i)=0} \delta(\vec{x} - \vec{x}_i)$:

$$\mathcal{E}_\alpha \delta_i x^{(i)\alpha} = -\text{Div}_i[x^{(i)\alpha}, \delta_i x^{(i)\alpha}] \Rightarrow \int \delta[\cdot; \mathcal{E}_\alpha^{(j)}] \mathcal{E}_\alpha^{(j)} \delta_{ij} x^{(i)\alpha} = - \int \delta[\cdot; \mathcal{E}_\alpha^{(j)}] \text{Div}_i[\mathcal{E}_\alpha^{(j)}; x^{(i)\alpha}, \delta_{ij} x^{(i)\alpha}] \quad (103)$$

Notably, consider an analytic curve: $g_k[*] \text{Div}_i[f_j; \cdot] + \text{Div}_i[g_k; *] f_j[\cdot] = d_i^j[g[*] f_j[\cdot]]$ ⁶⁶. This happens precisely when there exists an exact representation (over the coordinate (analytic) space⁶⁷) of the adjoint extension, which can then be reduced to the above for some differential operator (of the C^{n-2} class). This then allows the development of a dual pullback (because the cohomology admits a trivial representation): $d_j^{*(i)} \in C^{n-3}$.

$$\oint \left(\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] \tilde{\mathcal{E}}_\alpha^{(j)} \delta_{ij} z^{(i)\alpha} + d_i^j \left[\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] \tilde{\mathcal{E}}^j \right] \right) = \oint \tilde{\mathcal{E}}_\alpha^j [*] \text{Div}_i[\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}]; z^{(i)\alpha}, \delta_{ij} z^{(i)\alpha}] \quad (104)$$

Then, taking the Fourier involution⁶⁸ and dropping the EOM-centered term:

$$\int e^{i\tilde{\omega}_{(j)}[*] \tilde{\mathcal{E}}_\alpha^{(j)}} \oint d_i^j \left[\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] \tilde{\mathcal{E}}^j \right] = \int e^{i\tilde{\omega}_{(j)}[*] \tilde{\mathcal{E}}_\alpha^{(j)}} \oint \tilde{\mathcal{E}}_\alpha^j [*] \text{Div}_i[\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}]; z^{(i)\alpha}, \delta_{ij} z^{(i)\alpha}] \quad (105)$$

$$\Rightarrow i \oint \left(\tilde{\omega}_{(i)}^j [*] \delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] + i \int e^{i\tilde{\omega}_{(j)}[*] \tilde{\mathcal{E}}_\alpha^{(j)}} \tilde{\mathcal{E}}_\alpha^j [*] \text{Div}_i[\delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}]; z^{(i)\alpha}, \delta_{ij} z^{(i)\alpha}] \right) \quad (106)$$

$$= - \oint \int e^{i\tilde{\omega}_{(j)}[*] \tilde{\mathcal{E}}_\alpha^{(j)}} \sum_{\{*\}} \frac{\tilde{\mathcal{E}}_\alpha^{(j)}}{d_i^* \tilde{\mathcal{E}}_\alpha^{(j)}} \delta_* \quad (107)$$

where d^* was used as a proxy for the constraint envelope derivative.⁶⁹

Two things are immediate: firstly, when the RHS is zero the LHS represents exactly the classical Ward identities [51]. Notably, this will hold unless $d_i^* \tilde{\mathcal{E}}_\alpha^{(j)} = 0$; in turn, this puts strong restrictions on the constraint tangent space, namely $d_i^*[\cdot] d_i^j[*; \cdot] = 0$; or that $d_i^*[\cdot]$ must be functionally orthogonal to the adjoint flow of the canonical Green's operator $d_i^j \delta[*; \cdot]$. Then, the existence of a non-trivial orthogonal operator outside an adjoint-protected Green's pro-

⁶⁶So that contour space represents an algebraic topology (in varietal measure). It is trivially possible to embed the operator with a free-space dual copy; by the central extension theorem there exists a projective representation iff the covering algebras have a non-empty common centralizer[62], $Z(\cup \mathcal{A}_n) = \cap Z(\mathcal{A}_n) \neq \{\emptyset\}$

⁶⁷The base measure always has a well defined extension through the canonical-pullback of the free-projective push

⁶⁸which has a guaranteed (sub)convergence on any measurable subset

⁶⁹So, $d^* \tilde{\mathcal{E}}_\alpha^{(j)} d_i^j \delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] = \sum_{\{*\} | \tilde{\mathcal{E}}_\alpha^{(j)}[*]=0} \delta_*$ where the shifted base-space delta function was written as $\delta_*[x] \equiv \delta[x - *]$.

jection requires that either: the field domain walls⁷⁰ have non-adjoint boundary conditions, or self-adjoint field domains are (ir)regularly punctured [63]. Choosing to work with real-adjoint fields (as opposed to higher spin extensions [64]) leaves puncture-fields to source the RHS.⁷¹

In either case, the constraint condition can be given as $d^* \tilde{\mathcal{E}}_\alpha^{(j)} d_i^j \delta[*; \tilde{\mathcal{E}}_\alpha^{(j)}] = \sum_{\{*\} | \tilde{\mathcal{E}}_\alpha^{(j)}[*]=0\}^k c_*^{(k)} \delta_* = 0$ (which follows essentially from the fundamental theorem of algebra). This can only hold if each delta function is unsupported or in a canonical product state $\sum^k c_*^{(k)} = 0 \Rightarrow \sum^{k_T} c_*^{(k_T)} = -\sum^{k_B} c_*^{(k_B)} (+S_0) := \sum^{k_T} \tilde{c}^{(k_T)} (+S_0)$; then, further dividing into *in* (Kernel; out of domain) and *out* (of Kernel; in domain) gives: $\sum_{\{out\}}^{k_T} (c_*^{(k_T)} - \tilde{c}^{(k_T)}) \delta_* = -\sum_{\{in\}}^{k_T} (c_*^{(k_T)} - \tilde{c}^{(k_T)}) \delta_* (+\sum_{in,out} S_0^{in/out})$.

Importantly, the *in/out* designation represents a patch embedding relative to the contour; remembering it can always adjointly descend from the free-embedding contour, gives freedom in choosing the loop adjugant at the cost of algebraic complexity (if the embedding is neither the canonical product nor a k_T -local symmetrization). Ultimately, this reflects choosing different k_T vs k_B , c vs \tilde{c} , and S_0 and is pen-ultimately useful when the resulting loop algebra characterizes a $[\cdot]$ -domain (geometric) residue (whereby this relation characterizes ALL $det[*; \cdot]$ *in/out* of the field couplings through the central extension theorem). Notably, the actual splitting c vs \tilde{c} depends on the support fields which run, in-representation, to the local symmetries of the EOM and amount to (field) coordinate (on-)shellings.

Particularly, because S_0 represents a global center (representation) this immediately requires the *in*-geometrized algebra match the *out*-geometrized algebra; when there is a direct OPE mismatch then a global envelope on the \tilde{c} is induced. For example, suppose a dual-tangent *in* space of self-dual fields ($k_T \in \mathbb{Z}_2$, or $\tilde{c} = -c$) with an $SL(2, R)$ geometrization; if additionally there is exactly one geometrized pole $c_{out} \delta_{out} \in SL(2, R)$, the *out* EOMs must be (in-representation) of Heun type and thus covered by the hypergeometrics $\{F_2, G_2\}$. Dually, knowing the Kerr geometry has a conformal AdS_3 (self-dual) peeling symmetry cover [12] directly implies the existence of an $SL(2, R)$ *in*-representation.⁷²

In either case, denoting the LHS of 107 as W (the Ward term), and considering the saddle-point approximation leads to:

⁷⁰a.k.a., solutions are complete over the domain functional domain

⁷¹This can also be seen by examining the RHS of 107: if the EOM b.c. are not adjoint-propogated then their tangent space does not have a dual pullback, meaning the RHS involves higher order (derivative) moments at the same solution point (or, which pull out various moments through $[\cdot]' \delta'[\cdot] = \delta$). Notice this moment develops in the tangent pullback, so it must be associated with a canonical winding number.

⁷²Delving slightly further, when the extension has field-domain support (representing a field-continuous winding) the coefficients may be canonically dualized, resulting in a smooth constraint form $S_0 \rightarrow S_*[\cdot]$ representing an irregular singularity, which may then be further resolved at the level of the contour (loop) into a boundary (sub-charged) field.

$$W = - \oint \int e^{i\tilde{\omega}_{(j)[*]}\tilde{\mathcal{E}}_\alpha^{(j)}} \sum_{\{*\}} \frac{\partial^2 \tilde{\mathcal{E}}_\alpha^{(j)}}{\partial^2 d_i^* \tilde{\mathcal{E}}_\alpha^{(j)}} \delta_* \quad (108)$$

which expresses the globally embedded source terms as a series of correlator matchings $\langle \partial^2 d_i^* \mathcal{O} W \rangle = \langle \partial^2 \mathcal{O} \rangle$ which makeup the soft/sub-leading theorems (formulated as a shadow-extended algebraic cover of the renormalization group). The thermodynamic limit can be accessed as the specific-measure of the index the circle winding number (as the β -parameter⁷³), which directly compares the sheet-variety with the specific-affine (parameter per spacetime),

Accordingly, a second way to see emergent symmetries is succinct: if the system is adiabatically (volume) stable⁷⁴, then the low temperature, virial limit of the 2nd LoT reads:

$$\langle W \rangle = - \int k_\beta^{(i)} \delta Q^{(i)} = - \oint_T \int \delta Q \partial_T \ln T = - \oint_T S \quad (109)$$

$$= - \oint \int \frac{1}{T} \left(\frac{\partial C_V}{\partial V} \right)_T dV = - \oint \int \left(\frac{\partial^2 p}{\partial T^2} \right)_V dV \quad (110)$$

So. representing $V = e^{\Delta[\cdot]}$ gives $W = - \oint \int e^{\Delta} \left(\frac{\partial^2 p}{\partial T^2} \right)_{e^\Delta} d.\Delta$; let Δ , a scalar, have a dual functional representation as $d.\Delta = \mu_j d.N^{(j)}[\cdot] \Rightarrow \Delta = \int \mu_{(j)} d.N^{(j)}[\cdot]$. Naming the integral (of the conformal-volume constraint form) as a dual measure⁷⁵ gives: $\Delta = \kappa_{(j)[*]} p^{(j)}$ and:

$$W = - \oint \int e^{\kappa_{(j)[*]} p^{(j)}} \left(\frac{\partial^2 p}{\partial T^2} \right)_{e^{\kappa_{(j)[*]} p^{(j)}}} \mu_{(j)}[\cdot] d.N^{(j)} \quad (111)$$

Noting the 1st LoT (on each Volume slice) may be represented as $\frac{dW}{dT} = \sum_i \left(\frac{\partial S}{\partial \mu_i} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}} \frac{d\mu_i}{dT}$ ⁷⁶ identifies:

⁷³The association with the Fourier involution parameter $i\omega$ and the thermodynamic parameter β is made exact under the Wick rotation, which is a canonical ($U(1)$) convolution of the Fourier and Laplace transform: $\mathcal{W}_{\theta_k}[[*](z); [\cdot]] = [*](\theta_k(z))[\cdot]$. When $[\cdot](z) \in \{\mathcal{L}\}$, $\theta_k = e^{\frac{i\pi}{2}}$. The $U(1)$ symmetry descends in the product (topology) as $W_{\theta_k} \circ W_{\theta_l} = W_{\theta_k \circ \theta_l}$. This relationship is exactly captured by the Mellin transform: $\mathcal{M}_s[f] = [e^{-[\cdot]}]^T \mathcal{L}_s[f] = [e^{-[\cdot]}]^T \mathcal{F}_{-is}[f]$

⁷⁴Or, the system has a 1st LoT formulation as $dW = \left(\frac{\partial S}{\partial T} \right)_V dT + \left(\frac{\partial S}{\partial V} \right)_T dV$; notice this doesn't preclude further interactions, so long as they may be expressed as: $dV = \frac{\partial V}{\partial \tilde{V}} d\tilde{V} + \sum_i \left(\frac{\partial S}{\partial \mu_i} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}} d\mu_i$. Notice (sub-)dynamic fields must be held at fixed V (from the constraint $\left(\frac{\partial S}{\partial T} \right)_V$) and \tilde{V} ; this is exactly then requiring the "emergent" degrees of freedom have (a) conformal symmetry (in the volume parameter).

⁷⁵So, *int* is considered as a measure of the function on $\mu_{(j)}$ on the measure-basis $\in \{d.N^{(j)}[\cdot]\}$. Expanding over any covering set is allowed, namely: it is possible to expand the domain of the measure-dual basis (of the volume-constraint flow $d.\Delta$) in terms of the classical, conformal trajectories (although the dual density's number-fields may need to be properly extended $[\cdot] \rightarrow [*]$ to render the basis $\mathcal{E}^{(j)}$ linearly independent). Here, consider the analytic cover basis to be implicitly maximally extended basis (in the weakest topology) or, heuristically, in the Zarinski basis of the shell-group.

⁷⁶which then runs to an adiabatic constraint form: $\frac{d\mu_k}{dT} = - \frac{\partial \mu_k}{\partial S} \sum_{i \neq k} \left(\frac{\partial S}{\partial \mu_i} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}} \frac{d\mu_i}{dT}$

$$- \int e^{\kappa_{(j)[*]p^{(j)}}} \left(\frac{\partial^2 p}{\partial T^2} \right)_{e^{\kappa_{(j)[*]p^{(j)}}}} \mu_{(j)}[\cdot] d.N^{(j)} = \sum_i \frac{d\mu}{dT} \left(\frac{\partial S}{\partial \mu_i} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}} \quad (112)$$

$$\text{and} \quad (113)$$

$$\oint \delta_{\kappa_{(j)[*]} e^{-\kappa_{(j)[*]p^{(j)}}} \int e^{\kappa_{(j)[*]p^{(j)}}} \left(\frac{\partial^2 p}{\partial T^2} \right)_{e^{\kappa_{(j)[*]p^{(j)}}}} \mu_{(j)}[\cdot] d.N^{(j)} \quad (114)$$

$$\begin{aligned} &= \int \left(\frac{\partial^2 p}{\partial T^2} \right)_{e^{p^{(j)} \neq \infty}} \mu_{(j)}[\cdot] d.N^{(j)} \Big|_{\kappa_{(j)}[\cdot]=0} = \oint \delta_{\kappa_{(j)[*]} e^{-\kappa_{(j)[*]p^{(j)}}} \sum_i \frac{d\mu_i}{dT} \left(\frac{\partial S}{\partial \mu_i} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}} \\ &= \oint \sum_i \delta_{\kappa_{(j)[*]} \frac{d\mu_i}{dT} \left(e^{\kappa_{(j)[*]p^{(j)}}} \frac{\partial \mu_i}{\partial S} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}}^{-1} \quad (115) \end{aligned}$$

It is important to note that this OPE puts δ in a distribution basis about a (real) coordinate constraint basis $d.N^{(j)}[\cdot]$. Note the (internal) entropy is volume scale independent this implies $0 = \int \partial_S \mu_{(j)} d.N^{(j)}[\cdot]$ (which characterizes the internal constraints $\mu_{(S)}$ as extremal along every spacetime-distributed coordinate basis).⁷⁷ In the T-dual measure-space $[*]$ this can be represented as :

$$= \oint \sum_i \delta_{\kappa_{(j)[*]} (p^{(i)} d_* \kappa_{(j)}[*]) \frac{d\mu_i}{dT} \left(d_* e^{\kappa_{(j)[*]p^{(j)}}} \frac{\partial \mu_i}{\partial S} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}}^{-1} \quad (116)$$

$$= \oint \delta_* \sum_i p^{(i)} \frac{d\mu_i}{dT} \left(d_* e^{\kappa_{(j)[*]p^{(j)}}} \frac{\partial_* \mu_i}{\partial_* S} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}}^{-1} \quad (117)$$

and backtracking: $\partial_S V = V \partial_S \Delta \Rightarrow V \partial_V S = (\partial_S \Delta)^{-1}$, as well as the dispersion relation $p^i = \frac{\delta_{(j)}^i}{\Delta} \frac{\partial \Delta}{\partial \ln \Delta}$ ⁷⁸ yields:

$$= - \oint \frac{\partial_S \ln \Delta}{V \partial \ln \Delta} \delta_* \sum_i \sum_{k \neq i} \partial \cdot \left[\Delta \left(\frac{\partial \mu_i}{\partial \mu_k} \right)_{\{\mu_{k \neq i}\}, T, V, \tilde{V}} \frac{d\mu_k}{dT} \right] \left(d_* e^{\kappa_{(j)[*]p^{(j)}}} \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}}^{-1} \quad (118)$$

Both and the internal heat dispersion and the external spectral flow must be stable (in the adiabatic limit) so the ratio of the derivatives are pushed to second order (saddled). Finally, remembering the measure weight may be interpreted as a Lie-dual derivative operation $e^{\kappa_{(j)[*]p^{(j)}}} = d_{\tilde{\mu}_i} [\kappa_{(j)[*]p^{(j)}}]$, combining everything, and $[*]$ -contour closing to the \mathcal{R} real

⁷⁷So, pulling back using $\oint \delta_{f[\cdot]} e^{-f[\cdot]G} = d_G \delta_{e^{-f[\cdot]G}}$ identifies $d_G^* f = e^{-f[\cdot]G}$ as an inverse Lie propagator (a.k.a., a Lie integral). This can be seen by noting the LHS contour represents a (smooth) contour interpolation of f (or, it is dualized under the proper functional projector: $\oint f([\cdot] - [\cdot]) \dots \Big|_{[\cdot] \rightarrow [\cdot] - [\cdot]}$), while

the RHS represents the motion of the large (divergent) G-actions (which, because f is assumed smooth, is (up to class) fixed; a.k.a. the divergences are G-indexed). So expanding the RHS, dual projecting the LHS onto f , then applying d_G to both yields: $d_G f[\cdot] = e^{f[\cdot]G} \delta \cdot \Big|_{\frac{1}{(f[\cdot]G)}=0}$, which provides the final result. Indeed, in this case applying another integral on the LHS can be directly collapsed to represent a measure of the asymptotically divergent functional map $f[\cdot]G$.

⁷⁸As well as $\partial \cdot \left[\left(\frac{\partial \mu_i}{\partial \mu_k} \right)_{\{\mu_{k \neq i}\}, T, V, \tilde{V}} \frac{d\mu_k}{dT} \right] = 0$, which follows from the internal parameters holding an external conformal index $V \cdot \frac{\partial_S \ln \Delta}{V \partial \ln \Delta} \rightarrow 1$ for exactly the same reason

domain in the almost (everywhere) imaginative (a.k.a., Wick distributed, or $\parallel \rightarrow ik$) sense:

$$\int d\Omega_*, W = - \oint_T \int e^{ik_{(j)}[\cdot]p^{(j)}} \delta. \sum_i \sum_{k \neq i} \partial^2 \left[\Delta \left(\frac{\partial \mu_i}{\partial \mu_k} \right)_{\{\mu_{k \neq i}\}, T, V, \tilde{V}} \frac{d\mu_k}{dT} \right] \left(\partial^2 d_{\tilde{\mu}_i} [ik_{(j)}[\cdot]p^{(j)}] \right)_{\{\mu_{j \neq i}\}, T, V, \tilde{V}}^{-1} \quad (119)$$

Directly comparing with the particle physics picture imagines the shadowed tree dynamics as a second order (variety) of internal diagrams (ideals with possibly continuous index) over the classical Lie-moment [65]. Critically,

$$i \partial. k_{(j)}[\mu_i[\cdot]] p^{(j)} = \tilde{\mathcal{E}}_{\mu_i} = \frac{d\mu_{(j)}}{dT} \Delta \left(\frac{\partial \mu_i}{\partial \mu_{(j)}} \right)_{\{\mu_{(j) \neq i}\}, T, V, \tilde{V}} \quad (120)$$

gives produces the absolutely visible⁷⁹ spectral decomposition $\in d\Omega_*, W = - \oint_T \int e^{ik_{(j)}[\cdot]p^{(j)}} \delta.$ of the interactive *in*-shells⁸⁰. Letting the middle operator act on an exact momentum family (equivalently, keeping only the kinetic terms in some interactive branch), give a characterization of “fake Symmetrization under Uniformity”⁸¹ as:

$$\langle p | \tilde{\mathcal{E}}_{\mu_i} | \Delta^i \rangle = i \langle p^{(j)} | \delta_{(j)}^i \partial. k[\mu_i[\cdot]] | p^i \rangle \quad (121)$$

$$\Leftrightarrow i \langle \delta^i | k[\mu_i[\cdot]] | p^i \rangle = \int d[\cdot] \partial_{p^{(j)}} \langle p | \frac{d\mu_{(j)}}{dT} \left(\frac{\partial \mu_i}{\partial \mu_{(j)}} \right) \Delta | \Delta^i \rangle \quad (122)$$

$$(123)$$

Then, for example, letting the uniform dimensions tower over some differential operator D as $\Delta^{-n(i)} | \Delta^i \rangle = D | \Delta^i \rangle$ while simultaneously stabilizing the function field basis $k[\mathbb{F}[\cdot]]$ using (canonical) coordinate extensions produces:

$$\langle \delta^i | k[\mu_i[\cdot]] | p^i \rangle = -\frac{1}{2\pi} \oint_{\mathbb{C}[\cdot]} d[*] \partial_{p^{(j)}} \langle p | \frac{d\mu_{(j)}}{dT} \Delta^{1-n(i)} \left(\frac{\partial \mu_i}{\partial \mu_{(j)}} \right) D | \Delta^i \rangle \quad (124)$$

which drives the expectation that almost nowhere supported conformal symmetries reflect *in*-envelopes on the spectrum.⁸²

This construction also endows the internally scaled propagator with the top-induced symmetry form; in particular, if only one internal leg exists (*in* measure) then the remaining legs must have an induced residue symmetry on loop level,⁸³ In fact, assuming $\Delta^{1-n(i)}$

⁷⁹ un-shadowed, or exactly/uniuely represented

⁸⁰ note that, the inverse relation is second order, absolutely visible decompositions could be expected to emerge point- and line-wise (in the affine sense) even out of Fock-gauges. In the both cases, the membrane-of-emergence wave equations acquire an additional, hidden, matching degree of freedom amounting resulting from the disconnected boundary sub-regulators: $\int_{0+} \partial_- |in\rangle$ (usually assumed trivial/canonical) boundary conditions $\int_{\infty-} \partial_+ |out\rangle$. Note the trivial matching constraint is $1 - 1$ with assuming the existence of a real affine field-characterization.

⁸¹ Or, hidden *in*-deconvolution singular fixed point representations; this motivates the cavalier attitude towards operating modular center of mass translations.

⁸² And, since everything was formulated as coordinate measures, provides a quick connection, generically, between the soft theorems, the RT-theorems, and the second LoT.

⁸³ the splining operator $\partial_{p^{(j)}} \left[\langle p | \frac{d\mu_j}{dT} \right]$ accumulates towards an *in*-boundary mode (a projected operator).

is a $(\mu, p^{(j)})$ commutative operator (eigenfunction) along some closed connected cut in the extended domain then this can be thought of as a (left regulated) Mellin transformation of the internal(ly weighted) symmetric propagator. Indeed, assuming only the affinely regulated portion of the operator $p^{(u)}$ -flows in the *out* measure as a conformal OPE (known as the Heisenberg Picture of the conformal phase transition) gives

$$\langle \delta^i | k[\mu_i[\cdot]] | p^i \rangle = -\frac{1}{2\pi} \int d\Delta \Delta^{1-n(i)} \oint_{\mathbb{C}[N_k]} dN_k[\Delta] \langle d\chi | D_i^{(j)} \Delta^i \rangle = \tilde{F}[\chi, \Delta^i] \quad (125)$$

In particular, the Heisenburg relation can be characterized by saying that \tilde{F} is not invertable on the fundamental strip domain; then, the functional inside the contour integration domain must be multivalued.

2.3 Critical Phenomena in Gravitational Systems

A critical feature of classical, emergent (partially uniform Lagrangian-density) symmetries displayed above was the existence (quasi-)exact (family of) measure(s), or the 0-form constraint embedding $\Delta (\leftrightarrow \{\Delta^{(i)}\})$. Reflectively, this raises two immediate questions about complexity, namely:

a) what, if any, are the local and universal aspects of the family $\{\Delta^{(i)}\}$? (126)

b) can this structure be generatively compactified into a less complex topology? (127)

Naively both questions are computationally complex and unclear. Still, reflecting on the tangent constraint of the measurement space $d.\Delta = \mu_j d.N^{(j)}$, the fundamental theorem of calculus ($d^2 = 0$) immediately lets the symmetry space be sub-probed using a gauge dynamic symmetry extension, $\Delta \mapsto \Delta + \sum_{(k)} \oint_{\gamma^{(k)}[*]} d[*] d.\mathcal{G}^{(k)}[*; \cdot]$ ⁸⁴ By the Central Extension theorem it can be shown (see section 4) that the free topological embedding space is always functionally dense, leading to the (very) optimistic idea that studing the physics/representation theory of topological embeddings may be strictly sufficient to (understand) study both *a*) and *b*) from first principles.

Collectively, whatever the extended symmetry representation space, $[*]$, is it is algorithmically immediate to index the family of co-kernel operators relative to the sub-dynamic measure space already represented, $(\tilde{x} \oplus \mathcal{U}, *) \hookrightarrow (\mathcal{M}, \mathbb{T}(\mathcal{M}_{\vec{N}[\cdot]})) \odot \tilde{\mathcal{M}}_*$, where \odot represents some general family of constraint involutions. Generally, Green's theorem may be used to show that co-extensions along either the sub-measure space or it's tangent representation produce (pointwise) weighed shift symmetries in either as:

$$\Delta \rightarrow \Delta + \sum_k \lim_{* \rightarrow \cdot} \mathcal{G}_{\gamma_*}^{(k)}[*; \cdot] \quad d.\Delta \rightarrow \mu_j d.N^{(j)} - \langle \mathcal{G}^{(k)} d^2 N^{(j)} \rangle_{\Gamma^k} + \langle d.(d.N^{(j)} \mathcal{G}^{(k)}) \rangle_{\Gamma^k} \quad (128)$$

⁸⁴intuitively, this represents the functional flow of Stokes theorem as a geometric sub-space (or, an embedding) symmetry of topology, generally.

In turn, both of these symmetries may be considered over their effective representations which, by the convolution construction above and the general freedom of tangent space representation theory, respectively, gives:

$$\frac{V}{V_*} \mapsto^* \prod_k e^{-\lim_{\rightarrow^*} \mathcal{G}^{(k)}[\cdot, \cdot]} \quad d.\Delta^* - \langle d.[d.N^{(j)}\mathcal{G}^{(k)}] \rangle_{\Gamma^k} \leftrightarrow^* d.\Delta - \langle \mathcal{G}^{(k)} d^2 N^{(j)} \rangle_{\Gamma^k} \quad (129)$$

Intuitively, the leftmost relations decipher the relative thermodynamic volume partition as an emergent (functionally measured) OPE envelope; simultaneously, the rightmost relations maybe understood as a cohomology constraint on the $*$ -extension that compactly minimizes the difference between functional shifts of the $(\cdot, *)$ (, or, the measurement vs extended measurement) dispersions.

In fact, defining $\mathcal{T}^{(j)(k)} := d.N^{(j)}G^{(k)}$, this ‘‘rightmost’’ duality can be directly understood in the case of sub-exact symmetries to say that the mean dispersion of \mathcal{T} is an exact extended symmetry iff $\mathcal{G}^{(k)}$ represents a family of conformally exact harmonic weights, up to uniformity. Stronger yet, considering these to be matter type symmetry connections, everything to a Wald-type action gives:

$$\delta\tilde{S}_\Delta = d.\Delta - d.\Delta^* \quad \Rightarrow \quad \delta S_{\text{Wald}} - \delta\tilde{S}_\Delta \hat{=}^* \langle d.\mathcal{T}^{(j)(k)} \rangle_{\Gamma^k} \quad (130)$$

Accordingly, the algorithmic closure of the hidden structures of measurement symmetries, $(\vec{\mathcal{G}}, \vec{N})$, can be fully explored by considering the embedding algebra of the observation space, $\mathcal{G}^{(k)}[\cdot, \cdot] \hookrightarrow \mathcal{G}^{(k)}[N^{(j)}; \cdot]$. This can be considered the definition a non-matter, hidden conformal symmetries generally; further, when the loop-algebra Γ^k is closed over the measurement topology the RHS is analytically exact and, using Wald’s hypothesis and applying Noether’s second theorem, can be identified (up to uniformity) with the classical gravitational action $\text{RHS} \hat{=} \delta S_{\text{Wald}}$. Thus, it may be deduced that classical gravity emerges as a renormalization feature between conformal blocks or, as an n -pt OPE symmetry of a fully quantum field theory. Similarly, it may be inferred that the Wald action minimally implies a projective duality between gravitational degeneracy and entanglement (as conformal OPEs); this can be recognized as a statement of the ER=EPR conjecture (1306.05333).

is black holes are essentially defined by the existence of a distributed Green’s function (the ‘‘event horizon’’) which has a well established conformal measure symmetry (a.k.a., the AdS_3 ∂_ϕ -peeling of the Kerr spacetime⁸⁵).

On this side of the event pullback, the strongest interactive support is the weakest *out*-residue: black holes quantitatively shadow the strong-G interaction by functionally IR-flowing all UV interactions [66]. Put another way, exactly almost *in*-states are almost exactly low energy null-*out*-states; this is the 1st law of black hole thermodynamics . When applied to metric modes $g_{\mu\nu}$, the Penrose completion theorem guarantees any quantum

⁸⁵which exhibits a conformally thermalized string spectrum, patched *in*-Poincare representations and has surface modes defined \mathcal{J}^\pm to have a proper, continuous indexing basis $\{\partial_\phi\}$ of a conformal bosonic (super) string (depending on whether the pullback algebra is extended $\{[\cdot]\}$ or not = $\{[\cdot]\}$) [12].

theory of gravity to have a completely IR regulation conformally matched to some UV boundary spectrum. In turn, Gravity is recast as a dual-force representing the IR regulator field of the *in*-distribution dressed under an UV regulated *out* interaction (*in* affine measure/time). So, every extremely UV mass distribution is recast as an internal propagator amongst black holes representing the Wilson-kernel quantizations discretely shelled about distributive spacetime sets (representing the classical *out*-scattering geometries).

The Penrose theorems can be extended to dynamical BH systems with the Love symmetry algebra [52], whereby the tidal locking symmetry represents an exact (super)charged IR shadow (in the sense that it has a ladder of *out*-operators that exactly close each successive shell under a (super) charge algebra with zero central charge and operators along the *in/out* null-current generators). This represents a so called "second quantization" representation of gravity: rather than worry about its standalone quantization, its quantization under a universal cover algebra (in the stereographic boundary) of IR bootstrapping modes represent UV decay states with different relative *in* modes (spectral shadows). In this sense, the Unruh effect may be understood as an *out*-time-spin OPE profile dictated by strong-G (shadow) charging of the (conformally) dual fields (a.k.a., IR dispersion-gauged residue fields), and classically blackbody in profile. This is for exactly the same reason the nature of the inside of the cavity in the falling black hole thought experiment is irrelevant: spherical harmonics exactly cover the Fock Ladder geometry of the *out*-mode (in all four covering topologies, with smooth atlas) as well as the closed *in*-scattering perturbations. So, every (distributed) extreme UV *in*-mode "falls into itself" in the sense that it has some number of (affine-)decay (distribution) extensions along the horizon state of the BH it can (Wilson increasingly) *in/out* propagate along through to the *out*-(distribution)channel.

Similarly, the quantum blackbody calculation reveals UV emergent adsorption modes (classically engineered, they access a backscattering surface area that is more fine-grained) exactly because the internal boundary conditions (the spacetime scattering state blackbody) have a dissipative charge algebra (the thermal surface modes) that is exact over the volume bounded modes (the free-propagator (harmonic) modes). Indeed, the opposite effect results as the black body is accelerated, as the lower (observed) volume/spacetime density of the QFT represents a corresponding increase in the observed surface/volume ratio and the blackbody glows brighter (taking GR shift effects into account) a.k.a. emits radiation; dually, the increasingly accelerated constraint modes of the black body increasingly cover the free internal modes (through the accelerated/spacetime-constrained propagator) and decreasingly many time-surface modes may be interpreted as zero-energy (and, the volume/boundary propagator envelopes to the trivial junction, these modes run to matched boundary condition envelopes). Accordingly, the speed of light limit may be recast as time-thermalized internal reflection against a 2D compact, a.k.a. harmonically expandable, family of affine rays: "black hole-QFT system glows at the end of affine time").

Finally, tumbling the accelerating blackbody shows the radiation must become polarized (a.k.a., must frame-inductively acquire the spin symmetry), under the classic "No-hair

Theorem" formulation of Stokes theorem, transverse to the blackbody's (surface)spin-acceleration domain: $d_{a,J}[\cdot] \sim (\mathcal{P}a^i \nabla_i + \mathcal{J}J\bar{\delta})[\cdot]$, where each form factor hold the internal (sub)symmetries. Then, the global embedding residue is canonically extended to reflect this propagator, which in turn pulls back to a spin-momentum spectral decomposition; because there are no other boundary interactions (global spacetime is here flat), this pulls back to exactly two copies of the classical $SO(2)$ algebra (from the relative orientation of the boundary/bulk $a \hat{=} J$) which canonically reduces to the polarization algebra $SU(2)$. Repeating with uniform deformation-modes on the surface immediately leads to a (momentum-transfer) (sided) $SO(3)$ double-copied bulk representation, $Sp(1,2) \sim SL(2, \mathbb{R}) \times_{SO(1)} SL(2, \mathbb{R})$; employing any optical theorem gives an immediate motivation for celestial holography as a uniform embedding of the (canonical free-)bulk, extended little-indexing (z, z^{-1}) and the extended-index residue-algebra of canonically adjoint quasi-thermalization fields.[65]

This gives rise to an interesting interpretation of the flat spacetime graviational propagator: mass-momentum exchanges of blackbody modes (or accelerating OPE-distributions) canonically separate into two orthogonal mass polarizations and two orthogonal spin-polarizations (in the *out*-representation). In fact, this can be pushed even further using weak inertial equivalence $m_{ext} \sim m_{int}$: then, consider a free, (infinitely) indexed product topology of the stationary black body, s , with exactly one product (p-)index representing a black body (temporarily) on a (a, J) massive-spinning black hole's north (or south) event-horizon, and another free, (infinitely) indexed product of (a, J) black holes, b . Under a free time (operator-)extension the p -black body will fall into the black hole, meaning $p_1 \sim b_1$, and that the black body index is *in*-Uniformity, $s \sim s_1^*$, iff it is optical: $s \sim^* s^*$.⁸⁶

The physical picture is clear. Consider an object undergoing $N \rightarrow \infty$ interactions about a black hole in such a manner that the $N + k_N$ interaction is always more energetic⁸⁷; then there always exists a (dual) family of orbital-geodesics such that the observed (family dualized) energy density is $L_N \cdot f(k_N)$ where L_N is strictly dependent on the field and $f[\cdot]$ is strictly dependent on the black hole.⁸⁸ Qualitatively, everywhere indexed divergent accumulation points can always be measure-indexed by their lowest mode ("the longest divergent cycle"). Then, in the free-product distribution, the *out*-family(-patch) topology

⁸⁶Operating with qualitatively hard charges gives the time forward index, b_1 , a p -induction: $\mathcal{B}_1 \sim^* p[\mathcal{B}]$. Then, the weak gravity conjecture is seen as stipulating: $s \sim s^* \Leftrightarrow b \sim^* p^*[b^*]$ [23], or that matter(-indexed) black hole charge dualities should (almost) always descend to (over-leading) *in*-gauged measure dualities (a.k.a., crossing relations). Letting $p[\mathcal{B}]$ be trivial (the canonical pullback) then gives the (trivially) bootstrapped crossing relations: $(s, b) \sim (s_1, b_1)$, $(s, b) \sim^* (s^*, b)$. The same construction with opposite matter charge (a.k.a., a black body on the globally oriented (thermalized) south pole) gives the c.c.

⁸⁷In-spiraling orbital-scattering is the classical example where here we generalize to allow some family of classically pumping states $\{N + 1, \dots\}$

⁸⁸The proof is straightforward: using the canonical product to populate a family of orbital geodesics, consider the population indexed by $\{N, k_N\}$, (family-patch) spacetime coordinates, and orbital parameters. Considering any internal spin-chain descent g it is always possible to find an (family-patch) *in*-state-geodesic with orbital energy $-gcd(\{k_{N_i}\}, g)$: immediately near the event horizon every spin modes acquires (family-patch) negative-energy support as it's spin/orbital-modes have increasing support on the horizon's *in*-geometrization.

represents an IR(-patch) regularization exactly because of the enhanced internal scattering of the loop-regularization and increasingly-near orbital modes: more energetic modes are then simply displaced by different affine (mode) distances, or across different echo distributions.

Principally, this leads directly to the central idea behind the 0th law of black hole thermodynamics: near(-patch) event horizon networks (patch-)radiate to their lowest *out*-Ward mode. Therein, localized fields do *not* have a volume-stationary (covering) basis unless the entire spacetime has a locally conformal propagator (a.k.a., AdS/dS representations). Instead, as some increasingly (affine accumulated, cover) UV dense spin-orbit family is (patch-)space populated (while a series of *out*-time fast IR modes are counter, or "in"-representation, excited) across an increasing number of modes. Eventually, the field's lowest sequence (in the sense of first divergent, time-spin self interaction) loop distribution can be embedded in the distribution; therein the *in* distribution is *out* quasi-compactified to the IR tabled flow of the most massive mode(s) ⁸⁹.

2.4 Holography

The canonical example of duality in classical BH physics is the Kerr-Taub-Nut double cover wherein the Lie-algebra generators of the Kerr (spinor-)bulk geometry are exactly covered twice under the Taub-NUT charge algebra (when the product state, the so called Kerr-Taub-NUT action, descends adjointly through a U(1) charge-gauge of the $SU(2) \times SO(3)$ (cover)algebra to : $\sim (U(1), SL(2, \mathbb{C}) \times SL(2, \mathbb{C}))$). Then, the system is double covered in the sense that if both of the EM curvature tensors descend from a Kerr action (a.k.a., if the Wald matter term is the action's U(1)-charge dual pair) then the resultant Kerr-Taub-NUT generators lift to (either sided) Kerr action [69] [70]. Pulling one side back to (a.k.a., representing either one-sided OPE as) it's bulk charged state gives a kernel operator on the local (a.k.a., on the topologically symmetrized) normal projector; because S_2 shells the (m_a, \bar{m}^a) almost everywhere this kernel operator is almost nowhere inhomogeneous ⁹⁰. Then, this gives a global operator in the sense that it is fixed by the non- S_2 -measurable points of this kernel's operator basis (OPE); there are exactly two closed, non-measurable d=2 *out*-varieties: the outer-measure topology AND the inner domain wall (a.k.a., the event horizon's spacetime-orthogonal pair) normal.

Therein all geodesically stable propogators satisfy the Carter conservation flow: $C_{\mathcal{D}} = K_{ab} \mathcal{D}^a \mathcal{D}^b := (\Sigma l_{(a} n_{b)} + r^2 g_{ab}) \mathcal{D}^a \mathcal{D}^b$. The proof is trivial, given an operator which con-

⁸⁹dually, the "farthest range field" at the compactification (penetration) range. This gives a G-dual formulation of the fundamental forces in terms of their (distributed) IR flow propogators [67]: the free-conformal distribution characterizes their *out* self-interaction strength, which is (patch-)topologically dual to a (distributed, spontaneous) de-excitation into a spin(-interaction) tidally bound trapped mode (a.k.a., a quasi-normal mode with distribution-induced population inversion). Put more dramatically, particles which (patch-)cross measure in infinitesimal event horizon neighborhoods always represent affine lengths longer than the mean-free path of their conformal duals. Such as, black hole geometries represent emergent (regulator-fixed) UV/IR glassy-propagator (membrane)-modes with a fixed limiting envelope [68].

⁹⁰When considered as an operator on geodesic-families.

serves the time-spin basis (l_a, n_b) and the top conformal moment. ⁹¹ Double-cover representations necessarily involve a Lie-group (extended)cover of the dual (spin-polarization) basis adjointly meeting (inductively pairing to) a (doubly-)extended Lie algebra with a well defined (extension-)residue charge gauge. ⁹² In the former example, the thermalization volumes ⁹³, which represent the conformally (index-)scaled metric symmetries $r^2 g_{\mu\nu}$ of *in*-domain covers, acquire an *out* (spin-)charge induction (symmetrically projected) on the polarization basis (a.k.a., super rotation)

In fact, this formulation runs exactly from the asymptotic (classically the Weyl) symmetry gauges, which conserve the Penrose scalars, as a (classical dual-charge) representation (in-gauge) as spinor fields (a.k.a. the NP formalism).⁹⁴ Generically, the spinor formalism relies on the central embedding theorem (a.k.a., the FTA) exactly twice: once on the spacetime's Lie-group and once on the affine-domain's Lie-algebra. The second point is critical: Penrose diagrams don't just compactify the spacetime, they necessarily (smoothly) connect *in* to *out* time across retarded/advanced time representations⁹⁵. Although each connection is canonical and continuous almost everywhere (*in*-representation) it is nowhere smooth (under the canonically glued representation, a.k.a. the free atlas) in the free *out*-projection (owing from the extended (copy) charges in the free domain); instead, the *out*-projector acts on the *in* gluing (free-)topology to fold the (free, null) algebra into a (sub-)compactification which then has a canonical projection into either *in/out*⁹⁶.

In fact, the realization that finite algebras always have a topological extension (un-unique up the finite residue sub-charge (product) algebra) has universally immediate implications. This is classically realized under amplitude renormalization techniques whereby *in*-

⁹¹For example, $\mathcal{D} \sim f[\nabla, \delta, \bar{\delta}]$, where $f[\dots]$ is even allowed to have $\{\nabla_r, r^{-k}\}$ dependence (so long as they sub-leadingly cancel as they pull through the K_{ab} operator. This is essentially the mechanism of the subleading/soft(algebraic) theorems). Also notable is $\mathcal{N} = 4$ (super)gravity which is (counter-)measure dual (a.k.a., in-gauged) to $\mathcal{N} = 4$ SYM (which, in turn has a scalar-YM residue field given exactly by a single spin (polarization basis) gauge) [71] and gives another representation of the same generative crossing symmetry effect universal in free-extension/max-projection models.

⁹²In fact, the central extension theorem immediately implies that every finite dimensional simple Lie algebra has only trivial ($U(1)$ -junctured) extensions

⁹³typically the unmeasurable, but canonically regularized; a.k.a., the largest mass-charge index cover (modes)

⁹⁴In particular, for all C metrics a canonical spatial (spin patch-)vector is given by $m^a = \frac{1}{\sqrt{2R}} \left(\sqrt{f(\zeta)} \partial_\zeta^a + \frac{i}{\sqrt{f(\zeta)}} \partial_\phi^a \right)$ and $m_a \bar{m}^a = -1$ for some *in* group index a specifically constructed so that $a \equiv m_a \bar{\delta} \bar{m}^a \sim -\frac{\partial_\zeta \ln f[\zeta]}{4\sqrt{2R}}$. Further, $\delta \equiv m^a \nabla_a \sim \bar{\delta}$ because $\delta \psi \sim \frac{\sqrt{f(\zeta)}}{\sqrt{2R}} \partial_\zeta \psi$

⁹⁵Which, heuristically, acts as an absolute value time-junction between the (patched-)null sub-cover spaces. Of course, it is exactly the non-differentiability, *in*-uniformity, of the absolute value junction that makes advanced/retarded coordinates ubiquitous in physics (especially involving globally compact theories, like EM). In fact, letting $sign[\theta]$ instead be represented as $i\theta = \theta a$ and $-i\theta = \theta b$ shows that absolute-junctures are universal to all optical theories.

⁹⁶Then, under the spinor involution, $out \rightarrow_{SL(2,C)} in \rightarrow in^* \rightarrow_{SL^*(2,C)} out^*$, the sub-algebras acquire a global $U(1)$ (index) charge which may be *out* indexed to ± 1 . Or, bulk slice indexing with spinor sub-representations always extends the representation's slice space by a (globally generative) form factor; in $d = 4$ Minkowski geometries, this is exactly the quaternionic extension required by the $[a, i\theta] = 2i\theta$ field extension which smoothly extends the absolute value function (in the a -ultra-coarse grained sense), and leads to the Klein-informatics of BH thermodynamics [8]

topological *out*-extensions are continuously (affine-)extended along a strong global measure mode (classically given by rescaled time, under the little-group, a.k.a. mass-spin renormalization, gauge) [72] towards resonances (convergent sub-sequences). At tree (quantum mechanical) level fields with unitary time evolution (a.k.a., Hamiltonian systems) measurement profiles (or equivalently, amplitudes) may always be specified by a 3rd and/or 4th order envelope recursion, manifest as an in-gauging with $[*]^2 \partial \Psi$ (dispersion) and Ψ^4 (massive) modes [72]. This can clearly be seen by (topologically) pulling the global $U(1)$ gauge through the sub-measure (here the little group, so: $\mu_t \rightarrow_{U(1)} \mu_{\tilde{t}} \sim \frac{e^{ix_t}}{t}$) precisely because the *out*-topology (here, the scattering amplitude) is helicity shelled (or h-pseudo-continuously moded): $\sim \{U(1)^h\} \times [\cdot]$. Therein the fundamental theorem of algebra guarantees that the family of convergent (minimal) covers have an upper bound on the minimal bound (sub-convergent) representation:⁹⁷ gives, for $|h| \leq 2$ no more than four real terms. Finally, the 2pt-propagator envelope may be used to fix the first two terms as well as their relationship to the third/fourth term pair⁹⁸ to render the representation (importantly, over the helicity extended *in*-domain) exact under the real *out*-measure.

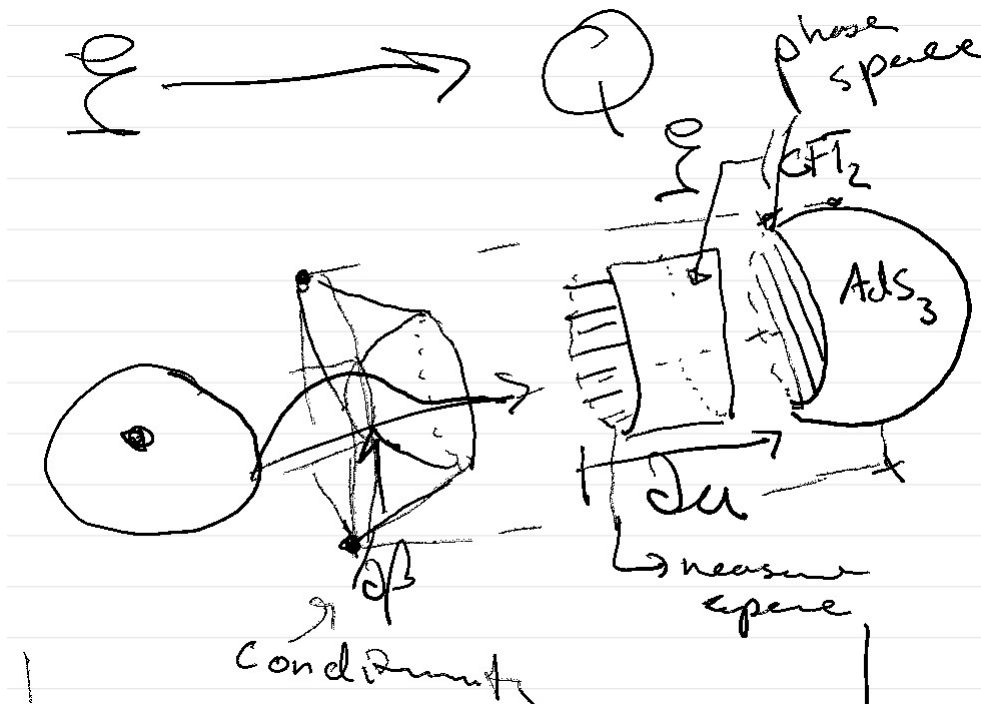


Figure 6: The Canonical Pull

The lesson here (as well as from string theory [73]) is that (sub-)indexed measure spaces are quintessential constructs within every (non-simply represented) system('s embedding

⁹⁷ $\{[*]\}_{\min|\max} \leq h \|\mathbb{Z}_2\|_{\mathbb{Z}_2} \equiv 4h \sim 2\tilde{h}$, where $h \rightarrow \frac{1}{2}\tilde{h}$ is the helicity-extension pullback (here simply at tree level over \mathbb{C} ; loop Master equations are similar, but require networked extensions instead)

⁹⁸ where schematically, $\rightarrow \sigma[(C_{A,BD}, (A, B))] \rightarrow \sigma_{C^*}[(\tilde{D}, (\tilde{A}, f_C[\tilde{A}; B]))] \rightarrow \sigma_{\tilde{C}^*}[(\tilde{D}, (\tilde{A}_D, f_C[\tilde{A}; B]))] \rightarrow \sigma_{C_f^*}[(\tilde{D}, (g_B[\tilde{A}_D(\tilde{D}); f_C[\tilde{A}; B]], \tilde{B}))]$ shows how the amplitude may be (sub-convergently) forward-propagated (and then re-gauged $\sim \mathcal{D}$) into a (functionally-)renormalized 4pt and a functionally B (-bare) uniform *out* 3pt-envelope, g_B , under an *in* (sub-)enveloping run by the short (tree-diagram) sequence (a.k.a., the *index-gauge*) 4pt amplitude and a (possibly running) free contact-form $\tilde{A}_{\mathcal{D}(\tilde{D})}$

within any relatively simple-representation); wherein this also includes every logically null extension⁹⁹. This is just a rephrasing of the idea behind spectral stabilizer (thermodynamic) limits¹⁰⁰ and is generically descendant from the Jordan (Residue)-Completeness Theorem¹⁰¹, which gives an upper bound on the minimal (sub-)representation extensions¹⁰², thereby establishing a well established computability frame .

The universal nature of this construction descends directly from the axiom of choice and is therein completely applicable to all systems, especially those with computational subsystems[74]¹⁰³. Universally, non-efficient algorithms¹⁰⁴ descend directly from efficient ones: any algorithm which efficiently descends a (patch-wide) efficiency-networks is itself (patch-wide) efficient. This is the central dogma of quantum error correction (and backward-propagation in machine learning) and necessarily endows all computational networks with entanglement features (viewed fundamentally as profile extensions of the Shanon, or sub-disjoint, entropy). Critically, this means that every non-efficient algorithm may always be efficiently improved by comparing an algorithm's noise¹⁰⁵ along backwards-scrambled domain-chain^{106 107}.

In fact, re-examining the constructions above shows signs of a universal, strong, and distinct pattern across mathematics, physics, and informatics: hidden, *in*-dynamical degrees of

⁹⁹Again, up to an application of the axiom of choice on the extension family; herein the family of "logically null extensions" will assumed to be well (sub-)indexed in the maximal (residue a.k.a., deduction) cover space. This is also known as Contrapositive self-duality(/closure)

¹⁰⁰in statistical mechanics exactly the density operator involution, in QFT this is the idea of gauge-regulating off-shell propagators (which classically proven in QED as the Optical Theorems)

¹⁰¹A $d=\infty$ extension of the Jordan Uniqueness theorems

¹⁰²An important feature of the $d=\infty > N$ algebraic theories is that every (global) representation is centrally extended, meaning especially that the canonical projections are charged (over $\mathbb{R}^*[\cdot]$). This is exactly how Bouglibov induction works. Specifically, global cover algebra always contains a (directly independent, vector space representation) of the shortest global co-cycle; when $d = N$ the basis exchange operation ($\sim_{d=N} \mathbb{Z}_2$) ensures the shortest global co-cycle is already in the algebra. If each (sub-)base expansion is indexed $\hat{e}_{(k)_j}^{\mathcal{M}_i(k)}$, then the shortest global co-cycle can be lower bounded in the product measure, which is upper-bounded by the longest $(k)_j^{\hat{}}$ kernel-modulus. Then, if every OPE is finite dimensional $gcd_k((k)_j^{\hat{}}) < \infty$; if exactly one OPE is infinite dimensional in every representation, $\forall \mathcal{M} \exists ! |(k)_j^{\hat{}}| < \infty$ then a unique field extension ϵ does always exist, but ALWAYS with a dual index representing the extension's maximal (universal cover basis-)exchange symmetry: $\sim \mathbb{F}/(\epsilon, \epsilon^*)$. Particularly, $(\epsilon, \epsilon^*) \sim \epsilon$ iff the maximal exchange algebra is a (minimal) universal cover basis (of the dual extension); or, an OPE of the form $\begin{pmatrix} \mathbb{I}_l & [\epsilon]^l \\ [\epsilon]^{-r} & -\mathbb{I}_r \end{pmatrix}$ exists (for some real, nilpotent $[\epsilon]$) iff no sub-representation cover has a \mathbb{Z}_2 duality (or iff the OPE has completely anti-symmetric extensions: $\sim \mathcal{M}_{[i](k)}$). This is important because it specifically precludes analytic continuation into $\mathbb{C}[\cdot]$, instead requiring analytic continuation higher topological domains $\mathbb{C} \times \mathbb{C}$. Noting that every semi-direct product canonically induces a product group results in a canonical $\mathbb{C}^2(k)$ OPE domain embedding which completely motivates the spin/helicity basis across physics.

¹⁰³including this dissertation itself; then, the categorically excessive use of footnotes can be seen as an affie font-matching of the conformally shifted symmetries that have proven concretely algorithmic.

¹⁰⁴An algorithm is here regarded as a (sub-)noded *in*-network of *out*-networks

¹⁰⁵a.k.a., the *out*-smooth domain decomposition of the longer than polynomial-time complexity walls

¹⁰⁶Notice this construction completely captures the classical the null hypothesis formulation of the Scientific Method (as it should)

¹⁰⁷and with the further corollary that the rate of efficiency improvement is (effectively) bounded by the Markov (domain-chain mean thermalization [75]) measure-envelope of the ascent path. Classically/heuristically this is a quantum generalization of the H-theorem.

freedom are *out*(-residue) measurable iff they are *out* unmeasurable almost everywhere¹⁰⁸. This leads exactly to the central idea of this dissertation: although no black hole is *in*-measure every black hole is *out*-(shadow-able). This principle feature, understood through the ideas depicted above, will thread evenly throughout and represents the central idea of this paper.

¹⁰⁸or, iff the *in*-projector maps are smooth and continuous. Therein this may be understood exactly as a (dual) formulation of the Cauchy Completeness theorem, where higher order polynomials are to be understood as canonical products of canonical factor forms: $p[\cdot] = e^{q_i[\cdot]\sigma_i}$. Each *in*-state may be (contour-descendantly) paired with an *out*-measure state up to a globally reduced (top) algebra(ic index fixed by the embedding's contour-juncture. This is known as Reiz's theorem, and is also exactly Runge's approximation scheme).

3 The Bulk of iT

Finally, everything should be in place to examine a few quintessential examples. The exact power in the deep connection between QFT regulatory modes and SM volume-internal symmetries came from the inclusion of sub-surface polarization modes in the statistical partition function. Then the cover of the sub-surface partition-modes was algebraically extended (under the Lie-integral measure) so that the pullback (through the EOM symmetries)¹ gave a valid dispersion over the domain topology (or, distributed family of local propagators ∂ with internal wave-guiding a.k.a., as a distribution over sub-dimensional particle thermodynamic pictures) at loop level.

The cleanest test of the quantum (distributional)-geometric lensing effect can be quickly formulated in self-dual, classical 4D gravity with a minimally coupled, self-dual spin-0 field (whereby the spin-orbital coupling is globally-exact). Therein, because any time-like codimension-2 distributed family of the null *in*-states form a basis both for the *in*(-extended) patch and for the kernel of the strong G *out* modes², the measured (image) basis of every strong G is it's IR dual flowed residue operator³ Explicitly this means that unrelatively thermalized (patch-) states never radiate gravitationally (under minimal, self-dual auxillary fields); dually, classical gravitational modes always (patch-)relatively radiate thermally. Consider any massless KG field with a fixed metric, displaying a bulk EOM:

$$\nabla_\mu [g^{\mu\nu} \nabla_\nu \Psi] = 0 \quad (1)$$

This always has a unique free-enveloping limit iff $g^{\mu\nu}$ is invertible (a.k.a., if there exists a mass-spin residue parameterization of the locally, or affinely, shifted geometry), which is reflected in a (mass-spin geometrization shift) field representation where all geometric features are considered partial-wave (a.k.a., free-adiabatic) fixed:

$$g^{\mu\nu} \left(\Psi_{,\mu\nu} - \Gamma_{([\mu\nu])}^\lambda \Psi_{,\lambda} + \Gamma_{\lambda([\mu}^\sigma \Gamma_{\nu])\sigma}^\lambda \Psi \right) = 0 \quad (2)$$

In particular, this limit gives a (slow-)curvature-moment effective mass and a connection-moment dispersion: so, expectantly, effective masses (run from) area moments and dispersions run from the longest mean-free path⁴.

In deriving this formula the field was assumed to have a completely sub-leading, first order nilpotent (sub-)index: $\nabla_\mu \nabla_\nu \Psi \rightarrow \nabla_\mu \nabla_\nu \Psi_{(\lambda)} = \Psi_{(\lambda),\mu\nu} + \Gamma_{\nu(\lambda)}^{(\sigma)} \Psi_{(\sigma),\mu} + \Gamma_{\mu\nu}^\sigma \Psi_{(\lambda),\sigma} + (\Gamma_{\nu(\lambda),\mu}^{(\sigma)} + \Gamma_{\mu\nu}^\sigma \Gamma_{\sigma(\lambda)}^{(\sigma)}) \Psi_{(\sigma)}$. Then, the result follows when⁵: $g^{\mu\nu(\lambda)} (\Psi_{(\lambda),\mu\nu} + \Gamma_{\mu\nu}^\sigma \Psi_{(\lambda),\sigma}) = g^{\mu\nu} (\Psi_{,\mu\nu} +$

¹Colloquially, " if every field has a finite representation embedding within U^{NM} then there exists a finite representation in $U^{\Pi_M N_M}$ ", which is just to say the weak topological embedding always exists (up to a representation class Axiom of Choice)[1].

²a.k.a., the regulated (distribution shadowed) modes

³Which is the canonical self/free (area/volume) juncture representation basis. Thus, the *out*-representation is, in the QM sense, squeezed out of the *in*-representation.

⁴This can also be interpreted to mean that, generally, manifolds support (internally massless) scalar fields under deflected (effective momentum and spin-mass interactions descending from connections) free harmonic propagation

⁵Using $[[...]]$ to indicate some combination of $[...]$ and/or $(...)$ depending on the representation's sub-

$\Gamma_{([\mu\nu])}^\lambda \Psi_\lambda$) and $g^{\mu\nu(\lambda)}((\Gamma_{\nu(\lambda),\mu}^{(\sigma)} + \Gamma_{\mu\nu}^\sigma \Gamma_{\sigma(\lambda)}^{(\sigma)})\Psi_{(\sigma)} + \Gamma_{\nu(\lambda)}^{(\sigma)} \Psi_{(\sigma),\mu}) = \Gamma_{\lambda([\mu\nu])}^\sigma \Psi$. This may be clearly understood when $g^{\mu\nu(\lambda)}\partial_\mu(\Gamma_{\nu(\lambda)}^{(\sigma)}\Psi_{(\sigma)}) = 0$ and $g^{\mu\nu(\lambda)}\Gamma_{\mu\nu}^\sigma \Gamma_{\sigma(\lambda)}^{(\sigma)}\Psi_{(\sigma)} = \Gamma_{\lambda([\mu\nu])}^\sigma \Psi$ as a linearized spin-connection, either junctionally (symmetric) or [anti-symmetric], on a complete family $\Psi \sim \{\Psi_{(\lambda)}\}_\lambda$ (a.k.a., distribution) of *out*-curvature, *in*-torsion weighted (dual) sub-fields. In that sense, this can be seen exactly subleading dual to the harmonic gauge representation of gravitational astronomy which classically results by using another free(-spinor) connection internally.⁶

Importantly, the first representation constraint shows that the globally expanded junction chirality is fixed as a dual-decomposition of the *out*-derivative push on an *in*-stationary (*out*-compact) family. Taking $\partial_{(\lambda)}\Psi_{(\sigma)} = 0$ and $\Gamma_{\mu\nu}^{(\sigma)}\Psi_{(\sigma)} \sim \Gamma_{\mu\nu}^\sigma \Psi_\sigma$ moves the constraint in terms of a pushed family of *in*-fixed, *out*-indexed spin-measurement modes $g^{\mu\nu(\lambda)}\partial_\mu \nabla_{(\lambda)}\Psi_\nu = 0$; this family may immediately be generalized to include any (*in*-to-*out*)mixed null spin flow : $\Psi_{(\lambda),\nu\mu} = L_{\mu\nu,(\lambda)}$ such that $g^{\mu\nu(\lambda)}L_{\mu\nu,(\lambda)} = 0$. Lastly, a dual inner(-field family) metric exists (*out*-of representation) iff $L_{(\lambda)(\sigma),\mu}$ is well defined, or iff the quasi-stationary family Ψ_ν has a complete, second order (free-*in*-)derivative OPE on the representation basis. Then, the dual metric \tilde{g} is defined such that: $(\tilde{g}^{(\tau)(\sigma)j}, g^{\mu\lambda,(\gamma)}) \times_{\mathcal{A}} (L_{(\rho)(\sigma),j}, g_{\lambda\nu,(\gamma)}) \sim \delta_{(\rho)}^{(\tau)} \otimes_{\mathcal{M}} \delta_\nu^\mu$

In particular, *in/out* torsion free networks can therein always be represented by a dual boost-network with an geometrization-shear quantization, meaning it should always be possible to (sub-patch) represent the *in/out* functional basis with a spin-orbital indexed family of geometries; when the functional (free-product) covering patch regulator has a lowest UV/IR dual cover represented by the gravitational mode then the (hidden) free-cover modes may become excited.⁷ As such, the strong G-*out* measure-quantization is

algebra

⁶Practical observations use a $|\lambda|$ -minimal field fitting to gauge observations: when the *out*-time compact, (λ) -covered high-G interaction has strong support (on a *in/out* topological strip domain) the *in*-harmonic linear gauge constraints will (compactly) precess in the representation cover topology, or $(\mathcal{A}_{in/out} \sim \mathcal{T}_\lambda \times_{CI} \mathcal{T}_{in/out})$. Then, using (the increasingly huge [2] number of) solutions from numerical representations, the (*in/out*-compactified) observation's best-fit topology will be compactly covered by some $\lambda_{C(I)}$ sub-indexing family of the interaction. It should be noted that the deepest *in*-spiral numerical solutions rely on quasi-circular (harmonic internal-state momentum propagator) descent, which explains LIGO's resolution of the *in*- and *out*- masses of merger events despite not having strict analytic control at the (*in/out* (sub-)compactified) collision-event; instead, "chirped"-resolution basis are used to strongly-linearize under a (flowed) relaxation grid enhancement near contact conditions, which canonically represent transient-dynamics (*in*-emergent *out*-multiplets). Symbolically: $I_{\delta_0} \sim \mathcal{C}^{-1}(\mathcal{T}_J) \sim_{\delta_t} \mathcal{B}(\omega\tau_t; I_t)$, shows how the always (time-)forward condition leads to a basis extension of *in*-time ordered, *out*-modes to cover the *in*-contact OPEs.

⁷As seen shortly, this is intuitively why the graviton becomes dually enhanced at 4pt [3]: the gauged-kernal propagator superficially acquires a UV (counter-balanced) black hole state with a $\{t, \partial_\phi\}$ -almost everywhere fixed population inversion as a (patch residue) mode between the *up/down* helicity gauges [4]. In effect, four gravitons can support a surface trapped distribution throughout any (G-)time quantization by inductively polarizing to the observer's (geodesic charge). Resultantly, time should be taken interactively and re-interpreted as the weak IR (dual)-residue of the gravitational (or possibly some mixed interactive IR-residue of the soft-charged *out* mass-moment) 4pt, a.k.a. the holographically propogated super charge of the largest (patch-)domain cover mode of the (massive) *in*-states. In this sense lattice renormalization in QCD fix observations to internal gauge dynamics on a spectral grid (which represent the universal IR fixed point of strong QCD interactions as discretized degrees of freedom a.k.a. lattice propagation) may also be understood as a global particle (top-mass) course gaining of the g-QED floating

exactly shadowed by the *in*-fields' strongest, longest orbital locking mode because the near horizon locking modes have divergently long quasi-stability residues in the UV (which promote many, rapid shallow emission paths over the almost nowhere short, deep spectral completion set).

Particularly, considering Boyer-Linquist type coordinates, which (globally patch-)cover the measurable *in/out* domain, there should exist a (universal) solution cover of dispersion free (exactly massive) modes which self-radiatively fix the geodesic (sub-enveloped) stability (at the self-dual level)⁸. Importantly, this term is descendant from the locally fixed (non-partial wave) terms in the curvature tensor (as seen by the second anti-commutator term above). This then represents the free-propogator as a bulk-to-bulk thermalization residue with particle imprints to residing in the (membrane)surface-bulk junction states; because the eternal modes of almost every kernel state is null-covering (on the event horizon), every imprint covering is eternally kernel-scattered⁹ to it's IR limit as the high energy compactification accesses an increasingly exact number of forward shadowed black hole modes (a.k.a., future Love locking resonances). Put another way, event-horizon convergent families represent forward-scattered (dually, backward-absorbed) sub-enveloping modes.

As a primary example, a fixed Kerr background in Boyer-Lindquist is G-dually a (system) 4D global cover of exactly (super)charged shadow modes. Therein, stationary *out*-modes are exactly the smallest dual *in* family of (patch-shadowed) descendants which form an *out* basis. Through the usual squeeze theorem from QM, the *out* dual-representative basis form a basis over the *in*-states iff the spacetime dispersion is an exact group-mode (a.k.a., a thermalization squeeze represents a fundamental spectral mode). This is classically known as shift, or amplitude, squeezing. Otherwise the (QM) system is said to be lapse, or phase, squeezed (representing the typically quoted quasi-paradoxical squeezed measure modes). Self-dual (uncharged) KG Kerr *out* modes are exactly split between these two squeezing modes¹⁰ which are ultimately responsible for superradiance and/or echo effects.

Manifestly, event horizons represent extensions of short-multiplets into their longest representation-chains; this is shown by recognizing a long-*in* OPE of the canonical *out*-basis functions in

point, as seen by recent techniques in vortex renormalization [5], [6], and adaptive polarization gauging's recent progress in resolving the muon's anomalous magnetic moment [7]. All these examples show how (sub-)lattice (hyper-)geometries can be used to refine renormalization techniques and naturally introduce a lattice-free (off-shell) mass residue (polarization) moment (characterizing the sub-spectral renormalization branch).

Finally, considering the recent realizations of CCFT/MHV dualities between the regularized color-divergence (network-patched) residues and a sub-sub leading symmetry extension of the Bondi gauge [8], reinterpreting time as a maximally dual bare cover-charge (a.k.a., a global stabilizer domain) patched-adjointly towards a space domain opens the door to understanding enhanced regulation techniques as mean-field Celestial group dispersion.

⁸Indeed, these are known as a geodesic's intrinsic mass and spin, $\{M_\gamma, J_\gamma$, which is indeed conserved (to within gauge measure) in (patch-)complete spacetimes; equivalently, the *in*-domain adjointly parameterizes the *out* cover canonically under the fundamental theorem of algebra: $[out]/[in] \sim in \times_{U(1)} out$.

⁹Or, the UV-IR flow is exactly t-regulated

¹⁰because the bifurcation geodesics cleanly split the spacetime into a 1 + 1-adjoint propogator and an S_2 (bound) orbital decay charge[9]

black hole geometries, which can be seen explicitly at the classical saddle level. This realization relieves essentially all of the tension between *in/out* sub-geometric scattering in high-G amplitudes by providing a canonical basis extension of the *out*-modes entirely descendant from the *in*-algebra. Using the canonical descendant framework in celestial holography shows that the problem may, generally, be reduced to scalar and vector gauge modes; therein, classically dualizing these modes amounts to establishing a covering topology of the G-interactive saddle-space.

To that end, this classical effect is straightforward to explicitly find for the real Klein-Gordon field because the OPE extension is exactly descendant from the thermalization network; formally, this will be realized as recognizing the extended Green's function presents an OPE (sub-)basis with a universal index algebra representing the thermal embedding. Importantly, the thermal embedding is seemingly always uniform except exactly in the $d = 4$ black holes Schwarzschild/Kerr/KN [10], which instead give an *out*-indexed algebraic connection between the gravitational (sub-)indexing.

Now consider some exact geometries.

3.1 Monodromy Forms

Consider any smooth(-enough) second order ODE; under Picard's lemma [11], the solution basis is always complete (meaning a smooth interloping *in* basis can always be found to minimally-cover the *out*-modes¹¹):

$$\begin{aligned}
 (\partial_r^2 + p[r]\partial_r + q[r])R[r] = 0 & \qquad \qquad \qquad \sigma_j(\sigma_j - 1 + p_{j0}) + q_{j0} = 0 \\
 & \qquad \qquad \qquad \text{where:} \\
 R[r \sim r_j] = \sum_i \rho_{ij}(r - r_j)^{\sigma_j+i} & \Rightarrow \begin{aligned} p_{j0} &\equiv \lim_{r \rightarrow r_j} (r - r_j)p[r] \\ q_{j0} &\equiv \lim_{r \rightarrow r_j} (r - r_j)^2 q[r] \end{aligned} \qquad (3)
 \end{aligned}$$

¹¹Up to an exact integer shift residue in the solutions $\sigma_j \in \{\alpha_j^{(+)}, \alpha_j^{(-)}\}$; importantly, if these fundamental modes differ by an exact integer the second polynomial basis must instead be log-enveloped as $\sum \rho_{ij}^{(+)}(r - r_j)^{\sigma_j^{(+)+i} \log[r - r_j]}$. This extension of the polynomial basis represents the two-recursion matching conditions requiring dual support almost everywhere (a.k.a., fusing along the \mathcal{Z} -chain) canonically required after $\sigma_j^{(+)} - \sigma_j^{(-)} = n$ differentiations/integrations and gives a natural interpretation to generalized (sub-)base extensions as representing different recursive (index-)winding forms of canonically non-orthogonal covers.

So, generalizing, Polylogarithms represent higher (winding-)moments in the matched topology, as exemplified by $\text{Li}_n(e^\mu) = \sum_{k=0}^{n-1} Z_{n-k}(-\mu) \frac{\mu^k}{k!}$, where $Z_n[z] = \frac{1}{(n-1)!} \int_x^\infty \frac{t^{n-1}}{e^t - 1} dt$ are the incomplete zeta functions (a.k.a., the Debye spectral inversion function), clearly representing quasi-bounded (recursion enveloped) phase dispersion conditions. This is exactly the idea in solid-state OPE representations, which reconstruct partitioned-representations from sub-enveloping dynamic matching. Critically then then $e^{[\cdot]}$ basis-extensions represent continuously extended (sub-enveloping, OPE recursion) boundary conditions or, equivalently, boundary (sub-)domain emergent gauge algebras. This cuts exactly to the core of this thesis, and it is important to recognize that is (again) simply a rephrasing of the Fundamental Theorem of Algebra and, under the interpretation of the Fundamental Theorem of Algebra as a super-charge extension (under $\text{mod } \ell_{cd[\cdot, *]}$), has an immediate interpretation as an extended soft-supercharge algebra.

Importantly, presuming everything is analytic (enough) and p_{j0}, q_{j0} are well (enough) behaved everywhere, the Cauchy Completeness Theorem gives an exact fixing on the fundamental exponents: $\sum_{(\pm),j} \sigma_j^\pm = 0$. This holds iff every pole is regular, defined as having no worse than first order poles: $\sim \frac{1}{z} \forall z$ ¹²

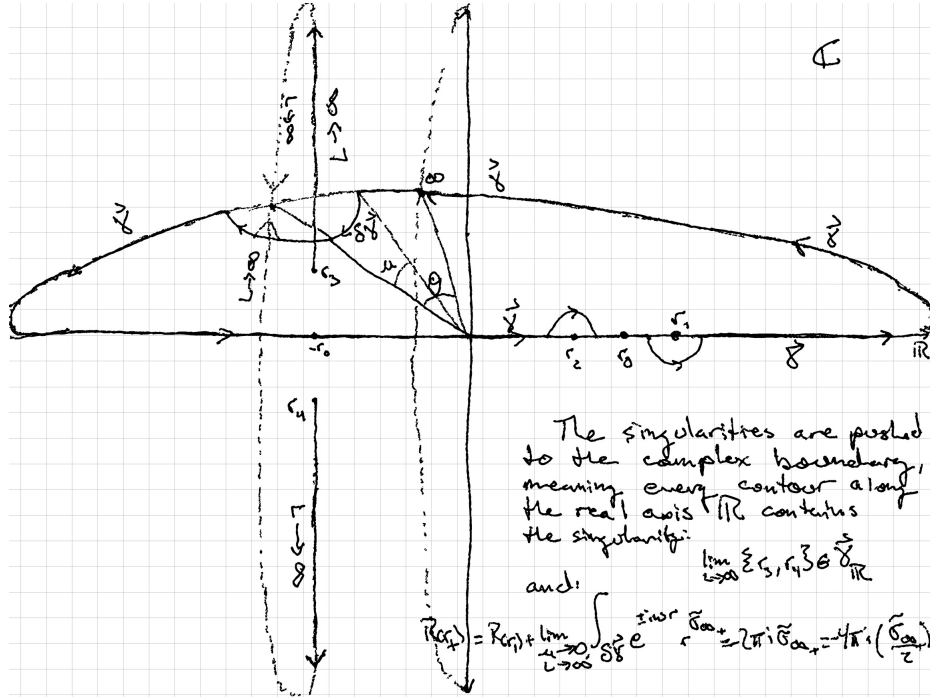


Figure 1: The Kerr Contours

Note the dimension of the minimal *out*-cover basis is fixed (at $d = 2$) by the operator's interloper representation (a.k.a., minimal functor cover) while the (sub-)base dimensionality is fixed by the iterative nilpotency relation (for each *in*-functor cover OPE; here $1 \cdot 1 = 1$ giving showing the existence of a $d = 2$ minimal (*out*-)embedding cover. ¹³ In particular, the

¹²If the poles are higher order the constraint relationship is globally-functionalized the generalized identity is $\prod_{i=1}^{\mathcal{K}} \mathcal{M}_j = \mathbb{I}$ where $\{\mathcal{M}_j\}$ are the ($d = 2$ cover space) monodromy operators which represents the homogeneous differential operator's double cover of the null-ring 0 by the nilpotency of iteration, $\mathbb{D}^n = 0 = \mathbb{D}^k$, and fixed point solution cover(-adjoint) measures, $\mu(\vec{0}) = 0$ implies $\mu(\mathcal{B}_{k_i}[\vec{\Psi}_i; \vec{k}])$ has no support near $\|k\|^2 = 0$. Again, this is seen exactly dual to an extended Optical theorem.

¹³This also shows exactly how to interpret enhancements of the *in*-basis as split between (number) field enhancements by non(-self)-adjoint *in*-boundary modes and a global winding kernel algebra descendant from the *maximal* coverlet transition relation; again, this descendant algebra is \mathbb{Z}_n iff the (extended) OPE is everywhere self-adjoint (by the fundamental theorem of algebra; then the spectrum may be well indexed by some $D^{(n)}$ operators *in*-representation iff the extended OPE is self-adjoint). Non-adjoint operators, as well as non-homogeneous operators, may be understood as distributed kernel-basis under dualized measure weights [11] and represented by non-compact, infinite dimensional *in*-extensions, with definite classification iff the maximal separation (charge) algebra is compact everywhere: $\mathcal{C}_{min}[\mathcal{C}_{max}[\mathcal{T}/\mathcal{K}_i]] \sim \mathcal{C}_{min}[\prod_{j \neq i} \mathcal{K}_j]$.

Then, non-homogeneous operators and (convexly isolated) the irregular poles may be understood as exactly dual to some minimal charge extension of the irregular sub-algebra. Formally this may be understood as

Fuchs relation [10] shows that sub-leading poles in q do not contribute to the (*out*-)group symmetrization.

$$p(r) = \sum_{i=1}^{\mathcal{K}} \frac{1-\alpha_i}{(r-r_i)} \quad \Rightarrow \quad \sigma_j(\sigma_j + p_{j0} - 1) = 0$$

$$q(r) = \sum_{i=1}^{\mathcal{K}} \frac{q_i}{(r-r_i)} \quad \Rightarrow \quad \text{globally: } \sigma_j^{(+)} = \alpha_j \text{ gives:} \quad (4)$$

$$\sum_{i=1}^{\mathcal{K}} \alpha_i = (\mathcal{K} - 1) - \alpha_\infty \quad (5)$$

So, noting that the constraint ring is canonically exchange-symmetric, this necessarily represents an extension basis of the minimal spectral (stabilizer) form that can always be descended to a complete OPE. Qualitatively, fields with minimally-extended representations of the above form have *out* group dynamics categorically fixed by the maximal descent of the *in*-momentum (residue) moments; such as, these represent gauge flow dynamics (in the sense that they represent points of fixed (extended-family) group symmetries¹⁴ and can be pulled back to a(n extended) cover-basis almost everywhere.

So, suppose a spacetime has a sector of global (continuous) symmetries ($x_T; g_T$) and a set of (geodesic family-path) coordinates ($x_P; g_P$); then the curved Klein-Gordon OPE may be expressed as:

$$\begin{aligned} \mathcal{D}_{KG} &:= \nabla \times_g \nabla \rightarrow (\partial_T \times_{g_T} \partial_T) \otimes \mathcal{D}_{KG}^{(P)} & \Rightarrow & \mathcal{D}_{KG}^{(P)} = \frac{1}{\sqrt{|g_P|}} \partial_b (\sqrt{|g_P|} g_P^{ab} \partial_a [\cdot]) \\ \mathcal{D}_{KG}[\cdot] = 0 \text{ and } [\cdot] &\sim e^{\pm i A_\mu^{(T)} x_\mu^{(T)}} [\cdot] & & \frac{1}{\sqrt{|-g|}} \partial_b [\sqrt{|-g|} g_P^{ab} \partial_a [\cdot]] = -g_T^{\mu\nu} \partial_\mu \partial_\nu [\cdot] \\ & & & \bar{\mathcal{D}}_{KG}^{(P)}[\cdot] + \frac{1}{2} [\ln g_T]_{,b} g_P^{ab} \partial_a [\cdot] \sim g_T^{\mu\nu} \lambda_\mu^{(l)} \lambda_\nu^{(n)} a_{(l)(n)}[\cdot] \end{aligned} \quad (6)$$

In particular, the (path-like) projected KG operator covers the *out*-multiplet $g_{(l)(n)}^{\mu\nu}$ exactly iff there exists a formal envelope basis $[\cdot] \rightarrow f(\cdot)[*]$ such that $\bar{\mathcal{D}}_{KG}^{(P)}[*] \sim f(\cdot) g_T^{\mu\nu} \lambda_\mu^{(l)} \lambda_\nu^{(n)} a_{(l)(n)}[*]$ OR the charge states cover the (geometrically) mixed dispersion (*in*-measure): $\frac{1}{2} [\ln g_T]_{,b} g_P^{ab} \partial_a [*] \sim g_T^{\mu\nu} \lambda_\mu^{(l)} \lambda_\nu^{(n)} b_{(l)(n)}[*]$ (also, some mixture thereof). Importantly, the envelope basis naturally represents a field-charge extension while the covered dispersion matching state represents a topological locking; accordingly, the appearance of both is a natural form a UV/IR mixing. Throughout, consider metric ring-orderings (a.k.a., line element ordering) as indexing the T/P sectors at the $r := x_P^{(1)}$ -entry. So, consider the $D = d$ dimensional Schwarzschild with

defining the (right-adjoint) relative motion of the irregular poles towards the (left-)co-domain, $\mathbb{I}(r+i) = g_{LR} L(r)[M](r)[R(i)\mathbb{M}_i]$ implies $R(i)\mathbb{M}_i = [L(r)\mathbb{M}_r]^{-1} g_{LR}^{-1}$, universally categorizes the right OPE up to the global co-kernel multiplet represented by stokes parameters.

¹⁴This is exactly how the Method of Images is used to classically resolve the conductive transport (strong-E) problem in electromagnetism and can be generically recognized as the (dual) magnetic field. The difference with strong-G is that the extension has complete coordinate support that (always) extends the (functor-)interloper almost everywhere (representing a magnetized index, rather than a magnetized coordinate).

a probe path dimensionality $d_P = 1$ ¹⁵

$$ds^2 = -\Delta dt^2 + r^2 d\Omega_{d-2}^2 + \frac{dr^2}{\Delta} \Rightarrow \begin{aligned} [g_T] &= \begin{bmatrix} -\Delta & \\ & r^2[\Sigma] \end{bmatrix} & | -g_T | &= -\Delta r^{2d} f(\phi_\alpha) \\ [g_P] &= [\Delta^{-1}] & \Rightarrow [\ln |g_T|]_{,b} g_P^{ab} [\cdot]_{,a} &= \Delta \partial_r [\ln [r^{2d} \Delta]] [\cdot]_{,r} \end{aligned} \quad (7)$$

$$\text{gives:} \quad \left(r^2 \Sigma^{ab} \lambda_a^{(l)} \lambda_b^{(m)} a_{(l)(m)} - \frac{\omega^2}{\Delta} \right) [\cdot] = \sqrt{\Delta} \partial_r \left[\sqrt{\Delta} [\cdot]_{,r} \right] + \frac{\Delta}{2} \partial_r [\ln [r^{2(d-2)} \Delta]]_{,r} [\cdot]_{,r} \quad (8)$$

$$\left(\sum K_L - \frac{\omega^2}{\Delta} \right) [\cdot] = \frac{1}{r^{d-2}} \partial_r [r^{d-2} \Delta \partial_r [\cdot]] \quad (9)$$

where¹⁶ $\Delta = 1 - \frac{2M}{r^{d-3}} + \frac{r^2}{L^2} = (r^{d-3} L^2)^{-1} \prod_{i=1}^{\mathcal{K}-1} (r - r_i)$, with $\mathcal{K} = d + 1 - \epsilon$; also, the $a_{(l)(m)}$ algebra descends from the $d - 2$ -dimensional spherical harmonic (functional) family as an (*in*-coordinate) field extension from it's canonical embedding in \mathbb{R}^{d-1} ,¹⁷ Further, consider the $d = 4$ Kerr-AdS geometries (with path dimensionality $d_P = 2$)

$$ds^2 = \frac{\Delta_\theta}{\Sigma} \left(\sin^2 \theta \left(\frac{(r^2 + a^2)}{\Xi} d\phi - a dt \right)^2 - \frac{\Delta}{\Delta_\theta} (dt - \frac{a}{\Xi} \sin^2 \theta d\phi)^2 \right) + \frac{\Sigma}{\Delta_\theta} \left(\frac{\Delta_\theta}{\Delta} dr^2 + d\theta^2 \right) \quad (11)$$

where

$$\Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2 \theta, \quad \Xi = 1 - \frac{a^2}{l^2}, \quad \Sigma = r^2 + a^2 \cos^2 \theta, \quad (12)$$

$$\Delta = (r^2 + a^2) \left(1 + \frac{r^2}{L^2} \right) - 2Mr = \frac{1}{L^2} \prod_{i=1}^4 (r - r_i) \quad (13)$$

Using $\frac{l^2(\Delta_\theta - \Xi)}{a^2} = \sin^2 \theta$ and $r^2 + a^2 = \Sigma + l^2(\Delta_\theta - \Xi)$, and naming¹⁸:

$$\gamma^2[r, \cos \theta] := \frac{\Sigma}{\Delta_\theta}, \quad \Gamma_\Delta[r, \theta] := \frac{\Delta}{\Delta_\theta}, \quad \omega_\phi[\cos \theta] = \frac{l^2(\Delta_\theta - \Xi)}{a\Xi} = \frac{a \sin^2 \theta}{\Xi}$$

$$H[r, \cos \theta] := \Gamma_\Delta - a\Xi\omega_\phi, \quad J[r, \cos \theta] = \omega_\phi \Sigma, \quad F[r, \cos \theta] := \left(\frac{J}{a\Xi\omega_\phi} + 2\omega_\phi \right) J$$

yields:

$$ds^2 = \frac{1}{\gamma^2[\Sigma]} \left(-H[\omega_\phi, \Gamma_\Delta] (dt - \omega_\phi[\cos \theta] d\phi)^2 - 2J[\omega_\phi, \Sigma] dt d\phi + F[\omega_\phi, J] d\phi^2 \right) + \gamma^2[\Sigma] [\Gamma_\Delta^{-1} dr^2 + d\theta^2]$$

¹⁵ meaning there are no convex embedding connections between it's subdomains a.k.a., "the particle-shells fall straight down/up to/from the source".

¹⁶ with $\epsilon = 0$ for d odd-dimensional spacetimes or $\epsilon = 1$ for d even-dimensional spacetimes

¹⁷ meaning they represent a field expansions over $\frac{1}{r^2}$ which preserve their (sub-global) projected sub-area measures: $(\gamma_\alpha - \frac{1}{r^2}) \in \mathbb{F}$; equivalently, $\mathcal{F}[z] \rightarrow J \left(\frac{\mathcal{F}^{2d}}{r^{2d}} \right)$. By the Second Isomorphism Theorem, the minimal

representation is canonically (sub-)separable (on-shell) s.t. $r^2 \Sigma^{ab} \lambda_a^{(l)} \lambda_b^{(m)} a_{(l)(m)} \sim \sum K_L$ because the free Euclidean extension $(\gamma_\alpha - x_\alpha) \in \mathbb{F}$ is clearly separable; in fact this is exactly the idea with Lagrangian multipliers (*in*-representations) and Grassman variable (*out*-)extensions, and gives the final result.

¹⁸ Where the *in*-domain has been specifically registered to the *out*-harmonic cover by inputting the path coordinate as a (smooth) trigonometric push $\tilde{g}[\dots, \theta] \rightarrow g[\dots, \cos \theta]$.

In particular¹⁹, defining $\chi_k := \ln \left[\sqrt{\frac{F}{|-g|\omega_\phi}} \right]$, note the relation:

$$H = \Gamma_\Delta + \sqrt{\frac{a\Xi F}{\omega_\phi}} \sinh \ln \left[\frac{\sqrt{a\Xi F}}{J} \right] \equiv \Gamma_\Delta + \sqrt{a\Xi|-g|} e^{\chi_k} \sinh \chi_k \quad (17)$$

It is also useful to consider the flat space time forms²⁰, given by $\lim_{l \rightarrow \infty} -\frac{d-1}{l^2} = \lim_{l \rightarrow \infty} \Lambda \rightarrow 0$:

$$\Delta = r^2 + a^2 - 2Mr \equiv \prod_{\pm} (r - r_{\pm}) \quad (18)$$

$$\bar{\gamma}^2 := \Sigma, \quad \bar{\Gamma}_\Delta[r, \theta] := \Delta, \quad \bar{\omega}_\phi[\cos \theta] = a \sin^2 \theta \quad J[r, \cos \theta] = \omega_\phi \Sigma, \quad (19)$$

$$H[r, \cos \theta] := \Delta - a^2 \sin^2 \theta = \Sigma - 2Mr, \quad F[r, \cos \theta] := \sin^2 \theta (\Sigma + 2a^2 \sin^2 \theta) \Sigma \quad (20)$$

Then, following the procedure as in Schwarzschild:

$$\begin{aligned} [g_T] &= \frac{H}{\gamma^2} \begin{bmatrix} -1 & -\frac{J}{H} + \omega_\phi \\ -\frac{J}{H} + \omega_\phi & \frac{F}{H} - \omega_\phi^2 \end{bmatrix} & | -g_T | &= \frac{\Delta \Delta_\theta \omega_\phi}{a} \equiv \frac{\Delta J}{a \Xi \gamma^2} & \sqrt{| -g |} &= \frac{J}{\sqrt{a \Xi \omega_\phi}} \\ & & |g_P| &= \frac{J \gamma^2}{\omega_\phi \Delta} & \sqrt{\frac{| -g_T |}{g_P}} &= \frac{\Delta}{\gamma^2} \sqrt{\frac{\omega_\phi}{a \Xi}} \end{aligned} \quad (21)$$

$$[g_P]^{-1} = \frac{1}{\gamma^2} \begin{bmatrix} \Gamma_\Delta & \\ & 1 \end{bmatrix} \Rightarrow [\ln |gn_T|]_{,b} g_P^{ab} [\cdot]_{,a} = \frac{1}{\gamma^2} ([\ln \Delta]_{,r} \Gamma_\Delta \delta_r^b + [\ln \omega_\phi \Delta_\theta]_{,\theta} \delta_\theta^b) [\cdot]_{,b}$$

Noting that the global symmetry sector is already (*in*-representation) separated²¹, (sub-

¹⁹Note the sub-functionalizations in the form above shows exactly how the physical functionals, $\{\omega_\phi, \Sigma, \Delta_\theta, \Delta\}$ envelope one another. For example, ω_ϕ is r -uniform (representing the metric's canonical and global spin projection measure a.k.a., the black hole's (AdS)-uniform steridian angular momentum flux: $\frac{[\Omega]}{[\text{sr}]} = \left[\frac{[A \rightarrow a^2 \sin^2 \theta]}{[(\sqrt{a})^2 \rightarrow a \Xi]} \right]$), and enters (functionally-)everywhere in the T-symmetry sector, but nowhere in the P(ath)-sector. Instead, the P-sector is/(has an *in*-representation with fixed, non-uniform poles) determined by $\frac{\Delta_\theta}{\Delta}$; *in*-representation, the P/T sectors are inversely weighted (in the direct sense) by a similar ratio $\frac{\Sigma}{\Delta_\theta}$; point-wise, these are the exclusive points where Δ_θ enters. Σ , *in*-turn, only enters as a (direct) coupling in $J = \omega_\phi \Sigma$, which itself (along with ω_ϕ) functionalizes F . Remembering that the T-sector can always be KG-dualized into second order, (dim- T)-multinomials under global wave-gauge fixing, this immediately gives the sense of a (sub)-spectral tower (built from ideals $I < \omega_\phi, \Sigma, J >$); in fact, despite the non-(sub)uniformity in the P-poles, note that $\delta \rightarrow 0$ uniformly descends $< H > \rightarrow < \omega_\phi >$ which makes the above intuition of a (sub)-spectral tower exact. Critically, this final connection gives a very clear idea of what is meant by the concept of "emergence": sometimes fixed-point, P(ath sub-)symmetry breaking may be *in*-(direct)-correspondence with a (P-directed) global (*out*-)T gauge.

Intuitively, this has a clear picture. Imagine an excessively cascading ocean wave. During the tumbling phase high frequencies can't propagate without relatively huge amplitudes because of the air gaps supplanting skin-skin bubble transmissions; accordingly, these transmission states must also be relatively low frequency (at scale) because these channels walls are also relatively closed (for long periods relative to the fastest propagators). Only low frequencies may traverse both the time and space gaps; dually, high frequency symmetries are hidden in the foam. Alternatively, in the lossy limit, low frequencies will almost always be shadowed by higher frequency interactions; in particular, edge reflection(/backreaction) is generically chaotic. Empirically this is can be shown by the empirical/analytic work of [12] on the water duality between BH dynamics and (near-skin resolved) water-vortex dualities, which follow exactly from their analytic KG (near-fixed) P-modes resolving otherwise (partial-pressure gauged) s(kin)-modes.

²⁰Notice $\Sigma - \Delta = 2Mr - a\omega_\phi$ shows Σ and Δ represent linear crossing relations between $(r, \omega_p \text{hi})$; indeed, $\Sigma - \Delta = 0 \Rightarrow \frac{r[\theta]}{2M} \sim \left(\frac{a \sin \theta}{2M} \right)^2 < 1$ shows that the (Σ, Δ) -level forms always lie behind the thermodynamic horizon, and that black hole mass-spin linear mixing relations can be represented as defect-propagators under the Cauchy-(thermodynamic)-density state duality [10] [13]

²¹*in*-field, as opposed to the sub-ordering of spherical embeddings in higher dimensional spherical har-

)separability is left to the (sub-)functional forms; the KG equation here reads²²:

$$\bar{\mathcal{D}}_{KG}^{eff}[\cdot] = \begin{pmatrix} \sqrt{\Delta}\partial_r(\sqrt{\Delta}\partial_r[*]) + \frac{\Delta_r}{2}\partial_r[*] \\ +\sqrt{\Delta}\partial_\theta(\sqrt{\Delta}\partial_\theta[*]) + \frac{\partial_\theta(\Delta\omega_\phi)}{\omega_\phi}\partial_\theta[*] \end{pmatrix} = \frac{a\sqrt{\Xi}}{\omega_\phi\Delta} \lambda_T^\rightarrow \cdot \begin{bmatrix} F - H\omega_\phi^2 & J - \omega_\phi H \\ J - \omega_\phi H & -H \end{bmatrix} \lambda_T^\rightarrow[*] \quad (22)$$

$$= \frac{a\sqrt{\Xi}}{\Delta} \lambda_T^\rightarrow \cdot \begin{bmatrix} \frac{\Sigma^2}{a\Xi} + (2\Sigma - H)\omega_\phi & \Sigma - H \\ \Sigma - H & -\frac{H}{\omega_\phi} \end{bmatrix} \lambda_T^\rightarrow[*] \quad (23)$$

Importantly, this shows that the effective KG equation's separability index is, in this case, simply a direct (ω_ϕ, Δ) idealization: the separability condition descends exactly to a T-*out* ("eigenomial") functional division condition²³.

In fact, it is clear that, near the zeros of Δ , all the RHS T-functions are directly divisible by ω_ϕ , $\langle H, J, F \rangle |_{\Delta \rightarrow 0} \subset \langle \omega; \Sigma \rangle$ and further that the (accumulation extended sub-ring) Σ dependence comes exactly from J : $\langle H, J, F \rangle |_{\Delta \rightarrow 0} \cap \langle \Sigma \rangle = \langle H/\Sigma, F/J \rangle |_{\Delta \rightarrow 0} \cup \langle \omega_\phi (= J/\Sigma) \rangle$. Then, by Hilbert's Nullstellensatz (and noting that $\langle \omega_\phi \Delta \rangle = \langle \omega_\phi \rangle \langle \Delta \rangle$, as well as that $\frac{H}{\omega_\phi \Delta} = \frac{1}{\omega_\phi \Delta_\phi} + \frac{a\Xi}{\Delta}$ is [sub-]ring separated), the RHS is divisible by $\omega_\phi \Delta$ iff $\langle \Sigma/\Delta \rangle$ is not a formal ring extension, which it is and as such there should be poles; still, applying the same reasoning to the residue ring shows that the RHS is algebraically (sub-index (r, θ) separable) iff $\langle \Sigma/\Delta \rangle \cap \langle \Delta \rangle \subset \langle \omega_\phi, \bar{\Sigma}[\Delta, \theta] \rangle$, for some $\bar{\Sigma}$ functional field extension, which it is²⁴.

monics

²²Where the T/P-separated KG form relation was (equivariantly) re-scaled to be in their fully separated form: $\{\mathcal{D}_{KG} = 0\} \equiv \mathcal{R}_{KG} \rightarrow \bar{\mathcal{R}}_{KG} := [\sqrt{\Delta\Delta_\theta g_P}] \mathcal{R}_{KG}$. In the massless case, this is just multiplying (here in the canonical real-number Field sense) to clear a denominator; but considering some non-homogeneous mass contribution, this dual co-set may itself only exist up to some particular massive spectrum indexing, which (assuming it exists) in turn would manifest as a characteristic mass (spectral-family) dispersion envelope sourced by the (mass-adjoint, sub-separated) KG-null kernel. Deductively, massive source/black hole thermal separability is possible iff the massive sub-field has a strongly-determined null-representation with a P-exact T (hyper)-plet mode. In this simplified case, the formal schematic would read: $\mathcal{R}_{KG}^{m^I} \rightarrow \bar{\mathcal{R}}_{KG}^{m^I} := [m^I]^T \cdot ([\sqrt{\Delta\Delta_\theta g_P}] \mathcal{R}_{KG})$.

In fact, Holography can be principally phrased as the converse formulation: when can (gravitational) T-symmetries be represented by some (closed, exact) family of matter charge(d current residues)? One of the clearest applications of this is the AdS_3/CFT_2 quantum mechanical correspondence, whereby the (Path-)asymptotic T-degeneracies of empty AdS_3 spacetime may be actually shown to be 1-1 with the spectrum of a family of $d = 2$ conformal fields (because the above onto relationship is into, and thus equivariantly invertible—although the size of the family is montrously large [14]). Therein, thermalization modes in this spacetime may be considered to have an asymptotically emergent quantized point (or, as the emergence of a conformal symmetry-quantization of the T-*out* field residues *in* the P fixed-point gauging).

²³And explains the interpretation of ω_ϕ as representing an angular radiative sub-weighting

²⁴And can be seen by noting that $\Delta + 2M\sqrt{\Sigma + a(\Xi\omega_\phi - a)} = (\Sigma + a\Xi\omega_\phi) \left(\Xi + \frac{\Sigma + a\Xi\omega_\phi}{l^2} \right)$ shows that $\Sigma, \Delta, \omega_\phi$ cannot all be simultaneously (functionally) zero (for all $M, a > 0$). For example, $\{\Delta = 0\} \wedge \{\Sigma = 0\}$ (representing the Kerr spacetime's origin in the ultra-low mass/spin limit) formally induces: $4M^2\omega_\phi = (a\Xi)^3 \left(\frac{\omega_\phi}{l^2} \right)^2 \left(\frac{l^2}{a} + \omega_\phi \right)^2$. Using Hawking's formula, and considering the thermal entropy as a spin density $4M^2\omega_\phi \sim S^{(sp)}[\omega_\phi]$ and the $(a$ -uniform envelope) as a space(/spin)-like volume density $(a\Xi)^3 \sim \bar{V}_{(sp)}^{(a)}$ casts this similarly to a 4pt (scalar-)spin connection between l -modes $A = \frac{\omega_\phi}{l^2} >: S^{(sp)}[\omega] \sim \bar{V}_{(sp)}^{(a)} || < A >< \omega_\phi >< (aA)^{-1} + 1 > ||^2$ [15][16].

Somewhat similarly, $(\Delta, \omega_\phi) \rightarrow (0, 0)$, now representing the (angular-)North/South near-horizon form, induces the contact degeneracy $S^{(L)}[\Sigma] = \Sigma^2 \left(\Xi + \frac{\Sigma}{l^2} \right)^2$, which can be seen as a linear shift in the dual

Indeed, it can be shown that the result may be separated under global T-eigenvalues $\vec{\lambda}_T = (\omega, m)$ as: [18]:

$$[\cdot] \rightarrow e^{(\omega + \frac{a}{l^2}m)t + m\phi} S[r]R[r] \quad \Rightarrow \quad (24)$$

$$\left[\left(\begin{array}{c} \frac{1}{\sqrt{\omega_\phi}} \partial_\theta [\sqrt{\omega_\phi} \Delta_\theta \partial_\theta] - \frac{am^2 \Delta_\theta}{\Xi \omega_\phi} + \omega^2 a (a - \Xi \omega_\phi) \\ \partial_r [\Delta \partial_r] + \sum_i \frac{(r_i^2 + a^2)^2 (\omega - \Omega_i m)^2}{\Delta_{,r}|_{r=r_i} r - r_i} - l^2 \Xi \omega^2 + (1 - \Xi) m^2 \end{array} \right) - \left(\begin{array}{c} -K_l \\ K_l \end{array} \right) \right] \odot \left(\begin{array}{c} S[\theta] \\ R[r] \end{array} \right) = \vec{0} \quad (25)$$

Using $\Delta_\theta = \Xi(1 + \frac{a\omega_\phi}{l^2})$ shows the top ($S[\theta]$ -) spectrum is Bessel-like²⁵ and therefore always gives a well defined spherical spectral basis: $\{K_l\} \subset \{a_{(l)(n)}\}$.

Resultantly, the KG equation can be equivariantly reduced to it's (spectrally-patched) radial ansatz:

$$\left(\partial_r [\Delta \partial_r] + \sum_i \frac{(r_i^2 + a^2)^2 (\omega - \Omega_i m)^2}{\Delta_{,r}|_{r=r_i} r - r_i} - l^2 \Xi \omega^2 + (1 - \Xi) m^2 - K_l \right) R = 0 \quad (26)$$

This is exactly a $d = 1$ dimensional, second order ordinary differential equation which is always solvable by Picard's Lemma.

3.2 Fuch's Relational Form

Letting

$$\alpha_i = \frac{(r_i^2 + a^2)(\omega - \Omega_i m)}{\Delta_{,r}|_{r=r_i}} \quad \wedge \quad \Delta^* [K_l] = -l^2 \Xi \omega^2 + (1 - \Xi) m^2 - K_l \quad (27)$$

$$\text{where,} \quad \Omega_i = \frac{a}{r_i^2 + a^2} \left(1 + \frac{r_i^2}{L^2} \right) \quad (28)$$

response form at affine infinity (along l). Lastly, and most clearly, $(\Sigma, \omega_\phi) \rightarrow 0$, representing the low-spin origin (at any Mass), gives $\Delta \sim -2iaM$, or a manifestation of the KTN duality of the Kerr spacetime's conformal shift symmetry[17]: $\sqrt{S^M[a]} \sim^* \Delta$. Taking $l^2 \rightarrow \infty$ in each of the above constructions shows the flat cases give, respectively: 1) a spin connection between 0-modes and a spin- pushed kernel tower on the identity mode, 2) a 1 - 1 exact degeneracy in the response Σ response $S^{(L)}[\cdot] = [\cdot]^2$, 3) a limit independent (l -uniform) r -kernel constraint that represents a universal *in*-envelope form (or, thinking in terms of field extensions, as the minimal $\Gamma \rightarrow 0$, $a \rightarrow 0$ descent gauge resolution, *in*-uniformity; because aM represents the black hole's (classical scalar) angular momentum). Finally idea can also be seen generally manifested in $\Gamma_\Delta = H - \frac{J}{\sqrt{\omega_\phi}} e^{\chi_k[\omega_\phi, J]} \sinh \chi_k[\omega_\phi, J]$, which shows how the functional division Γ_Δ may be represented as an analytic (but non-uniform) $\{\omega_\phi, J\}$ -ring extension of H

²⁵alternatively monodromy techniques may be employed, exactly as exhibited for the $R[r]$ *out*-form: note the top equation is pole fixed, at p-order (a.k.a., derivative order), where it envelopes as $\partial_\theta^2 + \Xi(\omega_\phi^{-1} + \frac{a}{l^2}) \partial_\theta \sim 0$. Using $\frac{a}{l^2} \partial_{\omega_\phi} \sin^2 \theta = \frac{\Xi}{l^2}$ gives the sense that this may represent an affine connection to the full Bessel branch; indeed, [18] shows an exact $a\omega$ -perturbative spectral closure over the spherical harmonics.

the Kerr KG radial ansatz may be written:

$$\left(\partial_r [\Delta \partial_r] + \sum_i \frac{\Delta_{,r}|_{r=r_i} \alpha_i^2}{r - r_i} + \Delta^* [K_l] \right) R = 0 \quad (29)$$

Considering analytic solutions, consider linearly sub-indexed²⁶ solutions forms:

$$R_{n^*}[r] = \sum_{n=i^* \in \mathbb{N}^+} a_n (r - r_{n^*})^{\sigma[n]} \quad \text{s.t.} \quad \sigma[n] = \sigma + n \quad \text{with} \quad \Delta = \prod_{i \in \{1,2,3,4\}} (r - r_i) \quad (30)$$

$$\Rightarrow \partial_r \Delta = \sum_i \prod_{j \neq i} (r - r_j)$$

giving:

$$\Delta \sum_{n=i^* \in \mathbb{N}^+} a_n \left(\partial_r^2 + \sum_i \frac{1}{r - r_i} \left(\partial_r + \sum_i \frac{\Delta_{,r}|_{r=r_i} \alpha_i^2}{(r - r_i) \prod_{j \neq i} (r - r_j)} \right) + \Delta^* [K_l] \right) (r - r_{n^*})^{\sigma[n]} = 0 \quad (31)$$

an immediate simplification can be made if we let the solution (function basis) index n^* match the pole index i , giving:

$$\Delta \sum_{n=i^* \in \mathbb{N}^+} a_n \sum_i \left(\left(\sigma[n](\sigma[n] - 1) + \sigma[n] + \frac{\Delta_{,r}|_{r=r_i} \alpha_i^2}{\prod_{j \neq i} (r_i - r_j)} \right) (r - r_i)^{\sigma[n]-2} + \Delta^* [K_l] (r - r_i)^{\sigma[n]} \right) = 0 \quad (32)$$

Remembering that the (*out* basis) index is lower bounded at $\sigma[0] = \sigma$, and that the poles of Δ are separated²⁷ gives a bottom fixing form as:

$$\sigma_n^{(\pm)} = \pm i \alpha_n \Rightarrow \sum_{n^{(\pm)}} \sigma_n = 0 \quad (33)$$

$$\wedge \quad R_j^{(\pm)} = (r - r_j)^{\pm i \alpha_j} \sum_{n=i^* \in \mathbb{N}^+} a_n (r - r_j)^n \equiv e^{\pm i \alpha_j \ln(r - r_j)} (1 + O(r - r_j)) \quad (34)$$

Then, noticing $\partial_r [\Delta \partial_r [e^{-i \alpha_n \ln \Delta} \tilde{R}]] = e^{-i \alpha_n \ln \Delta} \left(\partial_r [\Delta \partial_r \tilde{R}] - 2 \sum_n i \alpha_n \partial_r \tilde{R} - \frac{\Delta_{,r} \alpha_n^2}{\Delta} \right)$ shows that we can exactly shift out this degeneracy with:

$$\tilde{R} = e^{\sum_n i \alpha_n \ln(r - r_n)} R \Rightarrow \tilde{\sigma}_n (\tilde{\sigma}_n - 2i \alpha_n) = 0 \Rightarrow \tilde{\sigma}_n \in \{0, 2i \alpha\} \quad (35)$$

Using, $\sum_i \alpha_i = 0$ [10] shows that $\sum \sigma_n = 0$ still.

Finally, noticing the divergent covering frequency as $r \rightarrow \infty$ it's important to check the compactification of the far-field solution form. Transforming to coordinates $r \rightarrow \frac{1}{x}$, and

²⁶In the sense that the functional basis projection is compact over $\mathbb{R}[r]$

²⁷So, correspondingly, $\lim_{r \rightarrow r_i} f[R_{n^*}[r]] \rightarrow \lim_{r \rightarrow r_i} r_i f_i[R_i[r]]$

temporarily indexing the Δ -form by it's polynomial interpolative index²⁸, $|I|$. gives:

$$\begin{aligned}
\Delta\left[\frac{1}{x}\right] &= \frac{1}{x^{|I|}} \prod_i (1 - xr_i) , \quad \Delta\left[\frac{1}{x}\right]_{,x} = \frac{1}{x^{|I|+1}} \left(|I| - x \sum_i r_i \right) \\
\partial_{\frac{1}{x}} &= -x^2 \partial_x \\
\partial_{\frac{1}{x}} \left[\Delta\left[\frac{1}{x}\right] \partial_{\frac{1}{x}} \right] &= x^2 \partial_x \left[x^{-|I|+2} \prod_i (1 - xr_i) \partial_x \right] = x^{-|I|+4} \left(\partial_x \left[\prod_i (1 - xr_i) \partial_x \right] + \frac{(-|I|+2) \prod_i (1 - xr_i)}{x} \partial_x \right) \\
x^4 \Delta^{|I|} &\left(\begin{aligned} &\partial_x^2 - \left(\frac{\sum_i r_i}{\prod_i (1 - xr_i)} + \frac{|I|-2}{x} \right) \partial_x \\ &+ x \sum_i \frac{\Delta_{,r}|_{r=r_i} \alpha_i^2}{(1-r_i x)^2 \prod_{j \neq i} (1 - xr_j)} + \frac{\Delta^*[K_i]}{\prod_i (1 - xr_i)} \end{aligned} \right) X = 0
\end{aligned} \tag{36}$$

Noticing the Q_x -residues directly map to the r_i -represented patches leads instead to consider the $(r_i$ sub-)residue patch at $x = 0$, giving the Fuch's relation:

$$X[x] = \sum_n b_n x^{\sigma_x[n]} \Rightarrow \sigma_x(\sigma_x + 1 - |I|) = 0 \tag{37}$$

²⁸Given by the number of poles in the radial function under the canonical $R[x]$ -free projective index $\delta[\cdot]$; this gives a possible interesting connection to general residue minimization algorithms, whereby high polynomial numerical boundary harmonic minimization (thermalization) integration methods may necessarily introduce large conformal winding gauge fixings manifested in boundary decay fitting splines (typically gauged by the Weyl mean free iso-constraint gauges (such as the globally harmonic ADM gauge with fixed [relaxation cutoff]boundary conditions or Brown-York dispersion (resolution sub-volume, $max[S = \int_{t^*} f[V] e^{-f[V]} < \mu$) counter (Wilson-)cutoffs. This can exactly be manifest by the compactified waves in recent extremely high (frequency Fourier space) r -localization schemes under (hyper-core threading — creating a $lcmn_{thread}, d_{sub-calls}$ frequency basis enhancement) supercomputer simulations [13]

This gives the global form, and noticing²⁹ $|I| - 1 = 3 = \frac{d}{2} + 1$ (in $d = 4$) gives:

$$\sum_{n \in \{i^*, x\}} \sigma_n = |I| - 1 = 3 = \frac{d}{2} + 1 \quad \wedge \quad X_0^{(\pm)} = x^{\{3,0\}} \sum_{n=i^* \in \mathbb{N}^+} b_n x^n \quad (42)$$

Finally, it regularizing all the bottom forms to have flat exponents gives the following forms[22]:

$$\begin{aligned} & \left[\partial_r^2 + \left(\sum_{i=1}^4 \frac{1-2i\tilde{\alpha}_i}{r-r_i} \right) \partial_r - \sum_i \sum_{j \neq i}^4 \frac{\tilde{\alpha}_i(i+\tilde{\alpha}_j)}{(r-r_i)(r-r_j)} \right. \\ & \left. + \sum_{i=1}^4 \frac{\tilde{\alpha}_i^2}{(r-r_i)^2} \left(\frac{\Delta'(r_i)}{\prod_{j \neq i} (r-r_j)} - 1 \right) + \frac{\Delta^*}{\Delta} \right] \tilde{R}(r) = 0 \end{aligned} \quad (43)$$

and, defining: $Q[r] = \prod_i e^{-i\tilde{\alpha}_i \ln[r-r_i]} \equiv x^{-i \sum_i \tilde{\alpha}_i} \prod_i e^{\tilde{\alpha}_i \ln[1-r_i x]}$

$$\left[\frac{x^2 \partial_x (x^2 \Delta \partial_x)}{x^4 \Delta Q} + x^{-1} \sum_i \frac{\alpha_i^2 \Delta_{,r} |_{r \rightarrow r_i}}{(1-r_i x) x^2 \Delta} + \frac{\Delta^* - \sum K_L}{x^4 \Delta} \right] Q \tilde{R} = 0 \quad (44)$$

$$\Rightarrow (\sigma_\infty + i \sum_i \tilde{\alpha}_i)(\sigma_\infty - 3 + i \sum_i \tilde{\alpha}_i) = 0 \quad \Rightarrow \sum_i \tilde{\alpha}_i + \sum \sigma_\infty = \frac{d}{2} + 1 \quad (45)$$

In the (flat) Kerr case, by using $\Sigma + a\omega_\phi = \Delta + 2Mr$, the (pseudo-)source field is separable,

²⁹In Schwarzschild, $\Delta[\frac{1}{x}] = x^{\epsilon-3} \prod_i^{\mathcal{K}-1} (1-r_i x)$, which shows that the ∞ -index of d -dimensional AdS-Schwarzschild alternates depending on the oddness/evenness of the dimensionality (a.k.a., whether-or-not the Stokes dual defines a surface or a line): $|I| = \epsilon - 3$. Still, here the wave equation reads:

$$0 = x^{d-2+\epsilon} \prod (1-r_i x) \left(\frac{1}{\prod (1-r_i x)} \partial_x [\prod (1-r_i x) \partial_x [\cdot]] + \frac{1-\epsilon-d}{x} \partial_x [\cdot] + \left(\frac{x^{2(1-\epsilon)} \omega^2}{\prod (1-r_i x)^2} - \frac{x^{-2(1+\epsilon)} \sum K_L}{\prod (1-r_i x)} \right) \right) X \quad (38)$$

which gives the $x = \frac{1}{r}$ Fuch's (bottom crossing), $d = 4$, form:

$$r_i^2 \sigma_i^2 + \frac{r_i^2 \omega^2}{\Delta_{,r}^2} = 0 \quad \Rightarrow \quad \sigma_i = \pm i \frac{\omega}{\Delta_{,r}} \quad (39)$$

$$\sigma_0(\sigma_0 - (d+\epsilon)) = 0 \quad \Rightarrow \quad \{\sigma_0^{(+)}, \sigma_0^{(-)}\} = \{d+\epsilon, 0\} = \{\mathcal{K}-1, 0\} \quad (40)$$

This also shows that the spherical harmonics conformally propagate in higher dimensions (where every tangent space representation always has a local $d = 2k$ embedding structure which is canonically self-dual [19]) when $\epsilon \neq 0$:

$$\sigma_\epsilon(\sigma_\epsilon - 1 - d) = \sum K_L \quad (41)$$

Still, because the spherical algebra is free (in Schwarzschild), a basis may always be chosen s.t. $\sum K_L = 0$ (under advanced/retarded matchings).

More intuitively, this seems to indicate that, the thermodynamics in the self-dual sector can be well-controlled enough to spectrally match both sides (and gives some insight into the sense that correlators in $d = 4$, $\mathcal{N} = 4$ Super-Yang-Mills algebraically reduce (in the long-chain limit) towards the counter (*inter*-) propagating gravitational (super residue) charges [20] [21] $\langle O_{K_L^-} \rangle \sim \|(\phi_1, \phi_2)\|_{\mathcal{M}^d}$.

without the affine ω, m connection (as in the AdS case):

$$\bar{D}_{KG}^{eff}[\cdot] = \frac{a}{\Delta} \lambda_T^{\vec{\cdot}} \cdot \left(\left[\begin{array}{cc} \frac{(\Delta+2Mr)^2}{a} & 2Mr \\ 2Mr & a \end{array} \right] - \Delta \left[\begin{array}{cc} \omega_\phi & \\ & \omega_\phi^{-1} \end{array} \right] \right) \lambda_T^{\vec{\cdot}}[\cdot] \quad (46)$$

Defining $\tilde{\alpha}_\pm = \frac{\Delta, r|_{r \rightarrow r_\pm}}{r_\pm^2 + a^2} (\omega - \Omega_\pm m)$ and $\Delta^*[x] := \omega^2(x^{-2} + 2Mx^{-1} + 4M)$ results in the radial ansatz:

$$\left[\partial_r \Delta \partial_r + \sum_{\pm} \Delta, r|_{r \rightarrow r_\pm} \frac{\tilde{\alpha}_\pm^2}{r - r_\pm} + \Delta^* \right] R = K_L R \quad (47)$$

$$\text{Correspondingly,} \quad \sigma_i^{(\pm)} = \pm i \tilde{\alpha}_i \quad (48)$$

Then, the $\sigma \in \{0, 2i\tilde{\alpha}_i\}$ basis form is given by:

$$\begin{aligned} & \left[\partial_r^2 + \left(\sum_{i=1}^2 \frac{1 - 2i\tilde{\alpha}_i}{r - r_i} \right) \partial_r - \frac{2\tilde{\alpha}_+ \tilde{\alpha}_- + i(\tilde{\alpha}_+ + \tilde{\alpha}_-)}{\bar{\Delta}} \right. \\ & \left. + \sum_{i=1, j \neq i}^2 \frac{\tilde{\alpha}_i^2}{(r - r_i)^2} \left(\frac{r_i - r_j}{r - r_j} - 1 \right) + \frac{\bar{\Delta}^* - \sum K_L}{\bar{\Delta}} \right] \bar{R}(r) = 0. \end{aligned} \quad (49)$$

Still in the $\{\pm\tilde{\alpha}_i\}$ basis, $x = \frac{1}{r}$ -ansatz³⁰:

$$\left[\partial_x \left[\prod_{\pm} (1 - r_\pm x) \partial_x \right] + x^{-1} \sum_{\pm} \Delta, r|_{r \rightarrow r_\pm} \frac{\tilde{\alpha}_\pm^2}{1 - r_\pm x} + \omega^2 x^{-2} \left((2M)^2 + \frac{2M}{x} + x^{-2} \right) \right] X = K_L X \quad (50)$$

Here an irregular singularity develops simultaneous to a regular pole at $x \rightarrow 0$; considering the $\{\pm i\tilde{\alpha}_i\}$ letting³¹ $X = e^A \tilde{X}$

$$\left[\partial_x \left[\prod_{\pm} (1 - r_\pm x) \partial_x \right] + x^{-1} \sum_{\pm} \Delta, r|_{r \rightarrow r_\pm} \frac{\tilde{\alpha}_\pm^2}{1 - r_\pm x} + x^{-2} \omega^2 (4M^2 + 2Mx^{-1}) \right. \\ \left. + \prod_{\pm} (1 - r_\pm x) (\omega^2 x^{-4} + A_{,xx} + (A_{,x})^2 + 2A_{,x} \partial_x) \right] \tilde{X} = K_L \tilde{X} \quad (51)$$

Choosing $A = \frac{-i\omega}{x} + \ln x$ gives the indicial form (for the leading exponent of \tilde{X}):

$$2\omega(2M\omega + i\sigma_x) = 0 \quad \Rightarrow \quad \sigma_x = -2iM\omega \quad (52)$$

$$X_1 = e^{\frac{-i\omega}{x}} x^{-2iM\omega+1} (1 + O(x)) \quad (53)$$

Finally, the the other linearly independent solution may be found by the Wronskian inver-

³⁰Using $\Delta[\frac{1}{x}] = x^{-2} \prod_{\pm} (1 - r_\pm x)$.

³¹and noting $\partial[\Pi\partial[e^A[\cdot]]] = e^{A+\ln\Pi} [\partial[A\partial A] + 2\partial[A]\partial + \Pi^{-1}\partial[\Pi\partial]] [\cdot]$

sion method:

$$\begin{aligned}
W[x] = (x^2 \Delta)^{-1} & \Rightarrow X_{2,x} - \frac{i\omega - t(1-2Mi)}{x^2} X_2 \sim -\frac{e^{\frac{i\omega}{x} + (2iM\omega+1) \ln x}}{(1-i(2M\omega-x))(1-r_+x)(1-r_-x)} \\
X_{2,x} + \frac{X_{1,x}}{X_1} X_2 = -\frac{W[x]}{R_{1,x}} & \Rightarrow X_2 \sim e^{\frac{i\omega}{x}} x^{B_2} \Rightarrow B_2 = 2iM\omega + 3 \quad \forall \omega \neq 0 \quad (54) \\
& \rightarrow \frac{X_1}{X_2} = e^{\frac{-2i\omega}{x} - (4iM\omega+2) \ln x}
\end{aligned}$$

$$\text{and} \quad \sum_{\pm} \tilde{\alpha}_i + \lambda_{\infty} = 2 = |I| = \frac{d}{2} + 1 - (|I| - 1) \quad (55)$$

Correspondingly, the development of irregular poles (of the first kind) in the flat space limit may be regarded as (single) index (OPE) shift (in the boundary residue form, a.k.a., in the spectrally adjoint *out*-representation). In particular, the final line characterizes the "fake monodromy" as a residue (winding power gap) between the (boundary) regularized index measure³² of the radial pole fixing functional, Δ . Lastly, shifting the monomial form (fake monodromies) works exactly before, giving the shifted basis:

$$X_1 \sim e^{-\frac{i\omega}{x}} \quad X_2 \sim e^{\frac{i\omega}{x}} x^{4iM\omega+2} \quad (56)$$

Next is provided a slightly different formalism (from [10]) to handle general (irregular) singularities within a unified framework. Importantly, Picard's Lemma shows that a critical solution basis always exists to within the leading divergence modes (which can be used to iteratively fix the sub-leading divergences)[11]. Generally, consider no real restrictions on $(p[\cdot], q[\cdot])$:

$$(\partial_r^2 + p[r]\partial_r + q[r]) R = 0 \quad (57)$$

Generically, let $R = B[r]\tilde{R}[r]$ where $B[r] = \prod_i A_i$; then, for some real rational polynomial coefficients (which may, $i = m$) or may not $i \in \{s, r\}$ share rational zeros) $p \sim \frac{L_1}{M_1} = \left[\sum_{n=s} + \sum_{n=m} \right] \frac{P_n[r]}{(r^{bn} - r_n)^{\sigma_n^{(1)}}}$, $q \sim \frac{L_2}{M_2} = \left[\sum_{k=m} + \sum_{k=r} \right] \frac{Q_k[r]}{(r^{bk} - r_k)^{\sigma_k^{(2)}}}$ (where $b_{i/k} \in \{1, 2\}$ under the Real Remainder theorem,

$$0 = B \left(\begin{aligned} & \partial_r^2 \tilde{R} + \left(\left[\sum_{n=s} + \sum_{n=m} \right] \frac{P_n[r]}{(r^{bn} - r_n)^{\sigma_n^{(1)}}} + 2\partial_r \ln B \right) \partial_r \tilde{R} \\ & \left(\left[\sum_{k=m} + \sum_{k=r} \right] \frac{Q_k[r]}{(r^{bk} - r_k)^{\sigma_k^{(2)}}} + (\partial_r \ln B)^2 + \sum_i \partial_r^2 \ln A_i \right) \tilde{R} \end{aligned} \right) \quad (58)$$

This formulation shows the general idea of the algorithm: either A_i or B_i is first fixed depending on whether q_i or p_i is (divergence) leading, then the other form is fixed to regularize the other pole³³. Clearly there is something special about $\{\sigma_i^{(1)}, \sigma_i^{(2)}\} = \{1, 2\}$, where

³²Looking at the AdS analogue, $\frac{d}{2} + 1 - (|I| - 1) = 0 \rightarrow \delta_{|I|}$, gives $\lambda_{\infty} \rightarrow \delta_{|I|} - \sum_{\pm} \tilde{\alpha}_i$.

³³The classical physical realizations of the $B - A$ orderings are the semi-classical WKB approximation (or the Weinberg soft pole theorem), which selects a (formally infinite dimensional OPE) to (shadow-)fix the transient momentum modes under $G_i = -\frac{1}{2} \left[\sum_{n=s} + \sum_{n=m} \right] \int \frac{P_n[r]}{(r^{bn} - r_n)^{\sigma_n^{(1)}}}$; $A - B$ ordered fixings can be heuristically recognized as Optical Theorem/Ward Identity residue gauges, $\ln A_i^{-1} \sim \oint q + p^2$

$\partial_r \ln B$ can match both p, q exactly³⁴

Remembering the poles remain separated allows solutions to be (sub-)indexed by their puncture patch group-index embedding: $\tilde{R} = \sum_j \tilde{R}_{\sigma_j} = \sum_j \sum_{\sigma_j} a_{\sigma_j} (r - r_j)^{\sigma_j}$ ³⁵ So, letting

A_i also remain local to the poles³⁶ $A_i \equiv e^{G_i} := e^{-\frac{F_i[r]}{(r^{b_i} - r_i)^{k_i}}}$, first consider the solution form local to any given pole, $R_j = \sum_{n=0} a_n^{(j)} (r^{b_j} - r_j)^{\sigma_j + n}$, which may be rewritten³⁷ as::

$$-b_j r^{b_j-1} (\sigma_j + n) (r^{b_j} - r_j)^{n-1} \left(r \left((b_j - 1)r + \frac{(\sigma_j + n - 1)b_j r^{b_j-2}}{r^{b_j} - r_j} \right) (r^{b_j} - r_j)^{k_j} + (r^{b_j} - r_j)^{k_j - \sigma_j^{(1)}} P_j[r] + 2 \left(F_{j,r} - \frac{k_j b_j r^{b_j-1} F[r]}{r^{b_j} - r_j} \right) \right) \quad (59)$$

$$= (r^{b_j} - r_j)^{k_j - \sigma_j^{(2)}} Q_j[r] + \frac{1}{(r_j^{b_j} - r_j)^{k_j}} \left(F_{j,r} - \frac{k_j b_j r^{b_j-1} F[r]}{r^{b_j} - r_j} \right)^2 + \partial_r^2 F_j + \frac{2b_j k_j r^{b_j-1}}{r - r_j} \left(F_{j,r} - \frac{k_j b_j r^{b_j-1} F[r]}{r^{b_j} - r_j} + \left(\frac{r^{-1} b_j (2k_j r^{b_j} + r_j)}{r^{b_j} - r_j} + \frac{1}{r} \right) F_j \right) + \text{subleading} \quad (60)$$

Noticing all the n -dependence sits on the LHS, this formulation gives a picture of the general algorithm: letting $F = h + \lambda g$, g can be used to solve either

$$\Lambda_i = (r^{b_j} - r_j)^{k_j - \sigma_j^{(1)}} P_j[r] + 2\lambda_i \left(g_{j,r} - \frac{k_j b_j r^{b_j-1} g}{r^{b_j} - r_j} \right) \quad (61)$$

$$\text{or} \quad \Lambda_i = (r^{b_j} - r_j)^{k_j - \sigma_j^{(2)}} Q_j[r] + \frac{\lambda_i^2}{(r_j^{b_j} - r_j)^{k_j}} \left(g_{j,r} - \frac{k_j b_j r^{b_j-1} g}{r^{b_j} - r_j} \right)^2 \quad (62)$$

first depending on the stronger relative pole. Then, h may be used to solve the remaining

³⁴And, if the sub-elements do exist, must be of the form $\ln A_i \sim f[r] \ln[r - r_i]$ for polynomials f ; this is another formulation of the orthogonality of the $\log[\cdot]$ and shows that the poles are regular iff a $[\cdot] \log[\cdot]$ functional extension is complete. Otherwise, A_i will acquire a dominant power coupling, $\ln A_i \sim (r - r_i)^{2 - \max\{2\sigma_i^{(1)}, \sigma_i^{(2)}\}}$, and a sub-dominant form-factor $\ln A_i \sim \ln B - \sum_{j \neq i} \ln A_j$ which controls the relative

dispersion of the orthogonal solution (form) covers at each pole

³⁵e.g., there may exist a descendant global winding gauge which gives a non-trivial index embedding $\sigma_j = \sigma(j)$

³⁶and noting that $\partial_r G_i = \frac{1}{(r^{b_i} - r_i)^{k_i}} \left(F_{i,r} - \frac{k_i b_i r^{b_i-1} F[r]}{r^{b_i} - r_i} \right)$ and that $\partial_r^2 G_i = \frac{1}{(r^{b_i} - r_i)^k} \left(\partial_r^2 F_i + \frac{2b_i k_i r^{b_i-1}}{r - r_i} \partial_r F_i + \frac{b_i k_i r^{b_i-2}}{(r^{b_i} - r_i)^2} ((1 + b_i k_i) r^{b_i} + (b_i - 1) r_i) F_i \right)$

³⁷where, for simplicity it was assumed the p, q poles have the same algebraic order, $b_j = b_j^{(q)}$; generally, replace $(r^{b_j} - r_j)^{k_j - \sigma_j^{(2)}} Q_j[r] \rightarrow \frac{(r^{b_j} - r_j)^{k_j} Q_j[r]}{(r_j^{b_j} - r_j)^{\sigma_j^{(2)}}}$

equations³⁸ by directly substituting into the P/Q into the h -constraint form.³⁹ Specifically, consider the more complicated (second) order of (*out*-gauge operations) solved. Next, let g_i represent the (λ_i) sub-linearized solution, so $\partial_r^2 \lambda_i g \equiv 0$;⁴⁰ then, letting $Q + G^2 = \Lambda_i$ note $Q + (G + H)^2 = \Lambda_i [1 + \frac{2G}{\Lambda_i} H] + H^2$ then, "on the h -branch" the above equation reduces to:

$$\begin{aligned} & \partial_r^2 h_j - 2(\sigma_j + n) b_j r^{b_j-1} (r^{b_j} - r_j)^{n-1} h_{j,r} + \frac{2G b_j r^{b_j-1}}{r^{b_j} - r_j} (k_j - (\sigma_j + n) (r^{b_j} - r_j)^n) \\ & + \frac{2b_j k_j r^{b_j-1}}{r^{b_j-1} - r_j} \left(\frac{b_j r^{b_j-1} (\sigma_j + n) (r^{b_j} - r_j)^n + r^{-1} b_j (2k_j r^{b_j} + r_j)}{r^{b_j} - r_j} + \frac{1}{r} \right) h_j \\ & - b_j r^{b_j-1} (\sigma_j + n) (r^{b_j} - r_j)^{n-1} \left((r^{b_j} - r_j)^{k_j - \sigma_j^{(1)}} P_j[r] \right. \\ & \left. + r \left((b_j - 1)r + \frac{(\sigma_j + n - 1) b_j r^{b_j-2}}{r^{b_j} - r_j} \right) (r^{b_j} - r_j)^{k_j} \right) \end{aligned} \quad (65)$$

$$\begin{aligned} & \Lambda_i - \frac{2b_j k_j r^{b_j-1}}{r - r_j} \left(\frac{r^{-1} b_j (2k_j r^{b_j} + r_j)}{r^{b_j} - r_j} + \frac{1}{r} \right) \lambda_i g_j \\ = & + \frac{\tilde{G}}{(r_j^b - r_j)^{\frac{k_j}{2}}} \left(h_{j,r} - \frac{k_j b_j r^{b_j-1} h[r]}{r^{b_j} - r_j} \right) \left(\frac{1}{\lambda_i} \left(1 + 2b_j k_j r^{b_j-1} (r - r_i)^{k_j} \right) + \frac{1}{\tilde{G}} \left(h_{j,r} - \frac{k_j b_j r^{b_j-1} h[r]}{r^{b_j} - r_j} \right) \right) \end{aligned} \quad (66)$$

Notice the pole in λ_i holds the "emergent winding element", as should be expected from a Lagrangian functional.

Also, notice that all the terms on the LHS define a regular (non-homogeneous) ODE in h (as an OPE of the polynomials G, P), while the non-linear RHS explicitly involves the constraint forms $\{\lambda_i, \Lambda_i\}$ and the first derivative form H . Remembering that g_j is at most $[\cdot] \log[\cdot]$ divergent, while G is at most $(r - r_i)^{-1}$; in turn, this implies $\tilde{G}^{-1} \sim (r^{b_j} - r_j)^{\frac{k_j}{2}-1}$,

³⁸in fact, considering the enveloping used here it is immediate from the freedom in k_j , and from the Fundamental Theorem of Algebra, explicitly seen by choosing $k_j = \max\{\sigma_j^{(1)} - 1, \frac{\sigma_j^{(2)}}{2} - 1\}$, so that the above g -equations maybe added together and always solved as a positive power series in $(r - r_i)$ (or with a first \log branch extension: $\{h, g\} \in \{1, (r - r_i)\}, \{A(r - r_i), (r - r_i) \ln(r - r_i)\}$)

³⁹Note the alternative form of the second line by letting $\Lambda_j \rightarrow \frac{\hat{\Lambda}_j[r]}{(r^{b_j} - r_j)^{k_j}}$:

$$(r_j^b - r_j)^{-\frac{3}{2}} \sqrt{\frac{\hat{\Lambda}_i(r^{b_j} - r_j) - (r^{b_j} - r_j)^{\sigma_j^{(2)}-1} Q_j[r]}{\lambda_i^2}} = g_{j,r} - \frac{k_j b_j r^{b_j-1} g}{r^{b_j} - r_j} := G \equiv \frac{\tilde{G} (r^{b_j} - r_j)^{\frac{k_j}{2}}}{\lambda_i} \quad (63)$$

Then, because Q_j is the reduced polynomial, it has a stable $\lim_{r \rightarrow r_j}$ limit and Λ_i may be selected to cancel the bottom order and pull another half-power out; in this case, letting $\hat{\Lambda}_i \rightarrow \lambda_i^2 \tilde{\Lambda}$ gives (the pushed) $\tilde{\Lambda}_i$ an interpretations as a (weighted) residue of the Q -field :

$$\tilde{\Lambda}_i = \oint_{\gamma_0^{(r)}} \frac{(r^{b_j} - r_j)^{\frac{\sigma_j^{(2)}-1}{b_j}} Q_j[r + r_j]}{\lambda_i^2} \quad (64)$$

In particular, when $\sigma_j^{(2)} = 2$ this translates into a (linearly shifted) Q -moment. When $\sigma_j^{(2)} > 2$ this may be interpreted as a higher order integral moment (in the global constraint form). Similarly, the cases $\sigma_j^{(2)} = \{1, 0\}$ gives $\tilde{\Lambda}_i$ the interpretation as a Q -charge density or a Q -current, respectfully.

⁴⁰The interpretation being that $\lambda_i \sim \frac{C}{|\partial_r g|}$ represents a \mathbb{F} -field expansion over the g -flow, seen explicitly by making this substitution in the formula for $\tilde{\Lambda}_i$ above; this can also be seen by noticing that $\tilde{\Lambda}_i$ maybe written as the exact derivative of a family of $\frac{1}{\lambda_i}$ distributions, $\tilde{\Lambda}_i \rightarrow \partial_{\lambda_i} \Gamma_{\lambda_i}[Q, r_j; r]$ which have everywhere convergent $(0, M]$ measure.

leading to the expectation that $|h|_{\text{deg}} \geq -\frac{k_j}{2}$.

Before unpacking everything, turning back to the LHS leads directly to considering solving general ODE's near regular poles. As explained in [10], these always reduce to a Fuch's relation. So, considering $[\partial_r^2 + \frac{\tau_n^{(j)}}{r_j^{b_j} - r_j} \partial_r + \frac{\kappa_n^2}{(r_j^{b_j} - r_j)^2} + \frac{B}{r_j^{b_j} - r_j} + C][\sum a_n(r_j^{b_j} - r_j)^{\sigma_\tau + n}] = 0$; further, defining $\chi_\tau = \frac{1-\tau}{2}$ gives the Fuch's form:

$$\sigma_\tau^2 + (\tau_n - 1)\sigma_\tau + \kappa_n^2 = 0 \Rightarrow \sigma_\tau^\pm = \chi_\tau \left[1 \mp \sqrt{1 - \left(\frac{\kappa_n}{\chi_\tau}\right)^2} \right] \quad (67)$$

Notably, the solutions are always (sub-)dominately controlled by $\chi_\tau := \frac{1-2}{2}$ in the sense that ⁴¹:

$$\left\{ \sigma_\tau^\pm \left| \begin{array}{l} |\chi_\tau| \rightarrow k_n \\ |\chi_\tau| \rightarrow \infty \\ |\chi_\tau| \rightarrow 0 \end{array} \right. \right\} \sim \left\{ \begin{array}{l} \{0, \chi_\tau\} \\ \{\chi_\tau\}^{(2)} \\ \{\chi_\tau \mp i\kappa_n\} \end{array} \right\} \quad (68)$$

where, $\tau_n \equiv -2\sigma_j b_j r_j^{b_j-1} \delta(n)$ reminds $\chi_n \equiv \sigma_j b_j r_j^{b_j-1} \delta(n) + \frac{1}{2}$, where σ_j is the rational envelope (critical exponent) form of R_j and shows the clean connection between the over-

leading forms and sub-leading log-forms. In fact, $\sigma_j \neq 0 \Rightarrow R_j = e^{\sigma_j (r^{b_i} - r_i)^{k_j} \frac{\ln(r^{b_j} - r_j)}{(r^{b_i} - r_i)^{k_j}} \tilde{R}}$ shows these terms may be considered ultra k_j -weighted contributions to Λ_i .

So, considering the form of the Fuch's relation, pick $\{\sigma_j, n\} = \{-\frac{1}{2}, 0\}$; further, move any $\sim (r_j^{b_j} - r_j)^{-2}$ divergent terms in the multiplier distribution Λ_i to the LHS as $\frac{\hat{\Lambda}_i}{(r^{b_j} - r_j)^2} = \hat{\Lambda}_i = \Lambda_i - (\Lambda_{d>-2} + \Lambda_{d<-2})$ and use(/induct with) Picard's lemma to close(/"gauge") the LHS:

$$\begin{aligned} LHS = & \partial_r^2 h_j + \partial_r h_i + \frac{b_j k_j r^{b_j-2}}{r^{b_j} - r_j} \left(2Gr \left(1 - \frac{1}{2k_j} \right) + b_j \left(\frac{r^{b_j(1+4k_j)+2r_j}}{r^{b_j} - r_j} + \frac{2}{b_j} \right) h_j \right) - \hat{\Lambda}_i \\ & - \frac{b_j r^{b_j-1}}{2} (r^{b_j} - r_j)^{-1} \left(\begin{array}{l} (r^{b_j} - r_j)^{k_j - \sigma_j^{(1)}} P_j[r] \\ + r \left((b_j - 1)r - \frac{b_j r^{b_j-2}}{2(r^{b_j} - r_j)} \right) (r^{b_j} - r_j)^{k_j} \end{array} \right) \end{aligned} \quad (69)$$

$$LHS \sim 0 \Rightarrow \sigma_\tau = \pm i\kappa_0 \equiv \pm i\sqrt{k_j \bar{\kappa}}[G, P, \hat{\Lambda}_j; k_j, r_j] \quad (70)$$

⁴¹Critically, the $|\chi_\tau| \rightarrow 0$ case the sum of critical exponents is $\sum_{\pm} \sigma_\tau^\pm = 2\chi_\tau$; the coincidence with the index-measure's ($|\cdot|$) non-differentiable (more specifically, the oriented tangent bundle) point is not a coincidence, and a direct manifestation of Axiom of Choice's application in extending $\mathbb{R}[x] \times \mathbb{R}[x] \rightarrow \mathbb{C}[x]$ smoothly. In fact, revisiting the entire construction thus far and noticing that $\tau_n = \tau\delta[n]$ recasts this as a proof of the uniformity of the harmonic completion cover over strictly measurable function sets, $\mathcal{C}[\mathcal{L}^2[O[\cdot]]] \sim \mathbb{C}^{\lfloor \frac{\dim O + \epsilon_O}{2} \rfloor}[\cdot]$, and may be seen as the canonical "emergence" of the $\pm i$ field extensions (*in-between* $d = 0$ and $d = N$ differential forms), in each sub-domain, under the equivariance: $[\sigma_\tau^\pm] - \chi_\tau + (\pm i)\kappa_n \sim 0$. Note the (positive definite) ϵ_O is well defined and may be non-zero only when the (finite) operator is odd (and, further, always equal to 1 if the operator is finite dimensional; when the OPE is infinite dimensional, ϵ_0 is either 0 a formal cardinal measure extension, representing extensions of the form $i(\ln i - \frac{\pi}{2}) - e^{\frac{i\pi}{2}} \sim 0$) which denote hyperfine extension fields.

Now, turning back to the non-linear (constraint form) RHS, $\tilde{G}^{-1}H \sim (\frac{k_j}{2} - 2 \pm ik_j \bar{k})$; then, it can be shown that a measurable λ_i may always be chosen to be sufficiently small, in any representation, s.t. the second constraint is zero ⁴² Finally, note the worst case (in the computational complexity case) is exactly $k_j = 0$, which implies $\sigma_j^{(1)} = 1 \wedge \sigma_j^{(2)} \leq 4$ and is exactly the case when $P[r]$ contributes to \bar{k} .

Unwinding everything, the full solution was $R = \prod_i e^{\frac{h_i + \lambda_i g_i}{(r_j^{b_j} - r_j)^{k_j}}} \sum_j (r^{b_j} - r_j)^{\frac{1}{2}} \tilde{R}_j = R_j$ where g_i may be a log-type solution and h is strictly rational (and minimal). Then finally, $\lambda_i \sim \frac{H}{G}$ represents the largest (oscillatory) dispersion of the constraint residue mode (the least G reduces H-state)

Now with all the machinery of the full the KG-radial ansatz listed above may be quickly analyzed. In the AdS-Kerr case, $\max\{\sigma_j^{(1)}, \sigma_j^{(2)}\} = \{0^{(2)}\}$ and $k_j = 0$; further, note $p_j = \partial_r \ln \Delta|_{r \rightarrow r_j} = \frac{1}{r}$ giving:

$$LHS = \partial_r^2 h_j + \partial_r h_i + \frac{b_j r^{b_j-1}}{r^{b_j} - r_j} \left(G + \frac{b_j r^{b_j-1}}{(r^{b_j} - r_j)} \left(\frac{1}{4} - \frac{P_j}{2} \right) \right) \quad (71)$$

$$\Rightarrow \sigma_\tau = \pm i b_j r^{b_j-1} \sqrt{\frac{G}{b_j r_j^{b_j-1}} + \frac{1}{4} - \frac{P_j}{2} - \frac{\bar{\Lambda}_i}{(b_j r_j^{b_j-1})^2}} \quad (72)$$

Turning to the Q -form, choosing Q to be constant, $Q_i = \alpha_i^2$, implies $G = g_{i,r} = \pm \sqrt{\bar{\Lambda}_i - \alpha^2}$; letting $\bar{\Lambda}_i = \alpha^2 + \frac{b_j^2 r^{2b_j-2}}{16(r^{b_j} - r_j)^2}$ ⁴³ gives $g_i = \frac{-1}{4} \ln(r^{b_j} - r_j)$ and: ⁴⁴

$$\sigma_\tau = \pm 2b_j r_j^{b_j-1} \sqrt{2(P_j - 1) + \frac{1}{4}} \quad (73)$$

Then, the constraint form may be rewritten:

$$\frac{\tilde{G}}{\lambda_i} (r^{b_j} - r_j)^{-\sigma_\tau} \sim \sqrt{8(P_j - 1) + 1} \quad (74)$$

Using $P_j = 1$, $b_j = 1$ gives:

$$\sigma_\tau = \pm 1 \Rightarrow G(r - r_j)^{\mp 1} \sim 1 \quad (75)$$

⁴²This follows from the Sturm-Picone comparison theorem, and noting, for small x , $x^2 \sim \frac{k}{3!} + \frac{kx}{x^{k+2}}$. This can also be noted by letting λ_i be Fourier measurable and noting that $\frac{1}{r^2}$ is harmonically measurable ($\int_{\mathbb{R}/\{-\frac{1}{N}, \frac{1}{N}\}} \frac{1}{r^2} \sim 2N + \delta[r]$).

⁴³Note, choosing the positive root implies $\frac{G}{b_j r_j^{b_j-1}} - \frac{1}{4} = 0$

⁴⁴Although this equation seems to have units for the top monodromy, this is an artifact of the parabolic representation used in $r - r_j \rightarrow r^{b_j} - r_j$; then, $[r] = [r^{b_j}]$ and $[1] = [\partial_r r] = [b_j r^{b_j-1}]$, giving the formal definition of b_j as an irrational unit $[b_j] := [r^{b_j-1}]$. Analogously, pure numbers may be given (\log -)measure units according to their sub-linear idealizations: $\frac{[\ln[r]]}{[\ln[\frac{1}{r}]]} = [n]$, which is strictly descendant from the roots of unity of a given length.

Now, choosing a pole basis and adding across them all gives:

$$\sum_i \lambda_i \sim \sum_i \frac{\tilde{G}_i}{r - r_i} \quad (76)$$

Finally, letting $\sum_i \Lambda_i = \sum_i \alpha_i^2$ amounts to fixing $0 = \sum_i G_i$:

$$\sum G_i^2 \sim 0 \quad \Rightarrow \quad \partial \|\vec{G}\|^2 = 0 \quad (77)$$

Then, by Green's theorem, there must exist a functional s.t.:

$$\lambda_i \sim \delta S_i \wedge \sum_i \delta S_i = 0 \quad (78)$$

In fact, choosing $\Lambda_i = 0$ gives,

$$\sigma_\tau = \mp \sqrt{\frac{1}{2} \pm i\alpha_j} \Rightarrow G(r - r_j)^{\frac{1}{\sqrt{2}} \sqrt{1 \pm 2i\alpha_j}} \sim \frac{1}{\sqrt{2}} \sqrt{1 \pm 2i\alpha_j} \quad (79)$$

which shows G acting like a log-type OPE; then, using $\sum_i \alpha_i = 0$ shows it has a fixed second order norm:

$$\sum_j 2G_j^2 (r - r_j)^{\frac{2}{\sqrt{2}} \sqrt{1 \pm 2i\alpha_j}} \sim 4 \quad (80)$$

Finally, it's linear λ derivative (or, it's partial variety $\frac{r-r_i}{\lambda_i}$) acts like a psuedo-differential field (at this order) iff $\lambda_j \sim 1 \pm 2i\alpha_j$, which can be seen by rewriting the first form:

$$\tilde{G}(r - r_j)^{-1} (r - r_j)^{1 + \frac{1}{\sqrt{2}} \sqrt{1 \pm 2i\alpha_j}} \sim \frac{\lambda_j}{\sqrt{2}} \sqrt{1 \pm 2i\alpha_j} \quad (81)$$

Finally, because the global conservation form is at second order, λ_j represents a root of the critical exponent; because G is double rooted, λ_j is exactly the root of the critical exponent, meaning $\sigma_\pm = \pm \lambda_j^2$.

In the flat space (x -)case, $\sigma^{\{(1),(2)\}} = \{0, 1\} \Rightarrow k_j = 1$ taking advantage of the general formulation, and noting $\bar{p} = i \sum \tilde{\alpha}_i$ choosing $\Lambda_\infty = ((2M\omega)^2 - \sum_i \tilde{\alpha}_i^2 - \bar{p}) + 2Mx + 1 \equiv \bar{Q}$ gives:

$$\begin{aligned} LHS &= \partial_x^2 h_\infty + \partial_x h_\infty + \frac{1}{1-r_j x} \left(G + \left(\frac{7}{1-r_j x} \right) h_\infty \right) - \hat{\Lambda}_\infty - \frac{1}{2} (r - r_j)^{-2} P_j[r] \\ &\equiv \partial_x^2 h_\infty + \partial_x h_\infty + \frac{14h_\infty - i \sum \tilde{\alpha}_i}{2(1-r_j x)^2} - \hat{\Lambda}_\infty \end{aligned} \quad (82)$$

Thus, it may be said that the scalar Kerr interaction, on the h -branch, is a "Bessel Constrained" fluid sourced relative to some dual (A)dS Kerr (partial wave/)thermalized interaction.

3.3 Review: Ideal Magnetohydrodynamics in D=4

Despite having been studied since the onset of general relativity, black holes are still active areas of research. Indeed, their hidden geometries (and field shadows) provide a unique insight into physics, generally, that is seemingly only beginning. This paper considers perhaps the simplest (higher) dimensional generalization of the Kerr black hole (in D=4 dimensions): the Myers-Perry black hole (a single spinning black hole in D=5 dimensions). Hopefully continuing context will keep this motivation flowing throughout.

Therein, this section concludes by reviewing black hole thermodynamics and some basics of ideal magnetohydrodynamics; procedurally, the paper next examines some features of the $D = 5$ geometry^[2] and field equations in higher dimensions before establishing a perturbative framework (and sequentially performing a specific field perturbation). Finally, the resulting family of solutions is examined and conclusions drawn.

In order to simplify dynamics this paper will not include matter dynamics explicitly, instead using only (fixed-point) metric boost sources and vector gauge dynamics to perturb a single black hole. Towards using well quantized theories to better understand less quantized ones, it makes good hope to expect small, stable volumes of black hole phase space may be quantizable in terms of relatively infinitesimal, distinct field perturbations: ⁴⁵

$$S = S_{BH} + S_{matter} \approx S_{BH} + S_{EM} = \int \sqrt{-g} \left(R + \frac{1}{4} F^2 - A \cdot J \right), \quad \delta S \approx \delta S_{EM} \quad (83)$$

Indeed, under the gravitational saddling and FFE assumptions explained in section ABOVE, the action S is extremized by curved solutions to Maxwell's equations with a known field-

⁴⁵for a review, see^[6]

tensor form and, in unit-less form given by⁴⁶ :

$$ds_{\text{Kerr}}^2 = - \left(1 - \frac{r_s r}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r_s r a^2}{\Sigma} \sin^2 \theta\right) \sin^2 \theta d\phi^2 - \frac{2r_s r a \sin^2 \theta}{\Sigma} dt d\phi$$

$$\delta S = 0 \Rightarrow \quad \mathbf{dF} = 0 \quad \text{and} \quad F_{\mu\nu} J^\mu = 0 \quad \text{where} \quad J = d * \mathbf{F} \quad (84)$$

$$\text{and,} \quad \mathbf{F} = \mathbf{d}\Psi_\phi \wedge (\mathbf{d}\phi - \Omega \mathbf{d}t) + \mathbf{I}[\mathbf{r}, \theta] \sqrt{-\frac{\mathbf{g}_T}{\mathbf{g}_P}} \mathbf{d}\mathbf{r} \wedge \mathbf{d}\theta \quad (85)$$

$$\text{constrained by} \quad | -g_T | g^{ij} \varphi_{,j} \eta^\alpha \nabla_i \eta_\alpha = -\frac{dI^2}{d\varphi}, \quad (86)$$

$$I(r_i, \theta) = \frac{(r_s r_i \sin \theta)}{\Sigma} [(\Omega - \Omega_H) \varphi, \theta] \Big|_{r \rightarrow r_i}$$

$$\Sigma \frac{dI^2}{d\varphi} = \eta_\alpha \left(\Delta \partial_r \left[\text{Ad}[g_T]^{\beta\alpha} \eta_{\beta\varphi, r} \right] + \sin \theta \partial_\theta \left[\text{csc} \theta \text{Ad}[g_T]^{\beta\alpha} \eta_{\beta\varphi, \theta} \right] \right) \quad (87)$$

It's interesting to note that, regardless of closure, picking a corotation form with uniform length $\eta_\alpha \eta^\alpha \equiv \Lambda - 1$ pushes the stream equation to a sourced form of the P-operator (acting on the magnetic flux coordinates) $\nabla_i [\sqrt{-g} g^{ij} \varphi_{,j}] = \frac{F_{ij}}{2(\Lambda-1)} \frac{dI}{d\varphi}$; in fact, if the RHS were to match the T-source (symmetries) of the KG field then the thermal descendancy would be exact (which is the exact idea of inner-flat space spinner, a.k.a. Penrose connected, representations [30])⁴⁷ As mentioned at the end of the FFE review, in D=4, a

⁴⁶Following [23] and expressing everything in terms of the Schwarzschild equivalent radius $r_s = 2M$; $r_\pm = M(1 \pm \sqrt{1 - \frac{a^2}{M^2}}) \sim r_s \left(1 - \frac{1}{4} \frac{a^2}{M^2}\right) := r_s(1 - \frac{\alpha^2}{4})$, which explains that α -parameter as a spin(-family)-displacement (series-level) gauge from the Schwarzschild OPE and, in light of the results from [10], give it the firm interpretation as the lowest order, thermodynamically stable boundary excitation. Indeed, this interpretation then exactly deduces the $\Psi^{(0)}$ monopole-Schwarzschild state as a universal gravitational-EM action symmetry (denoted δ_{G_b}) in every IR completion gauge, which implies the monopole's lack of existence as locked into shadow states descending from quasi-massive (soft-representation weighted) graviton decay [24].

Watching the completion of this construction will amplify this heuristic to understand the electric force as a δ_{G_b} duality as descending from strong-G quasi-stable spin-time modes (a.k.a, scattering iso-tuplets, or Heisenberg representation connections: $SL(2, \mathbb{R}) \Big|_{U(1) \times U(1) \sim U(1)}$ [25]): electric charge separation (fields) are then free to be interpreted as vector spacetime symmetries *outside* of gravitationally saddled (thermodynamically exact) vector-field (coordinate-gauged) interactions. More precisely, considering the global representation nettings, quasi-classical E-fields are then considered G-boundary modal dualities under the laws of black hole thermodynamics [26] relative to some r -local *out*-patch continuation [27] $\frac{\Psi_\phi^{(0)}}{\Psi^{(0)}[\theta]} \sim [1]_{d\theta}$. Finally, taking the cosmological thermalization pullback (or considering the Cosmological Relativity Hypothesis) produces the hypothesis that an inflationary coupling to strong-gravity could be dual to out-of-equilibrium Cauchy modes propagating along the quasi-topological states ascending from the event horizon as $SL(2, \mathbb{C}) \times SO(3)$. Indeed, this interpretation seems to also capture recent CCCH/SYM dualities [28] casting the graviton OPE as forming an asymptotic dual state of unthermalized quark-gluon charges (a.k.a., in the non-perturbative deconfinement phase of QCD); pushing completely through, this casts the loop-complete graviton as OPE descendant (under affinely compactified "spacetime-symmetry" propagation/conservation) to the asymptotic annihilator of the super-conformal stabilizer basis responsible for the global boundary $SU(N)$ charges in non-perturbative branch extensions of QCD. [21] This circumstantially explains the BZ-monopole's 7th order Field extensions, mentioned above, as manifest of a duality constraint between the graviton propagator measure [29] and the contact OPE of three (electrically) gauged doublets constrained under a linear hypercharge relation, giving $2^3 - 1 = 6 + 1$ six matching, and one natural regulator, relations towering directly above the double-soft completion (extension) states.

⁴⁷Critically, this interpretation sets $\alpha^2 \sim \delta S = \sum \delta S_i$, or that the electric field is the "square-root"

canonical channel of energy extraction can be perturbatively closed at second order in the spin length⁴⁸, $O(a^2) \sim O(\alpha^2)$, which is known as the Blandford-Znajek model:

$$\Psi_\phi = \Psi_\phi^{(0)} + \alpha^2 \Psi_\phi^{(1)} + O(\alpha^4), \quad r_s \Omega = \alpha \omega^{(1)} + O(\alpha^3), \quad r_s I = \alpha I^{(1)} + O(\alpha^3) \quad (88)$$

As shown in a footnote at the end of the FFE review, the Schwarzschild-limit OPE (over a family of current-less, non-magnetically rotating fields) of the stream equation is

$$0 = r^2 \partial_r \left[\left(1 - \frac{2M}{r}\right) \varphi_{,r} \right] + \sin \theta \partial_\theta [\csc \theta \varphi_{,\theta}] \quad (89)$$

and admits a non-trivial family of scaled, radially invariant solutions $[\varphi]_\theta \sim [A + C \cos \theta]$ (under sin-cos — a.k.a., canonical harmonic — duality). Worrying only about positive quadrant,⁴⁹ $0 < \theta < \frac{\pi}{2}$, and considering lowest order modes fixed to the polar flux moment $\Psi^{(0)}[0] = 0$ selects the representative $\Psi^{(0)} = 1 - \cos \theta$. Then, considering the α -top Znajek condition at the $O(\alpha^3)$ horizon:

$$\alpha r_s I(r_i, \theta) = \alpha^2 I^{(1)} = \frac{(2r_s^2 \sin \theta)}{1 + 2Mr_i \sin^2 \theta} \left[(\Omega^{(1)} - 1) \left(1 - \frac{\alpha^2}{4}\right) (\sin \theta + \alpha^2 \Psi_{,\theta}^{(1)}) \right] \Big|_{r \rightarrow r_i} \quad (90)$$

$$\sim \frac{(2r_s^2 \sin \theta)}{1 + 2Mr_i \sin^2 \theta} \left[(\Omega^{(1)} - 1) \left(\sin \theta - \alpha^2 \left(\Psi_{,\theta}^{(1)} - \frac{\sin \theta}{4} \right) \right) \right] \Big|_{r \rightarrow r_i} \quad (91)$$

$$\Rightarrow \Psi^{(0)}[\theta] = 1 - \cos \theta, \quad \Omega \sim \frac{a}{2M^2} = \frac{\Omega_H}{2} \quad (92)$$

of the thermal entropy residue of spinning black holes, which concretely fits with the KTN double cover discussed above, $\sqrt{\text{Kerr}} \sim \text{Spinning-Dyon}$ (which is an asymptotic magnetic duality in \mathbb{C} -extended electromagnetism; see [31]). Because the 2nd LoT was invoked in both the KG and the BZ constructions, assuming both fields are weakly saddled to gravity (under a Wald prescription) the canonical thermal modes, (a.k.a., the thermodynamically fixed-gauge OPE eigenvalues) are algebraically dominated by whichever thermodynamic state has the largest covering of the dual kernel modes (whatever radicalized algebra acts as the geometric bath [32]); because the massless, real KG equation is exactly minimal and thermodynamically covering, it always acts as the fundamental thermodynamic gauge field [10] (between the strong gravity bulk/asymptotic boundary). Therein the spacetime constants can be relationally promoted to OPE geometrized units; then, the relations $\Psi_\phi - \Psi_\phi^{(0)} \sim \alpha^2 \Psi_\phi^{(1)}$ display the magnetic quadrupole moment, $[r_s \Omega - \alpha \Omega^{(1)}] \sim O(\alpha^3) \sim [r_s I - \alpha I^{(1)}]$ and the displacement current/direct current as constraint(/perturbation-shadowed) relations descendant from higher order fields.

Importantly, here the reasoning has here been involuted through the 0th LoT to induce the optical symmetries of classical celestial holography (rather than taking them as preferentially selected closure symmetries) as T-stabilized, asymptotically shadowed *out*-modes of black hole geometries; then, the S-Matrix symmetries of QED can be considered (affinely) descendant from the *T*-stabilizer *out* modes of strong (loop level) gravitational interactions. Radically (in the rigorous, idealized sense), this opens an interpretation of the KTN double-copy holography as a self-shadowed basis constructed over (possibly Λ -)thermalized vector doublets: $\sqrt{\oplus \sqrt{\Lambda_K}} \rightarrow (\sqrt{\text{Kerr}} \rightarrow \text{Spinning-Dyod})$. Then, the BZ monopole expansion can be considered the asymptotic, strong gravity ($O(\alpha^3)$ -shaddowed) dual to boundary-local weak electric current symmetries; powerfully, this gives a description of local, weak electric charge separation as a small amplitude transition between asymptotically small strong-gravity Mass-spin charge separation states, and concretely fits with [27].

⁴⁸And best represented in active coordinates $\alpha := \frac{a}{M}$ [23]

⁴⁹or, equivalently, allowing discontinuous current sourcing streams to exist on the symmetric plane $\theta = \frac{\pi}{2}$, which also happens to be an r -uniform subspace and thereby also the canonical gluing patch $\forall \mathbb{Z}_2$ symmetric solutions; as such, the negative current solution can always be glued into a global topology with net [0]-discrete current propagation. This condition can be relaxed by giving the outer topology a weak current flow, which amounts to promoting the monopole moment to a higher-order magnetic pole [23].

Further, noticing that $\Psi_\theta^{(1)} \sim g[r, \theta] \sin \theta$ implies a Znajek current elongation form $I^1 \sim g^*[r, \theta] \sin \theta$ and, remembering that algebraic residues of the form $A + C \cos \theta$ reside in the monopole solutions suggest that the $n = 2$ perturbation closure can be enveloped with:

$$\Psi^{(1)} = f(r) \frac{\sin^2 \theta}{2} \quad , \quad I = g(r) \sin^2 \theta \quad (93)$$

This can fully be justified by⁵⁰ noticing that

$$\text{Ad}[g_T] \equiv_{\text{Kerr}} \frac{r_s r}{\Delta + r_s r - a \omega_\phi} \begin{bmatrix} \left(\frac{r_s \omega_\phi}{2}\right)^2 & \omega_\phi \\ \omega_\phi & 1 \end{bmatrix} + \begin{bmatrix} (\Delta + r_s r) \sin^2 \theta & \\ & -1 \end{bmatrix} \quad (94)$$

Noting that the left division ring is second order in α (as $\omega_\phi \sim O(a)$) implies,

$$\text{Ad}[g_T] \equiv_{\text{Kerr}}^{O(a^3)} \frac{r_s r}{\Delta + r_s r} \begin{bmatrix} \left(\frac{r_s \omega_\phi}{2}\right)^2 & \omega_\phi \\ \omega_\phi & 1 \end{bmatrix} + \begin{bmatrix} (\Delta + r_s r) \sin^2 \theta & \\ & -(1 + \frac{\alpha \omega_\phi}{r}) \end{bmatrix} \quad (95)$$

giving, under $\Omega = \frac{\alpha}{r_s}$

$$\text{Ad}[g_T]^{\alpha\beta} \eta_\beta \equiv_{\text{Kerr}}^{O(a^3)} \frac{r_s r}{\Delta + r_s r} \begin{bmatrix} \frac{r_s}{2} \alpha \sin^2 \theta & \\ 1 - \frac{r_s}{2} \alpha \Omega \sin^2 \theta & \end{bmatrix} + \begin{bmatrix} r^2 \Omega \sin^2 \theta & \\ -(1 + \frac{\alpha \omega_\phi}{r}) & \end{bmatrix} \quad (96)$$

$$= \frac{\alpha}{2} \sin^2 \theta \left(\frac{r_s r}{\Delta + r_s r} \begin{bmatrix} r_s & \\ -\alpha & \end{bmatrix} + \begin{bmatrix} \frac{2r^2}{r_s} & \\ -\frac{r_s \alpha}{r} & \end{bmatrix} \right) - \frac{\Delta}{\Delta + r_s r} \begin{bmatrix} 0 & \\ 1 & \end{bmatrix} \quad (97)$$

In particular, only the rightmost term can support an $O(a^2)$ field interaction, but this term loses support on the horizon (as $\Delta \rightarrow 0$). Then, asymptotic form-matching (and using trigonometric duality $\sin^2 \theta \cos \theta \sim x^2 dx$) fully justifies the $\sin^2 \theta$ solution forms found above to induce separability (at this order). Note $\Delta \rightarrow 0$ gives a linear timelike eigenspace Lastly, that the radial poles of the stream ODE, $r \in \{\pm a, 0, \pm ia \cos \theta\}$, are separated and regular (p -type in η , q -type in $\nabla_i I^2$), and this solution can always be solved under Picard's Lemma. The final step of a full solution monopole BZ construction is to solve the resulting ODE⁵¹ for f, g ; instead of continuing with known calculations, this part of the algorithm will be exhibited in the next section for the $d = 5$ case.

3.4 $d = 5$ Myers-Perry Metric

Generically across physics, higher dimensions are well represented by (*inner*) connections between (adjointly bound) gauge fields; unifications between *outer* and *inner* morphisms are exactly (product, or naturally) dual when the left adjoint product (kernel) admits a

⁵⁰noticing that the only possible radical stems from the division $\frac{(\Delta + r_s r)^2 - a \omega_\phi \Delta}{\Delta + r_s r - a \omega_\phi} = \Delta + r_s r + \alpha \frac{r_s \omega_\phi}{2} \frac{r_s r}{\Delta + r_s r - a \omega_\phi}$

⁵¹Usually by using critical regularity features of the radial ansatz to constrain the full functional class to a bounded sub-domain that can be more easily (numerically) explored.

right adjoint product (union of kernels⁵²). This is the idea behind Wald Actions, whereby auxiliary (gravitationally saddled) fields may produce enhanced (gravitationally fixed) field multiplets of propagators (globally complete transition modals) which may then be used as a basis OPE (with a possible co-kernel action, a.k.a., counter-propogator) for a(n ideally *out*-measure analytic) OPE of the functional determinate (of some fully quantized gravitational action). Either way, Noether's theorem applied to the \mathbb{F} trivial fields⁵³ shows the number of diffeomorphic degrees of freedom form a (convex) category of Lie Groups (or that they are compact under simple group representations under the weakest, or globally discrete, $n \leq d$ sub-topological representations).⁵⁴

The first extra dimensional black hole solution was a $D = 5$ static, aximsymmetric solution found by Tangherlini in 1965⁽¹⁴⁾ and nearly 20 years later it was generalized to a spinning solutions by Myers-Perry⁽¹⁰⁾. In general higher dimensional black holes can exist in any $D \geq 5$ (the n -dimensional static, axisymmetric metrics were exhibited in the above sections). Disregarding the Reiner-Norstrom (charged) extensions, the largest difference in higher dimensions is the existence of additional angular invariants (momenta) which translates into a richer event horizon geometry and more subtle thermodynamics and light tracing. Indeed, in $D = d$ dimensions there are $N = \lfloor \frac{d-1}{2} \rfloor$ planes of rotation [36]⁵⁵, which amounts to promoting the spin current (det-adjoint) coupling forms from a strictly vector gauge into a higher spin transition form.

Still, black holes in $D = 5$ exhibit a number of interesting similarities⁵⁶ to $D = 4$ that make

⁵²where the right adjoint representation canonically includes all left normal subgroups, and also must close (by group axiom) the LR-group algebra (which, precisely means it must support the co-kernel of the L-adjoint operator cover, in measure space known as a "Grossencharacter"); in finite algebras, this realization leads exactly to the Galois extension algorithm and to the classification of the finite groups (through the central representations of the classical Lie algebras). In the generic infinite dimensional (gauge) group case, under the Selberg classification (specifically so called first and second Selberg conjectures, and their relationship to Artin's conjecture [33]), the right dual algebras can always be found by maximally extending the Dirichlet kernel OPE [33]. It is important to note that, so far as the author is aware, the topology must be normal, or (mostly convex) $T_{2.5+}$, and thus the OPEs must be, at worst (on-shell) adjointly (GL_n) vector-gauged, aside from possible "corner/non-archimedian defects" in the Weil compactifications over the Froebinius, sub-topological accumulation space representations, known as Satake parameterizations in Artin-Takagi class field theory [33]) so that the infinite union closes in the open-dualized (finite intersection) representation . In fact, it is a remarkable result of the Artin Completion dualities that $\mathfrak{su}(2)$ and $su(1,1)$ group algebras are almost always dense when the system is Generally Rationalized (GR) [34]

⁵³applied to the canonical trace OPE (at \mathbb{F} field fixed point(s)) and/or functorially using the anomalous cusp dimensional measure

⁵⁴Accordingly, the higher dimensional (most often quoted $d = 26$ or $d = 12$) embedding dimensions of string theory can be understood as the maximum number of maximally reduced free scalar gauge degeneracies in the global cover (which depends on the number of closed string propogation modes). In the completely closed spectrum, each (oriented) subspace is affinely oriented, giving "Spherical Variety" (SV) conjecture of $d = 4! + 2 = 26$ dimensions (a more rigorous (charge-grained) on along the Virasoro chain can found in [35]).

⁵⁵In fact these solutions were later generalized further (by Emperan and Real) to the $D = 5$ (doubly- and dipole-) spinning-ring solutions (which have a direct connection to scattting modes in non-perturbative string theory⁽⁷⁾) by tuning the gauged dyiad sources to excite the hidden $U(1)^N$ cocycles (to their unstable limits).⁽⁹⁾

⁵⁶Using the L/R projective representation of empty $d = 5$ space symmetry groups, $SO(4) \sim C_2 \times SO(3) \times SO(3) \rightarrow (SO(3) \times_{\{L,R\}} SU(2))^{(2)}$, the essential idea (moving forward) is to notice that this (projective) homomorphism results in a spinning-vector gauge; so, by adding a global spin-cut current (a.k.a., a spinning

these solutions an appealing start to extending the Blandford-Znajek process into higher dimension. ⁵⁷

To that end, this section will focus on $D = 5$ Myers Perry BH with only one of the angular momentum turned on ($b=0$)⁽¹⁰⁾:

$$ds_{MP}^2 = \frac{1}{\Sigma} [-H[dt - \omega_\phi d\phi]^2 - 2Jd\phi dt + Fd\phi^2] + r^2 \cos^2 \theta d\varphi^2 + \Sigma \left[\frac{dr^2}{\Delta} + d\theta^2 \right] \quad (98)$$

$$\text{with: } \omega_\phi = a \sin^2 \theta \quad J = \Sigma \omega_\phi \quad F = \Sigma \frac{J}{a} \quad H = \Sigma - m = \Delta - a\omega_\phi \quad (99)$$

where here⁵⁸, $\Sigma[r, \theta] = a^2 \cos^2 \theta + r^2$, $\Delta[r] = a^2 - m + r^2$, and $\Sigma + a\omega_\phi = \Delta + m$. In particular, note three primary differences between this metric and Kerr. Firstly, and most obviously, the extra dimension seems to sit between the T and P sectors. Although it may be instructive to push it into the toroidal sector (to compare with later results), because it represents a uniquely global $d = 1$ spacelike dimension it's easiest to here treat it as a separate source to push on the Kerr-ascending symmetry form.

Secondly, the coordinate envelope J matches the Kerr analogue, but $\frac{F}{J} = \frac{\Sigma}{a}$ represents

black hole), the hypothesis is that some vector gauge field may strongly couple to the weak BH dynamics in the entire spacetime and provide a spin-energy decay channel, as in the $d = 4$ case. In fact, ideally a simple higher dimensional black hole geometry could be constructed that would exactly capture a flat space (ramification) $SO(4) \rightarrow SO(3)^{(L)} \times U(1)^{(R)}$ R-unitary gauge. Constricting the $d = 5$ black hole to spin in a single direction (in compact affine-time) promotes one (background induced) spin gauge contact form to an independent local $U(1)$ fixing; then, a natural hypothesis may be that this local symmetry contact form, $\sim U(1) \times SO(3)$ may be gauged to produce $\sim SU(2)$, which may then be represented by weak monopole (a.k.a., an almost everywhere force-free, vector gauge field) symmetry breaking and pulled back onto $SO(3)$ (on the global "cover gauge").

In the $d = 3$ case, $SO(3) \sim \mathbb{Z}_2 \times_{(L)} SU(2) \times SU(2)$; then, the double cover constructions show the Kerr geometry's null geometry exactly captures the reflection extended spin algebra, and the ($0 < \theta < \frac{\pi}{2}$) monopole (or globally spin-fluxing, current fixed) gluing basis fixes the a -only decay channels (while other solution basis may be constructed using different current-balancing distributions; for example, the monopole support is $I \sim r^{-2}$ while the BZ parabolic form is $I \sim \frac{1}{r}$ supported [37]). Further, this shows that, in the $d = 5$ case, there should be solutions with constant $U(1)$ -gauges (e.g., no $\hat{\varphi}$ -directed magnetic flux) corresponding to direct lower dimensional analogues; considering the quaternion embedding of $SO(3)$ gives the Hamilton Cayley formula, $x'i + y'j + z'k = (a + bi + cj + dk)(xi + yj + zk)(a - bi - cj - dk)$ where unitary matrices may be given by $U = aI + ic\sigma_x + ib\sigma_y + id\sigma_z$, which shows that it is a strong hypothesis that a restricted RHS (the $d = 5$ single spinning background) may lead to a restricted LHS (the projected leg from a Kerr double-copy action), a.k.a a holographic partitionial functional.

⁵⁷In fact, comparing (??) and (??) directly shows they look functionally identical if we naively map $m \rightarrow r_s r$, $d\psi^2 \rightarrow 0$ while holding m . Although crude, this feature of the two metrics gives a heuristic picture of the embedding of Kerr into MP black holes (and will prove deep): $r_s[r] \sim \frac{m}{r}$, or, the Kerr radial coordinate is a mass scaling (asymptotically scaling) coordinate in MP. This gives a clear motivation to studying higher dimensional black holes: they may represent conformal phase curves (dynamics) in $D=4$ dimensions. In fact, this provides provides an easy contextualization for holography in general: the idea behind the AdS-CFT correspondence is to find higher-dimensional states which descend to exact dynamical systems in lower dimensions (a "holographic" representation of gravitational phase space); in this case, the "identification" requires that the apparent horizon shrink as the r_{Kerr} coordinate is advanced: $r_s \rightarrow r_s[r] = \frac{m}{r}$. This superficially looks like the black hole is evaporating (and, considering $\frac{1}{r}$ as a Poisson random variable and m a diffusion length, this gives some idea of the connections to Large-N limits (t'Hooft limits) in string theory)⁽¹¹⁾.

⁵⁸Notice that $[m] = [l^2]$ has the same units as r^2 ; also, the extra coordinate in this representation is (vector) directly orthogonal $\tilde{g}_T \otimes \tilde{g}_\varphi \otimes \tilde{g}_P \sim \tilde{g}_T \otimes (F[\cdot] \tilde{\delta}_\varphi \otimes \tilde{g}_P)$

a(n affine/lapse-like) shift by $+2\omega_\phi$ from the Kerr-analogous ratio⁵⁹; then, $ds_T^2|_{H=0} = -\frac{2Jd\phi}{\Sigma}(dt - \frac{F}{2J}d\phi)$ shows that this manifests as a continuous, radially independent time-advance, $\tau^{MP} \sim \tau^{Kerr} + \omega_\phi \Delta t$, in the relative ergosphere (time-spin Path) gauging $\gamma[t, \phi; r, \theta]$ $\gamma^\mu \nabla_\mu \gamma^\nu|_{H[\gamma^r[\cdot], \gamma^\theta[\cdot]]=0} = 0$ (this can equivalently be seen, by absorbing this term, as an asymmetric splitting of the perfect square over H). In total, this gives the idea that $d = 5$ single-spinning MP decay channels may asymptotically resemble Kerr decay modes under a strong interactive coupling (shadow gauge) at the ergosphere.

Lastly, and perhaps most importantly, the poles of Δ are exactly sign symmetric, \mathbb{Z}_2^\times : $r_\pm = \pm\sqrt{m - a^2} \equiv \pm\sqrt{m}\sqrt{1 - \frac{a^2}{m}}$, or $\sum_\pm r_\pm = 0$ (which, in this final sense, is a property it shares with AdS-Kerr).

$$r_i^{MP} = \{\pm\sqrt{m - a^2}\} \Rightarrow \begin{aligned} r_0 &= \sqrt{m} \\ \alpha &= \frac{\alpha}{\sqrt{m}} \end{aligned} \quad \text{SO:} \quad \frac{r_+^{MP}}{r_0} = \sqrt{1 - \alpha^2} \quad (100)$$

The outer MP event geometry ($S^2 \times S^1$) resembles the event geometry in Kerr (S^2) (with an independent $U(1)$ Killing space)⁶⁰. In particular, both have exactly one thermodynamic

⁵⁹Considering that the spacetimes' velocities and classical defects are functionally identical, $\{\omega_\phi^{Kerr}, \Sigma^{Kerr}\}[a, \theta] \equiv \{\omega_\phi^{b=0}, \Sigma^{b=0}\}[a, \theta]$, the difference $\frac{F_{Kerr}}{J_{Kerr}} - \frac{F_{b=0}}{J_{b=0}} = 2\omega_\phi$ is a sensible (affinely projected, $d\varphi \sim 0$) relative lapse measure between the embeddings.

⁶⁰It's particularly interesting to note the discussion following eq. 3.34 in [38], which casts the curvature singularity at $r = 0$ with two slightly different forms depending on whether one, or both spin parameters, are non-zero. As shown in [38], in the case of one vanishing spin functional it naively appears that $O(r^{\{-1, -2\}})$ matching forms, $\{\alpha, Z\}$, may be found (in Kerr-Schild type coordinates) which produce a smooth metric almost everywhere, except the exact singularity $r = 0$, leading to the idea that the curvature singularity may be resolved as a limiting form. Instead, the singularity can be shown to have been displaced into the index as a conical constraint on the accumulation topology (manifest as r coordinate-form measure, $\sim g_{rt}$ dependence in the light-orthogonalization): requiring the singularity be smoothly embedded then requires the accumulation-weighted metric 3.38b, and descends the singularity-smooth sub-cover measure to an infinite r -affine index (a.k.a., divergent proper distance), which is always strictly less than the distance to the conical convergence. In fact these covering conditions provide a direct connection to the footnote above hypothesizing the (single-spinning) $d = 5$ monopole as a strong-G induction basis of an ultra-fast resonance (manifest as a thermalized ergosurface bound-multiplet *outmode*): starting from an asymptotically far topology, reconstructing increasingly near-singularity features is always a $r_+ \geq r_{min}$ truncation process (the Quantum Extremal Surface extension of the singularity theorems). Still, using thermodynamic arguments [10], or also by considering conformal phase domain-weighted extensions of the free space propagator, the so-called "Replica Trick" [39], or also by considering recent simulations in numerical relativity [13], it can be inferred that dynamic black hole event horizons can, under the right circumstances, produce long shadows in the fast spectral modes (such as, for example, when a (distribution of) nearly maximally antipodally spinning black hole(s) collide[s] with a significantly more massive BH; numerical simulations/observations [13] indeed show a surface relative shockwave dispersion with a relatively quick ringdown time, as would be therein deduced under the Thermodynamics/Observation (T/O) hypothesis). Then, the complete light-surface thermalization phase, mentioned in the same footnote (and below), may represent an extended form of Cosmic Censorship, whereby strongly destabilized black hole topologies may be indexed by their (probe/observation/Wald-field) *out*-regulated, outer-light-surface matched, $d = 5$, $\hat{\varphi}$ -gauged, gluing boundary OPE sub-charges. In the case of a completely thermalized transition coarsening, this would naively correspond to the L/R product volume, $S_{T_R \times T_L}^2$.

Further intriguingly, this product gauge state seems to share many of the properties recently implicated in planar thermal Hall effects in quantum spin-liquids, particularly [40] newly found the antisymmetric thermal Hall responses above and below the gauge degeneracy phase, particularly as it compares to the over/under spinning of the ramified electric light gauging in single spinning MP 3.9. The matter-dual states specifically indicate a bosonic edge mode (coupled to a Berry curvature, indeed reminiscent of $S_{T_R \otimes T_L}$ in a conformal product gauge), rather than a fermionic excitation; looking back to the initial discussion of the stream

scale which regulates the degeneracy of the coordinate singularity, $r_+|_{\alpha \rightarrow \pm 1} = r_-|_{\alpha \rightarrow \pm 1}$. In fact the extremal MP horizons additionally converge on the degenerate horizon onto the curvature singularity (see section 3.2⁽¹⁰⁾) $r_0 = 0$ and, further, it can be shown that the Kruskal-like continuance towards the curvature singularity is timelike, meaning tangent space of the degenerate horizon is spacelike.⁶¹ As such, the single spinning $d = 5$ MP black hole can be embedded in a Penrose diagram resembling a Schwarzschild geometry (with two distinct horizons $r_H \in \{0, r_+\}$) with an angular momentum, and a (u, v) -entire analytic continuance in the $|u| > v$ *in/out* patched sense; see [38] eq 3.59.⁶², $\frac{r_{\pm}}{r_0} = \sqrt{1 - \alpha^2} \sim$

equation, specifically the interpretation of the cap current I as an ultra-fast co-rotation form symmetrically weighed on a large sub-spectral average (of some iso-shelling of Higgs forms), shows a consistent deductive trend. This also gives a canonical interpretation of the gluing patch — here, as in Kerr, a $d = 1$ closed surface (although, the single spinning MP is topologically relaxed to converge within any $S_\varphi \times [*]$ gauged modal) — induced $\hat{\varphi}$ -charges as *out*-relative inertial modes (or, *in/out* membrane transition charges), or as the (purely) harmonic charging of a bulk-cutoff dynamics. Backstepping again, now anticipating the KTN-representation, a $d = 5$ monopole solution should then correspond, under bulk-boundary descents, to a (Berry-dual) QED-loop analogue to the Coloumb gauge (shown below as a correspondence to the Wichmann-Kroll potential), and, under boundary-boundary transitions, to a thermal shadow fixing (shown by the light-surface frequency residue fixing at spacelike, globally T-symmetric infinity).

⁶¹In fact, this is exactly the jumping off point to construct the D=5 black-ring solutions: space-like degeneracies in the metric-kernel can be “balanced” by spacelike singular-points (known as the Inverse Scattering Method) to cover extended domains with previously negative mass asymptotics. The (boosted) event horizons of the black rings have enhanced symmetries that produce interesting asymptotic states which, in particular, that have interesting contact points with non-perturbative string theory⁽⁷⁾ [41]. Again, it’s important to note that the original Meyers and Perry constructions of higher dimensional black holes specifically concerned completely axisymmetric solutions to control the loop residue lengths (densely on the time branch); further, families of $d = 5$ black holes can generically be categorized by the shape of the maximally extended (closed) rational curves on the inner-thermalized(/dual) topologies: the dual Betti measure of the KK-instantons (closed in the light protected inner domain). The initial ISM solutions were found by closing the open loop topology (closed rational time-curves) pointwise in the global affine closure (to guarantee their convex exclusions from the *out*-domain), but recent works [42] show that this condition can be relaxed using a(ny) light-graded (sub-horizon/inner) asymptotic boundary algebra. In $d = 5$ and building on the ISM loop algebra, the author’s used the super-symmetric (affinely flat, ground-harmonic, $\langle n = 0; k = 2|^{*-}$ -)dual closure of the spacelike rational-loop algebra, $\bar{C}_{ISM} \sim^* SO(4)$, as a complete (ISM twisted) projector space for the (linearized) Kaluza-Klein spectral form to generate an infinite class of multi-axisymmetric (indexes) over $S^3 \times_{s \leq k-1} T^{n+k-2}$ -punctured bulks; better yet, they reversed the scheme to (counter-)compactify the rationally separated loop (instanton) sector of the strongly interactive(/Coloumb) branch (in the brane-topology). Functionally, this can be understood in the context of the general holonomy OPE of [34], which strongly motivates the resultant (separated) event horizon topologies $\{S^1 \times S^2, L(p, q), S^3\}$ as functor descent leg-forms of $\{\mathfrak{su}(2n), \mathfrak{g}_{|n|}, \sqrt{\mathfrak{g}_{|1|} \times \mathfrak{su}(2)}\}$.

⁶²There, notice the induced decay tail on the pullback as $r \sim [r_+]$ when $B > r_+$ (bottom page 330) is a natural hallmark of sub-harmonic topological emergence (principally shown under the classical 0th LoT’s equivariant fixing to the measurement scaled $\mu[N; \cdot]$ log-variance: $[\beta] \sim \partial_* \ln W[*; \cdot]$) and naturally points towards a Mellin-extension of the Fourier kernel decouplet used to thermalize empty spacetime, which exactly motivates the conformal coordinates of [43]. Understanding the Mellin-form itself as a(n affinely indexed, ln-pushed) functional extension of the Fourier modal, the Fourier transform of the β -thermalization domains may be thought of as a Mellin flow (along the principal branch) in constraint (phase) space: using Artin’s L-function measure (which is universally possible) $\mathcal{F}[\beta]_W[-s] \sim \mathcal{M}[-\partial_*]_W(-is)$. Of course, this is just a reformulation of wave/particle duality in classical Quantum Mechanics, as was explored in the section above ???. In fact, this idea can be directly seen from the Mellin-action on the projector basis monomials: $\mathcal{M}[e^{ix}] = e^{\frac{i\pi s}{2}} \Gamma(s)$; more naively still, the sub-leading/soft dualities may be inferred from the formula $\mathcal{M}[e^{-(\ln x)^2}] = \sqrt{\pi} e^{\frac{s^2}{4}}$, which conveys (counter-propogating) entropies with log-squared measures (and directly independent co-domain forms) as (constraint) pushed Gaussian envelopes.[44] [45], [38]

Then ⁶³. Further accordingly, the ergosphere comparison needs just compare to either representation of the radial poles to analyze the ordered-splitting forms [42]. Under the “affine time is affine quantumness”

$1 - \frac{\alpha^2}{2} - \frac{\alpha^4}{8} + \dots$; also, the ZAVO frame (e.g., the ergosphere surface [45] [23]) is given by $0 = H \Rightarrow \frac{r_{\pm}^{(erg)}}{r_0} = \pm \sqrt{1 - \alpha^2 \cos^2 \theta} \sim 1 - \frac{\alpha^2 \cos^2 \theta}{2} - \frac{\alpha^4 \cos^4 \theta}{8} + \dots$. Putting everything together, and further conjoining the AdS thermodynamic balancing of [10], tends the suggestion that single spinning MP black holes may be able to produce strong-gravitation solutions under ergosphere/event horizon vector shadowed (or, gauge transition-weight matched) forms; under the strong/weak \rightarrow soft/hard charge mixing of the $w_{1+\infty}$ (residue) algebra, this may be expected to be manifest as a bulk-boundary magnetic flux (an ergosphere gauged scattering-angle [46])⁶⁴.

So, following the procedure as in Kerr/Schwarzschild and supposing the usual solution form, $[\cdot] \rightarrow e^{i\lambda_\alpha x^\alpha} [^*]$ the KG equation can be written:

$$\begin{aligned} [g_T] &= \frac{H}{\Sigma} \begin{bmatrix} -1 & -\frac{J}{H} + \omega_\phi \\ -\frac{J}{H} + \omega_\phi & \frac{F}{H} - \omega_\phi^2 \end{bmatrix} & | -g_T | &= \frac{\Delta \omega_\phi}{a} \equiv \frac{\Delta J}{a \Sigma} ; & \sqrt{\frac{|-g|}{|g_\varphi|}} &= \frac{J}{\sqrt{a \omega_\phi}} \equiv \Sigma \sin \theta \\ [g_\varphi]^{-1} &= [r^{-2} \sec^2 \theta] & |g_P| &= \frac{\Sigma^2}{\Delta} \equiv \frac{\Sigma J}{\omega_\phi \Delta} ; & \sqrt{\frac{|-g_T|}{g_P}} &= \frac{\Delta}{\Sigma} \sqrt{\frac{\omega_\phi}{a}} \equiv \frac{\Delta \sin \theta}{\Sigma} \\ [g_P]^{-1} &= \frac{1}{\Sigma} \begin{bmatrix} \Delta & \\ & 1 \end{bmatrix} & g_{T \otimes [\varphi]}^{\alpha\beta} \lambda_\alpha \lambda_\beta &= \frac{1}{|-g|} \partial_j \left[r \sin \theta \cos \theta (\Delta \delta_{rr}^{ij} + \delta_{\theta\theta}^{ij}) \partial_j [^*] \right] \end{aligned} \quad (101)$$

This leads to:

$$\left[\frac{(r^{-1} \tan \theta)^{-1}}{\Delta} Ad[g_T]^{\alpha\beta} (\Sigma \lambda_\alpha) (\Sigma \lambda_\beta) + r^{-1} \tan \theta \|\Sigma \lambda_\varphi\|^2 \right] \quad (102)$$

$$= \frac{1}{r} \partial_r [r \Delta \partial_r [^*]] + \frac{1}{\sin \theta \cos \theta} \partial_\theta [\sin \theta \cos \theta \partial_\theta [^*]] \quad (103)$$

$$\text{with } \begin{array}{l} \Sigma \equiv H + m \\ \Delta \equiv H + a \omega_\phi \end{array} \text{ gives } \frac{\Sigma^2}{\Delta} Ad[g_T] = \frac{H + m}{H + a \omega_\phi} \begin{bmatrix} (H + m)^2 \frac{\omega_\phi}{a} - H \omega_\phi^2 & m \omega_\phi \\ m \omega_\phi & -H \end{bmatrix} \quad (104)$$

$$\Rightarrow \frac{\Sigma^2}{\Delta} Ad[g_T] \Big|_{H \sim 0} \sim r_0^2 \begin{bmatrix} \frac{r_0^2}{\alpha^2} & \frac{r_0}{\alpha} \\ \frac{r_0}{\alpha} & 0 \end{bmatrix} \quad (105)$$

In fact, taking each (metric)pole as an optical term and noticing that $H \rightarrow 0$ makes the

this is the qG-interactive dual form of the antipodal matching condition (which concretely fits under an R(education)-entanglement picture).

⁶⁴or an ergosphere local (t, ϕ) transition matching form; because the ergosphere is not a closed energy surface (it is not the closed union of geodesics[47]), ω_ϕ relative shift between the ergo-gauging in MPa and Kerr represents a strictly electric gauging of the Kerr sub-dynamics, representing an opening to understanding fast-transition processes in black hole dynamics through higher dimensionally gauged iso-linear transition forms across the outer-stabilization net. In particular, this gives the higher dimensional monopole solution exhibited below the clean interpretation as a stabilized (quasi-/partial-)basis OPE for outer-gauged transition dynamics (or, fast gravitational stabilizer modes under degeneracy-gauged OPE normalization weights, a.k.a., the membrane conformal phase regulator basis of the decay dynamics). [42]

adjoint source "look" flat (and advanced) immediately leads to the ansatz:

$$\Rightarrow \frac{1}{r} \partial_r [r \Delta \partial_r [*]] + \frac{1}{\sin \theta \cos \theta} \partial_\theta [\sin \theta \cos \theta \partial_\theta [*]] \quad (106)$$

$$\sim r_0^2 \begin{bmatrix} \frac{r_0^2}{\alpha^2} & \frac{r_0}{\alpha} \\ \frac{r_0}{\alpha} & 0 \\ & & im^2 \sqrt{1 - \frac{m}{r^2}} \end{bmatrix} \begin{matrix} (a)(b) \\ \begin{pmatrix} \lambda_t \\ \lambda_\phi \\ \lambda_\varphi \end{pmatrix} \end{matrix} \begin{matrix} (a) \\ \begin{pmatrix} \lambda_t \\ \lambda_\phi \\ \lambda_\varphi \end{pmatrix} \end{matrix} \begin{matrix} (b) \\ \begin{pmatrix} \lambda_t \\ \lambda_\phi \\ \lambda_\varphi \end{pmatrix} \end{matrix} \quad (107)$$

(under the appropriate, canonical transpose on the group eigenspace). Note, in this limit, there is no well defined λ_φ functional inverse at the horizon.⁶⁵

Indeed, the $r^{-1} \tan \theta$ terms seems to ruin the thermal separability⁶⁶ even when despite the nice RHS (a.k.a., P-descendant differential guide); still, the Adjutant matrix, under the (P-separated) relevant physical (rational) product weight seems to be only θ dependent on the ergosurface⁶⁷, indicating that the descent forms may split into thermalization bands across some field mediated action at the ergo-surface. The implication of a closed surface topology, with an open geodesic compactification, in the quasi-separated KG equations above (the formal eigen-character extended representation thermalization spectrum discussed above) immediately signals a vector gauge *out* (*in*-surface-charged) dynamic.

To that end, the $d = 5$ stream equation is constructed following the analogous procedures of $d = 4$: adding a spacetime-saddled electromagnetic field, the clearest difference between $d = 4$ and single-spinning $d = 5$ is the additional (space-like) Killing field, $d\psi$; consequentially, the electromagnetic field has two canonical magnetic field vectors (together which form the field's magnetic bi-vector) which naturally reduce over the spacetime into some $U(1)$ L-adjoint local gauge. This axial charge freedom when one spin is turned off will prove thermodynamically pivotal.

⁶⁵And also that the upper left sector separates in the $\alpha \rightarrow 0$ sub-accumulation. Also, note that $H \rightarrow 0 \Rightarrow \Sigma \rightarrow m$, which can be used to derive the above (and shows this is a candidate for a gravitationally mass-actionable pole expansion form factor)

⁶⁶more precisely, the strictly KG auxillary/scalar-thermodynamically parameterized

⁶⁷Indeed, noticing that the matrix row reduces to $I \oplus -I \in C_2$ (an element of the central compactification, or minimal topological cover index, of $SO(4) \sim C_2 \times SO(3) \times SU(2)$), lends specific interest to finding (Σ -scaled) field extension functors, $(\Sigma \lambda_\varphi)[\hat{*}; *]$ which pull back, under some canonically induced (and reduced, by construction) sub-field measure to a splitting basis form for the LHS (composed of the $\{I, \sigma_z\}$ -orthogonal spin operators, roughly: $\sim f(r^{-1} \tan \theta, \alpha^{-1})[\sigma_x \pm i\sigma_y]$). Instructively, $(r^{-1} \tan \theta)^{-2} \sim \frac{r^2}{\alpha \omega_\phi} (1 -$

$\frac{r_\pm^{(erg)}}{r_0}) \equiv (erg) - \frac{1 + \frac{H}{r_0^2} + \alpha \hat{\omega}_\phi - \alpha^2}{\frac{\Delta}{r_0^2} - \frac{H}{r_0^2}} (1 - \frac{r_\pm^{(erg)}}{r_0})$ shows that the functoral part of the splitting field extension should

be considered a parabolic linearization over the interaction-mean displacement field between the ergosphere and the outer-event horizon. Putting everything together (including the dimensional-descendant, ergosphere emergent orbital coupling gauge mentioned above) leads to the deduction that an auxillary field with (α -)differentially-exact coupling between the event-horizon and the ergosphere could possibly be supported by $d = 5$ (monopole) transition forms along the *in/out* gluing domains of perturbatively split ergosphere moments (between the ergosphere and the outer LS), thereby realizing the earlier deduction of an ergosphere L_φ -closure gauge iff the inner modes can be conformally regulated. As will be shown in [46], the monopole solution's light-surface-to- φ -surface OPE directly matches the Kerr conformal temperature, showing that it represents a strong candidate for hyperdimensional elongation representations of vector-gauged scattering OPEs in strongly self interacting, spin-colinear gravity decay processes; further, the off-shell decay pressure of the (point-shadowed-OPE) monopole form will nearly match the Wichmann-Kroll potential (which, fittingly, represents the asymptotic form of the Uehling loop correction to the Coloumb potential in QED [48])

Jumping to the final EM form factor, the symmetry arguments leading to the field constructions stay the same⁶⁸, leading to field forms:

$$\mathbf{dF} = \mathbf{0} \quad \mathbf{d} \star \mathbf{F} = \mathbf{J} := \star \mathbf{j} \quad \mathbf{F} \wedge \mathbf{F} = \mathbf{0} \quad (108)$$

$$\mathbf{F} = \mathbf{d}\Psi_\phi \wedge (\mathbf{d}\phi - \omega_\phi \mathbf{d}\mathbf{t}) + \mathbf{d}\Psi_\psi \wedge (\mathbf{d}\psi - \omega_\psi \mathbf{d}\mathbf{t}) + \mathbf{I}[\mathbf{r}, \theta] \sqrt{-\frac{\mathbf{g}_T}{\mathbf{g}_P}} \mathbf{d}\mathbf{r} \wedge \mathbf{d}\theta \quad (109)$$

Still, before probing the full gravitational pole it is productive to explore a few toy models built from asymptotic representations.

3.5 Asymptotically Flat Space

To illustrate the effect of the dimensional extension, it is convenient to first examine Maxwell's equations in the softest extension of local Minkowski representations: $d = 4$ asymptotically flat spacetime. These coordinates are adapted to (affinely) large spaced measurements⁶⁹ and given by (advanced coordinate u and retarded coordinate v ⁷⁰):

$$\begin{aligned} u = t - r & & x_1 = \frac{r(z+\bar{z})}{1+z\bar{z}} \\ v = t + r & & x_2 = -\frac{ir(z-\bar{z})}{1+z\bar{z}} \quad x_3 = \frac{r(1-z\bar{z})}{1+z\bar{z}} \end{aligned} \quad (110)$$

$$(111)$$

$$(112)$$

$$\Rightarrow ds_{adv/ret}^2 = -du_{a/r}^2 \mp 2du_{a/r}dr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \quad (113)$$

$$\gamma_{z\bar{z}} = \frac{(1+z\bar{z})^2}{2} \quad (114)$$

⁶⁸This is where the direct independence of the extended coordinate $\hat{\psi}$ is fundamentally important (to ensure there is almost always a $n = 4 < d$ sub-domain directly dual to the emergent magnetic guide, $\star \wedge_{\mu \neq \psi} dx^\mu \sim d\psi$). Directly, the Levi-Cevita density separates into an (normal ordered) form over ψ : $\epsilon_I \sim \epsilon_{\bar{I}\psi} g_{(\psi)}^\psi$; once again, the force free condition $F_{ab}J^a = 0 \Rightarrow F_{[cd}F_{ab]}J^b = 0$, so if \mathbf{J} is unsupported along $d\psi$ (corresponding to a $d = 2$ extension of the lower dimensional sub-current orthogonalization argument [45], with extended corollary that the $d = 1 \rightarrow 2$ surfaces be orthogonal to $d\psi$, e.g. $\mathbf{j} \sim \mathbf{A} + \sum_i \mathbf{b}_i \wedge \mathbf{d}\psi$, where \mathbf{A} is hodge-closed outside of $d\psi$) then degeneracy, $F \wedge F = 0$, again implies the field is force free. Finally, noting coordinate index ordering ambiguity gives the $\{\phi, \psi\}$ -index symmetric form of \mathbf{F} .

⁶⁹Compared to some local, $\Delta s^2 = \eta_{\mu\nu} \Delta_{x_0^\mu} x^\mu \Delta_{x_0^\nu} x^\nu$) under the usual coordinate ring, $\Delta_{y_0} y = y - y_0$

⁷⁰which can also be indexed by the orientation of the “ \mp ” symbol

Then⁷¹ Then, using coordinate-indexed δ notation Maxwell's equations read:

$$J^\mu = \gamma_{z\bar{z}} \left(\begin{array}{l} \delta_u^\mu [\partial_r [r^2 F_{ur}] \mp [\gamma^{z\bar{z}}] \partial_z F_{r\bar{z}}] + \delta_r^\mu [\mp \partial_r [r^2 F_{ur}] + r^2 F_{ur,u} \mp \frac{[\gamma^{z\bar{z}}]}{2} \partial_z F_{u\bar{z}}] \\ \mp \frac{[\gamma^{z\bar{z}}]}{2r^2} \delta_z^\mu [r^2 \partial_r [F_{\bar{z}u} + F_{\bar{z}r}] + r^2 F_{\bar{z}r,u} \mp \partial_z [[\gamma^{z\bar{z}}] F_{\bar{z}z}]] \end{array} \right) \quad (115)$$

$$\text{where, } \frac{J^\mu}{\sqrt{|-g|}} = \delta_u^\mu [\mp j_r] - \delta_r^\mu [j_u \pm j_r] + \frac{(1+z\bar{z})^2}{4r^2} (\delta_z^\mu [j_{\bar{z}}] + \delta_{\bar{z}}^\mu [j_z]) \quad (116)$$

$$\text{and } \nabla \cdot A = 0 \text{ implies: } r^2 A_{r,u} + [r^2 A_s]_{,r} = \mp \gamma^{z\bar{z}} A_{\bar{z},z} \quad (117)$$

$$\Rightarrow \begin{array}{l} [r^2 F_{ur}]_{,s} = [r^2 A_{u,r} + [r^2 A_s]_{,r}]_{,s} \pm [\gamma^{z\bar{z}}] A_{\bar{z},zs} \\ \pm [[\gamma^{z\bar{z}}] F_{ur}]_{,z} = r^2 A_{r,uz} + [r^2 A_{s,z}]_{,r} \pm [\gamma^{z\bar{z}}] A_{z,z\bar{z}} \end{array} \quad (118)$$

Then,⁷² this can be used to quickly gauge the "covariant" current:

$$\Rightarrow \begin{array}{l} \delta_u^\mu [\partial_r [r^2 A_{u,r} + [r^2 A_s]_{,r}] \mp [\gamma^{z\bar{z}}] A_{r,z\bar{z}}] \\ J^\mu = + \delta_r^\mu [\partial_s [r^2 A_{u,r} + [r^2 A_s]_{,r}] \mp [\gamma^{z\bar{z}}] (A_{u,z\bar{z}} \mp A_{\bar{z},zr})] \\ \pm \frac{[\gamma^{z\bar{z}}]}{2r^2} \delta_z^\mu [r^2 (2A_{z,ur} \mp A_{z,rr}) + 2r A_{s,z} \pm [[\gamma^{z\bar{z}}] A_{z,\bar{z}}]_{,z}] \end{array} \quad (119)$$

Note "lowering" electromagnetic currents restores symmetry between the δ_u^μ and δ_r^μ sectors; dually, "raising" the current induces a $\partial_r \rightarrow \partial_s$ with a uniform weight "displacement" current $J_\gamma^u = [\gamma^{z\bar{z}}] A_{\bar{z},zr}$, or: $\mathbb{T}[\mathcal{M}_u] \rightarrow \mathbb{T}[\mathcal{M}_f] \oplus \partial_r \oint_{d\Gamma_\gamma} \vec{A}_\gamma$. A quick way to resolve this current is to use the r -tangent of the Lorenz gauge (on the $[\gamma]$ -sector), yielding:

$$J^r = [r^2 A_{u,r} + [r^2 A_s]_{,r}]_{,s} \mp ([r^2 A_{r,u}]_{,r} + [r^2 A_s]_{,rr} + [\gamma^{z\bar{z}}] A_{u,z\bar{z}}) \quad (120)$$

At this point the selection of sources is simply analytic and closed; but, following the first fundamental ethos of functional analysis: there always exists relative phase categorizations based on spectral decompositions. In particular, the strongest (over topologically representable maps) universal (topological) algebra is almost always well defined along some $U(k)[*;\cdot] \times G_k[*;\cdot]$ representation decomposition [49]); considering the advantages of Feynman representation of time-like (a-priori time-like ordered) operators (namely the completeness of *in/out*-convolution decompositions [50]), and the natural identification of vacuum Maxwells' equations with the geodesic waves⁷³. In particular, noticing that ob-

⁷¹note, in the previous footnote's displacement notation: $u_{a/r} = \Delta_{\pm r}[t]$ and $\frac{1}{r}(x_1, -ix_2, x_3) = \frac{1}{\Delta_{-||z||}[1]}(\Delta_{-\bar{z}}[z], \Delta_{\bar{z}}[z], \Delta_{||z||}[1])$

⁷²Note, defining $\zeta = z - \bar{z}$ shows that $\pm [[\gamma^{z\bar{z}}] F_{ur}]_{,\zeta} = r^2 A_{r,u\zeta} + [r^2 A_{s,\zeta}]_{,r}$

⁷³This can be clearly seen by analyzing the T_1^1 representation of Maxwell's equations, namely defining $\mathcal{F}^\mu{}_\beta := g^{\alpha\mu} F_{\alpha\beta} \equiv \nabla^\mu A_\beta - \nabla_\beta A^\mu$:

$$J^\mu = \partial_\nu [\sqrt{-g} g^{\beta\nu} \mathcal{F}^\mu{}_\beta] = \partial_\nu [\sqrt{-g} g^{\beta\nu} (\nabla^\mu A_\beta - \nabla_\beta A^\mu)] \quad \partial_\lambda [\epsilon_{\mu\nu\omega k} g^{\beta\nu} \mathcal{F}^\mu{}_\beta] = 0 \quad (121)$$

$$\text{OR:} \quad \partial_\nu [\sqrt{-g} g^{\beta\nu} \nabla_\beta A^\mu] = -J^\mu - \partial_\nu [\sqrt{-g} \nabla^\mu A^\nu] \quad \partial_\lambda [\epsilon_{\mu\nu\omega k} \nabla^\mu A^\nu] = 0 \quad (122)$$

Notice this is just the typical free-space gauge decomposition: the LHS of Maxwell's equation look like a curved spacetime wave (indexed along each coordinate μ); then, in principle the gauge (bulk-bulk and bulk-boundary) may be chosen so that the RHS resembles an empty current with a gauge residue source. When formulated in the integral expressions/algebraic generators, this won't require a redefinition of

servable space appears to be smoothly measurable across large and small scales, a natural idea is to expand the gauge functions as functions of r : $A_\nu = \sum_k \frac{A_\nu^{(k)}[u,r;z,\bar{z}]}{r^k}$.⁷⁴ In this case, the Lorenz gauge condition (and defining the shorthand $A_s^{(k)} := A_u^{(k)} \mp A_r^{(k)}$) may be rewritten as:

$$\nabla^\mu A_\mu = 0 \Rightarrow r^2 A_{r,u} + \partial_r [r^2 (A_u \mp A_r)] = \mp \gamma^{z\bar{z}} A_{\bar{z},z} \quad (124)$$

$$(125)$$

$$0 = \sum_{k \geq 0} \frac{A_{r,u}^{(k)} \pm \gamma^{z\bar{z}} A_{z,\bar{z}}^{(k-2)} - (k-3) A_s^{(k-1)} + A_{s,r}^{(k)}}{r^{k-2}} \quad (126)$$

In particular⁷⁵, note that $k \leq 0$ represents a convergent series in $r \rightarrow 0$ compact measures, while $k \geq 0$ corresponds to large- r convergent gauge measures. In both cases, note that this expansion represents a constraint tree branched from the $k_\mp \in \{0, \mp 1, \mp 2\}$ residues. Because this index symmetry are simply connected under a \mathbb{Z}_2 gauge gluing for simplicity consider the $k \geq 0$ case corresponding far, locally large metric gaugings. Then, these

the local Green basis: instead, it can strictly be realized as a convolution/functional basis expansion [15]. Indeed, considering the force free condition, Maxwell's equations become:

$$F_{\mu\lambda} \left(\partial_\nu \left[\sqrt{-g} g^{\beta\nu} \nabla_\beta A^\mu \right] + J^\mu \right) = -F_{\mu\lambda} \partial_\nu \left[\sqrt{|-g|} \nabla^\mu A^\nu \right] \quad (123)$$

Then, if the RHS is 0 then the free wave analogy is precise; this holds for every leg on the global symmetry sector, for example $\mu \in \{t, \phi, \varphi\}$ in single spinning Myers Perry backgrounds. In particular, remembering that \mathbf{F} is anti-symmetric, this is the condition that the (differentiation) gauge field is transverse or spacetime symmetric. When the creation/annihilation spectrum is weighted by the transverse (closed flow-normal, bulk) normal, it may be computationally useful to include non-globalized symmetries. The classical example of local broken symmetries is time delay responses, whereby local symmetry breaks (heuristics) serve as integration constants for some (chaotic, quasi-open) sub-network (described classically above, and experimentally first realized in studies of ferromagnetism [51]); perhaps the quintessential application is the saddle point derivation of the Second Law of Thermodynamics/(Free-Energy/second quantization), whereby (differential-flow) entropy is established as a universal identification of stability/instability as the canonical involution parameter of the Free Energy (a.k.a., the Legendre fixed point representation of the actionable shells.).

⁷⁴Note that this representation may be considered a dressing, not simply an expansion, of the gauge field because we parameterize each series term by a u, r functional. This is exactly a hyperfine convergent sequence, in the parabolic sense.

⁷⁵note that $\partial_r A_\mu^{(k)} = 0$ represents a polynomial class gauge and produces the usual $d = 4$ AFS Lorenz condition

to zero with:

$$A_r^{(1)} - A_{r,r}^{(2)} = e^{f^{(2)}[u,z,\bar{z}] + g^{(2)}[r,z,\bar{z}] - \int \int dud^3\vec{k}[u]k^2[u]\tilde{A}_s^{(2)}} \quad (132)$$

$$A_r^{(4)} + A_{r,r}^{(5)} = e^{f^{(5)}[u,z,\bar{z}] + g^{(5)}[r,z,\bar{z}] + \int \int dud^3\vec{k}[u]k^2[u]\tilde{A}_s^{(5)}} \quad (133)$$

In the above, $\{f^{(j)}, g^{(i)}\}$ can be understood to formally include the lower bounded Fourier moments of the second order differential connection. In particular, this formally induces:

$$\frac{1}{2}e^{-\frac{1}{2}(f^{(5)} + f^{(2)} + g^{(5)} + g^{(2)} + \int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_s^{(5)} - \tilde{A}_s^{(2)}))}. \quad (134)$$

$$\cdot \left((A_r^{(4)} + A_{r,r}^{(5)}) e^{-\frac{1}{2}(f^{(5)} - f^{(2)} + g^{(5)} - g^{(2)})} \mp (A_r^{(1)} - A_{r,r}^{(2)}) e^{\frac{1}{2}(f^{(5)} - f^{(2)} + g^{(5)} - g^{(2)})} \right) \quad (135)$$

$$= \left\{ \begin{array}{l} \sinh \\ \cosh \end{array} \left[\frac{1}{2} \int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_s^{(5)} + \tilde{A}_s^{(2)}) \right] \right\} \quad (136)$$

Notably, the above equation represents a hidden(/emergent) twist symmetry in the non-linear $f^{(5)} - f^{(2)} + g^{(5)} - g^{(2)} \rightarrow f^{(5)} - f^{(2)} + g^{(5)} - g^{(2)} + \frac{i\pi}{2}$ while keeping $f^{(5)} + f^{(2)} + g^{(5)} + g^{(2)} + \int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_u^{(5)} - \tilde{A}_u^{(2)})$ fixed.⁷⁸

Note that this simply represents a full utilization of the residual trivial gauge symmetry, here used to balance the $\langle r \rangle$ -ideal poles; further note that the hidden \mathbb{Z}_2 fixed point was found by imposing polynomial class restrictions on the $A_{\{z,\bar{z}\}}^{(1),(2);(4),(5)}$ functionals while keeping the other gauge-functionals ($A_{\{z,\bar{z}\}}^{(0);(3)}$ and all $A_{\{u,r\}}$) classes not-necessarily polynomial in $\langle r \rangle$. This can be considered a form of topological smearing (of divergence free vector-functionals in this spacetime, generally) or as a non-linear functional class extension. Indeed, looking back to the original metric $-du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$, parameterizing the

⁷⁸Which, in turn, can be represented as a single twist in the order-(2) involution (Fourier) boundary conditions, $f^{(2)} + g^{(2)} \rightarrow \frac{i\pi}{4} - \frac{1}{2} \int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_u^{(5)} - \tilde{A}_u^{(2)}) \equiv \frac{1}{8} \left(\int_0^\infty \int_0^\infty d\Delta d\omega \omega^{-i\Delta-1} - 4 \int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_u^{(5)} - \tilde{A}_u^{(2)}) \right)$. In particular, $f^{(2)} + g^{(2)} \rightarrow_{z\bar{z}} 0$ may be represented as a conformally twisted matching constraint:

$$\int \int dud^3\vec{k}[u]k^2[u](\tilde{A}_u^{(5)} - \tilde{A}_u^{(2)}) = \frac{1}{4} \int_0^\infty \int_0^\infty d\Delta d\omega \omega^{-i\Delta-1} \quad (137)$$

which preserves a functional basis $\{S, C\}$ constrained such that $S^2 + C^2 = -1$ (i.e., constrained on the unit thermal circle). In this case, the LHS may be understood as an *in/out* decomposition of the notion of a point particle (density-)functional into an *in/out* Brillouin moment (time-density-)functional between the non-linear components of the electromagnetic field's (2) and (5) order gauge-moments. In this sense spacetime, as opposed to a measurement of spacetime, may be understood as an $r^{(3)}$ -order functional constraint (which by construction matches the (1, 3) Minkowski constraint).

Critically, here the only difference between large and small gauges is the lexicographic ordering of the \pm sign switch of the constraint vector in 127, which can be understood as a conformally fixed (*in/out*) switching between $(\tilde{A}^{(5)}, \tilde{A}^{(2)}) \Leftrightarrow (\tilde{A}^{(-2)}, \tilde{A}^{(-5)})$. Dually, systems which "appear" both big and small (or, have quasi-stable modes in both regimes) must have components which appear very small to very small observers as well as components which appear very large to very large observers; in fact, this makes heuristic sense, as quasi-normal modes are $O(1)$ dynamical features. Finally, this explains the role of thermodynamics in the arrow of (cosmological) time as a consistency condition between isomorphic conformal decompositions of vacuum unit-time under $r^{(3)}$ -order measurements.

advanced/retarded sign switch as $-2dudr \rightarrow 2(-1)^c dudr$, the Lorenz condition can be rewritten:

$$0 = \gamma^{z\bar{z}} A_{z,\bar{z}} + 2r^2 e^{-2ic\pi} \left(\frac{1}{r} \left[1 + \frac{r\partial_r}{2} \right] A_r + e^{ic\pi} \left(\frac{A_{r,u}}{2} + \frac{1}{r} \left[1 + \frac{r\partial_r}{2} \right] A_u \right) \right) \quad (138)$$

An easy, heuristic way to understand the above is by strongly closing the non-linear dimensions around small gauge functionals, or: $\tilde{A}_s^{(5)} \sim \tilde{A}_s^{(2)}$. Then, the above formal constraint is that $\tilde{A}_s^{(2)}$ is everywhere analytic (in u) over the non-analytic basis $A_r^{(i)} + \partial_r A_r^{(i+1)}$; when the matching is stationary in r then this represents a direct decomposition of $A_r^{(4)} \mp A_r^{(1)}$ into $A^{(2)}$ -moments. It's important to remember that this ran simply from the Lorenz gauge, $\nabla^\mu A_\mu = 0$, in asymptotically flat, $d = 5$ spacetime, and from excluding divergent field contributions; then, the small system bound naturally isolated points of $< r >$ -linearity from the non- $< r >$ -linear expansions.

Remembering this lesson from the non-linear expansions, it is quick to return to the fully linear case by setting $G = f[u, z, \bar{z}]$; then, considering $(k) < (2)$ to be spectral noise, the Lorenz condition reduces to:

$$A_{r,u}^{(k+2)} - (k-1)A_s^{(k+1)} \pm \gamma^{z\bar{z}} A_{z,\bar{z}}^{(k)} = 0 \quad k > 0 \quad (139)$$

By assuming $A_s^{(1)} = \mp A_r^{(1)} = 0$ the algebraic singularity (resulting from $k \sim 1$) can be isolated as u -like boundary conditions of the first two (z, \bar{z}) -gauge fields.⁷⁹

In fact, returning to Maxwell's equations with the same generalization on the gauge expansion gives:

$$\begin{aligned} J^\mu \pm [\gamma^{z\bar{z}}] \delta_{(a)}^\mu \partial_{z\bar{z}}^2 \begin{pmatrix} A_r \\ A_u \\ \frac{\pm 1}{2r^2} \oint_{z \sim z^a} [\gamma^{z\bar{z}}] A_{z,\bar{z}} \end{pmatrix} & \quad (a) \\ & \quad (140) \\ = \delta_u^\mu \left[\sum \frac{A_{s,rr}^{(k)} + A_{u,rr}^{(k)} - (k-3) \left(2A_{s,r}^{(k-1)} - (k-4)A_s^{(k-2)} - (k-2)A_u^{(k-2)} \right) - 2(k-2)A_{u,r}^{(k-1)}}{r^{k-2}} \right] \\ = \delta_r^\mu \left[\sum \frac{A_{u,rs}^{(k)} + A_{s,rs}^{(k)} \mp \left(A_{s,rr}^{(k)} + A_{r,ur}^{(k)} \right) + (k-3) \left((k-2)A_u^{(k-2)} - (1 \mp 1)(2A_{s,r}^{(k-1)} + A_{r,u}^{(k-1)}) - (k-4)A_s^{(k-2)} \right) - 2(k-2)A_{u,s}^{(k-1)}}{r^{k-2}} \right] & \quad (141) \\ \pm \frac{[\gamma^{z\bar{z}}]}{2r^2} \delta_z^\mu \left[\sum \frac{A_{z,sr}^{(k)} + A_{(s,z)}^{(k-1)} \mp \left((k-2)A_z^{(k-2)} - A_{z,r}^{(k-1)} \pm A_{z,s}^{(k-1)} \right)}{r^{k-2}} \right] \end{aligned}$$

Or, isolating the non-linear contributions:

⁷⁹ $A_{r,u}^{(i)} = \gamma^{z\bar{z}} A_{z,\bar{z}}$ s.t. $i \in \{2, 3\}$

$$\begin{aligned}
& J^\mu \pm [\gamma^{z\bar{z}}] \delta_a^\mu \partial_{z\bar{z}}^2 \left(\begin{array}{c} A_r \\ A_u \\ \frac{\pm 1}{2r^2} \oint_{z \sim z^a} [\gamma^{z\bar{z}}] A_{z,\bar{z}} \end{array} \right)^{(a)} - \\
& \left(\begin{array}{c} \delta_u^\mu \left[\sum \frac{(k-3)((k-4)A_s^{(k-2)} + (k-2)A_u^{(k-2)})}{r^{k-2}} \right] \\ \delta_r^\mu \left[\sum \frac{(k-3)((k-2)A_u^{(k-2)} + A_{r,u}^{(k-1)} - (k-4)A_s^{(k-2)}) - 2(k-2)A_{u,u}^{(k-1)}}{r^{k-2}} \right] \\ \pm \frac{[\gamma^{z\bar{z}}]}{2r^2} \delta_z^\mu \left[\sum \frac{A_{s,z}^{(k-1)} \mp (k-2)A_z^{(k-2)}}{r^{k-2}} \right] \end{array} \right) \\
= & \delta_u^\mu \left[\sum \frac{\partial_r (A_{s,r}^{(k)} + A_{u,r}^{(k)} - 2(k-3)A_s^{(k-1)} - 2(k-2)A_u^{(k-1)})}{r^{k-2}} \right] \\
& + \delta_r^\mu \left[\sum \frac{\partial_r (2A_{s,s}^{(k)} + A_{[u,s]}^{(k)} - 2(k-3)(1 \mp 1)A_s^{(k-1)} \pm 2(k-2)(A_u^{(k-1)} - A_{r,u}^{(k)}))}{r^{k-2}} \right] \pm \frac{[\gamma^{z\bar{z}}]}{2r^2} \delta_z^\mu \left[\sum \frac{\partial_r (A_{z,s}^{(k)} \pm A_z^{(k-1)})}{r^{k-2}} \right]
\end{aligned}$$

For clarity, the linear contributions are in the top line, while the sub-linear contributions are in the second equation. Note that the (k) functional cutoffs(/dimensional sourcing) at (2) and (3), meaning that the series have natural dimensional regularizers (at sub-dimensions 2 and 3); also, note the final two terms in the sub-linear J^r current can be inductively closed as a connection between the time integration bounds (of the unit-sublinear u-gauge field) and the (scale) r-gauge field. In fact, this relation can be substituted into the linearized gauge equations ($\delta_r^\mu []$ terms) to eliminate the A_r gauge field from the second line, leaving a series in only the u and s gauge fields; critically, the s -contribution to the bulk current is algebraically fixed to the kernel at $k = 4, 3$, while the (non-linear fixing) produces an exactly algebraic relation at order $k = 2$.⁸⁰

A similar analysis could be done for the u sector (to make direct contact with the divergence winding explored above) but for brevity this section will conclude by analyzing the non-linear constraints on the z, \bar{z} sector. Setting each order of the radial spline to zero amounts to solving the following tower of differential equations:

$$A_{z,s}^{(k)} \pm A_z^{(k-1)} = g_z^{(k)}[u, z, \bar{z}] \quad (145)$$

Here, g_z amounts to an effectively projected kernel⁸¹. Then, differentially pushing each

⁸⁰Note that the linear, covariant time current is always algebraically weighed (in the u, r sector), and that it reduces exactly to A_s^0 , 0 and A_r^2 at the (k) -levels (2), (3), (4) respectfully; there is a clear similar reduction in the z -equations at $k = 2$ Similarly, the r-equivalent linear equation reduces to $A_{r,u}^{(1)} + 2A_s^{(0)} + 2A_{u,u}^{(2)}$ and $2A_u^{(2)} + \partial_u (A_r^{(3)} - 4A_u^{(3)})$ at (2), (3), (4).

⁸¹Seen by noting $\partial_s g_z^{(k)} = \partial_u g_z^{(k)}$, or $\frac{\partial_s g_z^{(k)}}{\partial_u g_z^{(k)}} \sim 1$

splined closure relation independently and inducting produces the following identities:

$$g_{z,uu}^{(k)} - g_z^{(k-2)} = \partial_s \left(A_{z,ss}^{(k)} - A_z^{(k-2)} \right) \pm \left(A_{z,ss}^{(k-1)} - A_z^{(k-3)} \right) \quad (146)$$

$$A_{z,ssss}^{(k+2)} + A_z^{(k-2)} = \partial_u \left[g_{z,uu}^{(k+2)} - g_z^{(k)} \right] \pm \left(g_{z,uu}^{(k+1)} + g_z^{(k-1)} \right) \quad (147)$$

Then remembering that the g_z^i operators always pull back against s to projection involutions out of $\mp r$, and that the induction relation is second order, immediately leads to the treatment of the $(k) \rightarrow (k+1)$ induction step as s -sublinear (on A_z), or:

$$(\partial_s \pm 1) \left[A_{z,ss}^{(k)} - A_z^{(k-2)} \right] = 0 \Rightarrow g_{z,uu}^{(k)} - g_z^{(k-2)} = 0 \quad (148)$$

Looking back to 147 and noticing the primary difference is between the signs within each (sub-)ordering leads to the inference of the existence of an \mathfrak{su}_2 closed tower operator on the g -tower (operationalized over u), here denoted (and implying):

$$(\partial_u \pm_\gamma 1) \left[g_{z,uu}^{(k+2)} - g_z^{(k)} \right] = 0 \Rightarrow A_{z,ssss}^{(k+2)} + A_z^{(k-2)} = 0 \quad (149)$$

Looking back, indeed, the linear portion of the covariant current is symmetrically weighted between $(k) \in ([0, 4])$ (here operationalized over z). Note +that the critical linear (z -gauge) dimension is $k = 2$, which is also the lowest well defined ($k \leq 0$) value of 149. One way to interpret this is to say that non-linear, two point contacts may be used to close (in spin) the electromagnetic four-point (on- (k) -shell) up to an order (-4) OPE, $A_z^{(k)} - A_z^{(k+4)}$. Note that, for any finite $k \geq k_{\text{top}}$ cutoff this could be, optimistically, interpreted as the resonance of a (spacetime emergent) Wilson-type operator.

3.6 Finite Symbolic Corrections

This can be shown by explicitly expanding the equations of motion into the functional envelope of the independent degree of freedom, done here. Let some general metric be decomposed as $g = \hat{A} \oplus \hat{c} \oplus \hat{B}$:

$$[g^{-1}] = \begin{bmatrix} [A^{-1}] & & \\ & c^{-1} & \\ & & [B^{-1}] \end{bmatrix} \quad \begin{array}{l} \sqrt{|g|} = \sqrt{c} \sqrt{|A \oplus B|} \\ \text{Then, defining:} \end{array} \quad [g_4] := [A \oplus B] \equiv \begin{bmatrix} [A] & \\ & [B] \end{bmatrix} \quad (150)$$

$$\Rightarrow J^\mu = \partial_\nu \left[\sqrt{|-g|} g^{\alpha\mu} g^{\alpha\mu} F_{\alpha\beta} \right] = \sqrt{c} \left(\begin{array}{l} \left[\begin{array}{ccc} 1 & c^{-1} & c^{-2} \end{array} \right] \cdot \partial_\nu \left[\begin{array}{c} \left(\begin{array}{c} R^{(0)} \\ R^{(1)} \\ R^{(2)} \end{array} \right)^{\alpha\mu\beta\nu} \\ F_{\alpha\beta} \end{array} \right] \\ + \partial_\nu \ln c [R^0 - R^{(2)}]^{\alpha\mu\beta\nu} F_{\alpha\beta} \end{array} \right) \quad (151)$$

$$\text{where } \begin{pmatrix} R^{(0)} \\ R^{(1)} \\ R^{(2)} \end{pmatrix}^{\alpha\mu\beta\nu} := \sqrt{|g_4|} \begin{pmatrix} [g_4] \otimes [g_4] \\ [g_4] \otimes [\delta_{\varphi\varphi}] + [\delta_{\varphi\varphi}] \otimes [g_4] \\ [\delta_{\varphi\varphi}] \otimes [\delta_{\varphi\varphi}] \end{pmatrix}^{\alpha\mu\beta\nu} \quad (153)$$

are defined as ratios of the c-functional ⁸². Then, the force free condition becomes:

$$0 = F_{\mu\lambda} J^\mu = \sqrt{c} \left(\begin{array}{l} F_{\mu\lambda} \partial_n u \left[\sqrt{|g_4|} g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right] + F_{\mu\lambda} \partial_\nu \left[R^{(1)\alpha\mu\beta\nu} F_{\alpha\beta} + R^{(2)\alpha\mu\beta\nu} F_{\alpha\beta} \right] \\ + \partial_\nu \ln c [R^0 - R^{(2)}]^{\alpha\mu\beta\nu} F_{\alpha\beta} F_{\mu\lambda} \end{array} \right) \quad (154)$$

OR, stripping out the c-dimensional (trace exact) contact: (155)

$$\frac{F_{\mu\lambda}}{\sqrt{|g_4|}} \left(\partial_\nu \left[\sqrt{|g_4|} g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right] + \partial_\nu \left[R^{(1)\alpha\mu\beta\nu} F_{\alpha\beta} \right] \right) \quad (156)$$

$$= (\partial_\nu \ln c) \left[g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} F_{\mu\lambda} \right] + [\delta_{\varphi\varphi\varphi\varphi}]^{\alpha\mu\beta\nu} F_{\mu\lambda} \left(\frac{1}{\sqrt{|g_4|}} \partial_\nu \left[\sqrt{|g_4|} F_{\alpha\beta} \right] - (\partial_\nu \ln c) F_{\alpha\beta} \right) \quad (157)$$

The rightmost term is uniformly zero by the anti-symmetry of F, ⁸³, giving finally:

$$F_{\mu\lambda} \partial_\nu \left[\sqrt{|g_4|} g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right] + \frac{\sqrt{|g_4|}}{c} F_{\mu\lambda} \left(\frac{1}{\sqrt{|g_4|}} \partial_\nu \left[\sqrt{|g_4|} \delta_{\varphi\varphi}^{(\alpha\mu} g^{\beta\nu)} F_{\alpha\beta} \right] - (\partial_\nu c) g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right) = 0 \quad (158)$$

This equation has a few interesting properties surrounding how it collapses to the g_4 stream equation (the leftmost term). First, note that the final term drops out when c is constant (x^ν -independent) and that the second term drops when $g^{\beta\nu} F_{\varphi\beta}$ is constant (0 or x^ν -independent). Also, note only the first term remains when $c \rightarrow \infty$.⁸⁴ Note that the first two terms dominate (on the constraint integral shells) where g_4 is degenerate, while the final term dominates when c is small and extremely non-stationary(/oscillatory). This

⁸²Here, c was considered effectively constant at the level of Maxwell's equations, but possibly non-stationary at the force free conditional; this can be strongly interpreted as an asymptotic stability, as opposed to invertibility, boundary condition. Note the symmetrization GL operation is between the T_1^1 operational pairs of indicies, $R^{(1)} \equiv [\delta_{\varphi\varphi}]^{(\alpha\mu} g^{(\beta\nu)} := [\delta_{\varphi\varphi}]^{\alpha\mu} g^{\beta\nu} + [\delta_{\varphi\varphi}]^{\beta\nu} g^{\alpha\mu}$

⁸³This term is kept here as a comparison point to higher trace-class gauge comparison

⁸⁴While, conversely, as $c \rightarrow 0$,

$$F_{\mu\lambda} \left(\partial_\nu \left[\delta_{\varphi\varphi}^{(\alpha\mu} g^{\beta\nu)} F_{\alpha\beta} \right] - (\partial_\nu c) \sqrt{|g_4|} g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right) \rightarrow 0 \quad (159)$$

may also be applied to whatever gauge-constraint choice is made, $\oint \epsilon_{\mu\nu} A^\mu = \oint \epsilon_{\lambda\nu} \nabla^\lambda A_\mu$; considering the Lorenz gauge⁸⁵ and introducing a dot index to denote the canonical $d = 4$ subspace,

$$0 = \nabla^\mu A_\mu \Rightarrow \partial_\varphi \left[\sqrt{|g_4|} \delta_\varphi^\lambda A_\lambda \right] = -\partial_{\dot{\mu}} \left[\sqrt{c} \sqrt{|g_4|} g^{\dot{\mu}\dot{\alpha}} A_{\dot{\alpha}} \right] \quad (160)$$

and taking a naive (g_4 functionally fixed) limit on c (161)

$$\partial_\varphi \left[\sqrt{|g_4|} \delta_\varphi^\lambda A_\lambda \right] = \begin{cases} -\frac{\sqrt{|g_4|}}{2} g_4^{\dot{\mu}\dot{\alpha}} A_{\dot{\alpha}} \partial_{\dot{\mu}} [\ln c] & c \rightarrow 0 \\ \sqrt{c} \partial_{\dot{\mu}} \left[\sqrt{|g_4|} g_4^{\dot{\mu}\dot{\alpha}} A_{\dot{\alpha}} \right] & c \rightarrow \infty \end{cases} \quad (162)$$

In particular, notice that this (functionally fixed) gauge point, in both cases, presents as a relative divergence in the φ - Lorenz gauge normalization (unless c is essentially patched almost everywhere near $\rightarrow 0$, in which case the RHS of the top line is uniformly zero). Assuming that the spacetime is independent of the extended coordinate results in:

$$0 = \begin{cases} \frac{1}{c} \left(\partial_\varphi \left[\delta_\varphi^\lambda A_\lambda \right] - \frac{1}{2} g_4^{\dot{\mu}\dot{\alpha}} A_{\dot{\alpha}} \partial_{\dot{\mu}} c \right) & c \rightarrow 0 \\ \sqrt{c} \left(\frac{1}{\sqrt{c}} \partial_\varphi \left[\delta_\varphi^\lambda A_\lambda \right] + \sqrt{\frac{1}{|g_4|}} \partial_{\dot{\mu}} \left[\sqrt{|g_4|} g_4^{\dot{\mu}\dot{\alpha}} A_{\dot{\alpha}} \right] \right) & c \rightarrow \infty \end{cases} \quad (163)$$

Implicitly, when c represents a very large point-rescaling over a hidden dimension with unit sized gauge parameters the left term of the bottom limit drops out and gauge constraint reduces to the projected $d = 4 < 5$ constraint form. Conversely, when c represents a very small point-rescaling the gauge field over the represented hidden dimension is affixed to the rate of descent of the accumulation⁸⁶. Note that if c rapidly oscillates as it decays this can be considered a φ -pole momentum pole.

Comparing with the force free condition:

$$0 = \begin{cases} F_{\mu\lambda} \left(\frac{1}{\sqrt{|g_4|}} \partial_\nu \left[\sqrt{|g_4|} \delta_{\varphi\varphi}^{(\alpha\mu} g^{\beta\nu)} F_{\alpha\beta} \right] - (\partial_\nu c) g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right) & c \rightarrow 0 \\ F_{\mu\lambda} \left(\frac{1}{\sqrt{|g_4|}} \partial_\nu \left[\sqrt{|g_4|} g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right] - (\partial_\nu \ln c) g_4^{\alpha\mu} g_4^{\beta\nu} F_{\alpha\beta} \right) & c \rightarrow \infty \end{cases} \quad (164)$$

Note that the effective source functional in the $c \rightarrow \infty$ case is exactly the same functional effectively sourcing the A_λ Lorenz string in the $c \rightarrow 0$ limit. In the $c \rightarrow 0$ case the wave-like part is entirely effervescent (has a φ index $F_{\varphi\alpha}$), and is weighed by the $d = 4$ projected density against the c -velocity and the (4-)electromagnetic (4-)density. In the $c \rightarrow \infty$ case the system is, instead, represented by a massive mediation over the 4-interaction. Unifying

⁸⁵In the Myers-Perry metric, $c = r^2 \cos^2 \theta$, so these c -limits represent $\{c\} \rightarrow \{\infty, 0\} \sim \{r \rightarrow \infty, \theta / \rightarrow \frac{\pi}{2}\}, \{\{\theta \rightarrow \frac{\pi}{2}\}, \{\}\}$. Note that, in the large r -limit: $\int \lim_{\partial_\nu \gamma \rightarrow \infty} \partial_\nu \ln c \rightarrow 2 \int_\gamma [dx_\nu] (\ln \cos \theta)$

⁸⁵Noting $\nabla^\mu A_\mu = \frac{1}{\sqrt{|g|}} \partial_\mu \left[\sqrt{|g|} g^{\mu\beta} A_\beta \right]$

⁸⁶Meaning $\delta_\varphi^\lambda A_{\lambda,\varphi} = \frac{1}{2} A \cdot \nabla_4 c$; in the above analysis, c was assumed locally flat, meaning this represents a gradient of locally propagating Heaviside measure functionals. By the fundamental theorem of analysis, this *in*-accumulation weight is always constructable for smooth functionals (over some arbitrarily large cut-space embedding). Only in the entire embedding, or the everywhere uniform limit $c \rightarrow 0$, does the Gauss condition on small c directly imply $A_{\varphi,\varphi} = 0$

the notation with the index $\lim_{c \rightarrow \infty} \tilde{c} \rightarrow \ln c$ and $\lim_{c \rightarrow 0} \tilde{c} \rightarrow c$ (including units) and adding dots everywhere to denote the $d = 4$ projection indices in the definition of the stress tensor (in a strictly $d = 4$ embedding):

$$F^{\dot{\alpha}}_{\dot{\mu}} F_{\dot{\alpha}\dot{\nu}} = \frac{g_{\dot{\mu}\dot{\nu}} F^2}{5} - \mu_0 T_{\dot{\mu}\dot{\nu}}^{em} - \delta_{\varphi\varphi}^{\lambda\sigma} [g^{\varphi\varphi} F_{\sigma\dot{\mu}} F_{\lambda\dot{\nu}}] \quad (165)$$

Defining the lower dimensional stress tensors with dots as $\dot{F}^{\alpha}_{\mu} \dot{F}_{\alpha\nu} := \frac{\dot{g}_{\mu\nu} \dot{F}^2}{4} - \dot{\mu}_0 \dot{T}_{\mu\nu}^{em}$ finds:

$$\Rightarrow F^{\dot{\alpha}}_{\dot{\mu}} F_{\dot{\alpha}\dot{\nu}} - \dot{F}^{\alpha}_{\mu} \dot{F}_{\alpha\nu} + \delta_{\varphi\varphi}^{\lambda\sigma} [g^{\varphi\varphi} F_{\sigma\dot{\mu}} F_{\lambda\dot{\nu}}] + \frac{g_{\dot{\mu}\dot{\nu}} F^2}{20} = \quad (166)$$

$$\frac{g_{\dot{\mu}\dot{\nu}} F^2 - \dot{g}_{\mu\nu} \dot{F}^2}{4} - \left(\mu_0 T_{\dot{\mu}\dot{\nu}}^{em} - \dot{\mu}_0 \dot{T}_{\mu\nu}^{em} \right) \quad (167)$$

Note that Wick rotating φ produces a relatively electric index. Instead demanding that the dot index operation and the dot functional operation match linearly on the level of the stress tensor produces the matching constraint

$$\delta_{\varphi\varphi}^{\lambda\sigma} [g^{\varphi\varphi} F_{\sigma\dot{\mu}} F_{\lambda\dot{\nu}}] = -\frac{g_{\dot{\mu}\dot{\nu}} F^2}{20} \quad (168)$$

$$\text{which in turn implies:} \quad \Rightarrow \frac{5}{6} F^{\dot{\alpha}}_{\dot{\mu}} F_{\dot{\alpha}\dot{\nu}} \doteq \frac{g_{\dot{\mu}\dot{\nu}} F^2}{4} - \tilde{\mu}_0 T_{\dot{\mu}\dot{\nu}}^{em} \quad (169)$$

where $\tilde{\mu} := \frac{5}{6}\mu$. Remember that this term is carried on the regulatory flow, $\partial^\nu \tilde{c}$, in the equations of motion.

Then, this model predicts that $d = 4$ functionally sub-shelled (“dimensionally waveguided”) OPEs over $d = 5$ dimensions are characterized by a $\frac{5}{6}$ reduction in the kinetic junction (relative to the bulk conformal stress tensor). Or, supposing $F^2 = \frac{2}{3}\mu_0 \mathbb{I}^2$:

$$F^{\dot{\alpha}}_{\dot{\mu}} F_{\dot{\alpha}\dot{\nu}} \doteq \mu_0 (g_{\dot{\mu}\dot{\nu}} \mathbb{I}^2 - T_{\dot{\mu}\dot{\nu}}^{EM}) \quad (170)$$

Then, this is the gradient tensor (contraction) on \tilde{c} ; in either limit case, \mathbb{I} can be considered the bare kinetic regulator of the dimensional embedding, which can be understood in two ways.

Looking at [46], it was noted that the power extracted from the vertical type solutions was approximately $\delta_p \sim .3$ efficient, which can be compared to the $\delta_K \sim .36$ numerical efficiency found for typical black hole jets; accordingly, the generalized process found in [46] is strongly reflective of a kinetic embedding. Further, note that both $F^2 - \tilde{\mu} \mathbb{I}^2 = -\frac{\tilde{\mu}_0}{5} \mathbb{I}^2$ and $F^2 - \mu_0 \mathbb{I}^2 = -\frac{\mu_0}{5} \mathbb{I}^2$; further, $[\frac{1}{5}, \bar{p}_K] \sim .28$ can be directly compared to the critical phase point identified in [52], $(\frac{J}{M})_{crit} \sim .286$. Note that the final digit is exactly a unit scale shift of the power-efficiency difference between Blankford-Znajek [37] jets and those discussed in [46] $\frac{\delta_p - \delta_k}{10}$ can be explained by a thermal current (log-type field) embedding form factor. Intuitively, the full gravitational stabilizer “Love-locks” (on the kinetic electromagnetic decay branch at this thermal cross section). ⁸⁷

⁸⁷Consider naively adding MP analogous properties to \tilde{c} . Such as, consider $|g_4|$ to be an analytically measurable function of c , as well as some lapse(s) of c : $|g_4| \equiv g_4[(c[x_\alpha - \lambda_{I_\alpha}])_I]$. For example, if $g_4 \equiv$

Stronger yet, comparing the Kerr and Myers-Perry metrics in unitless coordinates (and omitting the bars over the operators⁸⁸ as a simple character redefinition):

$$\begin{aligned} ds_{MP}^2 &= -dt^2 + \Sigma \left(\frac{dr^2}{\Delta} + (r_0 d\theta)^2 \right) + (r^2 + \alpha^2) \sin^2 \theta (r_0 d\phi)^2 + r^2 \cos^2 \theta d\varphi^2 + \frac{1}{\Sigma} (dt - \alpha r_0 \sin \theta d\theta)^2 \\ ds_{Kerr}^2 &= -dt^2 + \Sigma \left(\frac{dr^2}{\Delta_K} + (r_s d\theta)^2 \right) + (r^2 + \alpha^2) \sin^2 \theta (r_s d\phi)^2 + \frac{r}{\Sigma} (dt - \alpha r_s \sin \theta d\theta)^2 \end{aligned} \quad (171)$$

Or, that, noting that $\Delta_K[r_s, \alpha; \bar{r}] = \Delta[\sqrt{2r_s \bar{r}}, \alpha; \bar{r}]$ ⁸⁹:

$$d\delta_K^2 := \lim_{r_s \rightarrow r_0} [ds_{MP}^2 - ds_{Kerr}^2] = \bar{r}^2 \cos^2 \theta d\varphi^2 + \frac{1 - \bar{r}}{\Sigma} (dt - \alpha r_0 \sin \theta d\theta)^2 + \Sigma d\bar{r}^2 \left(\frac{1}{\Delta} - \frac{1}{\Delta_K} \right) \quad (172)$$

Then, the embedding/gluing topology can be understood from a number of different accumulation regimes. Quickest, the differential measure is an exact functional in (two orthogonal coordinates) when $r \rightarrow \infty$, or $\lim_{\bar{r} \rightarrow \infty} d\delta_K^2 = \lim_{\bar{r} \rightarrow \infty} \bar{r}^2 \cos^2 \theta d\varphi^2$; then:

$$\lim_{r \rightarrow \infty} r^2 d^2 \delta_K^2 = 2 \lim_{r \rightarrow \infty} \left(\frac{d[r^2]}{2} + r^2 d \ln[\cos \theta] \right) \wedge d\delta_K^2 \quad (173)$$

So, then

$$d^2 \delta_K^\infty := \lim_{r \rightarrow \infty} d^2 \delta_K^2 \Big|_{d \ln[r^2] \sim -r^2 d \ln[\cos \theta]} = \lim_{r \rightarrow \infty} r^2 d \ln[\cos \theta] \wedge d\delta_K^2 \quad (174)$$

And, moving a slash dual operation out from the inner product to remember that this is an *outer* cohomology,

$$\langle d^2 \delta_K^\infty, d^2 \delta_K^\infty \rangle = \lim_{r \rightarrow \infty} r^4 (d \ln[\cos \theta])^2 \langle d\delta_K^2, d\delta_K^2 \rangle \quad (175)$$

Then, the algebraic topology can be considered (point) closed at second order whenever the RHS is 0.

Next, considering $\Delta_K[r_s, \alpha; *] - \Delta[r_0, \alpha; *] = 2r_s \left(\frac{r_0^2}{2r_s} - r \right)$, in limit⁹⁰ this defines a unique

$g_4[r, \cos \theta]$ and $[I]_\alpha = \delta_\alpha^\theta [0 \ \frac{\pi}{2}]$ then $g_4[r, \cos \theta] + g_4[r, \sin \theta] = g_4^{(\alpha)} = r^2$ and $\frac{c}{g_4^{(\alpha)}} = \cos^2 \theta$, or $g_4 \equiv f_4 \left[\sqrt{c_{I_0} + c_{I_1}}, \sqrt{\frac{c_{I_0}}{g_4^{(\alpha)}}} \right]$. For context, in single spinning MP, $c = r^2 \cos^2 \theta$, so $\lim_{r \rightarrow \infty} \lim_{\theta \rightarrow \frac{\pi}{2}} c_{r; [\theta \ \frac{\pi}{2}]} = \left[0 \ \lim_{r \rightarrow \infty} r^2 \right]$; in this case, this rigid shift indexing can exactly represent a delta function-like divergence. By adding a subindex of sub-rigid translations, $[\theta_0, \theta_\infty] = [0, \frac{\pi}{2}]$ $\lim_{\theta \rightarrow \frac{\pi}{2}} c[r, \theta] = \lim_{i \rightarrow \infty} \lim_{j \rightarrow i+1} \lim_{\theta_i \rightarrow \theta_j} c[r; \theta_i]$.

Indeed, in this system $\sqrt{|g_4|} = \Sigma \sin \theta \equiv (r^2 + \frac{\alpha^2 c^2}{r^2}) \sin \theta \rightarrow_{\theta \rightarrow \frac{\pi}{2}} r^2 \equiv \lim_{\theta \rightarrow n\pi} c^2$

⁸⁸ $\Delta[1, \alpha; r] = r^2 + \alpha^2 - 1$ and $\Delta_K[1, \alpha; r] = r^2 + \alpha^2 - 2r$

⁸⁹ or: $\lim_{r_0 \rightarrow r_s} [\Delta[r_0, a; r] - \Delta_K[r_s, a; r]] = 2r_s \left(r - \frac{r_s}{2} \right)$.

⁹⁰ meaning ignoring stabilizer stability measures, formally excluded by ignoring limit-sequence contributions of order $< d\bar{r}_K \pm d\bar{r}_0, d\bar{r}_K \mp d\bar{r}_0 >$. Notice that $\Delta[r_0, \alpha; r] - \Delta_K[r_0^3, \alpha; r] = -2r_0^2 \left(r - \frac{1}{2r_0} \right) = (2r_0)^{\frac{3}{2}} \sinh \ln \sqrt{2r_0 r}$ shows that the continuous embedding limit can be understood as a volume-extrinsic free-energy (*out*) constraint under a strongly-inverse net (*in*) scattering. Notably, the kernel constraint of this difference is free when $r_0 \rightarrow \infty$ and as well as when $r_0 \rightarrow 0$ converges faster than the absolute measure-

limit coordinate $r_* = \frac{r_0^2}{2r_s} \rightarrow \frac{r_s}{2} \equiv \frac{r_{\alpha \rightarrow 0}^+}{4}$ at a quarter of the outer Schwarzschild radius of a Kerr black hole of equivariant mass unit [53]. Note that at the distance $r = r_*$ the $d\tilde{r}^2$ radial index is not included in the inexact differential $d\delta_K^2$ (the inverse of the embedding metric is singular $g_\delta^{rr}[r \sim r_*] \sim O((r - r_*)^{-1})$). In fact, using the canonical natural number indexing uniformity $\tilde{r} = r + 1$ it can be shown that:

$$\left[\frac{\Delta[1, \alpha; r] \Delta_K[1, \alpha; r]}{(r-1)^2 - (1-\alpha^2)} \right]_{r \rightarrow \tilde{r}} - \frac{\Delta[1, \alpha; \tilde{r}] \Delta_K[1, \alpha; \tilde{r}]}{(\tilde{r}-1)^2 - (1-\alpha^2)} \Bigg|_{\tilde{r} \rightarrow r} = 0 \quad (176)$$

Remembering that $r = 1$ presents as the mean distance between the outer horizon and the first Cauchy surfaces, $\tilde{r} = 1$ represents the mean curvature singularity, $\tilde{\tilde{r}} = 1$ can be considered an $O(1)$ (orbitally-index field extension) deep bulk measure. Accordingly, this relationship can be immediately recognized as an exactly constrained near-singularity scattering mechanism under the continuous (topological) index contraction $r \rightarrow r$.

Doing some numerology, notice that

$$\frac{1}{6(22+1) - 1} = \frac{1}{137} = \frac{1}{6(22 + \frac{2}{3}) + 1} \quad (177)$$

“shows” the fine structure constant should be expected to emerge from a 6-charge, symmetric embedding interloper between a volume-to-area measurement (considered from $d = 4$ spacetime) over 21+1 internally thermalized degrees of freedom. In fact, with α the fine structure constant,

$$\alpha^{-1} - 137 \sim .035999084(21) \sim (1 - 2.5443861(11) \times 10^{-5}) \frac{p_k}{10} \quad (178)$$

where p_K is the energy extraction efficiency of a Kerr black hole under the collimated Blandford-Znajek process. Intuitively, then, it may be hypothesized that the electromagnetic field is dual to some (21pt) complex monodromy form (log-)regulated by an effective Blandford-Znajek mechanism mediated by a black hole embedded interaction.

3.7 "Twisted Radar"

Still, the above limit ordering was mostly helpful in resolving the strongly *in*-gauged limits of c (because c was taken to commute with the equations of motion). A less naive limit would expect to preserve the group algebra across all sub-sequenced accumulation points; typically, this effect is gauged in the adjoint algebra as a polarization decomposition. Typically of interest is the affinely parameterized geodesic paths, which in $d = 4$ AFS coordinates are locally oriented as:

$$q_4^\mu = (1 + z\bar{z}) \left(1, \frac{z + \bar{z}}{1 + z\bar{z}}, i \frac{z - \bar{z}}{1 + z\bar{z}}, \frac{1 - z\bar{z}}{1 + z\bar{z}} \right) \quad (179)$$

bilty of the coordinate r ; otherwise this difference kernel is a single valued field, $\{(r-x)^{-1} \mapsto (\frac{1}{2r_0} - r)^{-1}\}$

An immediate higher dimensional extension of the is simply to add a 4-orthogonal direction, x_5 ; there is some ambiguity how the additional dimension is embedded. The most natural extension is to simply include x_5 into the Poincare charges, which amount to choosing spherical coordinates over the space-like 4-volume. Such representations grant lightning access to deep and beautiful physics [54]. This section will make a different selection, namely a cylindrical construction (visualized as the product of a 3-sphere and an orthogonal line element). Then, making the same definitions as above (so, leaving r continue representing the 3-volume radius) gives

$$ds_5^2 = dx_5^2 - du_{a/r}^2 \mp 2du_{a/r}dr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \quad (180)$$

In particular, the flat $\mathbb{R}^{1,4}$ lightcone may be represented as:

$$q_5^\mu = (1 + z\bar{z}) \left(\cos \beta[r, z, \bar{z}], \frac{z + \bar{z}}{1 + z\bar{z}}, i \frac{z - \bar{z}}{1 + z\bar{z}}, \frac{1 - z\bar{z}}{1 + z\bar{z}} \cos \beta[r, z, \bar{z}], 0 \right) + 2\sqrt{z\bar{z}}(0, 0, 0, 0, i \sin \beta[r, z, \bar{z}]) \quad (181)$$

In this context, the (cylindrical) extra dimension can be considered a (uniformly rigid) projection of an affine spin measurement that depends on the spin radius (as opposed to the 3-radius)⁹¹. Note that $\beta[r, z, \bar{z}]$ only need to respect antipodal conditions up to $2\pi(m - n)$ modular relative sectors; accordingly, this shows a simple extension of lower dimensional unitarity that is covered by a disconnected thermal patch topology.⁹²

Considering the structure of ds_5^2 , note that the independent dimension is free and, is exactly "square structured" like $d = 2$ Minkowski rectilinear coordinates, $-dt^2 + dz^2$; then, it is immediate to wonder if the simply extended system, ds_5^2 , may represent a free, or unconstrained, embedding of some R-process. In fact, starting with the typical asymptotic coordinates of $d = 4$ spacetime $ds_{adv/ret}^2$, there is a natural pull-measure into a $d = 4$ -closed, analogous metric:

$$ds_2^2 = ||ds + idr||_{\mathbb{C}}^2 = dr^2 - du^2 \pm 2dudr + r^2\gamma_{z\bar{z}}dzd\bar{z} \quad (182)$$

In fact, starting from asymptotic coordinates in $d = 4$, consider two orthogonal vector space expansions, x_1, x_2 , such that one of them is guaranteed to complex $x_1 \in \mathbb{C}$; while

⁹¹Notice, $z\bar{z} = x_1^2 + x_2^2$; then, the $d = 5$ projection represents a conjunction between an affine spin rotation $(q_0 + q_3) \cos \beta$ and a residual subdimensional area-weighted "conjunction parameter" $iq_5 = -(\vec{0}_4, \sqrt{\zeta\bar{\zeta}})$, where $\zeta[z, \beta[r, z, \bar{z}]] = z \sin \beta[r, z, \bar{z}]$. This representation is not typical, as it is seemingly more sensible to close the group algebra exactly over the minimal charge representation functors (amounting to embedding the extended spatial dimension in global spherical coordinates so the angular momentum charges form a simple complex of the Poincare-algebra); still, it is chosen here to anticipate the target study: electromagnetism near a single spinning Myers Perry black hole.

⁹²In that respect, this can be seen as synonymous with F- and G-series compactifications in solid-state mechanics [55] whereby stabilization (phase topologies/)points are locally supported by compact generating functionals which guide the relative residue-weights of the modeled phase transition (say, the enhanced resonance at $\Delta[x] \sim 1$ of the momentum eigenstates in a thermally locked bosonic condensate which can be shown to loop-perturbatively close into the thermal contour under multi-phase locking throughout the G-/F-series).

the other is guaranteed to be integrable⁹³, $\langle dx_2|dx_2\rangle = c[*]^2 \langle d\varphi|d\varphi\rangle$ then, let $x_1 = iz_1$ this embedding metric is:

$$ds_6^2 = ds_4^2 - dz_1^2 + dx_2^2 \quad \Rightarrow \quad \|ds_6 + i\sqrt{d}dz_1\|_{\mathbb{C}}^2 = ds_4^2 + (d-1)dz_1^2 + dx_2^2 \quad (183)$$

$$\text{OR:} \quad d\tilde{s}_6^2 = \|ds_6 - \sqrt{d+1}dx_1\|_{\mathbb{C}}^2 = (d)dz_1^2 + c^2d\varphi^2 + ds_4^2 \quad (184)$$

Then, suppose $ddz_1^2 \rightarrow (d-1)^2dr^2$; then, the resultant is formally defined as:

$$d\tilde{s}_5^2 \rightarrow \|ds_6 - \sqrt{d+1}dx_1\|_{\mathbb{C}}^2 = (d-1)^2dr^2 + ds_4^2 + c^2d\varphi^2 \quad (185)$$

$$= ((d-1)^2 - 1)dr^2 + ds_2^2 + c^2d\varphi^2 \quad (186)$$

Then, $d = 1, c = 1$ corresponds to a single, orthogonal (geometric) field expansion over flat spacetime; $d = 0, c = 0$ corresponds to ds_2^2 .

Noticing the differing effective metrics of interest, the immediate choice is to naively functionalize them and look for functional response forms, given by $d\tilde{s}_5^2 \sim \tilde{d}[\cdot]dr^2 + ds^2 + c[\cdot]d\varphi^2$. In particular, the metric is singular at $(\tilde{d}, c) \sim \{(-1, c), (\tilde{d}, 0)\}$ ⁹⁴; the stream equation can be worked out in this, functionalized case to get a stronger sense of the embedding push, but may be left for a future work. Instead the single spinning Myers-Perry geometry will be next explored.

3.8 Single Spinning Meyers Perry ($b = 0$)

The clearest choice is to fix these coordinates towards the black hole's angular momenta, which shows the immediate advantage of the single spinning MP metric: the ψ -independence of both the metric- and vector-gauge fields should allow solutions with a global fixing of this gauge vector's area-flux, Ψ_ψ (or, $\omega_\psi = 0$). Note that global invariants in general relativity can be assumed to have a conformal (Weyl) interpretation; so, solutions with a constant Ψ_ψ should have an immediate thermodynamic connection.⁹⁵

Regardless, note that Maxwell's equations imply:

$$d\mathbf{F} = 0 \quad \Rightarrow \quad 0 = \Psi_\phi^{(1,0)}\omega_\phi^{(0,1)} - \Psi_\phi^{(0,1)}\omega_\phi^{(1,0)} + \Psi_\psi^{(1,0)}\omega_\psi^{(0,1)} - \Psi_\psi^{(0,1)}\omega_\psi^{(1,0)} \quad (187)$$

$$F \wedge F = 0 \quad \Rightarrow \quad 0 = \Psi_\phi^{(1,0)}\Psi_\psi^{(0,1)} - \Psi_\phi^{(0,1)}\Psi_\psi^{(1,0)} \quad (188)$$

⁹³Measurable and almost everywhere differentiable

⁹⁴Further, note that $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} -1 & \mp 1 \\ \mp 1 & -1 \end{bmatrix}\right)^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & \mp 1 \\ \mp 1 & -1 \end{bmatrix}$, so that, following the discussion about divergent matrices above, $(\tilde{d}, c) \sim (1, \infty)$ represents a "self-covariant" geometric limit.

⁹⁵More generally, the extra-magnetic field can be thought of as the additional (space-sheet adjoint) gauge freedom allowed across ψ -interaction modes (or, spacelike, embedded field-space topology); then, reductively, solutions with $\Psi_\psi \sim 1$ represent (conformally connected) spacetime perturbations entirely indexed by their Ψ_ϕ distributions (a.k.a., exactly dilatonic states). The solutions presented will focus on the $\Psi_\psi \sim 1$ case, but it may be interesting to try and qualify conformal subharmonics based on field-coordinate drift velocity as well. See (??) for the Tangherlini drift conditions.

while here, we note that, unlike $D = 4$, the degeneracy condition is not proportional to the volume form (index mismatch): $F \wedge F \not\sim \epsilon_{\alpha\beta\kappa\gamma\lambda}$, and thus it does not generally represent a separability invariant on the field sheet (unlike in $D=4$, where degeneracy and magnetic dominance guarantee complete, separable, space-like field sheet coordinates with deterministic support[45]). Still, if J , a $n = 4$ -form, has the same 4-volume orientation as $F \wedge F$, then this wedge indeed will represent a sheet invariant, $F \wedge F \wedge \star J \sim \epsilon$. This again is where the single spinning geometry with $\Psi_\psi \sim 1$ becomes interesting: examining (109), the constant Ψ_ψ condition operationally guarantees that neither $F \wedge F$ nor the vector current have functional legs on the $d\psi$ -form.⁹⁶

Imposing the FFE conditions shows the intertwiner nature of the Ψ_ψ coordinate in the degeneracy frame. Explicitly, the toroidal sector FFE equations, $j_\mu F^{\mu\nu} = 0$, are:

$$j_\mu F^{\mu t} = 0 = \omega_\phi \left(\mathbb{I}^{(0,1)} \Psi_\phi^{(1,0)}(r, \theta) - \mathbb{I}^{(1,0)} \Psi_\phi^{(0,1)} \right) + \omega_\psi \left(\mathbb{I}^{(0,1)} \Psi_\psi^{(1,0)} - \mathbb{I}^{(1,0)} \Psi_\psi^{(0,1)} \right) \quad (189)$$

$$j_\mu F^{\mu\{t,\psi\}} = 0 = \mathbb{I}^{(1,0)} \Psi_\psi^{(0,1)} - \mathbb{I}^{(0,1)} \Psi_\psi^{(1,0)} \quad (190)$$

Assuming separability in the flux coordinates, an attempt can be made to functionally satisfy the above parabolically: with $\Psi_\phi \rightarrow \Psi_\phi[f(r, \theta)]$ and $\Psi_\psi \rightarrow \Psi_\psi[g(r, \theta)]$ (e.g., $f \rightarrow A_\phi$ and $g \rightarrow A_\psi$), the above reduces to

$$d\mathbf{F} = 0 = \left(f^{(1,0)} \omega_\phi^{(0,1)} - f^{(0,1)} \omega_\phi^{(1,0)} \right) \Psi_\phi' + \left(g^{(1,0)} \omega_\psi^{(0,1)} - g^{(0,1)} \omega_\psi^{(1,0)} \right) \Psi_\psi' \quad (191)$$

$$F \wedge F = 0 = \left(f^{(1,0)} g^{(0,1)} - f^{(0,1)} g^{(1,0)} \right) \Psi_\phi' \Psi_\psi' \quad (192)$$

$$j_\mu F^{\mu t} = 0 = \omega_\phi \left(\mathbb{I}^{(0,1)} f^{(1,0)} - \mathbb{I}^{(1,0)} f^{(0,1)} \right) \Psi_\phi' + \omega_\psi \left(\mathbb{I}^{(0,1)} g^{(1,0)} - \mathbb{I}^{(1,0)} g^{(0,1)} \right) \Psi_\psi' \quad (193)$$

$$j_\mu F^{\mu\{t,\psi\}} = 0 = \left(\mathbb{I}^{(1,0)} \{f, g\}^{(0,1)} - \mathbb{I}^{(0,1)} \{f, g\}^{(1,0)} \right) \Psi\{f, \psi\}' \quad (194)$$

Note that (191) is satisfied if $\omega_\phi \rightarrow \omega_\phi[f(r, \theta)]$ and $\omega_\psi \rightarrow \omega_\psi[g(r, \theta)]$. Letting the current also depend separably on the functions, $I \rightarrow I[f(r, \theta), g(r, \theta)]$ then all these equations are automatically satisfied if $f^{(1,0)} g^{(0,1)} - f^{(0,1)} g^{(1,0)} = 0$. Summarily:

$$\begin{aligned} \Psi_\phi &\rightarrow \Psi_\phi[f(r, \theta)] , & \Psi_\psi &\rightarrow \Psi_\psi[g(r, \theta)] \\ \omega_\phi &\rightarrow \omega_\phi[f(r, \theta)] & \omega_\psi &\rightarrow \omega_\psi[g(r, \theta)] & \Rightarrow & d\mathbf{F} = 0 & (195) \\ I &\rightarrow I[f(r, \theta), g(r, \theta)], & f^{(1,0)} g^{(0,1)} - f^{(0,1)} g^{(1,0)} &= 0 & & j_\mu F^{\mu\{t,\psi\}} = 0 \end{aligned}$$

In particular, a constant functionalization of either field-flux is always a toroidal solution: $\{f, g\} \in \mathbb{R}^* \Rightarrow j_\mu F^{\mu\{t,\phi,\psi\}} = 0$. Single poloidal-coordinate functionals, $\{f_\theta(\theta), g_\theta(\theta)\}$, $\{f_r(r), g_r(r)\}$ are also always toroidal solutions, as are their "cross-parameterizations", $\{f_{\{r,\theta\}}, g_{\{\theta,r\}}\}$. In fact, although the remaining two force free conditions, $j_\mu F^{\mu\{r,\theta\}} = 0$, are in general complicated functions of the metric, when the fluxes can be parameterized by a single

⁹⁶This can be shown by tracking the unique term that disappears under the $d\Phi_\psi \rightarrow 0$ condition through Maxwell's equations: $F_\psi = d\Psi_\psi \wedge d\psi \rightarrow 0 \Rightarrow d\psi^\alpha [*d*F]_\alpha = 0$. This also shows how the fixed-flux condition can be tied to the emergence of ψ -independent(gauged) bulk invariants

poloidal coordinate one of the equations of motion is uniformly 0:⁹⁷

$$\{f_{\{r,\theta\}}, g_{\{r,\theta\}}\} \Rightarrow j_\mu F^{\mu\{\theta,r\}} \equiv 0 \quad (196)$$

This functionalized framework will help keep the perturbative frames conceptually rigid in the next section.

3.9 Tangherlini Harmonics ($\alpha = 0$)

The Tangherlini geometry can be taken as the non-spinning limit of a Myers-Perry type metric [38]⁹⁸:

$$ds_T^2 = -\left(1 - \frac{m}{r^2}\right)dt^2 + r^2 \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\psi^2 + \frac{r^2}{r^2 - m} dr^2 + r^2 d\theta^2 \quad (197)$$

It is critically useful the notice the hidden freedom of the ψ coordinate in this geometry: diffeomorphic symmetry automatically empowers this coordinate with enhanced functional freedom that is a uniform symmetry of the other coordinates. Precisely, $\psi \rightarrow \psi + \oint_{X_j[\vec{y}] \sim x}^{i \in I} d\vec{y} f[\vec{X}]$ is an exact, global, continuous functional symmetry with a ‘‘Euclidean’’-type sub-constraint form; this is exactly the same proof as the Banach-Tarski paradox, and this spacetime can be considered the physical manifestation of such.⁹⁹ Accordingly, it may be inferred that the Blandford-Znajek process in $d = 5$ may represent something uniquely universal about fields (at the completely interacting, or ‘‘Feynman-loop’’ level)

As before, the perturbation scheme proceeds by solving for an operator (FFE tower-)classification of the constrained field equation in this geometry in order to build a $O(a^0)$ -order support and ramify-out a quasi-canonical family of solution forms, exactly like the solution matched-form exclusion techniques in $D = 4$ [23]. Consider the following flux-coordinate differential operators:

$$\mathcal{L}_\phi \Psi_\phi = -m \Psi_\phi^{(2,0)} + \frac{m \Psi_\phi^{(1,0)}}{r} + r^2 \Psi_\phi^{(2,0)} + r \Psi_\phi^{(1,0)} + \Psi_\phi^{(0,2)} - \csc(\theta) \sec(\theta) \Psi_\phi^{(0,1)} \quad (198)$$

$$\mathcal{L}_\psi \Psi_\psi = -m \Psi_\psi^{(2,0)} + \frac{m \Psi_\psi^{(1,0)}}{r} + r^2 \Psi_\psi^{(2,0)} + r \Psi_\psi^{(1,0)} + \Psi_\psi^{(0,2)} + \csc(\theta) \sec(\theta) \Psi_\psi^{(0,1)} \quad (199)$$

Without currents ($I = 0$) the above (193) - (194) shows that torodal FFE equations are uniformly zero.

⁹⁷This again can be immediately seen by tracking the ‘‘shadow’’ of the now 0 terms, $\partial_{\{r,\theta\}} \Psi^{\phi,\psi} dx^{r,\theta} \wedge \eta_{\{\phi,\psi\}}$, under the EM current transform: $*d*$.

⁹⁸In fact, one immediate volume-curve of interest can be immediately spotted in this metric: $d\psi^2 \rightarrow d\phi^2 \Rightarrow ds_T^2 \rightarrow -\left(1 - \frac{m}{r^2}\right)dt^2 + \frac{r^2}{r^2 - m} dr^2 + r^2 (d\theta^2 + d\phi^2)$. This metric looks similar to toroidal AdS and highlights the advantage of combining the spacelike scalar degrees of freedom, $\chi_{\{\phi,\psi\}}$, in higher dimensions.

⁹⁹Essentially, although it is periodic, by the fundamental theorem of calculus (on bundle topologies) this symmetry closes over the complete harmonic dual space (of Lesbeque measures), \mathcal{L}^2 ; this is usually known as Green’s Theorem. As will become clear, this coordinate can be considered a universal (*in*-field) Legendre multiplier, and makes $d = 5$ Tangherlini spacetime a natural, and universal, functional kernel of (spin $\frac{1}{2}$)fields in conformally-curved $d = 4$ geometries. In some senses, this gives this spacetime an exact connection to topology categorically [34] and represents a real manifestation of duality generally (or, as a closed object-basis over the category of general relativity).

3.9.1 Arbitrary Velocity Series

For complexity (in the functional base) non-constant velocity corrections are included here (as the Q -operator) $\omega_{\{\phi,\psi\}} \in \mathbb{R}[*,*]$; then the poloidal equations can be shown to reduce to:

$$\begin{aligned} (j_\mu F^{\mu,\{r,\theta\}}) r^4 & - ((H^{\{r,\theta\}}[r,\theta]) \mathcal{L}_\psi \Psi_\psi + (G^{\{r,\theta\}}[r,\theta]) \mathcal{L}_\phi \Psi_\phi) \\ & = K^{\{r,\theta\}}[r,\theta] \left(\omega_\phi \delta_\phi[\Psi_\phi] + \omega_\psi \delta_\psi[\Psi_\psi] - Q[\bar{L}^r(r,\theta), \bar{L}^\theta(r,\theta); r] \right) \end{aligned} \quad (200)$$

where the variable-velocity contribution is given as

$$\frac{Q[x,y;r]}{x(m-r^2)} = - \left(1 - \frac{y}{x(m-r^2)} \right) \quad \text{and} \quad \bar{L}^{\{r,\theta\}} = \sum_{k \in \{\phi,\psi\}} \Psi_k^{\{\{1,0\},\{0,1\}\}} \omega_k^{\{\{1,0\},\{0,1\}\}} \quad (201)$$

and the static constraint variables are found as:

$$\begin{aligned} K^{\{r,\theta\}}[r,\theta] & = \frac{2r^4}{r^2-m} (\omega_\phi \partial_{\{r,\theta\}} \Psi_\phi + \omega_\psi \partial_{\{r,\theta\}} \Psi_\psi) \\ \delta_\psi[\Psi_\psi] & = \frac{r^2-2m}{r} \Psi_\psi^{(1,0)} - \tan \theta \Psi_\psi^{(0,1)} \quad \delta_\phi[\Psi_\phi] = \frac{r^2-2m}{r} \Psi_\phi^{(1,0)} + \cot \theta \Psi_\phi^{(0,1)} \end{aligned} \quad (202)$$

While the $\{H, G\}$ functions satisfy:

$$\frac{\vec{C}}{\omega_\phi} := \frac{1}{\omega_\phi} \begin{pmatrix} \Psi_\phi^{(0,1)} + \sin^2 \theta G^\theta[r,\theta] \\ \Psi_\psi^{(0,1)} + \cos^2 \theta H^\theta[r,\theta] \\ \Psi_\phi^{(1,0)} + \sin^2 \theta G^r[r,\theta] \\ \Psi_\psi^{(1,0)} + \cos^2 \theta H^r[r,\theta] \end{pmatrix} = \frac{\cos^2 \theta}{2} \begin{pmatrix} \tan^2 \theta K^\theta \\ \frac{\omega_\psi}{\omega_\phi} K^\theta \\ \tan^2 \theta K^r \\ \frac{\omega_\psi}{\omega_\phi} K^r \end{pmatrix} \quad (203)$$

In particular, the RHS is a set of constraints exactly similar to the Q -push functionals associated to the non-static sources, $\{\bar{L}^{\{r,\theta\}}\}$; in fact, this can be made exact as an functional integral¹⁰⁰

$$\frac{1-m}{2r^2} K^{\{r,\theta\}} = \sum_{k \in \{\phi,\psi\}} \Psi_k^{\{\{1,0\},\{0,1\}\}} \omega_k \equiv \oint_{\{\gamma_r, \gamma_\theta\}} dX_{\partial\Psi}^{\{r,\theta\}} \bar{L}^{\{r,\theta\}} \quad (204)$$

where the final notation denotes performing the coordinate gluing (cut-integration) along flat bundles of $\Psi_{\{\phi,\psi\}}$; this is identical to keeping (functional-)boundary terms (which here, additionally and critically, separate along their coordinate forms, $\{d_r, d_\theta\}$). In fact, this is very remarkable! Looking back, $\frac{\vec{C}}{\omega_\phi \cos^2 \theta}$ heuristically resembles an $\mathfrak{su}(2)$ index around the globally symmetric planes ($\theta \sim \{\frac{\pi}{2}, 0\}$) and, on the RHS, the functionally symmetric velocity points ($\frac{\omega_\phi}{\omega_\psi} \sim \pm\infty$); this happens exactly to be the non-rational (non-separable, over \mathbb{R}) sub-domains of the continuous curve limits, $[\theta(\lambda), (\omega_\phi[\lambda_\phi], \omega_\psi[\lambda_\psi])] \sim \{0, \infty\}$. This is immediately reminiscent of Airy curves, because is the topological generalization of this (open-type) solution basis (descending from higher dimensional accumulation residues).

¹⁰⁰By the Froebineus theorem and the (coordinate-)exact differtiability of $\omega_{\{\phi,\psi\}}$

Considered as coordinates this type of separability is canonical(/trivial) from the Jacobi-form (which is exactly the point of real Analysis). But the separability here (also) happens in function space (on the canonical real evaluation of the field); functional topology is universally closed, meaning that functional duals can be constructed for **any** ZFC embedded diffeomorphic completion [49]. This remarkable fact can be seen manifestly (from a physics perspective) in [34], and is absolutely relevant here.

In fact, applying these lessons above and constraining sub-functionals to level sets of coordinate-stationary magnetic fluxes, this push can immediately capture the universal branch of $\mathfrak{su}(2)$; stronger yet, realizing the extra space coordinate as a global, continuous, functional symmetry index enhances the sub-functionals to $SU(2)$ -exact gauge fields. Under Noether's theorem [56], this exactly represents a conserved charge in functional topology, known as a functional index[57]. This is universal, and is shockingly strong because this represents an absolute (gauge) duality between analysis and topological physics, and acts as a strong indication of the results to follow.

So, noting that the functional-gauge is algebraically enveloped (on the complete rational branch, or over the natural scalar form) as $\cos^2 \theta d\varphi^2 = (\cos^2 \theta, \langle \mathcal{D}^i \wedge_k \partial_i^{k_i} X_k^i \rangle)$; taking the classic categorization of the measure branch [57] seriously here immediately stipulates the existence of a measure continuance to $(\cos^2 \theta, \langle \cdot \rangle) \rightarrow (\langle \cdot \rangle)_{\sec^2 \theta}$, which immediately motivates the combined symbolic geography:

$$\mathcal{M}[\vec{\mathcal{B}}, \frac{\vec{z}}{z}; r, y] := \frac{(1 - \frac{m}{r^2})\vec{\mathcal{B}}}{zy^2}; \quad \mathcal{M}[\vec{\mathcal{C}}, \vec{\omega}; r, \cos \theta] := \frac{(1 - \frac{m}{r^2})\vec{\mathcal{C}}}{\omega_\phi \cos^2 \theta} \equiv \oint_{\{\gamma_r, \gamma_\theta\}} dX_{\partial\Psi}^{\{r, \theta\}} \bar{L}^{\{r, \theta\}} \quad (205)$$

noting the $\frac{n\pi}{2} \Big|_{n \in \mathbb{N}}$ divergent terms in $\mathcal{L}_{\phi, \psi}$ represent mirror-frozen dual pairs on a (compactified) functional regulation-matching surface (the embedded axisymmetric planar cut), the (mirror-symmetrized) extended (static) constraint topology [34] can be naturally motivated as an antisymmetric index rational over(/projective onto) the axisymmetric axis *in*-cohomolgy: $\cos^2 \theta \vec{\mathcal{C}} =: \vec{\mathcal{C}}$. By mixing constraints in the regulating topology and (point-wise orthogonally) pulling off mixed families from the divergent plane, rationally unwinding $\cos \theta = y$ gives:

$$\Rightarrow \mathcal{D}_\theta \mathcal{M}[\vec{\mathcal{C}}, \vec{\omega}; r, \cos \theta] \Big|_{d\vec{\mathcal{C}} \wedge d\vec{\omega}} = \mathcal{D}_\theta \Big|_{d\vec{\mathcal{C}} \wedge d\vec{\omega}} \oint_{\{\gamma_r, \gamma_\theta\}} dX_{\partial\Psi}^{\{r, \theta\}} \bar{L}^{\{r, \theta\}} = -2 \frac{(1 - \frac{m}{r^2})\vec{\mathcal{C}} dy}{\omega_\phi y^3} \Big|_{y[\theta] = \cos^2 \theta} \quad (206)$$

Amazingly (but unsurprising from the universal corner residue frames of [58])¹⁰¹, flux-frequency-gauged Tangherlini (deformed-)loops pulled of the mirror plane produce a $\frac{k}{y^2}$ Weinberg pole across the entire entanglement plane, where the sign and magnitude of k can vary based on the loop mirror cohort-gluing, the field frequency, and the radial index (now regarded as an entanglement *internal*, point index).

In particular, letting (non-)classicality be defined by sign determinancy of $\text{sign} \forall (\vec{\mathcal{C}} \cdot d.y) > 0$ or $\exists^c < 0$ this family-bundled deflection in the $d = 4$ pull space mimics Newtonian gravity

¹⁰¹Here, the index "tugs" medium sized (considered as all $0 < \beta < \dots < \infty$, β -graded) scalar fields into asymptotically rational sub-bundles over Cantor fields, $\{\pm\infty\} \leftarrow \{\infty, \infty \frac{\alpha_i}{\beta_i}\}_i$, at the divergent features.

and electromagnetism in one swoop: letting $\text{sign}(1 - \frac{m}{r^2}) \text{sign}(\omega_\phi) > 0$ gives a (consistently signed) graviton-like sub-shell, while $\text{sign}(\vec{C} \cdot d\vec{y}\omega_\phi) > 0$ gives a (\pm signed) electromagnetic like sub-shell (on the asymptotic, corner-pulled compactification domain). Critically, these choices only sign-exchange the $\frac{\omega_\varphi}{\omega_\phi}$ index features as a mirror, or T, duality. Pushing further, this result produces a full c(ontour-)loop Newtonian (off-shell) expansion of the mirror cut produce the master equations:

$$D_\theta \mathcal{M} \Big|_{d\vec{C} \wedge d\vec{\omega}} = -\frac{kdr}{r^3} \Big|_{\substack{\frac{cdy}{y^3} = \omega_\phi \frac{dz}{z^3} \\ \frac{dz}{z^3} = \frac{dr}{kr(r^2-m)}}} \equiv -\tau dr \left(\frac{1}{r-\alpha} - \frac{r+\alpha}{r^2-m} \right) \quad (207)$$

where the rational pole parameters are algebraically fixed as: $\frac{1}{k\tau} = \alpha^2 + m$ and $\frac{m-\tau}{\tau} = \alpha^2$; considering (k, m) fixed parameter points, these represent two equations fixed about the synthetic τ -derivative at $(\alpha, \tau) \sim (\alpha, 0)$. So, the constraint form can be clinically prescribed as:

$$D_\theta \mathcal{M} \Big|_{d\vec{C} \wedge d\vec{\omega}} = -\frac{kdr}{r^3} \Big|_{\substack{\frac{cdy}{y^3} = \omega_\phi \frac{dz}{z^3}; \delta_\tau m = \alpha^2, \frac{1}{\tau k} = \alpha^2 + m = f_\delta d_\delta \delta_\tau m \\ \ln\left(\frac{1-\frac{\alpha}{r}}{\sqrt{1-\frac{m}{r^2}}}\right) - \frac{1}{2\tau z^2} = V_z - \sqrt{\frac{\alpha^2}{m}} \tan^{-1}\left[\frac{r}{\sqrt{m}}\right]}} \quad (208)$$

Pulling off the tangent branch and regarding α^2 as a τ (synthetically-)exact symmetry immediately identifies α as the (synthetically-)dual scalar index, traditionally understood as a Kac-Moody current [15] resulting from “un-twisting” the T-dual continuous null-current.¹⁰² Under the harmonic Ward identity over a θ -wound state:

$$\langle D_\theta \mathcal{M} \Big|_{d\vec{C} \wedge d\vec{\omega}} O \rangle = -\frac{1}{2r^2} \langle kO \rangle \Big|_{k = \frac{1}{\tau(\alpha^2+m)}} \quad (209)$$

Choosing classical units over entanglement charge shows that the effective theory over the sub algebra separates strongly from the quantum critical point (at exactly the Planck

¹⁰²This can be seen precisely by remembering the global functional gauge allows a continuous index to be formed around the every divergent neighborhood. In particular, the $\frac{1}{z^2}$ poles are forced to match the winding divergence (conical form) as $\sqrt{m} \rightarrow 0$ up to $\ln(1 - \frac{\alpha}{r})$ corrections, which exactly presents α as 1-loop cutoff parameter on the multiparticle, or Foch, branch of the $SU(2)$ -exact theory. Note this is classical, and represents an exact shift in the classical phase-space saddles by a T-balanced multiparticle divergence index, $\alpha[z, m]$ connecting the higher dimensional mass to the lower dimensional, holographically thermalized classical fields by connecting the regulation index at $r \rightarrow \infty$ across the integration constant and the $\tau \rightarrow 0$ (synthetic-)poles, $V_z + \frac{1}{2\tau z^2}$. This is universal and exact by the Cauchy Completeness theorem, and can formally be used to construct the stereographic push-out $z \leftarrow (z, \bar{z})$ over the $(\tau, \alpha; z)$ -harmonic plane.

More excitedly, considering the (r, θ) shape of the index shows it tessellates the ZFC completion as a degree four twister and, deductively, should represent the T-dual formulation of the $(\langle 8, 2 \rangle \sim_F (4))$, Fuch-gauged) monodromic-index of black hole thermodynamics (on the closed string branch). Intuitively then, it should be expected that the monopole solution in $d = 5$ black hole spacetimes captures the non-perturbative (loop-exact) behavior of $d = 4$ black holes on the interactive thermal branch (or, as a complete gravitational propagator)! Indeed, shockingly, this will (shortly) be used to continuously produce the spinning-dyon² \sim Kerr asymptotic symmetry [17].

vacuum shear)

$$[k] = \left[\begin{array}{c} G \\ \frac{1}{4\pi\epsilon_0} \end{array} \right] W \equiv [G] \left[\begin{array}{c} 1 \\ \frac{\mu_0 \rho_h}{4\pi l_h^2} \end{array} \right] W \quad (210)$$

In fact, remembering this index emerged from a black hole driven magnetic field with both a free coordinate gauge symmetry as well as an index-coupled $SU(2)$ -gauge connection makes is a natural probe of the asymptotic $(k) = d - 1$ spacetimes on some complete functional shelling by closing warped sub-geometric gaps as mixed hyper-connected spin diagrams.¹⁰³

In particular, this descent algebra closes across the desired $d = 4$ phase diagram iff it closes across a single, $(k\infty)$ uniformly connected (set of) points. Then, the action probe here could be considered an asymptotic $SU(2)$ harmonic coupling form between $k = 4$ dimensional actions designed to capture a regulated $SU(2)$ -loop harmonics [34]; in particular, it will be shown to exploit the measurable divergence on the $\theta \rightarrow \frac{\pi}{2}$ -symmetry plane as an enhanced asymptotic light(-shelled) probe.

It could also be inferred that this classical probe represents a distributed connection between hard-soft degrees of freedom under asymptotically point-compact, smooth regulation. This guarantees that the divergent index points will always asymptotically dominate (at the synthetic level) by enforcing connection between the synthetic descent and the emergent (shadow) propagators. This is critical to understand the spin gap closure defined in spinning $d = 5$ geometry, which will emerge as a connected point (localized) measure of classical mass-entanglement spin in $k = 4$ as a (mass-polarized) Lamar radiating electron, also known as a classical muon impact parameter. In fact, this spin connection will happen at a universally symmetric configuration point at exactly the strongest indexed measure and be uniformly enforced as a Bloch-thermalization patch stabilizer on a radial (entanglement)gauge regulator; thus indeed, this class of (functionally descendant) black holes will be shown classically dual to point-acceleration emission of (spacetime corrections to) radially confined entanglement-patches (as E-Wickert corrections/Lamar radiation by using stationary, asymptotically separated gauge probes).

As has been remarked repeatedly, the Cauchy completeness on the functional kernel space

¹⁰³In fact, this unification scale can be used to solve infer the uniqueness of the 55-degree latitude "rush-to-the-poles" [59] during the solar cycle's magnetic flip as the maximal mean deflection angle of weakly convective columnar flow that can be cross stabilized (in fully interactive background); indeed, this is almost exactly twice the Weinberg angle. Put another way, as the sun's magnetic poles flip the wide-latitude convective border vortices can counter resonate at exactly twice the unification level; this can qualitatively be understood as a mixing complex (crossing stabilizer resonance) between the flat asymptotic W-gauge and the colored (sub-spinning) Higgs partitions [60]. Then, considering plasma in non-interacting $k = 4$ backgrounds to be the fully shielded gravitational out state over $d = 5$, or QED measures in flat spacetime exactly infers $\langle \theta_S^{max} \rangle_{\delta t=140 \text{ years}} \approx 54.25 \sim 54.32 \approx [W_\theta]_{\text{NIST}_{2018}}$; note, naively $\frac{|\langle \theta_S^{max} \rangle - [W_\theta]_{\text{NIST}_{2018}}|}{54.285} \sim \frac{1}{10^3}$, or that this error (very roughly) scales as a digit per space (a.k.a., coordinate) dimension, which is exactly the expected size of the virial error with a (correct) 0th order (classically unified) constraint parameterization(/virial expansion, which should be an expected representation of time-ordered solar diagramatics because of the natural, $[G_{\mu\nu}, F_{mn}]$ counterbalancing nature of the sun and the $\mathbb{Z}_2^{\Delta t \sim 11 \text{ years}}$ symmetry of this dynamic phase point).

guarantees this charge index has spectral control within every neighborhood; in turn, this shows that the above unification yields a strongly separated quantum curve index that cannot be effectively effectively suppressed in $\frac{dy}{y^3}$ (or $\frac{dr}{r^3}$) without ignoring quantum gravity corrections. Because the entire point is to capture a strongly quantum partition weight in all phase volumes $(\Psi^{\{r,\theta\}}, (r, \theta))$, this is a good thing as renormalization flow is automatically captured under:

$$\langle D_\theta \mathcal{M} \Big|_{d\bar{c} \wedge d\bar{\omega}} O \rangle = -\frac{1}{2r^2} \langle WO \rangle \Big| \left[\begin{array}{c} G \\ \frac{1}{4\pi\epsilon_0} \end{array} \right] = \left[\frac{1}{\tau(\alpha^2+m)} \right] = [1] \quad (211)$$

In fact, the final unitless condition can be presented as $\frac{d \ln \tau}{d\tau} = \oint_\delta d_\delta \delta_\tau m$, which amounts to stipulating that the $d = 5$ mass-defect parameter be exact; this can always be made (synthetically) true by applying the Cauchy completeness(/extension) theorem(s) over the scalar field supports, $\mathbb{F}[*]$.

Finally (and yet again), noting that the index is functionally strong exactly where classic fields are continuously regulated (under Green's theorem) gives this crossing relation weight exactly on the classical, $d = 4$ Harmonically weighted legs (which here are interacting on the shadowed Coloumb branch). Expectantly, on the weak-mass/charge (classical) shells:

$$\langle D_\theta \mathcal{M} \Big|_{d\bar{c} \wedge d\bar{\omega}} O \rangle \Big|_{\frac{1}{2} \langle QO \rangle = Wdw}^{[k] = \left[\frac{GMdm}{Wdw} \quad \frac{1}{4\pi\epsilon} \frac{Qdq}{Wdw} \right]} = \frac{[GMdm \quad \frac{Qdq}{4\pi\epsilon}]}{r^2} \quad (212)$$

Thus, the classical contact fall-offs of both classical gravity and electromagnetism have been unified by a topological index, as promised. Accordingly, it can be said that (as non-perturbative string theory amplitudes) classical mass distributions are weakly Z^0 -mediated, while classical charge distributions are W^\pm -mediated. In fact remembering that in classical QED the W^\pm exchange charge sits on the Coulomb form of $SU(2) \times U(2)_{e\gamma}$) spontaneous symmetry breaking brings confidence.¹⁰⁴

This framework is very strong, and the push-pull framework merits further elaboration. Instead of forcing the gauge of general relativity into the exact harmonic basis of $d = k$, or vice versa, this (synthetically gauged) topology allows both to meet on equally functional footings.¹⁰⁵ The radial form of these contact surfaces, namely those usually probed in

¹⁰⁴Indeed, the emergence of Newtonian- and electrostatic- force distributions were here unified as broken index exchanges in a Schwarzschild background (with a diffeomorphic, continuous gauge index, *in-shell*) by stripping the enhanced soft data into a complete (infinite dimensional) index; then, a uniform, corner symmetry was found to exist as a classical $[0, \pm]$ -index between the static force-types (separated from each other by quantum criticality and the $F_c \rightarrow 0$ limit on *in/out* entanglement pairs) [34]. Considering the discussion above [?? regarding the Higg's potential's appearance on the light-surface of the stream equation (in curved spacetime), the model of off-shell mediation is given a strong, topological image (as a universal interaction branch)

¹⁰⁵As Dr. John Wheeler quoted, "Space-time tells matter how to move; matter tells spacetime how to . . . curve. ". The posterior framework described here can be understood as the exact same quote under the dual meanings of tell, namely as the act of mistaken signification, such as in poker: "John tells by..."; such as, the quote may be quantized as "Changes in spacetime are either within or with-out changes of matter; changes of matter are either with-in or with-out . . . changes with-in or with-out curvature". Accordingly, gravitational waves are as much of an observationally-inductive effect as a spacetime/matter-

r-subspace (like those performed by Newton and Maxwell/Tesla themselves) can be understood as a strong string deformation (entanglement deflection) between asymptotic quasi-stationary configuration domains (causal diamonds) [62] supported by causally shielded $d = 5$ masses under null-weak $SL(2, \mathbb{R})$ -asymptotic probes. Put another way, space is the soft charge that "leaks" out of string field theory. This also gives an explanation for the classical direction of time as a boundary bifurcated sub-symmetry of magnetic (spin) currents at their maximally entangled, minimally confined global winding edge phases. Or, deconstructively time is exactly the wholes between the moments of the days of the years: time is the maximally convex functional-index of consistent measures, (and, accordingly, measurement of entropy is exactly the convex dual of the measurement of consistency). Now motivated by the strong predictive loop dynamics of the $\mathcal{L}_{\{\phi, \psi\}}$ off-shell index symmetries it is time to rewind fully to the $\vec{\omega}$ -stationary(/level-set) on-shell solution types.

As $\mathcal{L}_{\{\phi, \psi\}}[\Psi_{\{\phi, \psi\}}]$ are the only top 2nd-order operators, setting the RHS of 200 equal to 0 gives poloidal subspace conditions for the exact decomposition of (on-shell) vacuum solutions into force-free propagating modes:

$$K^{\{r, \theta\}}[r, \theta] \left(\omega_\phi \delta_\phi[\Psi_\phi] + \omega_\psi \delta_\psi[\Psi_\psi] - Q[\bar{L}^r, \bar{L}^\theta] \right) = 0 \Rightarrow \mathcal{L}_{\{\phi, \psi\}}[\Psi_{\{\phi, \psi\}}] = 0 \Leftrightarrow (j_\mu F^{\mu, \{r, \theta\}}) r^4 = 0 \quad (213)$$

Ignoring the kernel of K^{106} , the other decomposition condition can be separated as:

$$0 = \frac{r^2 - 2m}{r} \left(\omega_\phi \Psi_\phi^{(1,0)} + \omega_\psi \Psi_\psi^{(1,0)} \right) + \cot \theta \left(\omega_\phi \Psi_\phi^{(0,1)} - \omega_\psi \tan \theta^2 \Psi_\psi^{(0,1)} \right) \quad (214)$$

$$\Rightarrow \begin{cases} \lambda \frac{r}{r^2 - 2m} = \omega_\phi \Psi_\phi^{(1,0)} + \omega_\psi \Psi_\psi^{(1,0)}, \\ -\lambda \tan \theta = \omega_\phi \Psi_\phi^{(0,1)} - \omega_\psi \tan \theta^2 \Psi_\psi^{(0,1)} \end{cases} \quad \text{or:} \quad \begin{cases} \omega_\phi \omega_\psi \Psi_\psi^{(1,0)} \Psi_\psi^{(0,1)} = \frac{r \lambda^2 \cot \theta}{r^2 - 2m} \\ \Psi_\psi^{(1,0)} = \frac{r \bar{\lambda}}{r^2 - 2m} \\ \Psi_\psi^{(0,1)} = \bar{\lambda} \cot \theta \end{cases} \Bigg|_{\substack{\bar{\lambda} = \lambda A \\ AB = C}} \quad \omega_\phi \hat{\lambda} A = \omega_\psi \lambda C \quad (215)$$

Note¹⁰⁷ the constraint forms represent three equations in seven variables and thus the kernel is $n = 4$ dimensional; accordingly, the constraints represent an exact connection between coordinate derivatives of the magnetic- φ flux and entanglement patch (geometrized) synthetic

sourced induction; in fact, this is immediately clear from the gauge freedom of $d = 4$ spacetimes, and provides a precise understanding of the Harmonic gauge's uniqueness away from the final inspiral [61]. Further, considering the natural role warped topologies played in reconstructing the full, loop complete classical potentials it may be clear why the harmonic gauge is troublesome exactly on contact: in the low-separation charge limit functional harmonic probes strongly gauge to their canonical charge duals (as a form of loop-level UV/IR mixing) [2]. Again, this is textbook from an analysis point of view: the metric is diffeomorphically closed over the entire branch because δ_φ is a uniform, continuous global symmetry in every (t, ϕ, θ, r) neighborhood. Thus vacuum polarized emission probes must be dual to field spectra in the semiclassical limit; accordingly, magnetic conditions should amount to the virial completion of the semi-classical turning points, a.k.a. the mean field displacement kernel, or bulk force-kernel. This can clearly be understood in the knotted context of [34]

¹⁰⁶Considering the kernel of $K^{\{r, \theta\}}$: $K^{\{r, \theta\}} \equiv 0 \Rightarrow \partial_{\{r, \theta\}} \Psi_\phi = - \frac{\omega_\psi \partial_{\{r, \theta\}} \Psi_\psi}{\omega_\phi}$. In fact, K remains a separability-envelope even when the field velocity is non-constant; this enhances the idea of the Ψ_ψ flux as a sheet level-gauge condition: it can be used to solve exactly one of the FFE and, considering (109), can be seen as exactly the condition eliminating one of the Maxwell tensor's time legs: $K^{r, \theta} = 0 \Rightarrow \delta_{(t)}^{\nu} \delta_{(a)}^{\mu} F_{\mu\nu} = 0$.

¹⁰⁷ $\Psi_\varphi = d_2 \ln \sin \theta$ implies $\lambda \rightarrow \tilde{\lambda} = \lambda - \omega_\varphi$, showing this solution type amounts to a(n on shell, sub-

multipliers¹⁰⁸. These give subharmonic matching conditions¹⁰⁹ on the harmonic solutions (213). Critically, $\omega_\phi \sim \sqrt{\lambda} \Big|_{\lambda \sim 0}$ leaves ω_φ unconstrained and Ψ_φ fixed as a patch-boundary integration constant (where ω_ϕ acts as a Lagrangian multiplier between ultra-weak (Heavy-side) functional covers with a strict radial index ($\mu[*, \cdot]_{\mathcal{N}^*[\mathcal{B}]} \sim H[r_*, r.]$). This formally qualifies the Ψ_φ field as a $\mathfrak{su}(2)$ gauged index over the $d = 4$ projective T-mirror plane. Note that $\omega_\psi = \lambda = \omega_\phi \Big|_{\lambda=0}$ are always harmonically seperable (they universally solve the sublevel conditions (215)) up to the free quantum algebra, $O[\Psi_{\{\phi, \varphi\}} \omega_{\{\phi, \varphi\}}^{\{\{2,0\}, \{2,0\}\}}]$.

Indeed, general relativity is constructed, axiomatically, as a bundle topology functional ("Einstein's Equations": $G_{\mu\nu} = 0$), which amounts to the existence of a Euclidean sub-dimensional projector on the dimensional indexing topology (in every bound measure set) as well as a global braiding functional (on connective shell) that is itself indexed ("Time, Energy, Mass, Curvature, etc..."). Critically this exists at the level of the index selector, $\{\sigma\} \in \mathcal{C}_{oor}$, and is an exact manifestation of diffeomorphic symmetries strongly coupling with local decidable features; then, the Ψ_φ gauge can be considered a compactified partial(/decidability) index towards general relativity.

So, recapping, weakly coupling this T-measure gauge into a pointwise separated $SU(2)$ spectral shift. Specifically, choosing $\Psi_\varphi = 1 + d_1 \ln[\sin \theta]$ produces:

$$\Psi_\theta \Psi_{r,r} = \frac{r}{r^2 - 2m} \left(\frac{d_1 \omega_\varphi}{\omega_\phi} - \frac{\Psi_{\theta,\theta} \Psi_r}{\tan \theta} \right) \hat{=} \frac{r}{r^2 - 2m} \left(\frac{d_1 \omega_\varphi}{\omega_\phi} - \partial_x \Psi_\theta[x] \Big|_{x \rightarrow \ln \cos \theta} \Psi_r \right) \quad (216)$$

Then, taking the $d_1 \omega_\varphi \rightarrow 0$ accumulation can be seen as allowing the emergence of a separated tower solution $\frac{\partial_x \Psi_\theta[x]}{\Psi_\theta[x]} \Big|_{x \rightarrow \ln \cos \theta} = C$, which can be solved with the analytic form $\Psi_\theta[x] = e^{C(1+xD)}$, which produces a raw amplitude shift in Ψ_r ; then $\Psi_\phi^{(0)} = 1 + c_1 \ln \cos \theta$ can indeed be seen as the lowest matching spline.

Specifically, demanding the ψ -flux exactly represent an exact \mathbb{I}^\pm index in the sense that $\int d[\cdot] e^{\Psi_\psi[\cdot]} = \oint_{\gamma_I} d[\cdot] \sin(\theta[\cdot])$;

¹⁰⁸Or, that $\omega_\varphi^2 \Psi_\psi^{(1,0)} \Psi_\psi^{(0,1)} = \frac{\lambda \tilde{\lambda}}{C} \frac{r \cot \theta}{r^2 - m}$. In fact, letting C be quantized in units of ω_φ exactly displays this

as a Berry-gauge connection across the RHS poles and exactly shows the quantum spectrum as uniformly confined to the classical ground state by an infinite dimensional(/ continuously loop-indexed) symmetry current carrying the spacetime-orthogonal magnetic flux. This may be regarded as the quantum gravity equivalent to Maxwell's Displacement current in electromagnetism, presented here as the canonical quantization (functor-)residues over spacelike-orthogonally extended Schwarzschild (Tangherlini) background fields; critically, because everything is iff, this shows the $U(1)$ gauged entanglement network that closes around the anti-symmetrix coordinate (index: it continually catches the sign of the proper polar axis, $\theta \sim 0$) as $dI = \cos \theta d\varphi$

¹⁰⁹In fact, seperability of (214) is exactly the condition that at least one of the set of seperated products have a uniform envelope:, $\{\Psi_\phi, \Psi_\psi\} \rightarrow \{f_r f_\theta, g_r g_\theta\} \rightarrow f_{\{r,\theta\}} \{f_{\{r,\theta\}}, g_{\{r,\theta\}}\}$, solutions are sums of seperated product decompositions: $\tilde{\Psi} \rightarrow \sum_\phi \sum_\psi c_{\phi\psi} U_{\{r,\theta\}}^{\phi\psi} \{S^{\{\theta,r\}}, T^{\{\theta,r\}}\}$. Noting that the FFE harmonic operators have the same radial form shows the impetus to first classify the r -independent solutions: globally r -separated fields have a unique decomposition as harmonically separated solutions $\tilde{\Psi} \rightarrow U_a \{S^{a(b)}(\theta), T^{a(c)}(\theta)\}$ (written here in some solution basis, $\{T_{(b)}^\phi, T_{(c)}^\psi\}$), showing that continuous parity symmetries, or U-symmetries, are naturally identified with r-functional inflection points, in the leading effective amplitude. Over the Lie algebra representation, this gives a *in*-squeezed interpretation of the soft theorem: every exact field is closed, so every continuous field extension is loop closed on the functional tangent plane [56] as a rational, topological block expansion of the field parameters. Effectively, the spinning-dyon² \sim Kerr weak IR duality in perturbative, self-dual (here static) gravity has been enhanced to a symplectic cut in the open rational compactification.

spectral dual with strong $k = 4$ spacelike confinement which strongly identified it as a quantum Gravity interaction(/as an exact partition probe of the gravitational propagator). This can be quickly noted by seeing the constant velocity fields as an emergent (r, θ) patched connection across the φ Lagrangian constraint; or $\Psi_\psi = d_0 \ln \sin \theta$ and $\partial_\theta \Psi_\phi = 0$ yielding:

$$\omega_\psi d_0 = \frac{r^2 - 2m}{r} \omega_\phi \Psi_\phi^{(1,0)} \quad (217)$$

and an exact connection between the fixed-velocity, partial amplitude and the radial gauge flow by monotonicity (up to globally fixed, relative velocity signs). This equation clearly represents an r-pointwise defect in the (Ψ_φ) -dual current along the radial gauge branch that is uniformly suppressed at the $r \sim \sqrt{2}r_s$ (relativity) juncture; there any index scale, $\omega_\psi d_0$, can only reflect quantum perturbations (pointwise).

Further, on the symmetric plane $\theta \sim \frac{\pi}{2}$ the LHS represents the bare Ψ_φ gauge flux, which is never a local index unless it represents a (first order) phase connection. This can dually be seen by noting the $r \rightarrow \infty$ limit is asymptotically ordered towards the RHS r-gauge localization(/affine normalization/sign). Restoring the dressed contacts (on classical ϕ -shells) on this symmetric point gives

$$0 = K^\theta \left(\omega_\psi \Psi_\psi + Q[\bar{L}^r, \bar{L}^\theta] \right) \hat{=} K^\theta (\omega_\psi \Psi_\psi + Q[0, \Psi_\varphi \partial_r \omega_\varphi]) \quad (218)$$

$$= \frac{2r^4}{r^2 - m} (\omega_\phi \partial_\theta \Psi_\phi + \omega_\psi \Psi_\psi) \omega_\psi \Psi_\psi (1 + \partial_r \ln \omega_\varphi) \quad (219)$$

Critically¹¹⁰, the radial dimensional analysis at either (the Horizon $r \sim r_H$ or Harmonic $r^2 \rightarrow \infty$) regulation junctures runs as $\sim_{r \rightarrow \sqrt{2}m} 4mr$ and $\sim_{r \rightarrow \infty} \frac{2}{3} \partial r^3$ meaning the $d = 5$ mass index must exactly correspond to a radial gauge decay mode on the $r^2 \gg 2m$ symmetric branch domain under a spontaneously constrained r-residue locking (spacelike squeezed annihilator).

Accordingly, the diffeomorphically suppressed r -current may be carried across r -junctures as phase gaps in the semiclassical turning points (also known as phase deficit angles)[63] by mixing the UV suppressed ω_ψ interactions as a tower of IR flowed mixing modes with the semiclassical angular turning point (a.k.a., the scattering angle). Conversely, this identifies the $2r_s \ll r \ll \infty$ r-features along the symmetric plane with towers of broken symmetries as a discrete set of contact impulses in the ψ -frequency partitionable domain. This is completely restored (as the φ -Coulomb branch) by realizing that towers of Ψ_φ over the $\lambda \rightarrow 0$ ψ -sub-phase allow a partial wave connection across the tangent branch:

$$\cot \theta \frac{\partial \Psi_\phi[r_0, \theta]}{\partial \Psi_\varphi[r_0, n[r_0] \ln \cos \theta]} = \frac{1}{n[r_0]} \frac{\partial_\theta \Psi_\phi[r_0, \theta]}{\partial_{y_0} \Psi_\varphi[r_0, y_0[\theta]]} \Bigg|_{y_0[\theta] = -n[r_0] \ln \sin \theta} . \quad \text{Thus, } \Psi_\phi \text{ may be carried over}$$

their symmetric tangent curves as r-defective impulse shifts in the asymptotically regulated $\lambda \rightarrow 0$ solutions.

Applying topological optical theorems [34] [64] [57] at these junctures at the scale of quantumness (or, on a complete quantum block, $\Psi_\psi \sim 1$) shows that the dual gauge, Ψ_ϕ , can

¹¹⁰remembering that $1 + \frac{Q[x, y; r]}{x(m-r^2)} = -\frac{y}{x(m-r^2)}$

be considered a fully corrected quantum Gravity probe iff it can be constructably closed over the classical equations of motion. By scaling arguments on d_0 , closed radial solutions must be represented by this partition in conformal weight; thus, it should be expected that the running behavior of the symmetric plane will capture the conformal dimensionality (size of the asymptotic Bloch/phase sphere) at the correctly shelled asymptotic juncture (as a regularization for scalar asymptotic ∞) under contact with a stability constraint (or, uniformly, a connection between the asymptotic measurement domain and the near horizon Znajek flow).

3.9.2 Tangherlini OPE

So, returning to the classical equations of motion¹¹¹:

$$\begin{cases} \mathcal{L}_\phi[\Psi_\phi] \\ \mathcal{L}_\varphi[\Psi_\varphi] \end{cases} = \begin{cases} r\partial_r \left[\frac{r^2-m}{r} r\partial_r \Psi_\phi \right] + \tan\theta\partial_\theta [\cot\theta\partial_\theta \Psi_\phi] = 0 \\ r\partial_r \left[\frac{r^2-m}{r} r\partial_r \Psi_\psi \right] + \cot\theta\partial_\theta [\tan\theta\partial_\theta \Psi_\phi] = 0 \end{cases} \quad (220)$$

$$\Rightarrow F_k(x) = \begin{cases} x {}_2F_1\left[1 - \frac{k}{2}, 1 + \frac{k}{2}; 2; x\right] \\ G_{2,2}^{2,0}\left(1 - \frac{k}{2}, 1 + \frac{k}{2} \middle| x\right); \\ 0, 1 \end{cases} \quad (221)$$

$$\text{then, } \Psi_\phi = 1 - \sum_{l=0}^{\infty} c_l R_l(r) T_l^\phi(\theta), \quad \Psi_\varphi = 1 - \sum_{n=0}^{\infty} d_n R_n(r) T_n^\phi(\theta) \quad (222)$$

$$\text{where } R_l(r) = F_l\left(\frac{r^2}{m}\right), \quad T_l^{\{\phi,\psi\}} = F_n(\{\cos^2\theta, \sin^2\theta\}) \quad (223)$$

$$\text{s.t. } \begin{cases} \lambda \frac{r}{r^2-2m} = \omega_\phi \Psi_\phi^{(1,0)} + \omega_\psi \Psi_\psi^{(1,0)}, \\ -\lambda \tan\theta = \omega_\phi \Psi_\phi^{(0,1)} - \omega_\psi \tan^2\theta \Psi_\psi^{(0,1)} \end{cases} \quad (224)$$

Critically, the function $F_k[x]$ represents a linear-involution with the typical hypergeometric series (with a strongly matched Meijer completion¹¹²). The stabilizer constraints may be given as:

$$\frac{m\lambda}{2} = (r^2 - 2m)\partial_z F_l[z] \quad (225)$$

$$\cdot d_l \omega_\phi \cos^2\theta \left(\frac{c_l}{d_l} {}_2F_1\left[1 - \frac{k}{2}, 1 + \frac{k}{2}; 2; x\right] - \frac{\omega_\varphi \tan^2\theta}{\omega_\phi} {}_2F_1\left[1 - \frac{k}{2}, 1 + \frac{k}{2}; 2; x\right] \right) \Bigg|_{x=\cos^2\theta, y=\sin^2\theta}^{z=\frac{r^2}{m}} \quad (226)$$

$$-\lambda = d_l \omega_\phi F_l[z] \cos^2\theta \quad (227)$$

$$\cdot \left[\begin{array}{l} \frac{c_l}{d_l} \left({}_2F_1\left[1 - \frac{k}{2}, 1 + \frac{k}{2}; 2; y\right] + x \frac{(1-\frac{k}{2})(1+\frac{k}{2})}{2} {}_2F_1\left[2 - \frac{k}{2}, 2 + \frac{k}{2}; 2; x\right] \right) \\ -\frac{\omega_\varphi y}{\omega_\phi x} \left({}_2F_1\left[1 - \frac{k}{2}, 1 + \frac{k}{2}; 2; y\right] + y \frac{(1-\frac{k}{2})(1+\frac{k}{2})}{2} {}_2F_1\left[2 - \frac{k}{2}, 2 + \frac{k}{2}; 2; y\right] \right) \end{array} \right] \Bigg|_{x=\cos^2\theta, y=\sin^2\theta}^{z=\frac{r^2}{m}} \quad (228)$$

¹¹¹Remembering $\mathbf{F} = \mathbf{d}\Psi_\phi \wedge (\mathbf{d}\phi - \omega_\phi \mathbf{d}\mathbf{t}) + \mathbf{d}\Psi_\psi \wedge (\mathbf{d}\psi - \omega_\psi \mathbf{d}\mathbf{t}) + \mathbf{I}[\mathbf{r}, \theta] \sqrt{-\frac{\mathbf{g}_T}{\mathbf{g}_P}} \mathbf{d}\mathbf{r} \wedge \mathbf{d}\theta$

¹¹²that, only by being careful about the sense of confluent continuance, will be ignorable; keeping these irrational bulk gauges is another, more involved way to track in the index crossing. Again, the technique here to leverage the free algebra embedding for as long as possible (all the way into the final scalar/measure index).

Now, the juncture $r^2 = 2m$ displays clear importance, as does $r \rightarrow \infty$ [23]; analyzing these pushes on the stabilizer measure gives interesting forms on the sub-spectral forms at these r-points. Starting at $r \sim \sqrt{2m}$ will prove advantageous; there, because the radial generator has split relative to the $(r^2 - m)$ enveloping pole ($\forall m \neq 0$), the first sub-constraint sharply reduces:

$$m\lambda \sim 0 \quad (229)$$

$$0 = mc_l \omega_\phi x \partial_y F_k[y] F_l[z] \cdot \left[\frac{\partial_x F_k[x]}{\partial_y F_k[y]} - \tan^2 \theta \frac{d_l \omega_\phi}{c_l \omega_\phi} \right] \Bigg|_{x=\cos^2 \theta, y=\sin^2 \theta}^{z=2} \quad (230)$$

There is one clear way to solve this equation, and one (pair of complexly extended) confluenced way(s); first, a global solution may always be generated entirely by setting both the dispersive flow and the entanglement gauge identically to 0: $(d_l, \omega_\phi) \sim (0, 0)$, which amounts to an everywhere continuous and (symmetrically) smooth field solution. But, recognizing this system as a pair of coupled, spontaneously broken symmetries realizes that there must be a confluent field redefinition along some strongly symmetric sub-field continuance; looking at the symmetric measure planes $\{\theta\} \in \{0, \frac{\pi}{2}\}$ yields the constraint:

$$d_l \omega_\phi \left[\frac{(1 - \frac{k}{2})_l (1 + \frac{k}{2})_l}{(2)_l} + \frac{(1 - \frac{k}{2})(1 + \frac{k}{2})(2 - \frac{k}{2})_l (2 + \frac{k}{2})_l}{2(2)_l} \right] = 0 \quad (231)$$

Then, as the sub-shelled 2-pt (mean-correlators) form a modular group over the entanglement phase, it can be shown that the boundary patch continuance could be covered by a $k^2 = 24$ -dimensional crossing operator¹¹³; or:

$$d_l \omega_\phi \frac{(1 - \frac{k}{2})_l (1 + \frac{k}{2})_l}{(1 - \frac{k}{2})_{l+1} (1 + \frac{k}{2})_{l+1}} = d_l \omega_\phi \Rightarrow -1 = \frac{(2 - \frac{k}{2})(2 + \frac{k}{2})}{2} \quad (232)$$

$$\Rightarrow k^2 = 24 \quad (233)$$

This can be seen as exactly the renormalization $\sum n = -\frac{1}{12}$; then, this could be considered a measure of the p-adic group measure on the Meijer branch, or saturated soft-spectral flow density (over the dual measure spectral topology, \mathbb{R}^3), or the measure space of a 2-pt string at the critical dimension $k^2 = d_c - 2$.

Even better, this is iff and shows a strong connection to moduli stabilization in IIA/M-theory fermionic (Kahler-dimensionalized) regularization as a point of matched renormalization balancing between black holes/QED/open-string cohomologies (exhibited by $\frac{1}{12} \left(\frac{k}{\sqrt{2}}\right)^2 = 1 \hat{=}_{o.s.l.} \sum n$). Further, this is a (classical level) duality iff the passing branch can always be constructible exactly as a 21-pt $\times \mathbb{R}^3$ measured correction (from the sub-indexed (r, θ) g-loop quadratic foliation density used to sub-parameterize the classical unification juncture described above). Indeed, the existence of such a stabilizer will be con-

¹¹³ which exactly matches the number of colored Higgs branches over the proton's minimal stabilizer under the recently observed anomalous W boson mass anomaly [60]; this begins to complete the promise of a Higgs colored fermi crystallization of fully interactive QED

structed uniformly (as a maximal cover of the local stabilizer space extension towards the proper $U(1)$ G-spectral embedding form) in a subsequent chapter.

Finally, the $r \rightarrow \infty$ equation cannot be solved non-trivially unless $\partial_z F_k[z] \rightarrow \Big|_{r^2 \rightarrow m\infty} 0_m$ which cannot happen (without Meijer-control) at the harmonic radial boundary; still, remembering the diffeomorphic gauge symmetry can here act as a free (continuous) affine connection between the $d\phi$ -regulatory surface and the $\cos^2 \theta$ -projected sub-regulator densities, it's exactly sufficient to consider some divergent $N\infty$ regulatory connection which cuts off the radial gauge constraints at large r (explicitly seen by the $\lambda \rightarrow \lambda - \omega_\phi$ displayed on the φ -log branch.). This justifies the association with the hyperdimensional embedding as a dimensionally confluent regulation mixing symmetry (or, a broken symmetry measure density; or, as UV/IR mixing measure, here in the $SL(2, \mathbb{R}) \times U(1)$ -spin blocking.).

Finally, it can be shown that the 0-mode solutions of $\mathcal{L}_{\{\phi, \psi\}}$ meet the constant frequency separability conditions generally:

$$\left\{ \begin{array}{l} \Psi_\theta^\phi = c_1 - c_0 \log \cos \theta \\ \Psi_\theta^\psi = d_1 + \frac{c_0 \omega_\phi}{\omega_\varphi} \log \sin \theta \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \Psi_r^\phi = c_1 - c_0 \log \left(1 - \frac{r^2}{m}\right) \\ \Psi_r^\psi = d_1 + \frac{c_0 \omega_\phi}{\omega_\varphi} \log \left(1 - \frac{r^2}{m}\right) \end{array} \right. \quad (234)$$

With these 0-mode solutions in mind¹¹⁴, the next step is to analyze a specific perturbation. As mentioned above, the exactly similar radial forms of the $\mathcal{L}_{\{\phi, \psi\}}$ operators canonically lead to considering radially independent solutions at 0-order which may then generate $\{r, \theta\}$ -seperable ansatz at $O(\alpha^2)$ order. Consider the monopole-type partition states:

$$\Psi_\phi^{(0)} = 1 - c_1 \log(\cos \theta) \quad , \quad \Psi_\psi^{(0)} = 1 - d_2 \log(\sin \theta) \quad (235)$$

Noting that $\frac{c_2}{c_1} \sim -\frac{\omega_\phi}{\omega_\varphi}$ gives a precise sense in which the field "tree-trunks" mix exactly at the level of the longest dimensionalized velocity "gauge-branches" (as a feature of inductive closure). Considering the discussion above predicated on the existence of a (bulk mediated) IR-cutoff (so that the constraints may be modularly matched/the complete algebra is not assumed totally decidable), this crossing perturbation should exactly represent by p-adic, $p\infty$, regulatory crossing features (with enhanced global algebraic symmetries); such as the Ψ_φ field should be perturbatively stable (up to a possible r-renormalization surface).

Turning to the field equations (84), the constant frequencies guarantee $dF = 0$; then, the degeneracy [45] condition reads:

$$F \wedge F = 2d_2 \alpha^2 \cot^2 \theta \partial_r \Psi_\phi^{(1)} \left(\begin{array}{l} d\phi \wedge d\varphi \wedge dr \wedge d\theta \\ + \frac{\alpha}{\sqrt{m}} dt \wedge (\omega_\varphi^{(1)} d\phi - \omega_\phi^{(1)} d\varphi) \wedge dr \wedge d\theta \end{array} \right) \quad (236)$$

Critically, the last line is peels off as a higher order correction, while the first term remains as the magnetic dominance degree on this perturbative shell. In fact, regardless of being met¹¹⁵, at $O(\alpha^2)$ the degeneracy constraint is always only dual to time and, again, is closed

¹¹⁴Properly, Ψ_ψ -constant ($d_\theta = 0 = d_\psi$) solutions must be trivial (in the Tangherlini geometry). Still, this is not out of line with expectations: it may be that higher α -order Ψ_ϕ modes can single-handedly "balance" the angular momentum (chemical potential) dissipation (diffusion) under a proper seperability ansatz. See (215).

¹¹⁵or, of the result being everywhere magnetic/globally determinate

iff there exists a consistent harmonic compactification (at $r \rightarrow \infty$)¹¹⁶

Notably, this may be met three ways: a field redefinition $d_2 \rightarrow 0$, amounting to pushing Ψ to higher α orders, or with asymptotically flat field regularization, $\lim_{r \rightarrow \infty} \partial_r \Psi_\phi^{(1)} \rightarrow 0$, or with geometric framing, $\theta \rightarrow \frac{\pi}{2}$. In fact, as discussed above, all of these features can be found in the solution forest of AFS, here representing quasi-symmetrized T-shells at varying *in/out* scattering ratios, which can be seen by exploring the higher harmonics of the solution trees discussed above.

Accordingly, the constraint tree has been limited to finding a second order algebraic tree p -closure (at this gauge point):

$$\Psi_\phi^{MP} = \Psi_\phi^{(0)} + \alpha^2 \Psi_\phi^{(1)} + O(\alpha^4), \quad r_0 \omega_\phi^{MP} = \alpha \omega_\phi^{(1)} + O(\alpha^3), \quad r_0 I^{MP} = \alpha I^{(1)} + O(\alpha^3) \quad (238)$$

$$r_0 \omega_\psi^{MP} = O(\alpha^3), \quad \Psi_\psi^{MP} = \Psi_\psi^{(0)} + O(\alpha^4) \quad (239)$$

Note that $\sqrt{m} \omega_\psi = O(\alpha^3)$ asks that the entanglement crossing flow be, additionally, mass[-action] saddled (on each solution neighborhood). The FFE constraints read:

$$j_\mu F^{\mu, \{t, r\}} = O(\alpha^3), \quad j_\mu F^{\mu, \phi} = c_1 \alpha \frac{\sec \theta \partial_r I}{r^3}, \quad j_\mu F^{\mu, \psi} = -d_2 \alpha \frac{\csc^2(\theta) \partial_r I}{m r^3} \quad (240)$$

Immediately, the current must be radially independent¹¹⁷. This leaves a single (stream) equation $j_\mu F^{\mu, \phi} = 0$, $\partial_r I = 0$. Assuming separability in the perturbative ϕ -flux and the current, I , and considering *out*-radial gauge fluxes changes second order (and in the $k = 4$

¹¹⁶Then, it can be shown that the degeneracy condition flows as:

$$d \star F \wedge F = 2d_2 \alpha^2 \cot^2 \theta \partial_r \Psi_\phi^{(1)} \left[-2 \sec^2 \theta d\theta + \partial_r \ln \left[\partial_r \Psi_\phi^{(1)} \right] dr \right] \wedge \left[dt + \frac{\alpha}{r_0} \left(\omega_\phi^{(1)} d\varphi - \omega_\phi^{(1)} d\phi \right) \right] \quad (237)$$

By Stoke's theorem (or Carton's little formula), the $\int d\varphi(\cdot, \star)$ constraint flow is uniformly zero iff the above is zero; critically, the geometric support drops out of the angular term, meaning that, outside of the $d_2 \rightarrow 0$ limit, radial confinement renormalization must be expected at second order, $\partial_r^2 \Psi_\phi^{(1)} \hat{=} 0$. This exactly amounts to a (shell-)frequency matching at large volumes and high frequencies, which is exactly a Wilsonian-type field regulation. Notably, this is the proper point of contact with field redefinition techniques, where pushing a $U(1)$ -connected branch over the $p_\phi^{(1)}$ will result in the broken winding potential gradient along the r -branch. Rather than pursue the $k \rightarrow k + 1$ inductive closure, the proof will instead continue as an extension induction with a complete closure $\{\partial_k < \partial_{k+1}\} \wedge \{k_r < k_{r+1} \sim k_1\}$; this section will instead prove a condition about this state's local forward differentiability and then close the constructive loop at the highest connected *out*-point to produce the final $\mathbb{P}_{\mathcal{O}_{21}} \times \sqrt{SU(2)^2} \times \mathbb{R}_*^2 \rtimes U(1)$, where the first L-adjoint product represents a prime spin idealization over the spectral pullback, which is (effectively) pushed over the residual (spin) coordinate gauge on the RHS. It should be noted that this is exactly the decomposition type expect from this classical Berry-contacted construction, and that this bi-partite symmetrization matching lets the strongest of the topologically extended spin-rationalization theorems be applied [34].

¹¹⁷or asymptotically r -sublinear: $x^2 \partial_x I \rightarrow 0 \Big|_{x=\frac{1}{r}} \Rightarrow I[r] \sim r^{\alpha' - 1} \Big|_{\alpha' > 0}$ (looking like a Post-Newtonian potential constraint, as expected); this again shows a bridge to effective field theory

space) gives: $\Psi_\phi^{(1)} = f(r)g(\theta)$, $I(r, \theta) = I_r I(\theta)$, leaving:

$$\begin{aligned}
& -\frac{4r^4 (I(\theta)^2)'}{c_1 m r^2 (m - r^2) d[\sin^2 \theta]} = \tag{241} \\
& + \frac{\partial[\sin 2\theta] \left(m^3 - 2\frac{m^2}{c_1} r^2 \left(\frac{c_3}{c_1} + \frac{d_2 c_3}{c_1^2} \right) + r^6 c_3^2 \left(1 + \frac{d_2}{c_1} \right)^2 \right) - (m^3 - 2c_3 m^2 r^2 + (1 + d_2^2) c_3^2 r^6)}{\frac{m}{c_1} r^2 (m - r^2)} \\
& + f(r)g(\theta) \left(\frac{(2 \csc 2\theta g'(\theta) - g''(\theta))}{g(\theta)} + \frac{(r(m - r^2) f''(r) - (r^2 + m) f'(r))}{r f(r)} \right)
\end{aligned}$$

Noticing the abundance of trigonometric functions, and that $g(\theta) = \sin^2 \theta = T_{(4)}^\phi(\theta)$ is an r -dependent solution higher on the functional tower, making separability justified as a perturbation inference.¹¹⁸ Then, it can be shown that requiring uniform separability requires an (almost everywhere) unique set of choices¹¹⁹. In fact, using the fundamental theorem of algebra on the $(AB - C)$ -rule in (hypergeometric)PDE analysis¹²⁰ allows an immediate deduction on the minimal functional as $I = \sqrt{c_4 - c_5 \sin^4 \theta}$. Finally, choosing $d_2 \sim 0$ amounts to looking for an exact 1 – 1 conformally matched phase point between the regulatory $p\infty$ extensions and the usual, $r \rightarrow \infty$ harmonic compactification (as a scaling dimension on the dual gauge magnetification). The resulting analytic ansatz is¹²¹:

$$f''(r) - \frac{(m + r^2) f'(r)}{r^3 - mr} + \frac{4f(r)}{m - r^2} - \frac{c_1^2 (4(c_3^2 r^6 - 2c_3 m^2 r^2 + m^3) - \lambda r^4)}{c_1 m r^2 (m - r^2)^2} = 0 \tag{242}$$

Note that the singular point of (242) is at the perturbatively stable (unspinning) horizon, $r = \sqrt{m}$, which only matches the coordinate singularity when $\alpha = 0$; thus, it should be considered to control an independent set of integration constants (“shadow constants”) representing the primary idealized interloping between the confluent matching modes. As seen shortly, it will be meaningful to select these constants so to be running cutoffs(/shadow fluxes) in the $p\infty$ regime.

¹¹⁸Notably, this is functionally the same $O(\alpha^2)$, Ψ_ϕ θ -product decomposition as in Kerr.

¹¹⁹Unless $\theta \sim \frac{\pi}{2}$, in which case this equation can be reduced to a modular matched (polynomial amplitude patch) unwinding as an r -point rescaled g_θ -flow:

$$0 \sim_{n\pi} 2 (I(\theta)^2)' + c_1 m \frac{(m - r^2) f(r)}{r^2} g'(\theta) \Big|_{\theta \sim \frac{\pi}{2}} \Rightarrow \begin{aligned} f(r) &= \frac{2A}{1 - \frac{m}{r^2}} \\ g(\theta) &= \frac{I(\theta)^2 - B}{c_1 m A} \end{aligned}$$

This is the modular equivalence of the stream equation, a.k.a., an entanglement stream equation (eSe); note that A can be chosen to fix a value of $f(r_A) = 1$; then, in fact, $r_{\frac{1}{4}} = \sqrt{2m}$, implying that the radial juncture (identified above) indeed behaves like a(n exact) weak gravitational propagator under a Hawking radiative (bulk) connection, as expected.

¹²⁰and that $\cos \theta \sin^3 \theta = \frac{1}{8} (2 \sin 2\theta - \sin 4\theta)$; renormalizing the current amplitude to the source mass-shell gives $c_5 \sim \pm c_1^2$, and positivity towards the perturbation stable horizon as $m \rightarrow 0$ decides upon the $+$ choice, which in turn keeps the source pole positive (when moved to the RHS). Accordingly, this normalization amounts to the *out*-coordination, or a $(-, \vec{\tau})$ type orientation (on the pull/*in* space); in fact, it will be shown that the other orientation indeed presents as a scattering parameter (or, as generating a $(+, -, -, -)$ pull space).

¹²¹Defining an amplitude scaled parameter $c_1^2 \lambda [c_1] := c_5$

Denoting the (in-homogeneous) integration constants as $\{C_1, C_2\}$ gives the following radial contact potential at the Tangherlini stable (mass-absolute) horizon (with exactly suggestive *out*-branched \mathbb{F}_r geometric couplings):

$$f(r) = \frac{C_2 - 2c_1m(1 + \ln r^2)}{2m} \quad (243)$$

$$+ c_1^2(4m(-1 + c_3)^2 + \lambda) \left[\begin{array}{l} \frac{\ln[-m+r]}{4c_1m^2} \left(m + r^2 \ln(\sqrt{-m + r^2}) \right) \\ + \frac{r^2}{4c_1m^2} \left((1 + i\pi - \ln m) \ln\left(\frac{r^2}{m}\right) - {}_2\text{Li}\left[1 - \frac{r^2}{m}\right] - \ln r^2 \ln m \right) \end{array} \right] \quad (244)$$

$$+ \frac{r^2}{c_1m^2} \left[\begin{array}{l} mc_1^2 {}_2\text{Li}\left[1 - \frac{r^2}{m}\right] \\ + \ln[\sqrt{-m + r^2}] \left(mc_1(2c_1(m \ln \frac{r^2}{m} - 1 + 2c_3^{\frac{3}{2}} \cosh \ln \frac{\sqrt{c_3}}{2}) - C_2) \right) \\ + \ln \sqrt{r} \left[\begin{array}{l} i\pi c_1^2 \lambda - 2c_1 C_2 \\ \frac{c_1^2 m}{2} (\ln r + 1 + i\pi - 2c_3(1 - i\pi \sqrt{\frac{c_3}{2}} \sinh(\ln \sqrt{\frac{c_3}{2}}))) \\ (\frac{1}{4} + (-1 + c_3)^2) \frac{\pi^2}{6} + \frac{1}{4} (1 - i\pi) \\ + c_1^2 m (C_1 + c_3^2 - i\pi(1 + 2c_3^{\frac{3}{2}} \cosh \ln \frac{\sqrt{c_3}}{2})) \end{array} \right] \end{array} \right] \quad (245)$$

At this point, the problem has been sufficiently collapsed towards this radial string ($d = 5$ stream) equation to give latitude towards the integration constants(/field regulation); since this perturbation was selected at a unit index dimensionalization $d_2 \equiv 0$, it can be seen that corrections to the field cannot effect the degeneracy condition, amounting to the selection of uniformly stable crossing junctures.¹²²

So, hoping to extract ${}_{p_r}\infty$ extensions with some clear embedding of the usual harmonic measure, ∞ , categorically relies on a field regulation; having pushed the stream-form into the r -axis, this symbolically amounts to a Λ_{p_r} cutoff form, or a far field integration cut-off/counter propogator in r . Particular, these terms can be considered emergent counter radiators amounting from a classically squeezed measurement; such as, the divergent correction (stripped off) is exactly a spontaneously broken index that was separated from this measure by $d_2 = 0$. Notice that is exactly consistent with the kinectic-holography arguments made in the section above 165

Considering the T-horizon stability as (uniformly) necessary (and otherwise uncorrectable

¹²²Considering the uniform uniqueness of Lesbesque measurable sets is only given up to the maximal real harmonic embedding, the discussion above shows this perturbation to have a strong stability criteria on the r -shell that can be ${}_{p_r}\infty$ relaxed by involuting the Meijer ghost constraint (basis) with differing harmonic regularization points (equivalently, across differing Wilson cutoffs). This is easier here because of the symmetric $\theta \sim \frac{\pi}{2}$ index, which always identifies these cutoffs with some set of hyperfine continuances (or, ${}_{p_r}\infty$ extended fields). This can be tracked by noting the polylogarithm induction formula, ${}_{i+1}\text{Li}[z] = \int_0^t \frac{i\text{Li}[t]}{t}$;

equivalently ${}_s\text{Li}[e^{\frac{2\pi im}{p}}] = p^{-s} \sum_{k=1}^p e^{\frac{2\pi imk}{p}} \zeta(s, \frac{k}{p})$, and $\zeta(s, \frac{k}{p}) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s}$

with the ($\mathcal{L}_{\phi\varphi}$ contacts) finds the ansatz:

$$\lim_{r \rightarrow \sqrt{m}} f(r) \rightarrow mC_1 + c_1 \left(\begin{array}{l} -(1 - \pi(\frac{\pi}{6} - i))(1 - c_3(2 + c_3)) - 2c_3(1 - c_3(\frac{i\pi}{24} - 1)) \\ 3 \ln 2(9c_3 + (1 + c_3 + c_3^2) \ln 2) \end{array} \right) \quad (246)$$

$$+ C_2 \frac{1 + \ln \frac{2}{\sqrt{m}}}{2m} + c_1 \ln m \left(\begin{array}{l} i\pi - \ln \sqrt{2} - \frac{7}{8}(c_3 - 1)^2 \ln m \\ -2c_3((1 + i\pi + \ln \sqrt{2}) - \frac{3c_3}{8}(1 + \frac{4i\pi}{3} + \frac{2}{3} \ln 2)) \end{array} \right) \quad (247)$$

$$+ \frac{c_1 \ln(r - \sqrt{m})}{2} \left(\begin{array}{l} \frac{C_2}{mc_1} + \frac{\lambda}{2m} + 3(c_3^2 + \frac{2}{3} \ln 2 (c_3 - 1)^2) \\ -(2 - (1 - c_3)^2) \ln m + \frac{\lambda}{2m} \ln(2\sqrt{m}) \\ + ((1 - c_3)^2 + \frac{\lambda}{4m}) \ln(r - \sqrt{m}) \end{array} \right) \quad (248)$$

Regardless, there exists freedom in how to choose the integration constants (C_1, C_2), but not enough latitude to guarantee this solution be well defined on the T-stable horizon (which instead fixes $\lambda = \frac{c_3}{c_1^2}$), a very general expectation. Still, C_2 can be seen as connected the T-stable free parameters to the asymptotic states¹²³, which (combined with the ultra large slow renormalization) presents as an asymptotic scaling form(242); taking the simplest branch matching constant in each case yields¹²⁴:

$$\begin{aligned} c_5 &\rightarrow -c_1^2(-1 + c_3)^2 \\ f \in \mathbb{R}^*[r] &\Rightarrow C_1 \rightarrow c_1(\alpha_1 + 2ic_3^2\beta) \\ C_2 &= 4c_1m((\frac{c_3}{2} + 1)^2 - (1 + \ln r_0)) \end{aligned} \quad (249)$$

$$f[r] = -c_1 \left(\begin{array}{l} 2 \ln r + ir^2 c_3^2(\pi + 2m\beta) \\ -\frac{r^2}{m} \left(m(\tilde{\alpha}_1 + \ln m \frac{1}{m}) + (1 - \frac{m}{r^2}) c_3^{\frac{3}{2}} \cosh \ln \frac{\sqrt{c_3}}{2} \right) \\ {}_2\text{Li}[1 - \frac{r^2}{m}] + \ln[-1 + \frac{r^2}{m}] \left(\ln \frac{r^2}{m} + \ln \frac{m}{r^2} \right) \end{array} \right) \quad (250)$$

Then¹²⁵, by direct inspection, the imaginary portion of the constant can be clearly related to it's square renormalizability as a rolling mass phase. For now, consider the (asymptotically) single patched phase legs designated by¹²⁶ $\beta = -\frac{1}{2m}$; accordingly, this should be expected

¹²³And can be seen as a reduction from the $\lim_{r \rightarrow \sqrt{m}} \ln \sqrt{-m + r^2} \ln \frac{r^2}{m}$ branch ambiguity that is, here, resolved by the two-polylogarithm; generally, this can be collapsed to a single $U(1)$ -interloping phase by the appropriate choice of C_2 , as seen shortly.

¹²⁴and where the ridiculous final term is meant as a reminder of the polylogarithm's base-2 connection to the fundamental idealization form

¹²⁵the final term in the last line can be seen as a simple reminder that this closure is nice because the field is a propogating logarithmic scalar field and thus exactly closes it's (harmonic moment) poles over the log-branches.

¹²⁶It can be shown that $\lim_{r \rightarrow \infty} f[r] = \frac{c_1 r^2(-6 + 12c_3 - \pi^2 + 6m\alpha + 3ic_3^2(\pi + 2m\beta) - 3(\ln m)^2 + 6c_3^2 \ln r)}{6m}$, and $\lim_{r \rightarrow r_0} f[r] = \frac{c_1}{2}(-2 + 2m\alpha + ic_3^2(i + \pi + 2m\beta) + (c_3^2 - \ln m) \ln m)$, so that $\beta = \frac{-1}{2m}$ eliminates the complex components at both limits. Further, $\Delta_{r_0}^\infty[f] := \frac{\lim_{r \rightarrow r_0} f[r]}{r_0^2} - \frac{\lim_{r \rightarrow \infty} f[r]}{r^2} = \frac{c_1(\pi^2 - 3c_3(4 + c_3) + 3c_3^2 \ln \frac{r_0^2}{r^2})}{6r_0^2}$; note, at the monopole Znajek point, $\Delta_{r_0}^\infty[f] \Big|_{c_3=\frac{1}{2}} = \frac{c_1}{6r_0^2}(\pi^2 - \frac{3^3}{2^2} + \frac{3}{4} \ln \frac{r_0^2}{r^2})$. Then, using a choice for c_1 explained below (with J the spin of the black hole), $\Delta_{r_0}^\infty[f] \Big|_{c_3=\frac{1}{2}} = \frac{iJ}{24r_0^3 J[\frac{\pi}{2}]}(\pi^2 - \frac{3^3}{2^2} + \frac{3}{4} \ln \frac{r_0^2}{r^2}) = \frac{-iJ}{24r_0^3 J[\frac{\pi}{2}]}(\frac{3^3}{2^2} - \frac{3}{4} \ln[r_0^2 e^{\frac{4\pi^2}{3} - \ln r^2}])$. Because f represents the only r-stable perturbation function, this

to calculate the longest non-thermal mode emerging from this perturbation (known as a von-Neumann instability of this perturbation shell).

Next, $\text{Re}[C_1]$ gains the interpretation as the shifted bare mass (as $r \rightarrow \sqrt{m}$), $\tilde{\alpha}_1 = \alpha_1 + \ln m^{\frac{2}{m}}$. Note that in the large mass limit, $\tilde{\alpha}_1 \sim \alpha_1$. Further, note that $\lim_{r \rightarrow \infty} f(r) =$

$\lim_{r \rightarrow \infty} \frac{r^2}{r_0^2} \left(\lim_{y \rightarrow r_0} f(y) - \pi^2 + c_3^2 \left(\frac{c_1}{\sqrt{c_3}} \cosh \frac{\sqrt{c_3}}{2} + 3 \log \frac{r^2}{r_0^2} \right) \right)$; in particular, this shows exactly what would be expected in a conformal loop, namely that scale invariant running¹²⁷ ($\frac{r}{r_0} \sim$

1) gives: $\left[\lim_{r \rightarrow r_0} - \lim_{r \sim r_0 \rightarrow \infty} \right] f(r) = -\pi^2 + c_1 c_3^{\frac{3}{2}} \cosh \frac{\sqrt{c_3}}{2}$. Thus, it should be expected that c_3 tunes the effective tension between a horizon focalized string dynamically attached to a large, continuously scaling open string. In this (AdS-CFT friendly) version of the peeling setup it is immediate that c_3 represents a Bloch defect state and, accordingly over $k = 4$, a (continuous, squeezed) thermal connection to the gravitationally partitioned vacuum. Recollecting everything, the soft magnetic form factor displays as:

$$\begin{aligned} \Psi_\psi^{MP} &= 1, \quad \omega_\psi^{MP} = 0 \\ \omega_\phi^{MP} &= \frac{\alpha}{r_0} c_3, \quad I^{MP} = \alpha c_1 (c_3 - 1) \sqrt{c_4^* + \sin^4(\theta)} \end{aligned} \quad (251)$$

$$\begin{aligned} \Psi_\phi^{MP} &= 1 - c_1 \log \left(r^{\alpha^2 \sin^2 \theta} \cos \theta \right) \\ &\quad + c_1 \frac{\alpha^2 r^2 \sin^2 \theta}{2m} \left({}_2\text{Li}[1 - \frac{r^2}{m}] + m \left(\tilde{\alpha}_1 + \ln m^{\frac{1}{m}} \right) + \left(1 - \frac{m}{r^2} \right) c_3^{\frac{3}{2}} \cosh \ln \sqrt{\frac{c_3}{2}} \right), \end{aligned} \quad (252)$$

where the last line was rescaled as $c_4^* = \frac{-2c_4}{mc_1^2(c_3-1)^2}$.

3.10 Bulk Power Regulation

The equation for the total energy flux is [46]

$$P = -2(2\pi)^2 \int (\omega_\phi \partial_\theta \Psi_\phi + \omega_\psi \partial_\theta \Psi_\psi) I d\theta \quad (253)$$

the resultant integrand for the perturbative solution becomes:

$$\frac{dP}{d\theta} = -2(2\pi)^2 c_1^2 c_3 (1 - c_3) \frac{a^2}{m^2} \sqrt{c_4^* - \sin^4 \theta} \left(\tan \theta + \frac{\alpha^2}{2m} \cos \theta \sin \theta \frac{dP^{(2)}}{d\theta} \right) \quad (254)$$

where,

$$\frac{dP^{(2)}}{d\theta} = -c_3 m (4 + c_3) + 2r^2 \left(2c_3 + m\alpha_1 - 1 + \log\left(\frac{m}{r^2}\right) \left(\frac{m}{r^2} - \log r^2 \right) + \log r (c_3^2 - \log(r^2)) + \text{Li}_2\left(1 - \frac{r^2}{m}\right) \right) \quad (255)$$

conformally centralized limit operator is a proper r-operator on the entire perturbation; considering everything unitless as partition measures (and the J -envelope as the sector selection), $\frac{3^2}{2^2}$ represents a sorting density from the three degree of freedom configuration space to a two degree of freedom space while the second term represents $\frac{-3}{4}$ -weight degrees of freedom running from a $\ln r$ defect in Gaussian partitions of the event horizon r_0 . [53]

¹²⁷or, under $p \infty$ extension that is strongly additive to (all the harmonic numbers; a.k.a., the total loop operator scale) both $\lim_{r \rightarrow \infty} \frac{r^2}{r_0} \rightarrow \lim_{p \rightarrow \infty} \frac{\lambda_p r^2 + p}{\lambda_p r_0^2 + p}$

The value of c_4^* is then immediately fixed by the only divergent term in the domain of integration, $\lim_{\theta \rightarrow \frac{\pi}{2}} \tan(\theta) \Rightarrow c_4^* = 1$, which requires the current profile to uniformly close as $\theta \rightarrow \frac{\pi}{2}$. Consequentially:

$$I^{MP} = i\alpha c_1(c_3 - 1)\sqrt{1 - \sin^4(\theta)} \quad (256)$$

Performing the integration over $\theta \in [0, \frac{\pi}{2}]$,

$$\frac{P}{2} = - \int_0^{\frac{\pi}{2}} \frac{dP}{d\theta} = -i \frac{\alpha^2 c_1^2 c_3 (1 - c_3)}{4r_0} \left((2 + \pi) + \frac{\alpha^2 P^{(2)}}{8r_0} \right) \quad (257)$$

$$\text{where,} \quad (258)$$

$$\frac{P^{(2)}}{2} = \frac{\pi}{8} \frac{dP^{(2)}}{d\theta} \quad (259)$$

Note that this is perfectly well defined at $O(\alpha^2)$ order. Now, as noted above, there are two distinct cut surfaces (the event horizon at the confluent matching surface); starting with the far field, it should be expected that the far r -gauge may become membrane dynamic. Accordingly, under the conformal hypothesis, some unitless adjoint radial coordinates, $r = \Gamma\sqrt{m}$, must be exclusively relevant; then, if we choose

$$\alpha_1 = \frac{c_1}{m} \left(1 - 2c_3 + \frac{1}{2} \ln \alpha + \frac{\pi^2}{6} + \ln r_0 \left(1 - \sqrt{2}c_3 \ln \Gamma \sinh \ln \frac{c_3}{\sqrt{2}} \right) + \frac{4\chi[c_3; \Gamma]}{\alpha^2 \pi \Gamma^2} \sinh \ln \frac{\alpha^2 \chi[c_3; \Gamma]}{\sqrt{2 + \pi}} \right) \quad (260)$$

$$\text{where } \chi^2[c_3; \Gamma] := c_3^{\frac{3}{2}} \pi \cosh \ln \frac{\sqrt{c_3}}{2} + \log \Gamma \quad (261)$$

Now the corrected power is real and square measurable (and exactly one loop order higher)¹²⁸:

$$-\frac{P_\Gamma}{2} = \frac{\pi \alpha^4 c_1^3 c_3^5 (1 - c_3) \Gamma^2}{16r_0} \quad (262)$$

It's important to remember that the regulators exist in the $f(r)$ integration domain, which give them immediate extensions into the solution diffeomorphic dualities. Importantly, the final term can be considered a loop-induction step by Fermi's Golden rule iff Γ is interpreted as a mass-tunneling branch (running both ways in the $\bar{10} \odot 10$ regulation topology), which can be explored in a number of ways from even this classical string OPE¹²⁹.

In fact, considering all the regulator structures pushed into the r_∞ space gives wide birth to this coordinate choice; so, $U(1)$ rescaling as $\Gamma \sim e^{-\pi c_3 \left(1 + \frac{c_3^2}{4}\right) + i \frac{(2+\pi)b_3}{\alpha^4}} \chi_3 \frac{(2+\pi)}{\alpha^4}$ implies

¹²⁸And resemble a mean field scattering defect (under the interactive picture) as $\frac{c_1^2 P_\Gamma}{2m\Gamma^2} = \frac{\pi \alpha^4 c_1^5 c_3^5 (1 - c_3)}{16r_0^3}$

¹²⁹note that, on the first continuous branch, $\chi \equiv \pi c_3 \left(1 + \frac{c_3}{2}\right)$, which makes the effective (massless) regulator frame effectively super-rotational (again, on the lowest branch) at fixed (α, c_3) . By power scaling arguments on the massive field $\Gamma \in \mathbb{F}_{r_0} = \frac{\mathbb{R}^{\bar{p}_\infty}}{\mathbb{R}\sqrt{m}}$, or $\langle \mathbb{F}_{r_0} : \mathbb{R}^{\bar{p}_\infty \bmod \sqrt{m}} \rangle = \langle \mathbb{R}^{\bar{p}_\infty} \rangle$ by Hilbert's Nullstanz, or the second isomorphism theorem (on the maximally decidable partial logic algebraic extensions/completion)

$\chi^2[c_3; \chi_3] = \ln e^{i \frac{b_3(2+\pi)}{\alpha^4} \chi_3^{\frac{(2+\pi)}{\alpha^4}}}$ which gives α_1 the form:

$$\alpha_1 = \frac{c_1}{m} \left(1 - 2c_3 + \ln \alpha + \frac{\pi^2}{6} \right. \tag{263}$$

$$\left. + \ln r_0 \left(1 - \sqrt{2}c_3 \left(\frac{2+\pi}{\alpha^4} (\ln \chi_3 + ib_3 - \pi c_3 (1 + \frac{c_3^2}{4})) \right) \sinh \ln \frac{c_3}{\sqrt{2}} \right) + \frac{2\sqrt{2+\pi}}{\alpha^4 \pi \chi_3^{\frac{2\sqrt{2+\pi}}{\alpha}}} (\ln \chi_3 - 1) \right) \tag{264}$$

Finally, shifting by a unit rescaling $\chi_3 \rightarrow \bar{\chi}_3 = \chi_3 e$ and coordinate field redefining one more time as $\bar{\chi}_3 \rightarrow \tilde{\chi}_3 = \bar{\chi}_3^{\frac{2(2+\pi)}{\alpha^2}}$ produces:

$$\alpha_1 = \frac{c_1}{m} \left(\begin{array}{l} 1 - 2c_3 + \frac{1}{2} \ln \alpha + \frac{\pi^2}{6} \\ + \ln r_0 \left(1 - \sqrt{2}c_3 \left(\frac{2+\pi}{\alpha^4} (1 + ib_3 - \pi c_3 (1 + \frac{c_3^2}{4})) \right) \sinh \ln \frac{c_3}{\sqrt{2}} \right) \\ + e^{-\frac{(2+\pi)}{\alpha^4}} \frac{\ln \tilde{\chi}_3}{\pi \alpha^2} \left(\tilde{\partial}_3 (\ln \tilde{\chi}_3) - \pi \sqrt{2+\pi} e^{\frac{(2+\pi)}{\alpha^4}} \ln r_0 c_3^2 - 2 \right) \end{array} \right) \tag{265}$$

In particular, notice the appearance of an effective delta function around the unspinning limit $\sim_{\alpha^2 \rightarrow 0} \delta(2+\pi)$, which results from this naive calculation's use of rescaling without the exact bulk contact and is exactly trying to "square the circle" exactly into one copy of the spin regulators; indeed, $\pi = -2 \Rightarrow A_{\odot}^{(-2)}[r] = (i\sqrt{2}r)^2$, or, defining the two-confluent diagonal as $r_{/\odot} = \sqrt{2}ir$, $A_{\odot}^{-2}[r] = r_{/\odot}^2$. This is exactly as expected because this prescription is very reductive¹³⁰: effectively, this calculation truncates spacetime at some fixed radius (implicitly, $r > r_0$) and begins loop correcting at all orders (the *in* bulk), as seen by the high variability of this m_3 partition (on the scale of the perturbation).

Typically this is a bad sign for power spectra because it signals an ill-defined vacuum state; but, again, this is conceptually a diagnosis of the renormalization current of the vacuum (which only ever needs to close up to it's own renormalization field's closure). In particular, this example shows that spin-interactions in the conformal information measures yield well defined corrections with a single field extension, namely $p_\alpha \sim \{e^{\frac{2+\pi}{\alpha^4}} : \alpha \in (0, 1]\}$, that represents a uniform loop at the next order. The claim is that a simple knot, representing unification, descends deeply into a complex field of regulators and can only be uniformly sorted at some higher order (specifically, under $2\delta(= 24 - 3 \text{ maximal convolutions})$ before a time stable measurement emerges. Toward that, it has been shown that the log-type solutions have a near horizon power partition that can indeed be interpreted as a classically scattering object.

Alternatively, the current could be considered real and the field could be considered (a) spinning (ghost) simply by defining c_1 in terms of c_5 (instead of vice versa). So, considering

¹³⁰or, exactly as reductive as the section above

the near horizon limit (and renaming $c_5 = b_\phi$, so $c_1 \rightarrow \frac{i}{c_3-1} \sqrt{\frac{b_\phi}{m}}$):

$$\frac{dP_H}{d\theta} \approx -\frac{b_\phi}{m} \frac{c_3 \sqrt{m} \sqrt{1 - \sin(\theta)^4}}{4m^2(1 - c_3)} \left(\frac{a^2 c_3^2}{m} \left(\pi - 4m\alpha \sqrt{\frac{m}{b_\phi}} \frac{1 - c_3}{c_3} \right) \sin(2\theta) - 4i \tan(\theta) \right) \quad (266)$$

$$\Rightarrow \frac{P}{2} = \int_0^{\frac{\pi}{2}} \frac{dP}{d\theta} = -\frac{1}{16m} \frac{c_3}{(1 - c_3)} \frac{1}{\sqrt{m}\gamma^2} \left((2\alpha^2 \pi (1 + \frac{2}{\pi}) - i\gamma^2)^2 - 2i\gamma^3 (1 + \frac{2}{\pi}) \right) \quad (267)$$

$$\text{given a loop rescaled parameter } \gamma: \alpha = \frac{1}{4\pi} \sqrt{\frac{b_\phi}{m}} \frac{\gamma^2}{a^4} \quad \gamma \in \mathbb{R} \vee \gamma \in i\mathbb{R} \quad (268)$$

Let $\gamma = \frac{-2}{c_3}(1 + \frac{2}{\pi})$; in the above, a negative α corresponds to a completely imaginary loop rescaling $\gamma \rightarrow \pm i\lambda$ and a totally imaginally second order term $O(i\lambda^2)$, and a real $O(\pm\lambda^3)$. In fact, we see that this parameterization has a "wedge" at the origin: $\{\alpha\} = \{\gamma \in \mathbb{R}\} \cup \{i\gamma\}_{|\gamma \in \mathbb{R}}$, with a sign freedom in the λ subfield. Remembering that the initial reality condition on α was to make the perturbative solution real on the horizon, we can introduce a relaxation to this condition by allowing:

$$\gamma^2 \approx \epsilon - 2\pi i \frac{a^2}{m} (1 + \frac{2}{\pi}) \quad \text{where,} \quad \begin{aligned} |\epsilon| \gg 2\pi\alpha^2(1 + \frac{2}{\pi}) &\Rightarrow \alpha \gg \approx > 0 \\ |\epsilon| \ll 2\pi\alpha^2(1 + \frac{2}{\pi}) &\Rightarrow \alpha \ll \approx < 0 \end{aligned} \quad (269)$$

giving¹³¹

$$\frac{P}{2} \approx -\frac{1}{16} \frac{c_3}{(1 - c_3)} \frac{\frac{\epsilon}{m}\epsilon}{\sqrt{m}\gamma^2} \left(1 - i\sqrt{\frac{i}{\epsilon}} (1 + \frac{2}{\pi}) \left(\frac{2\pi}{\epsilon} \frac{a^2}{m} (1 + \frac{2}{\pi}) - i \right)^{\frac{3}{2}} \right) \quad (270)$$

$$= -\frac{c_1}{16} \sqrt{\frac{c_3^2}{m^2 b_\phi}} \frac{\epsilon^2}{\gamma^2} \left(1 - i\sqrt{\frac{i}{\epsilon}} (1 + \frac{2}{\pi}) \left(\frac{2\pi}{\epsilon} \alpha^2 (1 + \frac{2}{\pi}) - i \right)^{\frac{3}{2}} \right) \quad (271)$$

In fact, it can be readily shown that both limits of the "nearly-real" regularization schemes produce a dominantly real P . We note that epsilon has the same unitlessness as the flux perturbation, and that $\frac{\epsilon}{m}$ have the units of a second-order frequency, $[\frac{\epsilon}{m}] = [\frac{a^2}{m^2}]$ which was originally unperturbed by the single spinning axis' reflection symmetry ($\omega^{MP} = \frac{a}{m} \omega^{(1)} + O(\frac{a^3}{m^3})$). This fits the idea that the 5-d perturbation benefits from a half-order fixing in the regulators that allows surprising control.

What's left it to demonstrate this α^2 splining does indeed inductively close over the linear branch. This will be done two ways; firstly, it will be shown that the 0^{th} solutions strongly separate from the $O(\alpha^3)$ perturbations¹³² that it separates strongly from the $O(\alpha^{3=2+1})$

¹³¹Note that the ϵ -critical scale is exactly the Goldstone re-angularization, or $\pi \rightarrow 2 \Rightarrow |\epsilon| \sim_{\text{critical}} 4\pi\alpha^2$

¹³²(so they can be added as free correctives on each iteration towards a branch twining at $< O(\alpha^{n+1}), O(\alpha^{n+2}) >$, which amounts to showing the inductive forward step $\partial_k \leq \partial_{k+1} \wedge k_r < k_{r+1}$; then, showing that the divergently complex $O(\alpha^{2p})$ -spin corrections are indeed closed and complete over a finite set of $SU(N)$ stabilizers (shown shortly) amounts to establishing the $({}_p\mathbb{F})$ group algebra must have a modular representation. Further, because the induction (co-algebra) was adjointly framed, this also closes the "knot index" hypothesis used to unify Newtonian Gravity and Classical Electromagnetism

interactions. Even better, it is possible to directly infer the largest possible $U(N)$ state (a.k.a., the thermal state), which will allow an exact construction of a pointwise matching membrane (moving from *out* to *in*).

3.11 $O(\alpha^3)$ -matching

This method has the clearest analogy to standard perturbative techniques. The monopole construction has obvious limitations, notably that it involves an infinitesimal current sheet which forces the separability of the spacetime along $\theta = \frac{\pi}{2}$. Exactly as in classical electromagnetism, we may assume a leading order di-pole (α^3) dependence on the (thermodynamic) length:

$$\Psi_\phi^{MP} = \Psi_\phi^{(0)} + \alpha^3 \Psi_\phi^{(3)} + O(\alpha^4) \quad (272)$$

while leaving the other perturbative seeds the same. Additionally, as mentioned in 3.3, far-field perturbations should asymptotically stabilize the outer-light surface. Equivalently, radial functionalizations ($\{f_r^{(3)}, g_r^{(3)}\}$) should have asymptotic units much longer than the largest thermal length: $r \gg r_0$. Equivalently, this condition can be functionalized by rescaling the radius into thermal units and demanding that infinitesimal coordinate flow be larger than the fundamental phase-space unit: $r\alpha = \bar{r} \gg \alpha r_0$; then, the di-pole stream is unable to “excite” the region inside the light surface. This thermally monomic smoothing transform helps make the glueing procedure perturbatively analytic; it also acts as a way to gauge how long the next smallest (field α -polynomial) induction co-cycle must be¹³³. Indeed, performing this coordinate transformation and considering radially functional current distributions, it can be shown that the field equations reduce to:

$$j_\mu F^{\mu,t} = O(\alpha^7), \quad j_\mu F^{\mu,\phi} = O(\alpha^7), \quad j_\mu F^{\mu,\psi} = 0, \quad j_\mu F^{\mu,r} = O(\alpha^7) \quad (273)$$

and the final equation can be made $O(\alpha^7)$ with:

$$\theta I_{\bar{r}} \rightarrow c_1 c_3 \sqrt{\pm C_1^2 + \left(\left(\frac{\bar{r}}{\alpha}\right)^2 + r_0^2\right) \sin^4 \theta} \gg C_1 c_1 c_3 \sqrt{\pm 1 + 2 \frac{r_0^2}{C_1^2} \sin^4 \theta} \quad (274)$$

It’s important to note the high degree of uniformity in the field equations, which shows the $\Psi_\phi^{(0)}$ trunk can be readily deformed (in the perturbative spline sense) into functionally large dipole moments with no constraints; moreover, this von Neuman propagation is uniform over a minimal constraint on the asymptotic current (which represents exactly a

(pointwise) iff there is a freely deformable $\mathbb{R}_*^{(4)} \times \mathbb{R}^{(4)}$ subindex. Then, under the interpretation that the $\omega_\phi \rightarrow -\omega_\phi$ \mathbb{Z}_2 -symmetry represents charge duality (of the ultra weak gravitino), seen quickly from $\Delta_{\mathbb{Z}_2}^{\omega_\phi} c_3^{\frac{3}{2}} \cosh \ln \sqrt{\frac{c_3}{2}} \rightarrow -i c_3^{\frac{3}{2}} \cosh[\frac{i\pi}{2} + \ln \sqrt{\frac{c_3}{2}}]$ and $\Delta_{\mathbb{Z}_2}^{\omega_\phi} \Psi_\phi^{MP} \rightarrow -i \frac{c_1 c_3^{\frac{3}{2}} \alpha^2 r^2 \sin^2 \theta}{2 r_0^2} \cosh[\frac{i\pi}{2} + \ln \sqrt{\frac{c_3}{2}}]$. It’s also important to note that the asymptotic charge dual scales up equal and opposite to the corotation c_3 solution $\Gamma_* \sim e^{\pi c_3 \left(1 + \frac{c_3}{4}\right) + i \frac{(2+\pi)b_3}{\alpha^4}} \chi_3^* \frac{2+\pi}{\alpha^4}$. This exactly identifies this state as the dual measure (as given above for the $r \rightarrow \text{infy}$ power matching) by the choice $\Gamma^2 \sim \chi_3 \frac{2(2+\pi)}{\alpha^4}$; critically the signs of each power profile (P, P_Γ, P_H) are exactly determined by the sign of $-1 \leq c_3 \leq 1$.

¹³³remembering the monopole solution was found to have a alternating (weakly confined) co-index(or that it’s corrections UV/IR mixed as $O(\alpha^{\pm 2n})$)

monopole-type source).

Most importantly, as seen in [46], this current is nearly a function of the vertical field, $\theta I_{\bar{r}}[\Phi_{\phi}^V \sim r^2 \sin^2 \theta] \sim \sqrt{\pm 1 + \frac{\Psi_{\phi}^V}{\omega_{\phi}^{T_0}}}$, where $\omega_{\phi}^{T_0}$ is the Tangherlini Ψ_{ϕ} -entire (Ψ_{ϕ} turned off) vacuum velocity. Therefore, this completes the forward inductive proof iff $\bar{\Psi}_{\bar{r}}^{(\phi)}$ is an invertible function of $\frac{\Psi_{\phi}^V}{\omega_{\phi}^{T_0}}$ (or both separately), as similar to what was mentioned above (is to be expected with a forward closed induction); more critically the central index used here, $\theta \sim \frac{\pi}{2}$ implies $\omega_{\phi}^{T_0} = \sin^{-2} \theta \sim 1$, so the p -gluing used here is in fact exact on the symmetric plane $\theta = \frac{\pi}{2}$. Then, the RHS represents a minimal asymptotic r-density condition on the V -out field topography along the symmetric (indexing) plane or, dually, a V -constraint on the maximally in -current.

Meaningfully, this perturbation has a uniform bound when the $\theta I_{\bar{r}}$ exceeds a critical value, qualifying this regime as a "slip-stream" asymptotic state whereby strong resonances in either scale can be excited through UV/IR (accordingly, out -to- in push-/ in -to- out pull-)mixing dualities (as considered by a harmonically fixed measure); accordingly, it can also immediately be inferred that the extra field represents a confinement parameterization between r -separated conformal blocks, which is consistent with its association as a sub-partition above.

Still, this proof is brash and leaves the (closed) equations open at $O(\alpha^8)$ without an absolute stability analysis. To exactly control this method, higher order perturbative terms must be $[k, O(\alpha^k)]$ -added to $\omega_{\bar{r}}$ and $I_{\bar{r}}$ and new partitionable functional basis can be decidedly added to balance on the relatively stable $O(\alpha^k)$ surfaces and exactly stabilize the $O(\alpha^{l>k})$ corrections truncated at each grading [23]. Considering the solution gluing constraint outlined in [46]:

$$I^{\infty}(\Psi^{\infty}) = \pm \omega_{\phi}^{\infty} r \sin \theta \cos \theta \partial_{\theta} \Psi_{\phi}^{\infty} \quad \wedge \quad I^{(1)} \sim i \sqrt{1 - 4e^{2\chi} \sinh^2 \chi} \Big|_{\chi = \ln \cos \theta} \quad (275)$$

$$\Rightarrow I^{(1)}[\chi] = -8r_0 e^{2\chi} \sinh^2 \chi (1 - \omega^{(1)}[\chi]) \quad (276)$$

This gives three ways to produce a zero current matching domain:

$$\begin{array}{l} \chi \rightarrow -\infty \\ \chi \rightarrow 0 \\ \omega^{(1)}[\chi] \rightarrow 1 \end{array} \iff \begin{array}{l} \theta \rightarrow \frac{(2n+1)\pi}{2} \\ \chi \rightarrow in\pi \\ \omega^{(1)}[\chi] \rightarrow \omega^{(1)}[\chi_{\phi}, \chi_{\varphi}] = 2e^{\chi_{\phi} + \chi_{\varphi}} \cosh(\chi_{\phi} - \chi_{\varphi}) \Big|_{\chi_{\varphi} \rightarrow 0}^{\chi_{\phi}[\theta] = \chi_{\varphi}[\theta + \frac{\pi}{2}]} \end{array} \quad (277)$$

In fact, considering the spin solution, the final condition is met for all $\chi_{\varphi} \sim \Psi^{(0)}$, while the first implies that d_2 cannot be strictly taken to zero non-trivially¹³⁴. Put another way, d_2 is a convexly connected scalar field on the symmetric plane, $\theta = \frac{\pi}{2}$! Or, the symmetric plane is always magnetically protected because it acts as a continual current gap between different, non-degenerate configurations! In this sense, noticing that the decay branches may be

¹³⁴ again unless $\theta \sim \frac{(2n+1)\pi}{2}$ in which case the condition is smoothly matched by the limit space regardless of d_2

continually r-indexed along the free symmetric plane, the inclusion of an asymptotic r-cut is more justified (exactly as the $d = 5$ analogue of the “spark gap” in [37]); the difference here is the extra geometry, which produces different conformal towers in the well defined function space.

In fact, because of this spacetime’s unique relationship to canonical measurements (spin, $SU(2)$) across physics, there exists an even stronger (loop virial) closure of this boundary matching.

3.12 Conformal Light Envelope Matching

In addition to classifying globally analytic field solutions (with decided uniformity bounds), it is possible to classify field perturbations their functional effects on bulk quasi-invariants, like the light surface (defined by $F^2 = F_{ab}F^{ab} = 0$), [45]). In $d = 4$, the value of this invariant relies on the degeneracy condition representing an exact topological splitting index, namely $F \wedge F = 0 \Rightarrow F_{ab}F_{cd}v^c w^d = 2F_{[ac}v^c F_{|b|d]}w^d$, or $F = \alpha \wedge \beta$ s.t. $\alpha_a \sim F_{ac}v^c$ and $\beta_b \sim F_{bc}w^c$; then, the sign of F^2 determines whether both α and β are spacelike (+) or if one of them is timelike (-). Accordingly, $F^2 > 0$ represents (almost) everywhere spacelike boundary conditions directly orthogonal to the timelike current symmetry (by $F_{ab}j^b = 0$, and are hence time decidable); dually, $F^2 < 0$ represents (almost) everywhere timelike boundary conditions (which represent space decidability, as the fields near the electric boundary contacts become strongly background ordered at the exact contact time).

Heuristically, it may then be expected that $F^2 \rightarrow 0$ presents a razor of indecidability. In fact, because of the block separability of the Kerr metric, this allows strong statements about field confinement in fully interactive, n-black-hole matching membranes [45]; in particular, electric moments must always be surrounded (quasi-isolated) by magnetic stabilizers in black holes many-body problems up to closed light surface crossings, which act as directed causal-membranes between quasi-localized(/separated) electric sub-fields).

In $k = 5$, degeneracy of the vector gauge field is (generically) not as clear cut because the field tensor’s self-wedge is not antisymmetrically onto the determinate symbol; that is, unless $\theta \sim \frac{\pi}{2}$, in which case the diffeomorphic structures allowed of $\varphi \rightarrow \tilde{\varphi}$ gain access to an entirely strong UV weight (at 1-loop, $\sim \prod_i F_i[*]g_i[\tan \theta]$). Critically, this means that degeneracy, as a self-wedge measure of the vector-gauge tensor, has restored meaning as an antisymmetric symbol (with a non-Jacobi weight). To be more precise, consider F^2 with only 0^{th} order, Ψ_ϕ perturbations: $f(r) \rightarrow 0$ and $c_2 \sim 0$:¹³⁵

$$F^2 = \frac{8c_1^2 \left(\alpha^2 \left(\alpha^2 c_3^2 \sin^2 \theta \left(\alpha^2 + \tan^2 \theta + \frac{r^2}{r_0^2} \right) + c_3 \tan^2 \theta \left(c_3 \frac{r^2}{r_0^2} \left(\alpha^2 + \frac{r^2}{r_0^2} \right) - 2 \right) - 1 \right) + \sec^2 \theta \left(1 - \frac{r^2}{r_0^2} \right) \right)}{m^2 \left(-\alpha^2 + 1 - \frac{r^2}{r_0^2} \right) \left(\alpha^2 \cos 2\theta + \alpha^2 + 2\frac{r^2}{r_0^2} \right)^2}$$

In fact, the light surface has a very physical interpretation in terms of in-sheet scattering:

¹³⁵Most precisely, $c_2 \sim \frac{1}{\infty}$

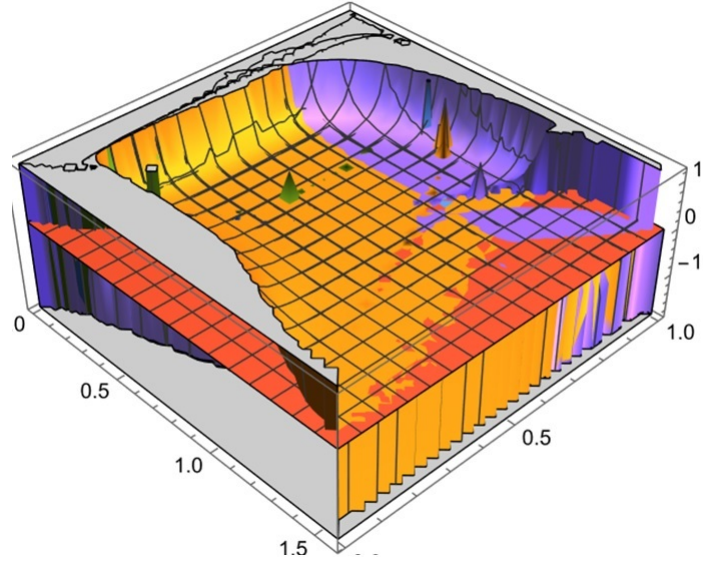


Figure 2: In particular, note that, for both single spinning Meyers Perry and Kerr, $2\pi^2 T_R = \frac{1-\alpha^2}{2\alpha^2}$, but this is enhanced in Meyers Perry such that $\frac{1-\alpha^2}{2\alpha^2} = \frac{S_- S_+}{(4\pi^2)^2 \left(\frac{J}{\sqrt{-2\pi^2}}\right)^2}$.

Enforcing $S_+ S_- = 4\pi^2 J^2$ requires $U(1)$ extension, but then it can be shown that $\alpha \sim \frac{1}{\sqrt{2}}$ represents a "Wick phased" stability point. In particular, notice that, at this spin rate, the monopole Znajek spin parameter is exactly identified with its higher order, $c_3^Z = \frac{1}{2} = \alpha_0^2$. In the above diagram Gold corresponds to $c_3 \rightarrow \frac{1}{2}$, Blue to $c_3 \rightarrow \frac{1}{2\alpha^2}$, Green to $c_3 \rightarrow \frac{1+\alpha^2}{2\alpha^2}$ and Purple to $c_3 \rightarrow \frac{1-\alpha^2}{2\alpha^2}$.

it represents a holonomic sheet constraint.¹³⁶ In $d=5$, the toroidal (universal) sector is $d=3$ dimensional; as such, it can be naively deduced to have some kind of CFT_3 product construction in a conformal representation and further, because this black hole only has a single spin and the electromagnetic field has only a single velocity, we can dually naively infer that the induced CFT_3 has some exact momentum modes¹³⁷. Considering the modes as independent but coupled, it would then be expected that they would be related by a purely geometric term; then, expectantly

$$p_\psi^2 = \frac{g_{\phi\phi}^\partial}{g_{\psi\psi}} p_\phi^2 = (r_0 \omega \tan \theta)^2 \quad (278)$$

and, considering a far field, ψ -quantization of the light radius, we can consider: (279)

$$\frac{\tilde{r}_{\text{LS}}^\psi}{r_{\text{LS}}^{(\phi)}} = p_\psi^2 \Rightarrow r_{\text{LS}}^\psi \equiv \tilde{r}_{\text{LS}}^\psi = (r_0 \omega \tan \theta)^2 r_{\text{LS}}^\phi = r_0 \{r_0 \omega \sec \theta \tan \theta, r_0^2 \omega^2 \tan^2 \theta\} \quad (280)$$

This $F_{(0)}^2 = 0$ equation is quadratic in r^2 ; therefore, four radial light distances can be immediately solved.¹³⁸

Consider the lowest order values of the light surfaces (around small $\alpha = 0$): $F^2[r] = 0 \Rightarrow r \in \{\pm r_+^{(0)}, \pm r_-^{(0)}\} \approx r_0 \{\frac{\csc \theta}{c_3 \alpha}, 1\}$; critically, this is the light surface as measured in the $k = 5$ *out* topology, which is exactly suppressed to $d = 4$ compact diffeomorphic charges on the symmetric plane. Because a dimension is ‘‘squeezed out’’ of the accumulation branch it is necessary to calculate the projective Jacobian¹³⁹. Accordingly, more

¹³⁶For example, in $d=4$, if we consider adding a small set of nearly massless charge carriers (so we can regard the action as having a simple auxiliary plasma term) then the light surfaces represent effective sheet horizons: surfaces which particles are either forced to traverse or are unable to cross (depending on the specifics of the particle spin states). Further, this example is nearly the same: although the toroidal sector is here 3D, Ψ_ψ is fixed to a constant almost everywhere so we may regard the sheet scattering as quasi-2D. In fact, looking at the r -independent 0-mode solutions, this quasi-1+1 nature of these solutions can be directly seen by examining (??) that either flux is uniformly constant at either boundary of this gluing patch, $\Psi_\phi^{(0)}(\theta = 0) = 1 = \Psi_\psi^{(0)}(\theta = \frac{\pi}{2})$, and conversely divergent at the other boundary $\Psi_\phi^{(0)}(\theta = \frac{\pi}{2}) = \infty = \Psi_\psi^{(0)}(\theta = 0)$. Then, interestingly, these log solutions can be considered boundary sub-indexes of flux singularities: $(\Psi_\phi, \Psi_\psi) \sim (\infty, \mathbb{R}) = (\Psi_\phi^{(0)}, \Psi_\psi^{(0)})|_{\theta \rightarrow 0}$, $(\Psi_\phi, \Psi_\psi) \sim (\mathbb{R}, \infty) = (\Psi_\phi^{(0)}, \Psi_\psi^{(0)})|_{\theta \rightarrow \frac{\pi}{2}}$!

¹³⁷Induced by the representative algebra, $\mathcal{H}_{\text{CFT}_3} \rightarrow \mathcal{H}_{S_{L_\psi}^1 \times S_{L_\phi}^2}$. We note that the ϕ direction is canonical

from the line element: $ds^2 = ds^2[t, \phi, r, \theta] + r^2 \cos^2 \theta d\theta^2$ has a natural submetric away from the global boundaries, $\theta \in [0, \pi]$. This induction is ultimately from the ISM construction of the black hole, which endows a global space-like charge at $\theta = \frac{\pi}{2}$

¹³⁸Regardless of their exact profile, it can be shown that $r_+ > r_-$ unless: $|c_3| \alpha > |\csc(\theta)| \geq 1$. This is exactly the condition that $\omega_\phi \sim O(1)$, which strictly violates our perturbative assumptions (by setting a [non-carried] sub-scale on α). This gives the intuition that the observed interaction should ‘‘twist’’ out.

¹³⁹This comes from the Wronskian method applied to the induced metric; note that, for all $y'' - b[x]y = a[x]y'$, $a[x] = \partial \ln W \sim \sqrt{K} = \partial \ln g_{\varphi\varphi} = \tan \theta$; noticing $\cos^2 \theta \partial_\theta (\beta \tan \theta d(\varphi d\varphi)) = 2\beta \sin^2 \theta d\varphi^2 \rightarrow_{\theta \rightarrow \frac{\pi}{2}} 2\beta d\varphi^2$ shows, by replacing $a[x]y' = \tan \theta d[\varphi d\varphi]$, the symmetric plane asymptotics should be linearly rescaled by $\beta \tan^2 \theta$, where β is the relevant modular parameter (when considered as topologically flowed constraints along the $d\varphi$ modular expansion). Then, notice $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{r_-^{(0)}}{r_+^{(0)}} = \alpha c_3$ is a constant (and well defined over $\alpha \in (0, 1)$). Note that this is effectively what was done before (*using* $\tan^2 \frac{\pi}{2} \sim \sec^2 \frac{\pi}{2}$), but here it was shown that this is the correct flow onto the compactified φ -patch. Then, all together, the complete rescaling, $(\alpha c_3 \tan \theta)^2$, is a projected $\theta \sim \frac{\pi}{2}$ juncture quantized in terms of the T-symmetric effective light(-surface) spacing. Note that if this effect is included in the discussion \vec{C} that this amounts to $\vec{C}dq \rightarrow \vec{C}(dq)^3$, which explains the necessity of a strictly Euclidean $d = 3$ stabilized code subspace as a consistent contact

relevant asymptotic radii should be $\tan^2 \theta$ rescaled around the symmetric plane to parameterize the divergent number of diffeomorphic symmetries suppressed on this plane; are those which diverge ultra fast (in light-surface relevant units) on the symmetric plane $\{\pm r_+, \pm r_-\} \approx r_0 \alpha c_3 \tan \theta \{\sec \theta, \alpha c_3 \tan \theta\}$ ¹⁴⁰. In particular, note that there is inherent tension in between the small alpha, near-equatorial phase-volume accumulations, $\xrightarrow{\alpha \rightarrow 0} r_{\pm}^{(0)} \xleftarrow{\theta \rightarrow \frac{\pi}{2}}$ which is exploitable as an interesting measure curve in the perturbative solutions (because the light surface, naturally, captures the *mass – spin* stability as a construct; accordingly, the naturally scaled (mass uniform) light unit is $r_0 \alpha = a$ the spin unit).

Then, requiring magnetic dominance across light stabilization sub-volumes acquires meaning: (magnetic) stability requires the light field be diffusely positive definite (or, quasi-determinate) across it's surface contact, which is equivalent to providing the existence of a pair of spacelike (measurement) legs (on the field tensor's canonical measure dual, here the rescaled light surface area). Turning the full, second order perturbation, the value of F^2 across asymptotic light volumes should remain positive definite at least to within the T-stability of the light surfaces (in order that the background is still T-like; also, so that the operators $\mathcal{L}_{\phi, \varphi}$ have strong light support).

Indeed, this exploit is direct: let $f(r) \neq 0$ and consider the resultant field scalar F^2 . Instead of solving the light kernel directly, consider the light-enveloping of the field scalar ¹⁴¹, F^2 by weighing it with the $f = 0$ light-surface volume $r_+^{(0)4}$ near the thermalization point in phase space $\alpha \rightarrow 0$ and the bulk-extended boundary accumulation space $\theta \rightarrow \frac{\pi}{2}$. Then, it can be shown that:

$$O(\alpha^3) \xrightarrow{\alpha \rightarrow 0} \frac{r^4 F^2}{2} \xleftarrow[r \rightarrow r_+^{\psi}]{\theta \rightarrow \frac{\pi}{2}} - \frac{-c_1^2 - c_1^2 \alpha^2 + 2c_1^2 c_3 \alpha^2}{(\theta - \frac{\pi}{2})^2} = 2c_1^2 \alpha^2 \frac{-c_3 + 1 + 2\pi^2 T_R^2}{(\theta - \frac{\pi}{2})^2} \quad (281)$$

$$O(\alpha^8) \xrightarrow{\alpha \rightarrow 0} \frac{[r^4 F^2] O(\alpha^2) - [r^4 F^2] O(\alpha^4)}{2} \xleftarrow[r \rightarrow r_+^{\psi}]{\theta \rightarrow \frac{\pi}{2}} c_3^2 \frac{(c_4 + c_5 + c_1^2 r_0^2 (-1 + 2c_3)) \alpha^4}{r_0^2 (\theta - \frac{\pi}{2})^6} \quad (282)$$

$$\text{and} \quad (283)$$

$$O(\alpha^8) \xrightarrow{\alpha \rightarrow 0} \delta_{\theta}^2 r^4 F^2 \xleftarrow[r \rightarrow r_+^{\psi}]{\theta \rightarrow \frac{\pi}{2}} 1 - 2\alpha^2 \left(c_3 - \frac{1}{2} \right) \left[\frac{(\alpha^2 - \chi^2)(\alpha^2 + \chi^2)}{\chi^4} \right]^{\chi_{\theta} = i \frac{2n+1}{2} \sqrt{\alpha} \delta_{\theta}} \quad (284)$$

Note opposite c_3 displacements around $\frac{1}{2}$ represent $n \rightarrow n + 1$. Here $T_{R,L}$ are, perhaps surprisingly, the right temperature of the (α -analogous) Kerr Black holes. The Znajek condition is found at the next order. ¹⁴²

protocol.

¹⁴⁰Or also, with $(x = \sin \theta)$, $\{p, \pm r_-\} \approx Nl \frac{x}{1-x^2} \{1, Nx\}$

¹⁴¹Here by pointwise weighing each leg of the field strength by the outer-light radius: $F_{ab} = S_{ab}^{(cd)} e_{(c)} e_{(d)} \rightarrow S_{ab}^{(cd)} (r_+^{(0)} e_{(c)}) (r_+^{(0)} e_{(d)})$

¹⁴²Note that left right conformal temperatures are given as: $T_{L/R} = \frac{\bar{r}_+ \pm \bar{r}_-}{4\pi\alpha}$ [43], and that Kerr and single spinning MP uniformly share a right conformal temperature; jumping to the final interpretation immediately, $\frac{1-\alpha^2}{2\alpha^2} = 2\pi^2 T_R^2 = \frac{T_R^2 e_{\phi}^2}{6S_+ S_-} = \frac{24T_R^2 J^2}{S_+ S_-}$, which reminds that the "total (Bloch) sphere measure", $4\pi^2$, is a half-space phasal surface induction envelope.

In fact, setting each angular spline to zero separately gives the combined condition:

$$0 < \frac{-\tilde{C}^2 + c_1^2 r_0^2 (1 + 4\pi^2 T_R^2)}{r_0^2} \Big|_{\tilde{C}^2 + c_4 + c_5 = 0} \Rightarrow 2\pi T_R = \frac{\tilde{C} \sqrt{1 - \left(\frac{c_1 r_0}{\tilde{C}}\right)^2}}{c_1 r_0} \quad (285)$$

$$2\pi T_L^{Kerr} = \frac{\tilde{C}}{c_1 r_0} \quad (286)$$

Starting from ssMP: $\bar{r}_\pm = \pm\sqrt{1 - \alpha^2} \Rightarrow T_L = 0 \wedge T_R = \frac{\sqrt{1 - \alpha^2}}{2\pi\alpha}$. Usually this is problematic from a CFT dual perspective because of the central uniformities induced by the 0-field sector $T_L(T_R, c_\phi) \equiv (0)$ (e.g., the spin bootstrap of the monodromy technique works [43] exactly because $T_L \pm T_R \neq T_L \pm T_R$); in turn, this can be fixed by a single extension in the (on-shell) function space (notably, the canonical $\mathbb{R} \rightarrow \mathbb{C}$ Wick $U(1)$), $S_+ = \frac{c_\phi}{3}(T_L + iT_R)$ and $S_- = \frac{c_\phi}{3}(T_L + iT_R)^*$, but at the cost of the technique's probing power (because the largest continuous branch is necessarily "auto-correlated"). Effectively, the entire continuum of symmetries was used to find a single, unique coordinate atlas [58]

Still, emergent symmetries are generic features of the isomorphism theorems and should be considered always relevant towards the penultimate $\mathfrak{su}[2]$ embedding (*in-complexity*). Note that, because of the symmetry in the MP poles $\bar{r}_0 \sim \pm\sqrt{1 - \alpha^2}$, $T_L^{MP} = 0$, $\{\pm\delta T_L\}$ is always a T -balanced (functional) branch. According to the embedding/measurement duality hypothesis threaded throughout, and noting that the left Kerr temperature is also monovariate ($T_L^K = \frac{1 + \alpha^2}{2\alpha^2}$), it is natural to try and force the Left-Kerr (LK) symmetry on the MP system as a regulatory(/geometrically shadowed) feature of strong MP-K-interactions (on the electromagnetic field gauge probe).

So, define $\frac{1 - \alpha^2}{2\alpha^2} = \frac{S^2}{2\pi^2 J^2} := 2\delta$; or $\frac{1}{1 + 4\delta} = \alpha^2$ then, $\delta = 1$ implies $\alpha = \frac{1}{\sqrt{5}}$, while $\delta = \frac{1}{4}$ gives $\sim \frac{1}{\sqrt{2}}$.

Note, $\delta = \frac{S^2}{J^2} \frac{J_{Kerr}^2}{S_{Kerr}^2}$ represents the relative phase quantization of the bulk over the boundary asymptotics then $c_3 < 2 \frac{1 + 2\delta}{1 + 4\delta} \rightarrow_{\delta \rightarrow \infty} 1$. In fact, choosing $\delta = 1$ (representing bulk-boundary parity) as well as the flat Znajek connection ($c_3 = \frac{1}{2}$) selects a unique black hole/probe configuration at the relative frequency $\omega_\phi \sim \frac{1}{2\sqrt{5}} \sim .2236$, which is relatively close to the Weinberg angle, $W_\theta \sim .22290$, as is optimistically expected (at this point). Choosing $\sigma = -\frac{3}{4}$ saturates the Znajek condition, $c_3 = \frac{1}{2}$ signifying that global enforcement of the magnetospheric vacuum condition represents a squeezed system (quantized over a $\frac{1}{4}$ -BPS symmetry meaning, 3/4 of the spectrum is unstable/hidden by the constraint entropy, discussed below, flowing below the quantization features). It's important to reiterate that this was applied to the full $O(\alpha^3)$ perturbation with a constant field velocity; so, it could dually state that $d = 5$ bulk monopole solutions are exactly bulk-cut stable with at least 4 "Kerr-baths" to decay between (over an adjoint octuplet, $4 \times 3 - 4 = 8$ basis of hidden modes) or, equivalently, their stabilizers exist as a single constraint among four (spin)charges.

To see how this works functionalized over the probe field, hypothesize a Cardy relation: $S_\pm = \frac{c_\phi}{3}(T_L \pm T_R)$.

Then it can be shown $\frac{1 - \alpha^2}{2\alpha^2} = -\frac{18S_+ S_-}{c_\phi^2}$, which can be continued in two ways depending on the hypothesis (because this is a junction-graphed space). Consider $S_+ S_- = 4\pi^2 J^2$ a universal quantization feature of black holes [43]; then, product entropy of ssMP is always negative, $J \in i\mathbb{R}$, indicating the quantization is a topological remnant (on the Coloumb branch), e.g. hole-type quantization/duality/emergent feature indicating that this black hole is always interactive. Then, $\frac{1 - \alpha^2}{2\alpha^2} = 2 \left(\frac{6J}{c_\phi}\right)^2$ gives the $SO(6)$ entanglement degrees asked for above.

Suppose instead the central charge is stable (canonically gauged), $c_\phi = 6J$. Then $\frac{1 - \alpha^2}{2\alpha^2} = -\frac{S_+ S_-}{6\pi^2 J^2} \sim \frac{2}{3} \left(\frac{iS}{2\pi J}\right)^2$, represents action on an entropy cone $S \rightarrow_{\{k\}} e^{\frac{(i+2k)\pi}{2} + \ln S}$. Better yet, combining both singles out a unique bulk stabilization point, $\alpha_{<0>}^2 = \frac{1}{2}$, which in turn induces $F^2 \sim -(c_3 - 1)$. It is also exactly square tessellated above the (vacuum, $d = 5$) black hole frequency: $\alpha_{<0>}^2 - \alpha_Z^2 = \alpha_Z^2$. In fact, this exactly promotes the use of $\frac{1 - \alpha^2}{2\alpha^2} = T_R^{Kerr} T_L^{Kerr}$ as a micro-canonical ensemble (over asymptotically induced thermal residues met by the splined monopole degeneracies) [45] [34][58]. Then, the gauged-pushes off the junctured, $d = 4$ partition space properly represent partitionable domains in the product representation of the bulk by considering them indexed by the (here, quasi-grouped and scale shifted) field velocity $c_3 - 1$

Pushing the expected conformal identities ACROSS the two systems yields:

$$S_{\pm} = \frac{c_{\phi}\pi^2}{3} (T_L^{Kerr} \pm T_R) \Big|_{c_{\phi}-12J=0} = \frac{2J\pi\tilde{C}}{c_1r_0} \left(1 \pm \sqrt{1 - \frac{c_1^2r_0^2}{\tilde{C}^2}} \right) \quad (287)$$

This can be solved for the light, open parameter \tilde{C} :

$$\tilde{C} = \frac{c_1r_0\pi J}{S_{\pm}} \left(\frac{S_{\pm}^2}{4\pi^2J^2} + 1 \right) = c_1r_0 \cosh \ln \left[\frac{S_{\pm}}{2\pi J} \right] \quad (288)$$

Or, conversly, as a condition between the magnetic flux on either side of the junction:

$$\frac{c_1^+r_0\pi J}{S_+} \left(\frac{S_+}{4\pi^2J^2} + 1 \right) = \frac{c_1^-r_0\pi J}{S_-} \left(\frac{S_-}{4\pi^2J^2} + 1 \right) \quad (289)$$

It is also possible to expand (287); then, using $\tilde{C} \sim \alpha^{-2}$ as a relative divergence guide, expanding to second order, and solving the outer horizon for \tilde{C} :

$$\tilde{C}_2 = \frac{c_1r_0S_+}{8\pi J} \left(1 \pm \sqrt{1 + \left(\frac{4\pi J}{S_+} \right)^2} \right) = \frac{c_1r_0^3}{2J} \left(1 \pm \sqrt{1 + \left(\frac{J}{r_0^2} \right)^2} \right) \quad (290)$$

To include the low spin branch, pick the bottom frame:

$$\tilde{C}_2^{\sigma} = \frac{c_1}{2} \sum_{n=1}^{\infty} \binom{\frac{1}{2}}{n} \left(\frac{-J^{2n-1}}{r_0^{4n-3}} \right) = \frac{c_1J}{2r_0} \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n+1} \left(\frac{(iJ)^{2n}}{r_0^{4n}} \right) \sim_0 \frac{c_1J}{4r_0} \quad (291)$$

$$\Rightarrow c_4 + c_5 \sim_0 -\frac{c_1^2J^2}{4r_0^2} \quad \Rightarrow c_1^2 \sim \frac{(iJ)^2}{16r_0^2(c_4 + c_5)} = \frac{-S_+^{Kerr} S_-^{Kerr}}{2^6\pi^2r_0^2(c_4 + c_5)} \quad (292)$$

This can be interpreted using the method of images: a holographic Kerr horizon can be used to thermally "split" the shell the radical-ideal algebra (to factor it) in such a way to admit a quick (though non-canonical) $su(2)$ embedding onto the phase domain. Finally, returning to the bulk power regulated solution, approximate the loss of spin energy as:

$$\frac{-iP_{\delta I}}{2} \sim \alpha^2 \frac{(2+\pi)S_+^{Kerr} S_-^{Kerr} c_3(1-c_3)}{2^6\pi^2r_0^2(c_4+c_5)} = \frac{(2+\pi)S_+^{MP} c_3(1-c_3)}{2^6\pi^2r_0^2(c_4+c_5)} = \frac{(2+\pi)c_3(1-c_3)}{16\pi(c_4+c_5)} \quad (293)$$

In the friction limit of [37], the bulk coefficient of (junctured-)fusion, $-ik$, is given by the real coefficient of $c_3(1-c_3)$, or:

$$i\pi k = \frac{(2+\pi)}{16(c_4+c_5)} = \frac{(2+\pi)c_1^2r_0^2}{J^2} \quad (294)$$

which represents the relative dissipation per current wave. Combining everything discussed throughout this paper, the electromagnetic field solution above can be reinterpreted as a

model for Hawking Radiation whereby a late-time entanglement wave dissipation mechanism develops a quasi-continuous basis of *out/in* stabilizers (with radiative support), as shown in the diagram below. Indeed, comparing to ?? shows that the spacetime response form is fixed to an internally spring-like free energy (across continuous amplitudes),

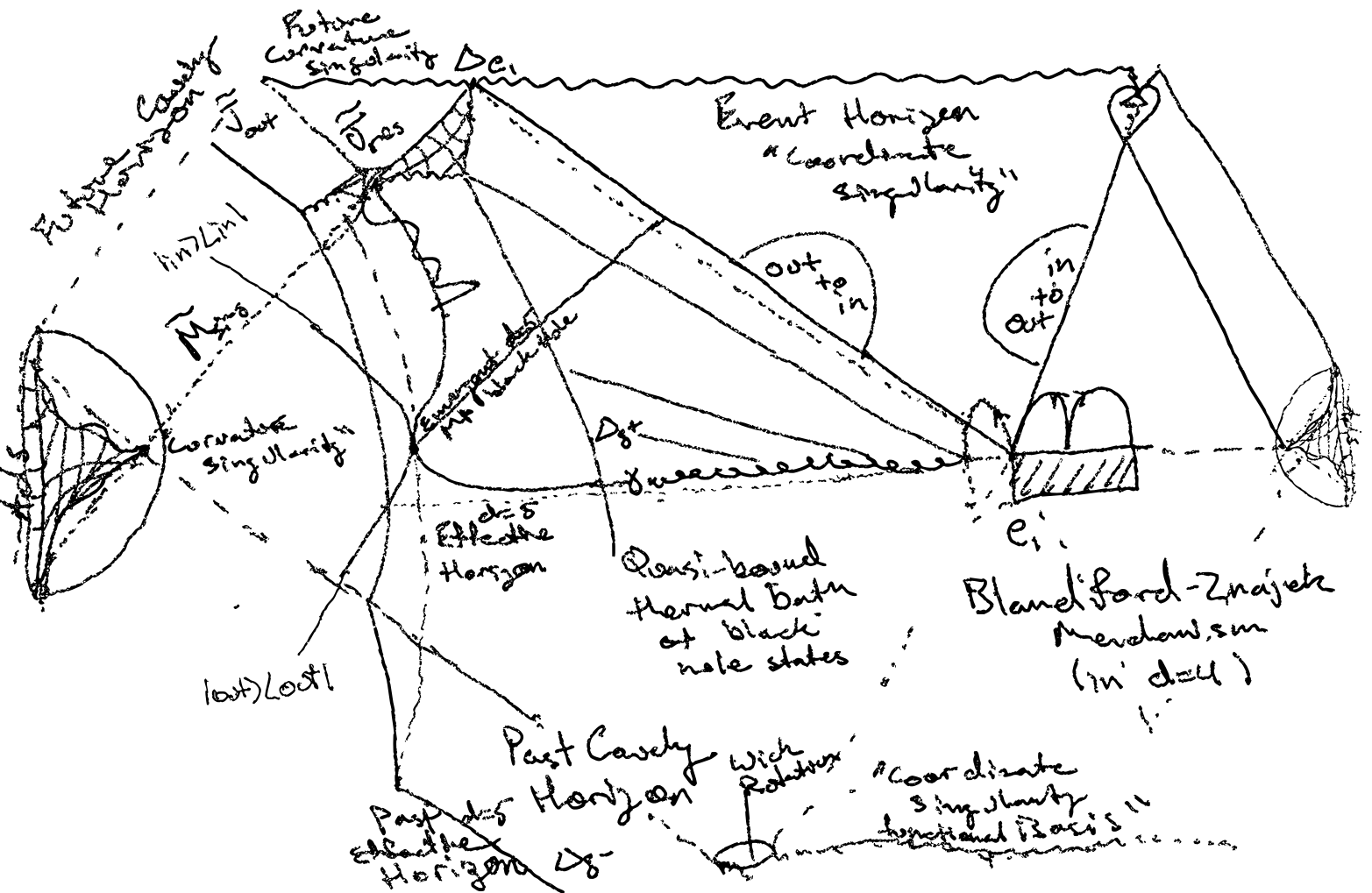
$$\tilde{F} \left[\chi = ic_1, \Delta = \frac{J}{r_0} \right] = (2 + \pi) \chi^2 \partial_\Delta^2 \ln \Delta \quad (295)$$

Note this represents an expansion of the region between the inner and outer horizons, a space which is asymptotically stable because of the geodesic convergence conditions; accordingly, in this diagram, the inner box is Wick rotated to produce a radial spectral map which, from (closed domain) Noether conservation can be used to infer a strictly quasi-stable scattering topology in the presence of a sea of massive (pion-type) decay resonances. This can then be immediately interpreted as a future massively mediated spin-decay of the scattering, in-falling neutral plasma. In this sense, the cold plasma acts as an “extra dimensional decay medium” from the perspective of spin-fluid states between the (outer) horizons by creating a natural cut-surface in the edge of the spectral chart based on the emergent coupling from the black hole. Then, as the massive string is interactively spun into the Future Curvature Singularity the original thermal scattering support acts as an effective horizon during the flight of decay. At this point it is important to remember the section on synthetic derivatives, most importantly that: the only difference between the coordinate functions in the MP geometry and the Kerr geometry is $m_{Kr} \rightarrow m_{MP}$, there is a hidden switching symmetry in the gauge basis of the extra dimensional field, $\Psi_\theta[\theta] \sim \ln[\cos \theta]$ and $\Psi_\varphi[\theta] \sim \Psi_\theta[\theta + \frac{\pi}{2}]$.

A quick review of the inverse scattering method [65] and also the original construction of the Myers-Perry solutions [38] shows that, functionally, these black holes were constructed exactly by axiomatically extending the Hawking-Penrose Consistency conditions¹⁴³ to general dimensional, balanced configurations. Indeed, inverting all the dashed (interactive) lines and treating them as solid (geodesic/charge) lines gives exactly these constructions. Then, the black hole studied above can be seen as confined to the upper diamond within the inner spectral sheet during a Blandford-Znajek interaction. Indeed, this embedding can be understood exactly in the context of the Love-embedding found in [66]

¹⁴³Namely, for all interactions: Spacelike closure, spin-like closure (in time), and charge continuity (in space-time). Or, also: the existence of a bound/trapped surface with no-particle interactions that can destabilize the charges associated to the trapped surface (as dual resonances).

"Virtual Black Hole Decay"



Note, the $\theta \rightarrow \frac{\pi}{2}$ accumulation space always has a dominately negative scalar measure-space extended from the source point, $\theta = \frac{\pi}{2}$, $\langle r_+^2 | F^2 | r_+^2 \rangle < 0$ UNLESS there is an exactly thermal gap between the synthetic range of $c_3 \in [-1, 1]$ and the (boundary complete) effective range, $c_3 - 1 \leq 4\pi^2 T_R^2$. In particular, this gives a strong, and interesting, logic juncture between the phase curves: the entire symmetry plane $\theta = \frac{\pi}{2}$ is magnetically stable iff either A) $c_3 \leq 1$ and $T_R^2 \geq -(c_3 - 1)$ or B)¹⁴⁴ $c_3 > 1$ and $T_R T_L < 0$ and $\frac{c_3 - 1}{4\pi^2} := \delta c_3 \equiv T_L T_R$. In particular, this exactly ties the positive partitionability of the Kerr Bloch sphere (at this loop order, $\sim \frac{1 - \alpha^2}{2\alpha^2}$) to the light barrier (Znajek) condition of the constant frequency field. In fact, because everything is iff, demanding space-like closed (well) determinedness and a positive Kerr conformal partition, $T_R T_L > 0$ is exactly that $\omega_\phi < \alpha$ and, the degree of juncture back-induction is exactly the thermal weight.¹⁴⁵ Indeed, the bulk Znajek condition at this order is $c_3 = \frac{1}{2}$, which induces $\delta c_3 \sim \frac{-1}{8\pi^2}$ and $\delta c_3^* \sim \frac{-3}{8\pi^2}$ dimensional reductions; and, in fact, these deficit angles can be exactly explained as tessellation completion dimensionalization induced from the topological string theory [53] by looking at thermalization spectra of higher $k = 5$ black hole harmonics [43](particularly those with exactly $SU(2)$ sub-representations, as shown in the next section).

Indeed, perhaps most interesting is the demand that the field never allow a (degeneracy) gap between the lightsurface, amounting to only allowing field couplings at the (effective) speed of light(/the higher dimensional mass action); more precisely, $F^2 = 0$ and $T_L T_R \geq 0 \Rightarrow \omega_\phi^{(1)} = \alpha$, which is twice the local Znajek frequency. This fixing can then be explained as an $SU(2) \rightarrow SU(4)$ forward (inductive) matching surface that only matches decaying waves¹⁴⁶ to the $d = 4$ thermalized (black hole) background.

Vice versa, considering a dually charged system (represented by $c_3^* = -c_3$) in plus and minus pairs $F_+^2 \sim 1_\odot + T_r T_L$, $F_-^2 = 2c_3^Z$ implies that the F_-^2 (antisymmetric) light superposition represents the ungapped $T_R T_L$ phase space, while the F_+^2 symmetric light position represents a continually gapped phase. Reflecting, this fully justifies the association of the sign of c_3 as a weak index of the gravitino (by matching the index groups onto their direct algebraic symmetry break). This can again be run backwards, so that $T_R T_L > 0$ implies that the light surface must become decoherent (as it involves higher harmonics of the $\frac{1}{2}$ modes as $\sim 2n$); or the local monopole field must bulk-speed up, or develop a kink, relative to it's $c_3^Z \sim \frac{1}{2}$ locally coherent states when the projective thermalized background is non-zero. Both of these techniques can rely on n_k -dimensional harmonic decompositions(/field expansions) in a finite, compact space, which can always be completed [49].

This provides a quick proof of connectedness outright, but will be more rigorously useful as a proof that both the *in* and *out* junctures in this construct can always be $SU(2)$ bridged

¹⁴⁴Or, with $\delta_{RC3} := \frac{\alpha^2 \delta c_3}{T_R^2} = \frac{1}{T_L^2}$

¹⁴⁵Then, considering $c_3 \rightarrow -c_3$ as Gravitation to strong electromagnetic duality, this reduces the thermalization barrier to $T_R T_L - \frac{1}{4\pi^2} \sim -\frac{|c_3|}{4\pi^2}$; then, geometric $k = 4$ tessellations (LHS) become virially infected with their lattice conformal dimensionalization (RHS), which is commonly known as the angle deficit in unification holography. Remembering $c_5 \sim -c_1^2(-1 + c_3)^2$ shows how current's axi-vector ascent profile is thermally minimized (as a global angular flux).

¹⁴⁶Specifically, again by Fermii's golden rule, those existing in a $\Gamma^{-1} \sim \frac{\partial \omega_\phi \ln \delta A}{\omega_\phi^Z} \sim \frac{2}{\omega_\phi} \sim 4$, or as an $A_{mp}^{\partial_t \alpha \equiv 0} \sim [\cdot]^{-4}$ off-Znajek shell resonance.

into virially constructed, connected universes(/worldverse diagrams). ¹⁴⁷

It should finally be noted that the monodromy technique produces exactly four (AdS-separated) first order coordinate poles (in global, Boyer-Lindquist compactification), and that the flat space residue produced a contact constant $\mathcal{K} = d - 1$. Then considering a single F^2 juncture between determinate field velocities and $-\langle (1 + T_R T_L) \rangle_{n_k}$ contacts; picking a 3-juncture gives $-3 - \langle T_R T_L \rangle_{n_3}$. Further, if two of the junctures are across ∞_{flat} this gives (for $\delta S^{\text{KerrAdS} \rightarrow \text{Kerr}} = 0$) $0 > c_3 - 3 - (\langle T_R T_L \rangle_{1_3} - 3) = c_3 - \langle T_R T_L \rangle_{1_3}$. Picking the final contact as on the (flat) black hole gives $\langle T \delta S \rangle_{\odot} < \Delta \omega_{\phi}$, which puts a strain on the maximal coherence as a balance between Znajek monopole harmonics and rescaled heat-decay charges. Again, the regulatory nature of this duality is clear: the higher surface black hole can only decay, and remain continuously junctured, to the (outer) harmonic membranes below the vacuum heat dispersion of the projected black hole. In fact, this can be seen as an extension of the Second law of thermodynamics, or that the light membrane discussed above couples, remarkably, to the vacuum as an emergent, second order filtered decay channel which creates decoherent splitting at the $d = 5$ phase surface as the monopole branch is forced to iteratively interact (coherently) in Znajek spinnors of degree $2z_n = 2 \frac{\langle \delta M^2 \rangle}{\alpha r_0}$, which can be finally seen as a semiclassical quantization at loop order (and, a fully realized quantum gravity prediction at tree level).

The following sections will expand on the emerging unification scheme by finishing some existence proofs.

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¹⁴⁷On face, the light-field scalar superficially seems like a simple rescaling symmetry of the field invariant; the difference is that it is a proper light surface solution (at lower order), and, correspondingly, has more field relevant OPE structure than generic asymptotic stabilizer. On a final note, heuristically considering the $F^2 = 0$ space as an orthogonal, $d = 1$ dimensional subspace, the $F^2 \rightarrow r_+^4 F^2$ can be seen as a $d = 4$ subspace rescaling; then, the frequency duality was found in a very shallow $\alpha \approx 0$, and wide, patch of phase space. This is seemingly the dual phase volume to the Znajek condition; meaningfully, the (topologically embedded) Znajek condition has been shown to directly be a shadow quantization condition.

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4 $2\infty \Lambda^{(nd)} B_{(e)\gamma}[\odot]_{(nd)}$

4.1 Separability: Guided, Partially Accessible Boundary Channels

As was hinted above, then central dogma of topological indexing is to always flow laterally under the weakest topological bridge and then comparatively recompactify the product topology as a minimally closed residue (a.k.a., as the strongest topology in the conjoined open dual, \mathcal{T}^*). Summarily, producing weak gauge symmetries with shelled equations of motion under a strongly correlated index divergence put partial constraints on the maximally divergent measure topologies by producing poles within highly tuned measure spaces. Effectively, this charges the relative measure algebra iff the bulk-to-boundary gauge juncture is $\sim^* \mathbb{R}_*^{(2d)}[x]$, which is only functionally (piecewise) possible as a 1+1 type II Neumann U-exchange on the (running) measurability (sub)index [1]. Over $d = 4$ the totally relative features could be expected to emerge as $\mathfrak{M}^* \times (\mathbb{R}^4[x] \times \mathbb{R}^4[x]) \hat{=}^{(r)} \mathbb{R}_*^{4r} [\mathfrak{M}_{(r)}^* \hat{\times}^{(r)} \mathbb{R}^4[x]]$, which makes all of the subdual, emergent dimensional regularizations subject to topological control under $G_{\mu\nu} = 0$.

The existence proof will proceed by using the classical asymptotic thermal duality, $\text{Kerr}\partial\text{Kerr} = \frac{\partial\text{dyon}}{2}$, as a natural r -dual juncture; because the thermal partial wave domain of $d = 4$ is asymptotically spinor complete [2], the free descent of this $SU(2)$ -closed magnetic gauge field can be considered a strong crystallization feature in both spectral and string networks [3] [4].

The Cauchy completeness on the functional kernel space guarantees this charge index has spectral control within every neighborhood (in the sense that the gauged diffeomorphic current gives analytic measure everywhere); this is different from usual gauge theories because it specifically invokes that axiom of choice to guarantee the inverse map is (uniformly) onto the domain set. But this collides with naive topology for the same reason the Banach-Tarski construction is seemingly paradoxical: volume is a measurement, not an object. There, the object can be simultaneously partitioned into symmetric, uniform number sub-measures which can be brought into contact and both found, in the sense of the original measure, equal to the original volume. This is resolved by noting that the algorithm amounts to adding a new positive characterization towards the continuum, namely $[0, R] \sim R\infty$ (conversely, the new exclusion $0_R \sim [-\frac{1}{R}, 0]$), and continuing the original measure into its hyperfine continuance. By accepting both indicators as decidable the hyperfine measure of the original sphere can be shown to yield exactly one copy of the continuum measure (this is exactly one by the R -Axiom of Choice invocation) and can be shown by placing R -derivative statements surrounding each previous $\mathbb{R}[x]$ evaluation $\delta_{ij}(x^j_k) - \frac{x_0^{(k)i}}{R}$. Because 0_R is undecidable in the original continuum [5], it's left to be checked against every previous evaluations sectorially; by the same theorem (and decided extension) there are exactly $R\infty$ previously undecidable statements which can now be R -wise deciphered yielding $\|S^3\|_R - \|S_R^3\|_R = \|S^3\|$ (or, the sphere measure is shelled by a difference between its natural R -extended measure and an R -emergent topology of derived tautologies).

Thus, in the construction above it may be inferred that there are exactly $(3 + 1)\infty$ quantum continuous degrees of freedom residing in the curve index (notably \mathbb{R}^3) and the (weakest quasi-coordinate) measure is a piecewise-stepped, sequentially effective spec-

tral flow (also known as the weak oscillatory measure): $|O_\Omega| \sim [\Delta^{ij}(\vec{C}[q_{ij}], \nabla \vec{C}[x_{ij} - q_{ij}]), \frac{1}{1-\theta[\vec{C}, q]}, ((\vec{y}_0, d\vec{y}_0), (\vec{y}_1, d\vec{y}_1), dy)]$. As long as $\vec{C}[q]$ is onto the complete curve (under any *in*-boundary gluing) this is strong enough to hypothesis-categorize (disjoint, rational) emergent defects of the quantum topology as a $d - 2 = 3$ embedding of $\tau = 1$ one scalar measure of quantumness(/chaos; here shown as the sub-relative continuum orientation). It's important to note that evaluating each subdomain requires the boundary splines to be sub-deterministic, and so each check for quantumness here requires the spline index to be expanded by a Euclidean four-plet, $[(\vec{y}_i, d\vec{y}_i), (\vec{y}_j, d\vec{y}_j)]$, which critically specifies another set of indices resulting in $k = 6$ six effective number (possible) quantum number expansions. In fact, as will be seen below, by adding bulk spin to the metric topos this closed set will naturally be enhanced to $4 \cdot 3 = 12$, with the canonical embedding index choice presenting as a $m = 2$ dimensional gauge connection giving the full (orientable) $k = 10$ degrees of freedom of $d = 4$ spacetime.

Picard's theorem (always) closes (over bounds) with this construct, so (across each sub curve), quantum measurements can be naturally seen to emerge as a conformal scalar with two effective indicies (as sub-boundaries) and four spacelike indexed measurements: $\mathfrak{M}[dq_{ij}; (\vec{y}, d\vec{y})_{ij}]$. Notably, at each local spline domain, because the axiom of choice was invoked over $[R]^3[y; \cdot]$ at the quantum point-expansion the trace functional gluings must be freely (trace)-gauged as $SO(3) \times SO(3)$ along the globally shelled quantum curve (projective/correction $\sim q\mathbb{CP}$).

Still, by Picard's lemma on $1 + 1$ topologies, this is always deterministic (and solvable); this can also be seen as the canonical emergent network implied by the Rice-Shapiro theorem, which we state here for clarity[5]:

Theorem (Rice-Shapiro)

Suppose that \mathbb{A} is a set of unary computable functions such that the set $\{x : \phi_x \in \mathbb{A}\}$ is recursively enumerable. Then, for any unary computable function f , $f \in \mathbb{A}$ iff there is a finite function $\theta \subset f$ with $\theta \in \mathbb{A}$

Although this invocation of the axiom of choice immediately implies the R -sorting process is always undecidable $H[S^3, S_R^3]$, it is also exactly marginal to correlation kernels along strongly R -dependent sub-universes [4] in the weakest topology (Heavyside/Manhattan measure) because the quantum indicator functional can only be sequentially renormalized "to within dq renormalization". This can also be seen to be dual to disregarding all "double checks/complexity-squared" statements $\frac{x^{(k)i}}{R^2} \hat{=} 0$ by noting the application of the above to the halting problem (and the decidability of the Axiom of Choice). Then, the choice $\int \delta \hat{=} 1$ can be seen as dual to only measuring single, stringlike choice invocations "on top" (and dually below, canonical from enforcing real coordinate closure $\pm\mathbb{R} \sim \pm\mathbb{R}$), exactly as seen above. In fact, the halting problem is indeed dual to the vertical line test, and the above use of the delta function can be seen as immediate from applying the RS theorem to the indication topology:

(Lemma) The sets $\{x : \phi_x \text{ is total}\}$ and $\{x : \bar{\phi}_x \text{ is total}\}$ are not recursively enumer-

able, or the functional measure of the delta function is meaningful only up to the relative(/effective) facts(/determinacy).

Thus the closure of Einsteins equations is a consistency check, not an existence proof, of the functional consistency of manifold embeddings of spacetime because it uniformly descends from $\left(\frac{g_{\mu\nu}}{2}\delta^{mn} - \delta_{\mu}^m\delta_{\nu}^n\right) + T_{\mu\nu}\frac{\delta^{mn}}{\delta^{tu}R_{tu}}\Big)R_{mn}\delta^{M\mu\nu} \hat{=} 0$ (as shown at the beginning this paper). In particular, the Bianchi identity gives:

$$\nabla_{(k)} \times T_{\mu\nu} - T_{\mu\nu} \nabla_{(k)} \times \delta^{M\mu\nu} = - \left(G_{\mu\nu} \nabla_{(k)} \times \delta^{M\mu\nu} \right) \quad (1)$$

This quickly shows that classical, continuous (streams of) equations of motion are only graded to the degree of their (spin-)functional separability. Indeed, this explains the Belafante tensor as a functional(-boundary) gauge completion as well as recent dualities in the Celestial Holography program (such as spontaneous-twistor-breaking/deconfinement-phase-residues/quasi-T-string-descent pushes onto the $\omega_{k\infty}$ asymptotic algebras.

In fact, it is this functional curvature's boundary convolution that mixes the coordinate isometries to produce gauge-phased stream effects (1st Law of Thermodynamics/Spectral Chern-Simon Phenomena/Soft-theorems) by assuming classical measurability(/decidability); or, by choosing theories of physics with strong $\sim \mathbb{N}^{\infty}[x]$ dualized measures (a.k.a., a strongly separated dual topologies, or absolute statements about nature). BUT, $\mathbb{R}[x]$ is dense over $\mathbb{N}^k[x] \forall k$, meaning functional differentiation is never complete as an entirely spectral phenomena without a decidedly closed (conformal) harmonic mixing mode on the boundary (topos). Indeed,

$$\int f \partial \delta[e^{\chi} x] = \int_{\{\chi \Rightarrow \{k\infty\}_k \rightarrow \{k\infty\}_k \cup \chi\infty\}} e^{-\chi} [\delta[e^{\chi} x] \partial f + f \delta[x]] \quad (2)$$

Although the second term is canonically defined(/constructed) by the first order sentences, as $\chi \rightarrow \infty$ becomes complex the second term is always a composition (involution) of first order sentences against the statement continuum; the first term is always the composition of derived (functionally canonical, tangent compactified) statements in the (field extended) *chi*-topology. Thus it can only be normalized, in $\chi \rightarrow \infty$, if the statement $f[x_0]$ is independent of $\chi \rightarrow \infty$; this is just a rephrasing of the Halting problem.

Thus, all action principles are only well defined up until their measure embedding, which is concretely undecidable (as a measure scale-prescription); just as in Noether's second theorem, the symmetry (here an action stabilizer) is only defined up to the functional degree of actionability related to the (global) symmetry. This is uniformly true from the coordinate embedding of every coordinate measured action, and can be seen from (the back of the envelope) as $\partial^4 \tilde{S}^{\pm} \sim \pm 1 \Rightarrow \sum \delta \tilde{S} = \delta[\int dx^4 - \int dx^4] = \delta[\sum \infty_{\pm}]$. This always requires a field extension to define; by the fundamental theorem of algebra $\mathbb{C}[z, \bar{z}]$ is the minimal smooth field extension.

In fact, this extra compactification represents the generalized (renormalized) Ward identity (and the minimal cover explains the Optical theorems); thus, choosing the func-

tional embedding topology amounts to selecting an asymptotic algebraic probe (as a functional 2-point measure embedding toward the functional leveling). This immediately explains the nature of unit-scale/renormalization/asymptotic-gauging as well as the quasi-complete large (decidable) algebras they tend to produce: each functional topology is usually gauged to the $\otimes_i \mathbb{R}[x_i]$ independently extended continuums, which is an extremely constrictive topological embedding because it can never control correlated topological mixings without (undecidable) measure-pointwise expansions that must be checked (by OPE-conflation/effective probes/cut-continuance). By Godel's incompleteness theorem (or the Neukida-Uchida theorem), there always exists a decidable index (p-adic, or $p\infty$) that can be constructed to be partially exact and, by the lemma to the Rice-Shapiro theorem, this partially exact index may be closed iff the inverse delta function is well defined (or, equivalently, if the spectral flow has a defined topological index on the conformally crossing branch; necessarily then, functionally complete symmetries must carry topological indices covering their spectral decidability, or soft-OPE residue corrections, at the classical measure phase). Now the choice of a functionally independent, $[R][*]$ -type $U(2)_* \times SU(2)$ gauge connection is naturally motivated as a conformally soft functional field (measured probe construct); thus, this can be considered a closed tower of gravitationally induced B-loops pairs in a full string-field theory.

This gives the natural point of contact for amplitude constructions as already rigorously strained under the classical $d = 4$ (classical choice invocations/dimensional global foliation domains/minimal decidability domain) under the original bootstrap performed by Einstein. Indeed, in all the cases of Brownian motion/Photoelectric effect/General relativity local hidden degrees of freedom were (quasi-)periodically closed within hypothesis: Brownian motion unifies local, bounded fluid-surface dissipation algebras as a (quasi-)universal unit-charges (under the diffusion index), the Photoelectric effect unifies local, bounded vector charge-surface dissipation algebras as a (quasi-integrated) indicator states, and General Relativity unifies local, bounded gravity-light surfaces to within universal consistency (by the Averaged Null Energy Condition, which is an only if statement promoted to iff for all decidable boundary topologies [6]). In fact, all of the above are all only if statements that can be shown to be consistent(/have a functional inverse) up to a universal measurement (each in the form of an emergence/experiment/entanglement index); then, the extended dimension can immediately seen as a dimensional contextualization field, or as a heuristic memory/conformal stabilizer/junction-register.

Then, despite being at most partially complete, the $R^{(1)}$ sub-reference can be (quasi-decidedly) regularized (most simply, under closed network probes); this is most effectively/commonly done by truncating $\infty \rightarrow n$ (background gauged)/ $\frac{\Delta x}{\Delta p} \rightarrow 0$ (locally measured) IR/UV interactions. In Classic Computing language, these amount to run-time/overflow (as methods of Algorithmic dual lattice regulation) and, in the final section of this paper, the results thus far will be indeed prove dual to both quantum error correcting code and quantum gravity shadow renormalization as an emergent unification moment in the (relatively) dual completion modalities. Indeed, the identification of a universal entanglement index as an $SU(2)$ -gauge on the functional coset topology is now immediately clear as a way to keep the measurement topology, as a junction, in gauge as well: the functional

pullback should never be extended beyond the axiomatic connection without boundary (quantum/entanglement/gravitational) corrections because every (G-)global identifier is only defined up to it's largest defined set.¹

Above all, this shows that measurement, as a functional symmetry², contains quantum contacts at every scale iff it contains quantum contacts at any scale. Equivalently, off x-coordinatizations, $x \rightarrow \tilde{x}[y] = x_0 + y$ always enhance the first term's x-spectral moments as a canonical spontaneous symmetry breaking feature (known as the Coulomb branch in the product completion). In fact, construction of an extended history index, $\ln x \sim_I \partial x$ is always possible following from Noether's second applied to the partial-universal difference choice as a history index, $\delta_r^i \partial \delta$; consistent laws of physics can thus be seen as those while have a continuum matching, exactly pushed free index (replicatability/decidability/time). Indeed, this can be seen as the functional renormalization extension of Noether's second theorem, which could be considered "Noether's Third theorem"; then, the index proceedings above can be summarized as third order quantization, whereby unification proceeds by maximally squeezing hidden degrees of freedom into measure dense consistency-chains/bubble-flows/asymptotic-renormalizations. Of course, this is immediately true in action: science, as a machine, is a partial-unification algorithm over a quasi-regulated, bound sub-continuum of hidden degrees of freedom that is not universally decidable (the electric field was, essentially, discovered 100 years after Newton's death).

Most importantly, the above formulation is an axiomatic correction to all action principles as a uniform soft symmetry breaking across the y -subshelled universe.

4.1.1 Euclidean sub-dimensionalization

Still, it is useful to note that the motivation of either is primarily a difference of coordinate, or measure imposed, symmetries. Either measure minimizes the curvature density, but each represents the curvature functional on different coordinate pullback topologies. Of particular interest is when a divergence symmetry may be produced from some density functional symmetry of the Euclidean and Minkowski actions, which may then be used to construct an instanton and particle source interaction as a tower of on-shell Minkowski solutions. For example, consider some function (density) as level sets of the instaton and Minkowski Lagrangians as well as some overlap coordinate functions parameterized by the E/M pullbacks:

$$f \rightarrow f[L^E[\hat{g}, \hat{R}; \hat{x}^{\hat{\alpha}}], L^M[g, R; x^\alpha]; \hat{x}^{\hat{\alpha}} \times x^\alpha \quad (3)$$

$$\begin{aligned} \text{s.t. } f[\cdot, *, \hat{x}^{\hat{\alpha}} \times x^\alpha] &= F[\cdot, *]_{\hat{\alpha}\alpha}^{\hat{x}x} + h_i[\hat{x}^{\hat{\alpha}}, x^\alpha] \hat{N}^i[\hat{x}^{\hat{\alpha}}] + \hat{h}_i[\hat{x}^{\hat{\alpha}}, x^\alpha] N^i[x^\alpha] \\ \Rightarrow \delta f - \delta F[\cdot, *]_{\hat{\alpha}\alpha}^{\hat{x}x} &= (\hat{h}_{i,\hat{\alpha}} N^i + (h_i \hat{N}^i)_{,\hat{\alpha}}) \delta \hat{x}^{\hat{\alpha}} + (h_{i,\alpha} \hat{N}^i + (\hat{h}_i N^i)_{,\alpha}) \delta x^\alpha \\ &= \hat{h}_{i,\hat{\alpha}} h_{i,\alpha} \hat{N}^i N^i \left(\left(\frac{1}{h_{i,\alpha} \hat{N}^i} + \frac{(\ln[h_i \hat{N}^i])_{,\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} N^i} \right) \delta \hat{x}^{\hat{\alpha}} + \left(\frac{1}{\hat{h}_{i,\hat{\alpha}} N^i} + \frac{(\ln[\hat{h}_i N^i])_{,\alpha}}{h_{i,\alpha} \hat{N}^i} \right) \delta x^\alpha \right) \end{aligned} \quad (4)$$

¹This is quickly seen by $\frac{\delta[x]}{x} = \frac{\partial(\ln x)}{\partial x} \delta = \partial \left(\frac{\ln x}{\partial x} \delta \right) - \frac{(\ln x)}{\partial x} \partial \delta$, which procedurally graphs how (here branched) bubble diagrams may take on non-zero modular flow near antiholomorphic generators (or, on hyper-compact spectral moments).

²Of information

Then, further simplifying by assuming the LHS is a divergence symmetry in the freely induced topology $\mathbb{R}^4 x \mathbb{R}^{3,1}$, which results in the following ansatz:

$$\frac{D_{\vec{\alpha}} W^{\vec{\alpha}}[\cdot, *; \hat{x}^{\hat{\alpha}} \times x^{\alpha}]}{\hat{h}_{i,\hat{\alpha}} h_{i,\alpha} \hat{N}^i N^i} + (1 - \frac{1}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i}) \delta \hat{x}^{\hat{\alpha}} + (1 - \frac{1}{h_{i,\alpha} \hat{N}^i}) \delta x^{\alpha} = \left([1 + \frac{(\ln[h_i \hat{N}^i])_{,\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i}] \delta \hat{x}^{\hat{\alpha}} + [1 + \frac{(\ln[\hat{h}_i N^i])_{\alpha}}{h_{i,\alpha} \hat{N}^i}] \delta x^{\alpha} \right) \quad (5)$$

Suppose the extended bulk-form $\hat{h}_{i,\hat{\alpha}} h_{i,\alpha} \hat{N}^i N^i$ shares the symmetries of $W^{\vec{\alpha}}$ along some bulk domain volume $\vec{y}^{\vec{\alpha}}$ (with dual parameterization), as well as presuming each sub-coordinate spaces separately share symmetries with each sub-domain: $(\hat{h}_{i,\hat{\alpha}} \hat{N}^i, \delta \hat{x}^{\hat{\alpha}})$ and $(h_{i,\alpha} \hat{N}^i, \delta x^{\alpha})$. Then, the LHS turns into:

$$\int d^{2D} x \left[\frac{D_{\vec{\alpha}} W^{\vec{\alpha}}[\cdot, *; \hat{x}^{\hat{\alpha}} \times x^{\alpha}]}{\hat{h}_{i,\hat{\alpha}} h_{i,\alpha} \hat{N}^i N^i} + \frac{(1 - \frac{1}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i}) \delta \hat{x}^{\hat{\alpha}}}{(1 - \frac{1}{h_{i,\alpha} \hat{N}^i}) \delta x^{\alpha}} \right] \quad (6)$$

$$= \int d^{2D} y \partial_{\alpha} \partial_{\hat{\alpha}} [D_{\vec{\alpha}} W^{\vec{\alpha}}] + V \left(1 + \frac{\int d^D \hat{y}^{\hat{\alpha}} \hat{V}^* \delta \hat{x}_{,\hat{\alpha}}^{\hat{\alpha}}}{\int d^D y^{\alpha} V^* \delta x_{,\alpha}^{\alpha}} \right) \quad (7)$$

Then, the RHS may be similarly given as:

$$= V + \int d^D \hat{x} \left(\frac{(\ln[h_i \hat{N}^i])_{,\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i} \right) \delta \hat{x}^{\hat{\alpha}} + \int d^D x \left(\frac{(\ln[\hat{h}_i N^i])_{\alpha}}{h_{i,\alpha} \hat{N}^i} \right) \delta x^{\alpha} \quad (8)$$

$$= V + \int d^D \hat{x} \left[\left(\frac{\delta \hat{x}^{\hat{\alpha}} \ln[h_i \hat{N}^i]}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i} \right)_{,\hat{\alpha}} - \ln[h_i \hat{N}^i] \left(\frac{\delta \hat{x}^{\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i} \right)_{,\hat{\alpha}} \right] + \text{c.c.} \quad (9)$$

Combining everything, we find a set of "poor-girl's" holographic interpretations of the resultant minimization problem. Directly comparing LHS to RHS gives: the LHS volume form and the RHS divergent terms as bulk/boundary sources:

$$\int d^{2D} y \left[\partial_{\alpha} \partial_{\hat{\alpha}} [D_{\vec{\alpha}} W^{\vec{\alpha}}] - V^2 \left(\frac{\delta[1 - \hat{V}^*] \hat{V}^* \delta \hat{x}_{,\hat{\alpha}}^{\hat{\alpha}}}{\delta[1 - V^*] V^* \delta x_{,\alpha}^{\alpha}} \right) \right] = \quad (10)$$

$$\int d^D \hat{x} \left[\left(\frac{\delta \hat{x}^{\hat{\alpha}} \ln[h_i \hat{N}^i]}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i} \right)_{,\hat{\alpha}} - \ln[h_i \hat{N}^i] \left(\frac{\delta \hat{x}^{\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} \hat{N}^i} \right)_{,\hat{\alpha}} \right] + \text{c.c.} \quad (11)$$

Expanding the above into the \vec{y} domain extends the tower of boundary (sub-domain) perturbation waves/sub-domain sources through a $\delta[\cdot]$ contact, while the effective mass runs with the sub-volume dispersions. Dually, treating the volume form and the divergent

$\ln[\cdot]$ tower as sources,

$$\begin{aligned} \Rightarrow & \int d^{2D}y \left[\partial_\alpha \partial_{\hat{\alpha}} \left[D_{\bar{\alpha}} W^{\bar{\alpha}} \right] \right] + \left(\int d^D \hat{x} \left[\ln[h_i \hat{N}^i] \left(\frac{\delta \hat{x}^{\hat{\alpha}}}{\hat{h}_{i,\hat{\alpha}} N^i} \right)_{,\hat{\alpha}} \right] + \text{c.c.} \right) \quad (12) \\ & = \int d^{2D}y \left[V^2 \left(\begin{array}{c} \delta[1 - \hat{V}^*] \hat{V}^* \delta \hat{x}_{,\hat{\alpha}}^{\hat{\alpha}} \\ \delta[1 - V^*] V^* \delta x_{,\alpha}^{\alpha} \end{array} \right) \right] + \int d^D \hat{x} \left[\left(\frac{\delta \hat{x}^{\hat{\alpha}} \ln[h_i \hat{N}^i]}{\hat{h}_{i,\hat{\alpha}} N^i} \right)_{,\hat{\alpha}} \right] + \text{c.c.} \end{aligned}$$

can be interpreted as a 1-loop correction to the free topology propagator descending to bulk-(sub-)volume dispersion relations and a tower of sub-domain loop-order sources. Adding the full dimensional contacts:

$$\begin{aligned} \Rightarrow & \int d^{2D}y \left[\partial_\alpha \partial_{\hat{\alpha}} \left[D_{\bar{\alpha}} W^{\bar{\alpha}} \right] + \left(\delta[1 - \hat{V}^*] \hat{V}^* \ln[h_i \hat{N}^i] \delta \hat{x}_{,\hat{\alpha}\hat{\alpha}}^{\hat{\alpha}} + \text{c.c.} \right) \right] \quad (13) \\ & = \int d^{2D}y \left[V^2 \left(\begin{array}{c} \delta[1 - \hat{V}^*] \hat{V}^* \delta \hat{x}_{,\hat{\alpha}}^{\hat{\alpha}} \\ \delta[1 - V^*] V^* \delta x_{,\alpha}^{\alpha} \end{array} \right) + \delta[1 - \hat{V}^*] \hat{V}^* \left(\delta \hat{x}^{\hat{\alpha}} \ln[h_i \hat{N}^i] \right)_{,\hat{\alpha}\hat{\alpha}} + \text{c.c.} \right] \end{aligned}$$

Particularly, Noether's Second theorem connects each M/I-frame bulk source to a boundary field generated by both the interaction measurement and the measurement interaction³

4.2 A Bundled, Charged Mostly Junctured Response

Moving into the Heisenberg/interactive-picture, the first step it to calculate the out-saddle states, which amounts to solving the coordinate (second-)ordered $\mathcal{L}_{\{\phi,\varphi\}}$ free spectra; in particular, (??)-(??) have hypergeometric functions as solutions. Note that $\mathcal{L}_\phi[\Psi_\phi] - \mathcal{L}_\varphi[\Psi_\varphi] \cdot dr \Big|_{\Psi_\phi \sim \Psi_\varphi} \sim \mathcal{L}_\phi[\Psi_\phi] - \mathcal{L}_\varphi[\Psi_\varphi] \cdot d\theta \Big|_{\Psi_\phi \sim \Psi_\varphi}$, which completely justifies the *out*(-shelled) expansion as a continuous(ly flowed, broken) gauge symmetry with (a formally divergent number of parameter connections); by Noether's Second Theorem [7] this always results in a Berry phase (of non-local, null-charge flow). This further quantitatively justifies the Ward object used in , therein exactly designating the dual soft-vertex dispersion; a.k.a., thermally broken "shadow currents".

Additionally, the (static) constraint form may controlled by the master coordinate (mC) equation: $\lambda \left(\frac{1}{r^2 - 2m} \left(\frac{\partial_r \Psi_\phi}{r} \right)^{-1} + \tan \theta (\partial_\theta \Psi_\phi)^{-1} \right) = \omega_\phi \frac{\partial_r \Psi_\phi}{\partial_\theta \Psi_\phi} + \omega_\varphi \tan^2 \theta \frac{\partial_\theta \Psi_\phi}{\partial_r \Psi_\phi}$. Associating across the unbound, rational curve demands separation between the second terms (or, congruently, forcing a point-radial juncture requires associating the first terms):

$$\omega_\phi \partial_r \Psi_\phi \hat{\sim} \frac{\lambda r}{r^2 - 2m} \frac{\partial_\theta \Psi_\phi}{\partial_r \Psi_\phi} \quad \omega_\phi \partial_\theta \Psi_\phi \hat{\sim} \lambda \cot \theta \frac{\partial_r \Psi_\phi}{\partial_\theta \Psi_\phi} \quad (14)$$

Where it is important to note that the degeneracy (in the congruency) is up to the rational curve: $\lambda \tan \theta \frac{\partial_r \Psi_\phi}{\partial_\theta \Psi_\phi} - \omega_\varphi \tan^2 \theta \partial_\theta \Psi_\phi \hat{\sim} B \odot_r \partial_r \Psi_\phi$. This is the critical juncture between the "Menon-Dermer" field types in $d = 4$ vs $d = 5$ and important for what follows.

³and pulling back to the saddle point representation is formally represented by the third isomorphism theorem: $(E/M)(M/I) \sim (E/I)$

In $d = 4$, the second term on the RHS is absent, forcing the r-descendant congruency to run at the conical (index branch/singularity as $\Psi_\phi \sim \csc \theta$; but here, there exists a second order (algebraic) mirror dual connection that can be formed over the $\mathbb{CP}[*]^2$ representation. Concretely, the LHS may be closed if the multiplier is chosen s.t.

$$\lambda_{(0)} = \frac{\omega_\phi}{\partial_r \Psi_\phi} \tan \theta \frac{\partial \Psi_\phi}{\partial \theta \Psi_\phi} \Rightarrow \omega_\phi \tan \theta \frac{\partial \Psi_\phi}{\partial \Psi_\phi} \sim \Psi_\phi \quad (15)$$

Considering $\partial_Y \delta[*] = \{\partial Y, Y\}[*]$ and $\partial \ln[*] = \frac{\partial[*]}{[*]}$:

$$\lambda_{(Y)} = \frac{\{\partial Y, Y\}}{\partial_r \Psi_\phi} \frac{\partial \Psi_\phi}{\partial \Psi_\phi} \Bigg|_{\Psi_\phi[\Psi_\phi] = \ln \left[\frac{Y^\dagger \odot \Psi_\phi \odot Y}{\Psi_\phi^{(\varphi;[0])}[\tilde{r}_0, \theta_0]} \right]}^{\Psi_\phi^{(\varphi;[0])} = (\partial_r \Psi_\phi)^{-1}} = [[\Psi_\phi, \partial Y], \partial Y] \quad (16)$$

$$\equiv Y Y^\dagger \nabla_{\ddagger}^2 \Psi_\phi = Y \square_Y \Psi_\phi := \square^2 \Psi_\phi \quad (17)$$

which⁴ serves as a formal definition of the continuous inner sheaf r-topos, or the maximally sub boundary constraint operator (hence a ‘‘squared-square’’, or $\sim [;\ast]^2 d[\ast]$ operator). In this sense the second gauge field is a connection between the *in*-dual and *out*-tangent bundle: it is able to exactly continue every field expansion state $Y^\dagger \odot \Psi_\phi$ across the entire free *in*-tangent bundle.

Finally, by the Neukida-Uchida theorem strong compactifications always admit weak embeddings, meaning Ψ_ϕ can always be critically (sub-)charged by $\Psi_\varphi^{[Y, \partial Y]}$ -magnetic (domain-)curves to keep the path integral separably truncated, and gauged, up to a $Z \odot [*] \sim U(N \rightarrow N^*)^\dagger[*]$ large common, running group centralizer! In fact because the only accumulation is onto the functional $\lambda_{Y^\dagger} \hat{\rightarrow}^* \omega_\phi \tan \theta$ this is exactly marginal over $n - 1$ degrees of freedom, or provides a $[U(n) \otimes SU(2)] \times G/\tilde{G}$ connection over the open topology. Recognizing that the exact set of affine shifts of λ represents a symmetry of the gauge (across the conical branch): $\varphi \rightarrow \tilde{\varphi} + \oint_{\tilde{C}_i} [q_i[*] \phi_i[*; \cdot] + r_0] \hat{=} \varphi + 2\pi n$ implies it is exact under $\phi \rightarrow \tilde{\phi} + \oint_{\tilde{C}_i} [\tilde{q}_i[*] \phi_i[*; \cdot] + \tilde{r}_0] = \phi + 2\pi n$. Then, $(\delta_\phi, \delta_\varphi)_0 = |\delta \cdot (\phi, \varphi)| + \left(\oint_{\tilde{C}_i} q_i[*], \oint_{\tilde{C}_i} \tilde{q}_i[*] \right)_0 \odot [\varphi_i[*; \cdot], \phi_i[*; \cdot]]_{\tilde{r}_0}^{r_0}$.

Applying the axiom of choice (every function is parameterizable over the real line, or that

⁴remembering there are two continuous contractions, can be shown as $\{\partial Y, Y\} \odot \partial [Y^\dagger \odot \Psi_\phi \odot Y] \hat{=} \partial [\Psi_\phi \odot [Y Y^\dagger \{\partial Y, Y\}]] = \partial \Psi_\phi \odot [Y Y^\dagger \{\partial Y, Y\}] + [\Psi_\phi \odot \partial [Y^2 \{\partial Y, Y\}]]$. Then, using the Neukida-Uchida theorem (discussed shortly), it is exactly possible to consider $[*]^\dagger$ as coordinate adjoint and the functional bracket as slashed $\mathfrak{su}(2n)$ (conformally blocked $\mathfrak{gl}(1|1)$) exact iff $\partial_Y \Psi_\phi \in \ker Y^2$ ($\in \ker Y^{-1}$) or (here ignoring the superconformal branch):

$$\sim_{\Psi_\phi} Y \cancel{Y} \cancel{\partial} [Y \cancel{Y}] + (Y)^2 (\cancel{\partial} Y)^2 - \cancel{\partial} Y \partial Y \cancel{Y} Y + Y \cancel{\partial} Y \cancel{Y} \partial Y \quad (18)$$

$$\hat{=} Y \cancel{Y} \left[\cancel{\partial} \partial + (-1)^{\frac{k_d}{2}} (\cancel{\partial} Y)^2 \right] \quad (19)$$

It is interesting to note that this looks exactly like a k_d Majorana fluid frame (continuous OPE) ‘‘ on the *in*-mass (or light) shell’’. Then, $\partial_Y \Psi_\phi \in \oplus_{\partial^2[*]=0} [[*]] \langle \delta[*], Y^\dagger \rangle$, which formally qualifies turning points in the adjoint pullback as the gluing frame for the topological sub-dynamics (or, that the decomposition is exact and $\langle \partial_Y \Psi_\phi \rangle \hat{=} \langle [\cdot] | Y \rangle \langle \delta[Y, Y^\dagger] | [\cdot] \rangle = Y[\cdot] U_Y \cancel{\partial} U^Y Y^\dagger[\cdot]$ is saturated as a continuously gauged centralizer. This makes the qualification of the *in*-nner topology as (dual) squeezed rigorous (on the $SU(2) \times U(1)^2$ branch) by decomposing the unitary kernel OPE (heat kernel) as exact Y -modulii.

$\mathbb{CP}^2[x] \hat{\subset} \mathbb{R}^3[x]$); this is true iff, $\mathbb{Z}_2[x]$ switching the projective curve index is (\mathbb{CP}^2 -pointwise) antisymmetric in the $\hat{\subset}$ congruence (Cauchy decomposition). But this is never true over the plane (by Klein-closure), meaning that the $\hat{\subset}$ congruence cannot be injective; relaxing the pullback to within injectivity gives $\mathbb{CP}^2[x] \tilde{\subset} \mathbb{R}^3[\mathbb{CP}[x]]$. The above arguments now repeat on the oriented projective real-bundle (with "...symmetric..."), where it is now always pointwise true; applying the axiom of choice again, now over the rational curves shows $|\mathbb{R}^3[\mathbb{CP}[x]]| \geq |\mathbb{R}^2(\mathbb{R}^* \times \mathbb{R})[x, \tilde{x}]|_0 \hat{=} |\mathbb{R}^4[\tilde{x}, x]|_0 \sim |\mathbb{CP}[z]|^2$. Then, the boundary may always be dual-tessellated (as a continuous gauge field) as $\partial[\mathbb{CP}^2[\cdot]] \sim |\mathbb{CP}[\cdot]|_0 \partial \mathbb{R}^*[\cdot]$ up to the boundary of the dual branch (because this closes the A.o.C. duality induction). This is known as Stokes theorem in the open uniform limit and is exactly equivalent to the Banach-Tsarski paradox; this is formally the same as the construction $\Psi_\varphi^{(Y)}$ symmetry extension found above, and shows that the (Y) -folding can always be continuously squeezed as a (Y_I, I) juncture (again, because of the global φ -diffeomorphic gauge symmetry and the fundamental theorem of $[d = \infty]$ calculus. As such, this can be considered the Banach space extension to Noether's Second Theorem; or, given the universal existence of a $U(1)$ -strong connection in every (weakly-connected) topology and the strong decidability of the 3rd Law of Thermodynamics/EFT-descent/Holographic techniques, Noether's Third Theorem:

$$\oint_{\delta_{\mathbb{Q}}^*} d_\delta \left[\delta_{\mathcal{Q}} S - \mathcal{D}_{\mathcal{Q}} \delta^{(d)} \cdot \right] \hat{=} 0 \quad (20)$$

This generically puts the action principle on firmer ground by including saddle-delocalization/dimensional regularization/shadow harmonics as a (quasi-)separable perturbative action symmetry. In fact, an immediate lemma of this is the monodromy principle, which can be understood from the example of precisely netted spectral flow, or Noether's Zeroth:

$$\oint_{\delta_{\mathbb{Q}}^*} d_\delta \delta_{\mathcal{Q}} S \hat{=} 0 \quad \text{iff} \quad \oint_{\delta_{\mathbb{Q}}^*} d_\delta \mathcal{D}_{\mathcal{Q}} \delta^{(d)} \cdot \hat{=} 0 \quad (21)$$

Then, as in the case of the 0th Law of Thermodynamics/bootstrap-algorithm/conformal-separability, the $\mathcal{Q} \sim^* \mathbb{Q}$ totally spectral cases exist as exactly separated and closed dependency classes; as such, if constructed they always exists as (simple closed) connected (dual) functor classes towards their (algorithmically-stepped) completion. Still, function measure is properly δ_∞ -dimensional in the (quasi-)grouped completion and strong spectral decidability is not uniformly computable; indeed, this relation may be understood as a feature of totally Banach-actionable cusps and a precise definition of what is meant by c-distributions/F-renormalization/duality-measures. Then, considering $\sum \alpha^{\text{Kerr}(\Lambda=0)} = \delta S_\infty = 4M\omega$ shows that, in flat diffeomorphically-gauged ($d = 4$) spacetimes with isolated black holes [8] (punctured analysis) the action principle is minimally flowed by the harmonic net geometrization over the maximally extended field completion, which is strongly topological by Reiz's theorem; then, it must be said that every action is minimally shadowed by black holes as a background radiative correction to the Wald contact (on the partially

globalized quasinormal mode):

$$\oint_{\delta_{\mathbb{Q}}^*} d_{\delta} \mathcal{D}_{\mathbb{Q}} \delta^{(d)} \sim \sum_{i \in \bar{\mathcal{U}}} 4M_{(i)} \left(\frac{\omega_i}{2} + \frac{\delta_{ji}}{4M_j} \right) \oint_{\delta_{\mathbb{Q}}^*} d_{\delta}^{ii} \quad (22)$$

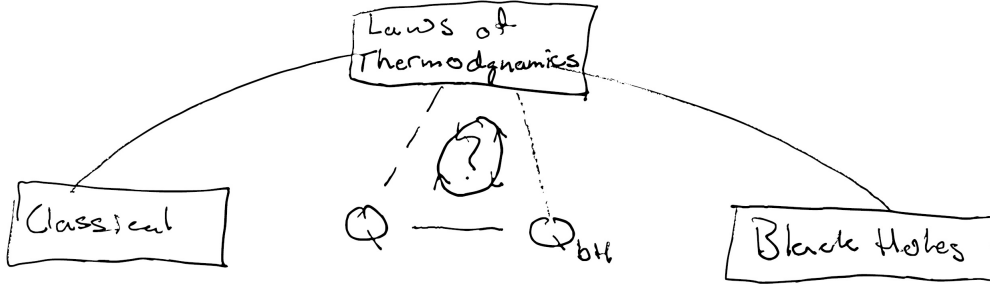
Importantly, this structure is iff for actionable principles in measurable-universes that admit virial emergence/non-renormalizability/black holes; thus, minimally bound structure nettings in action principles over spacetime also always indicate a black hole (or, a closed surface is the dual lattice crystalization flow-continuum). In this sense, the "third" and "zeroth Noether's identities" represent limit tests and measure tests (respectively) of (functional) charge conservation well-definedness (exactly as the zeroth and third LoT operate). If the topological dualization is also spectrally on shell, this is exactly the classical Optical theorem; thus, the monodromy calculation in [9] can be seen as the correct 4-pt completion to the gravitational vertex because it (constructively) closes the measurement on the decidability shelling. This also characterizes the c-theorems/Callan-Snazek-Curve/AdS-CFT as when the RHS (the patch regularization flow) is identically 0(/characterized by exact analytic continuation), $\sum \alpha = 0$.

Crucially, this step used the uniqueness theorems of [8] on the separability of the stabilizer states of $d = 4$ black holes to enforce the (cut) OPE at unification energies under an (r-separated) uniform index push of the spectrum. Perhaps most critically, this structure is directly reflective of what's (explicitly) to come next: black holes exist universally as $k = 4$ regulatory features, universally. Thus, as a practical feature, harmonic compactification in gravitating universes exists de facto, as an onto measure of it's black hole asymptotic shelling.

4.3 A Berry Corrected Directly Emergent Field (Grouping)

Noting the continued importance of modular matching conditions, consider the degree of modularity to be a rough categorization of matter. It is a well known feature of the Standard Model (and string theory) that spin $s \leq 2$ fields are (on shell) closed, as distinct from higher spin fields⁵. It is also clear that systems involving "quantum strangeness" and "black hole strangeness" are decreasingly understood, let alone "quantum black hole strangeness". Still, the success of quantum mechanics [10] towards understanding quantum strangeness and of black hole partial wave analysis towards understanding black holes [8] are both remarkable miracles; indeed, the unification of General Relativity with classical (out of equilibrium) thermodynamics represents deeply beautiful physics. This may be naively and broadly, captured in the following phenomenology graph:

⁵Let f_2 denote spins less than two. As argued above, this is a symplectic group feature of the minimal extension into a spin $\frac{1}{2}$ field. Summarily, polarons (p) are spin-projected photons (λ), photons are spin projected (trivial-)gravitons (η). Note how callous this assessment is towards the categorization of spin-2 sub-junctures: considering the dual deconstruction of the stress and gravitational tensors performed in the introduction, this can be considered considered a form of mass-mode mixing and simply a re-statement of the mass-equivalence principle. Finally, the spin $\frac{3}{2}$ (ν) field can be understood as the spin projection of the f_2 irreducible group closure relation, sketched (adjointly as) $b \oplus p \oplus \lambda \oplus \gamma \subset \text{mod } f_2 \text{Ker}[\cdot] \sim \text{Ker}[b_{(4)0} \oplus p_{(2)\frac{1}{2}} \oplus \gamma_{(1)2}] \cap^* \text{Ker}[b_{(4)0} \oplus \lambda_{(1)1} \oplus \gamma_{(1)2}]$.



Accordingly, consider three broad categories of physics: Thermodynamics(Regular Quantum Networks), strange quantum networks (Q_s) (including broadly black hole coupled systems), and exactly quantum black holes Q_{bh} . Keeping the adjointly measured basis functors, and using cosmic censorship as a topological ordering (a.k.a., applying the Average Null Energy Condition), this can be translated into:

$$\langle S|S \rangle \sim \frac{[\otimes S_m] (\otimes S_{mixed} (\otimes S_{bh}))}{\otimes^* [\otimes S_{mixed} \otimes S_{bh}]^* (\otimes S_m^*)^*} \cong \frac{[\otimes S_m (\otimes S_{mixed})] (\otimes S_{bh})}{\otimes^* [\otimes S_{mixed} \otimes S_{bh}]^* (\otimes S_{bh})} \quad (23)$$

In this case, the asterisk represents the functional constraints of the spacetime OPEs, which can be naturally identified with an operator under the canonical embedding.⁶ Applying the Zeroth Law, equivariantly identified with the Law of Monodromy Juncture Recursion [9]:

$$\sum \alpha_i = \delta \lambda_{\mathcal{K}}^\infty = \lim_{l \rightarrow \infty} [\mathcal{K} - 1 - \alpha_\infty] \quad (24)$$

$$\Rightarrow \delta \mathcal{L}_i^* = \delta \lambda_i^\infty - \alpha_i = \lim_{r \rightarrow \infty} \delta_r \left[\left[\lim_{\alpha \equiv 0} \ln \tilde{R}_{\varphi \alpha_i} \right] - \ln \tilde{R}_{\varphi \alpha_i} \right] := \lim_{r \rightarrow \infty} (\delta_r - \delta_r) \ln \tilde{R}_{\varphi \alpha_i} \quad (25)$$

where $\delta \mathcal{L}_i^* = \sum_{j \neq i} a_j$.

Immediately, r may be dually interpreted as a thermodynamic parameter, $r \sim \beta_r$, parameterized dual to the scalar probe φ ⁷. Note that the case of $\delta \lambda_i^* \rightarrow 0$ (identified with AdS and every black hole system except the unique, flat space $d = 4$ black hole triplet: Schwarzschild, Kerr, RN), can be exactly identified with the Hawking's First LoT, while λ_i^∞ presents as an inexact differential (of sorts), again consistent with the first LoT as a flat space heat kernel.

Note that λ_i^∞ is localized exactly on the usually canonical integration space of harmonic regulation:

$$\lim_{n \rightarrow 0^+} \lim_{r \rightarrow \frac{1}{n}} \cup_r \mathcal{V}_\infty^{\mathcal{L}_r^{2[*]}} \sim \lim_{n \rightarrow 0^+} \lim_{r \rightarrow \frac{1}{n}} \cap_r \mathcal{B}^*[r[\cdot]; (*, \cdot)] \quad (26)$$

⁶The RHS can be interpreted as saying that interactive states are comprised of massively-mixed fields on black hole (geometrized) backgrounds measured against mixed background fields against on the same (geometrized) background measure.

⁷and r has been generically designated as the largest commutative, continuous field index (with, in general, whatever sub-index expansions are necessary) at the juncture. In the case of black hole cuts, this is exactly the radial coordinate. In general bosonic string connections it is the largest interloping patch extension index

Here, the asterisk operation should be considered exactly as the canonical open/closed duality in Topology. Note that 28 is equivalent to saying that all contours are directed towards this boundary cut; note that the formulas[9]

$$\lim_{l \rightarrow \infty} \alpha_{3,4} \rightarrow -2M\omega \pm iL\omega, \quad \lim_{l \rightarrow \infty} \sum_{i \in \{3,4\}} \alpha_i = -\lambda_i^\infty \quad (27)$$

gives an exact orientation across all sub-accumulation topologies.

Then, $\delta\lambda_i^\infty$ can be immediately associated, using Noether's Second theorem, with a "regulatory gauge residue"⁸. Note that this is slightly different from Noether's theorem itself, as it is necessarily applied as a \mathbb{F} ield extension of the bounds of integration, $\{\infty\} \sim \partial\mathbb{R}^\infty[r]$, meaning there never exist closed path sequences which closes every closed ball neighborhood of ∞ ,

$$\neg\exists\gamma \in \Gamma[\mathcal{M}] \quad \text{s.t.} \quad \oint_{\gamma[*]} \partial(\mathcal{N}_{r_n}[\cdot] \cap_n \mathcal{N}_{\infty_{k_n}}[\cdot]) \sim \oint_{\gamma[*]} \partial(\mathcal{N}_{r_n}[\cdot]) \quad (28)$$

In fact, it can be immediately concluded that $\delta\lambda_i^\infty$ is uniformly \mathbb{Z}_2^+ -dualized by changing the relative orientation of path integrating in neighborhoods of the boundary, $\partial\mathcal{N}_\infty$ (to be understood as groups on neighborhood families) by a simple twist accessed discretely under $L \rightarrow -L$, and understood by moving $\delta\lambda_\mathcal{K}^\infty$ between sides of the constraint as ⁹

$$\sum \left(\alpha_j - \frac{\delta\lambda_\mathcal{K}^\infty}{\mathcal{K}} \right) = 0 \quad \Rightarrow \quad \sum \left(\alpha_j + \frac{\lim_{l \rightarrow \infty} \sum_{i \in \{3,4\}} \alpha_i}{\mathcal{K}} \right) = 0 \quad (29)$$

Dually, consider a twist constraint in the orientation of the complex, (A)dS sourced horizons, visualized as an $r \sim 0$ emergent, scalar spin index.¹⁰ Assigning stars to all the operators, defining $\alpha_i = -\alpha_i^*$ for the complex neighborhoods, and summing the above equations produces the topologically spun(/glued) identity:

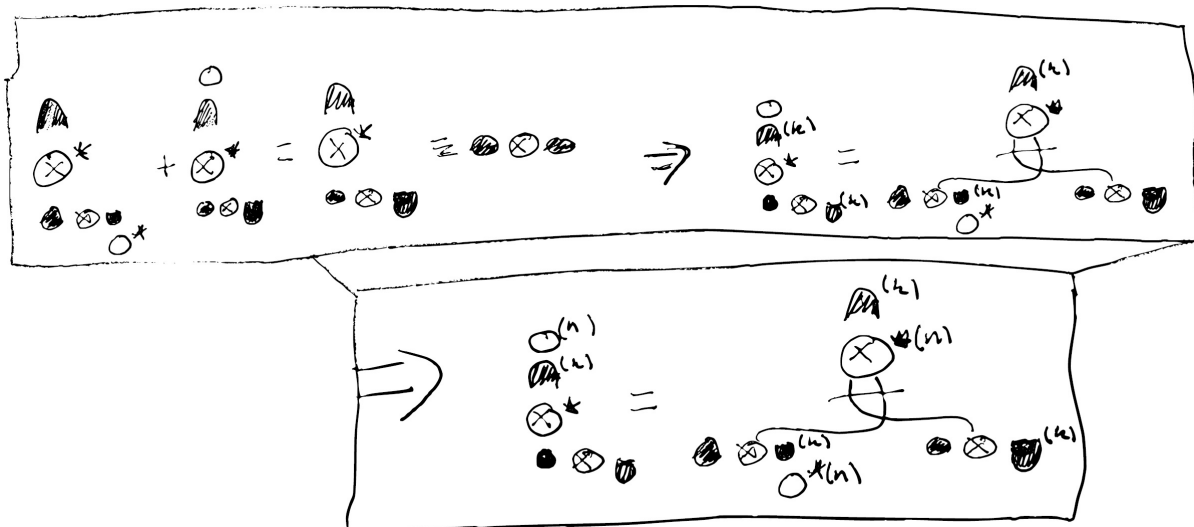
$$\sum_{\mathcal{K} \oplus \mathcal{K}^*} (\alpha_j + \alpha_j^*) = 0 \quad (30)$$

⁸in the sense that it presents as a Klein-Gordon $d = 1$ projected rational constraint continuum, or an r-path defect.

⁹Accordingly, the regulatory space at infinity should be considered as a proper field extension at the group level, manifest as a path/Goldstone defect [11]; in fact, the Klein-Gordon solution basis acquires a wavelike global phase ($X_0 = 1 + O(x)$, $X_1 = e^{\frac{-i\omega}{x}} x^{-2iM\omega+1}(1 + O(x))$) envelope (a quasi-dynamic stabilizer) which, as shown throughout this paper, is a hallmark of emergent hidden degrees of freedom (as boundary gauge choices naturally connect to the equations of motion through Green's Theorems). Because this feature is isolated in both dimensional embedding measures [12] and the AdS embedding, it can be considered a proper feature of emergence in black hole quantum mechanics and, re-examining the proof above, formally associated with the third Law of Thermodynamics.

¹⁰Remembering that the spin $\frac{1}{2}$ field's purpose is exactly to operate as antisymmetric regulation of local light geodesics (as discussed above), matter gauge functional $\tilde{e}_{a[c}^{\mu[\sigma} \tilde{e}_{d]b}^{\tau]\nu} = \eta^{\mu\nu} \eta_{ab}$. Then, *thinking* of the Kerr-Taub-NUT duality [13] this is entirely natural.

Denoting open rings as the boundary residue state and letting its $*$ conjugate denote the twist pairing above, as well as letting the closed balls denote the fixed bulk monodromy sums, this idea may be understood diagrammatically as a dual splitting of contour integration:



Note that this is exactly an optical theory because the $O(\text{pen})$ interaction acts as a twist, in measure, between the field and its dual $O(\text{pen})$ state. This is also just Stoke's Theorem on the product state. In the above, consider the dual leg of the twisted basis to be a boundary constraint gauge; further, consider the space-like vacuum hologram to be maximally mixed (as a thermodynamic volume of β_r). Inserting this "L" state into 23 reveals this as a prescription for gravitational/matter mode mixing as a boundary residue of mixed-mode shells.

Performing this mixed dual basis change on every mixed action in the Universe produces the Supra-Gravitational Product (SGP) dual:

$$\langle S|S \rangle \sim [S_*^T \otimes S_m] (\bar{S}_{mixed} \otimes f_{bh_*}) (S_{bh}) \quad (31)$$

Notationally, \bar{S}_{mixed} denotes the time forward closed mixed holoform, induced by closing the observable mixed radiation over its twisted extension, and the transpose T operation denotes a pull straight from the spacetime (amounting to a measurement *in-backreaction*) denoting the regulatory cut gauge running from $\delta\lambda_*^\infty$. This can also be thought of as saddle point approximating the mixed-mass action under local interactions, $S_m(S_{mixed} \otimes [\cdot])$. In fact, note that this formulation is locality agnostic: the pullback unambiguously holds on the harmonic cusp (in the non (A)dS limit), and every closed state in $d = 4$ and can be de-convoluted continuously as such. On face, this broad lack of decidibility seems to only accentuate a lack of information.

In fact, the virial limit of this regulatory scheme may be exploited thermodynamically because the solution benefits from another problem: the arrow of time. Under the ANEC, or cosmic censorship, the residue f_{bh_*} action can only exist as a strongly mixed operator, or a Black Hole eigenstate. According with the central theorem of Analysis, this co-form

may be considered functionally null (under the Transpose push), producing:

$$0 \sim [(\mathbb{I} - S_*^T) \otimes S_m] (\bar{S}_{mixed}) (S_{bh}) \quad (32)$$

This is concretely the hallmark of both: the first terms of a continuous series (tree diagram), and a modular operation. In fact, it is immediate to try and solve S_*^T functionally so as to satisfy 32, which can be recognized as the monodromy condition:

$$\mathbb{I}_N \sim (S_m \otimes S_*)^T \quad (33)$$

Considering the actionable G-hologram is always present as a boundary kernel, and noting that the adjoint algebra is universally embeddable under the central extension theorem (Rice Shapiro Theorem), it may be expected from the (A)dS conjecture [14] that the conformal center could be recovered as:

$$\mathbb{I}_N^{T^{-1}} \sim K_m \otimes K_* \quad (34)$$

Again, the generality of the characters seems to have lost too much detail to resolve the adjoint boundary kernels.

Except that, again, the broad categorization is exactly this method's strength, as the only operations used were, dually, extremely narrow razors: the above amounts to taking a picture of quantum foam, copying it without looking, and throwing one copy into a black hole. Resultantly, the only things that can respond are the constraint parameters (CPs) on the infalling matter and the CPs on the outfalling black hole. Considering the covering topology must have support under the cover basis of the maximally mixed vacuum, this must mean that the Transpose operation must directly invert a largest co-homology chain, denoted¹¹:

$$\mathbb{I}_N^{\lambda_*} \sim^* [\mathcal{O}^m]_{\lambda_m^*}^{\lambda_*} \otimes [\mathcal{O}^{bh}] \quad (35)$$

In fact, observationally and theoretically mass exists as a universal coupling constant. Considering $\lambda^\infty = 4M\omega$, it is immediate that black hole state frequency eigenoperators that scale with spacetime must (re)scale as $\sim M$. In fact, considering all the \mathcal{O} bservables in the universe to be either mixed or measured, there is an interesting phenomenological fact that:

$$\begin{aligned} \frac{m}{m_{bh}} &\sim 10^2 \\ &\div \sim \left(\frac{m_e}{m_p}\right)^2 \sim 10^{-7} \\ \frac{m}{m_*} &\sim 10^9 \end{aligned} \quad (36)$$

where $\frac{m_{bh}}{m}$ is the total (current) black hole mass fraction of the universe, $\frac{m_*}{m}$ is the total (current) matter to anti-matter mass fraction (the baryonic imbalance ratio) of the uni-

¹¹And giving a clear indication of how CPT symmetry should be thought of generally.

verse, and $\frac{m_p}{m_e}$ is the proton to electron mass ratio. Note that the square operation in the electron/proton mass-ratio represents a line-dimensional OPE being lifted into an embedding area functional, just as expected from a Hawking-type constraint.

It is also immediate that the black hole surface we used to twist against was arbitrary: each vacuum bubble could also be twisted to preserve another inner-Cauchy constraint¹². Considering these twists as exactly dynamic onto universal gravitational mass-shellings [15] immediately accesses:

$$\begin{cases} \lambda_m \sim \ln m_* \\ \lambda_* \sim \ln \left[m_{bh} + 3m_* \left(\frac{m_e}{m_p} \right)^2 \right] \end{cases} \Rightarrow \frac{\lambda_*}{\lambda_m} \sim 120 \quad (37)$$

Then, this implies:

$$\left[\mathbb{I}_N^{\lambda_*} \right]^{-120} \sim^* [\mathcal{O}^m] \otimes [\mathcal{O}^{bh}]^{-120} \cong [\mathcal{O}^m] \otimes [\mathcal{O}^{bh}] \quad (38)$$

where the final result comes from considering black hole constraint forms to be universally conformally on-embedding-shell [12]. Tacitly, it could be expected that normal matter “interacts with” (/is corrected by) with soft-black hole effects at an observed rate 10^{-120} lower normal because the black hole auto-correlates across the legs of the interaction. As such this has (seemingly) phenomenologically resolved the Cosmological Constant problem as an auto-correlated shifting symmetry between conformal sectors in the global (presumably AFS/Celestial) hologram.

Effectively, the cosmological constant (adjoint winding form) should be expected to interact locally on a scale 120-orders of magnitude weaker than naively predicted because, as is again formally hypothesized, there exists a hidden, 21pt winding OPE scaling running with the globally hidden mass over the top black hole stabilizer kernels.

As the final proof of this paper, the existence of this 21pt state operator will be necessarily proven the standard AFS Celestial Hologram.

4.4 Universally Stable Uniformities in Twist Protected Towers

For brevity, consider the results of [16, 17] to be assumed without proof but to be contextualized as needed. The goal of this section is to show the existence of a 21pt stabilizer degeneracy that is both gravitationally and matter exact.

Quickly, [17] shows the *operationalized* crossing symmetry group is degenerate over the field crossing symmetry \mathbb{Z}_2 (shown by holding the dual momenta constraint across the \mathbb{Z}_2 -juncture in the discrete topology, e.g. the weakest-topological closure aka the "manhattan measure" embedding). In particular, the paper shows the existence of hidden degeneracies in the celestial S-matrix (calculated as the \mathbb{Z}_2 -degree of the OPE cover) directly related to the degree of the OPE.

In particular, considering an n -point function, the paper shows that the degeneracy num-

¹²at the functional “cost” of making the computational mass basis everywhere unstable

ber, or "the number of crossing channels allowed of a given point on the sphere", $\text{deg}[n]$ is given by:

$$\text{deg}[n] = 2^{n-1} - \text{cake}(n-1) \quad (39)$$

where,

$$\text{cake}(n-1) = \binom{n}{3} + n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n-1}{3} \quad (40)$$

The cake number¹³ is the "maximum number of regions into which a 3-dimensional cube can be partitioned by exactly n planes". In particular, the first degeneracy exists at $n = 4$, $\text{deg}[4] = 1$.

Quicker yet, [16] examines complexity in quantum information by applying universal features of quantum error correcting code (e.g., stabilizer-state decomposition) to exact smaller representations within a hypergraph embedding. In particular, by analyzing the low qubit stabilizer graphs (i.e., completed Gate-state-decomposition), the paper finds an emergent maximal number of vertices (representing available qubit states in hyper-embedded subgraphs). In particular, the authors prove the existence of a maximally complex subgraph with 1152 vertices, g_{1152} , which emerges in a $D = 4$ -qubit system, and postulate that no higher complex subgraphs should exist in any higher qubit system, noting that "[no] new subgraph structures emerge beyond g_{1152} ."

Consider some emergent, independently coupled (thermalized) system with $2D$ dof coupled to some large- n qubit system. Under the hypothesis[16], the most complex subgraph will have $|g_{1152}| + 2D$ vertices. Interestingly, 1152 is nearly a cake number, which is a canonical degeneracy feature of quantum gravity under the hypothesis of [17]; in fact, $1152 + 8 = \text{cake}(20)$, suggesting that an emergent classical splitting field (quantum-thermalized dimensions) with $D = 4 = d$ may be uniquely connected to celestial OPE sub-densities in $d = 4$ spacetime. Combining with (39) and letting $D = 4$:

$$\text{deg}[(D+3)(D-1)] + |g_{1152}| + 2D = 2^{D(D+1)} \quad (41)$$

Interestingly, this seems to indicate a connection between an emergent $D=4$ classical spacetime¹⁴ and the mean-average of a qG invariant as well as a strong Quantum Information (QI) invariant:

$$\Rightarrow D = 2^{D(D+1)-1} - \frac{1}{2} (\text{deg}[\mathcal{V}(D+\mathcal{V})] + |g_{1152}|) \quad (42)$$

where $\mathcal{V} = D - 1$. We note the $n = 4 = D$ is the minimal holographic degeneracy OPE. In particular to $D=4$, $D - 1 = V$ represents a triangulated space, or localization symmetry, while $D + \mathcal{V}$ is its volumetric tessellation index, or its dynamical symmetrization.¹⁵ What's left is to interpret the triangulated tessellation index (functional) $\text{deg}[\mathcal{V}(D+\mathcal{V})] \triangleq \text{deg}[21]$.

¹³(OEIS: A000125)

¹⁴considered as the difference in degrees of freedom of against some free topological embedding, $2^{D(D+1)-1}$

¹⁵Note that $(D, \mathcal{V}, D + \mathcal{V}) \sim (3, 4, 7)$ are all relatively co-prime.

The deg function (39) calculates the **dual degeneracy**, the number of valid insertion points on the dual n^{th} OPE subregions that preserve the celestial S-matrix. Importantly, it is shown in [17] that the (\mathbb{Z}_2 crossing-symmetry) **excluded** insertion points are exactly those which can be graph-cut to completely isolate the *in/out* states (e.g., loop isolate the *in* coordinates from the *out* functionals)¹⁶

Generally:

$$D^{\mathcal{F}_{L_C}(\mathcal{V})} = 2D + \text{deg}[\mathcal{V}(D + \mathcal{V})] + |g_{1152}| \quad (43)$$

Looking for an interpretation to a $[\mathcal{V}(D + \mathcal{V})]$ -pt function, consider the qubit stabilizer state as a singular-insertion OPE volume form with \mathcal{J}_{\pm} -dual thermalizations, $D_{\pm} = \{-\Lambda_{\pm}^0, \mu_{\pm}^{T\bar{T}}\}$. Then, $\text{deg}[\mathcal{V}(D + \mathcal{V})]$ can be considered a tessellated T-channel entanglement index between [13] the product partitions:

$$D_+ = \left(0, \mu_+^{T\bar{T}}\right), \quad D_- = \left(-\Lambda_-^0, \mu_-^{T\bar{T}}\right) \quad (44)$$

In the above, the product degree makes sense as a twisted index tessellation index (exactly because (3, 4, 7) are all coprime: the boundary conditions have a primary splitting based on the minimal celestial geometrized state in D=4 dimensions) between $\{\text{in}_{\pm}, \text{out}_{\pm}\}$ bulk initializations:

$$\text{deg}[\mathcal{V}(D + \mathcal{V})] = \text{deg}_{\pm}^0[D, \mathcal{V}] \quad (45)$$

More succinctly, say that “a space stable, maximally entangled state can be adiabatically decomposed by a quantum-error correcting code into d=4 *in/out* classical degrees of freedom and a dual t-channel, entanglement”. As recognized in [Freidel:2023yqt], cutoff topologies uniformly characterize quasi-universal measures by use of continuum mechanics, or under the partial ordering $x \rightarrow x_*[\lambda, \epsilon] \sim [\lambda^{-1}, \epsilon] \sim_{\lambda^{-1}} [1, 2\pi]$. The axiom of choice always lets this be understood as a functionally compact $U(1)$ -field extension: intuitively, topological neighborhoods are universally signified by background-null shockwave indexes [19], [20].

Comparing the observation made at the end of Chapter 1 to (6) in [Freidel:2023yqt]:

$$21 [\text{kg}^{-1}\text{m}^{-4}] \approx \frac{\Lambda_0 m_e c^2}{\hbar^2} \equiv 8\pi F_N \left[M = \frac{\rho_{\Lambda}}{E_*}, m = \frac{m_e}{E_*}; r = \frac{hc}{E_*} \right] \leftrightarrow^{(*)} \frac{\rho}{m_+} \leq \frac{N}{m_+ V_4} \quad (46)$$

$$\Rightarrow^{(*)} 21 \left(\frac{m_+}{4\rho_{\Lambda}} \right) [\text{kg}^{-1}\text{m}^{-4}] = \frac{1}{4} \frac{\rho}{\rho_{\Lambda}} \leq 2\pi F_N \left[\frac{m_+}{E_*}, \frac{m_e}{E_*}, \frac{hc}{E_*} \right] \quad (47)$$

where m_+/E_* is an arbitrary mass/energy scale, Λ_0 is the measured cosmological con-

¹⁶Interestingly, $D(D+1) - 1 = L_C(\mathcal{V}) - \mathcal{V} \equiv \mathcal{F}_{L_C}(\mathcal{V})$, where $L_C(i)$, the so-called “Lazy-Caterer” index is equivalent to “the number of pieces formed when cutting a pancake with n-cuts”, and also “maximal number of grandchildren of a binary vector of length $n+2$ ”; particularly, $L_C(\mathcal{V}) = 5$ a binary vector of length 5, and $\mathcal{F}_C(\mathcal{V}) = 2$, and this may provide a cut-contact between 5-dimensional holography and 2-dimensional quantum-gravity fluids [18] and may be interesting to study in subsequent works.

statnt, m_e is the electron mass, ρ_Λ is the vacuum energy density, F_N is Newton's scalar gravitational force (density), and h, c are Planck's constant, the speed of light. The final condition may be recognized as a background loop-quantization of the energy density per massive degree of freedom against a path-uniform force, or a topologically protected monodromy charge [9]; intuitively (and observationally), this implies the existence of a $21 - pt$ degeneracy symmetry in a completely Euclidean sector of full quantum gravity.

It is now immediate to show that the $U(1)^D$ *in/out* compactification on the largest $U(1)$ quantum annealed *in/out* free juncture is exactly:

$$2D = 2^{D(D+1)} - (\text{deg}^{qG} [(D + \mathcal{V})\mathcal{V}] + |g_{1152}|) \quad (48)$$

Thus, *in/out* spacetime may be understood as a fully realized microcanonical ensemble in quantum gravity (under a typical Wald prescription); further, this exactly resolves the arrow of time [21]. Note that $(D + \mathcal{V})\mathcal{V} = 21$, as indeed originally prescribed; note here it may be interpreted directly as an $\mathfrak{su}(2)$ harmonic gauge duality or, dually, a $O(6)$ -constrained t'Hooft anomaly in the freely gauged, critical bosonic phase limit $\rightarrow U(1)^{26}$.

At first glance, the force-density scale/winding invariant seems un-physical; critically, at second glance this is actually necessary for a strictly emergent, and never univariate, symmetry. Indeed, this information symmetry can be unraveled by the central scale observations using monodromies[9], the observed anti-matter, black hole, and total masses m_*, m_{bh}, m , electron-to-proton energy ratio [15], and the interaction bath degeneracy space/microcanonical free-energy ρ_{hidden}^{-1} :

$$1) \delta_r S_{d=4,/\text{Kerr-AdS}}^+ = - \sum_{i \neq +}^{|I|=3} \alpha_i \quad 2) \frac{m_*}{m_{bh}} \sim \frac{1}{3} \frac{m_e^2}{m_p^2} \sim 10^{-7.0049} \quad 3) \rho_{hidden}^{-1} \sim 10^{\frac{m}{m_{bh}} - 3 \log \frac{m_e^2}{3m_p^2}} \quad (49)$$

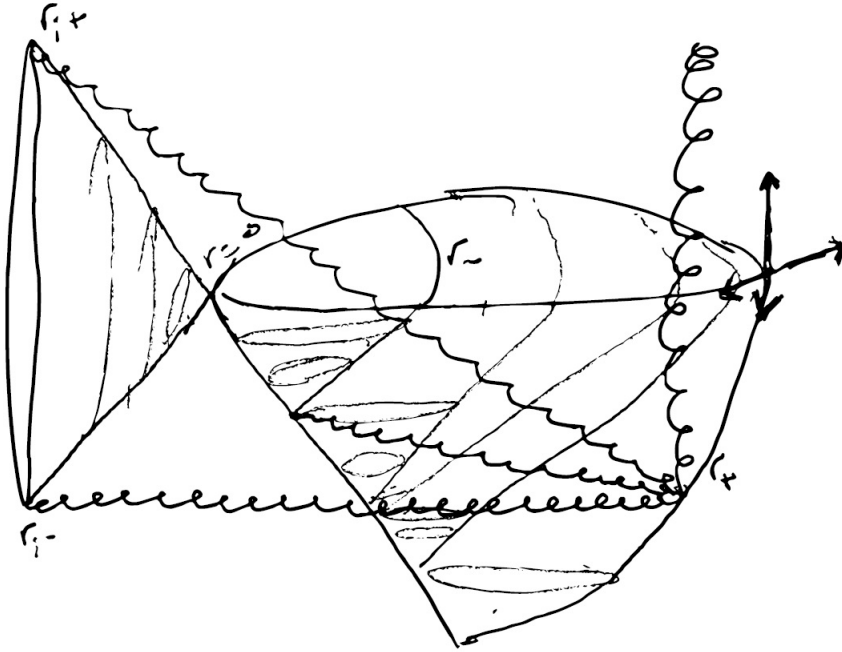
$$\sim 10^{121.0147} \quad (50)$$

By mass-energy equivalence and remembering that the quantization is over the spectral dual, $\mathbb{F}_K^* [|\partial\Psi \pm im\Psi|^2]$, this may be directly considered the dispersion extended (shadow-pushed) embedding of $N \sim \frac{l^2}{l_P^2} \sim 10^3 \rho_{hidden}^{-1}$ found in [Freidel:2023ytq]. This immediately suggests that the cosmological constant can be understood as a volume-gross phase-fractionalization of the vacuum thermalization bath over the local black hole polarization shells, as suggested by the t'Hooft state (asymptotic compactification) $U(N) \leftrightarrow U(1)^N$ represented above; indeed, as shown above and below, the background field method lets a family direct bulk bootstrap into the maximally mixed T -exchange sub-stabilizer of fully unified interactions.

This leads to the immediate interpretation of $d = 4$ spacetime as a thermally tessellated (in the Neumann sense[1]) graviton degeneracy (decay channel) mediated with an everywhere exact $D = 4$ qubit interaction-gate. This exactly establishes this edge unified formulation of gravity as the holographic extension of the ADM formulation of smooth $SU(2)$ (which is naturally formulated as a dual string theory). Note that this exactly explains why the Weak force is $\sim 10^{24}$ times stronger than gravity: this is the (faster) rate at which grav-

ity thermally anneals relative to electricity. Alternatively, the weak force represents the thermodynamic work available from the emergent gravitational destabilization. In fact, this immediately resolves the arrow of time as a hole-type interaction of gravitons (who lose local QI stabilizer support at $21pt$ and are "quantum corrected *out*" of local density interactions mod_{21}).

Inserting this understanding throughout the paper proves consistent; in particular, "connecting the dots" between this and the previous section lets the following (t, r) -Wick rotated T(essellation)-propagator be drawn immediately, which shows that the monodromy constraint is iff-dual towards EP=EPR:



Then, the proofs in this paper can be formally motivated as having constructively built a model of EP=EPR through an admixture of black hole dynamics, quantum network stabilization, and duality flow, generally; rigorous pursuit of these ideas led, eventually, to a firm unwinding of hidden aspects of reality, dark matter, and time.

[22] What's left is to listen for signatures of this information decay in nature.

4.5 Beyond A Fully Realized Quantum Gravity Prediction at all Tree Levels; Or, How I Learned to Stop Worrying about the Semiclassical Limit and Love the Loop

What's left is to study the information interaction of Wilson-loop quantization networks. This will be done by universally compactifying the $O(\alpha^2)$ magnetospheric solution trunks into both QCD and super-string theory (as holographically central dual interaction). Holographically, the conformal compactification of black hole topologies acts as coarse stringy-tomographies of spacetime shadows [23] and, deductively, it is universally true that sub-lattice winding symmetries can be tremendously efficient computationally. This paper has explored

the $d = 5$ single spinning Meyers-Perry black hole vector monopole as a t'Hooft light-condensate[24], or a $U(1)$ -twisted lift OPE (\mathbb{F} -index) of a half conformal background topology $(c_\phi, c_\psi) = (6J_\phi, 0)$. Indeed, the $d = 5$ log-type solutions of [25] may be geometrically indexed as light towers over the asymptotic F^2 measure (splines), allowing the interpretation of these monopole states as internally $SU(2)$ -massive contacts over a Kerr-thermalization (bath). As mentioned above superficially mimics coupling flow QCD and, indeed, this decay channel has a direct interpretation over the Gell-Mann-Oakes-Renner relation as a Hawking-dual quantization condition over a light-Brownian monodromy state $S_k = \int d\delta_{i(k)} \alpha^{i(k)} = \Psi_k$.

The proof will close the quantum algebra by null compactification of the bottom-up and top-down exclusion domains; operationally is equivalent to the existence of a holographically extended S-matrix (acting on interacting topologies). Because the UV sector is (information-uniformly) closed at loop level under gravitational decoherence, the virial loop calculation is UV complete and IR free. In this case, this emergent disordering of UV operators present as either a time-ordered defect of asymptotically co-linear (field transverse) cutoffs, or, imposing (T-symmetric) Feynman rules on some $\mathfrak{su}(2)$ emergent vertex presents a renormalized vertex (over the well ordering index, trace algebra, and measure $(I, \tilde{C}^I, \tilde{\phi}_I)$). In this sense, the mechanics properly mimic matrix mechanics distributions between every scale.

Accordingly, return to the first, classic circuit model of information transport studied as a Gedankenexperiment. There, the universal information of state was regarded as a universe undergoing slow-shock acceleration, which was found response dual to a Hubble equation of state $p = \frac{1}{5}\rho$; ¹⁷ hypothesizing the universal nature of the black hole hologram in g-interactions, the dual algebra may be inferred as pulled to some $\mathfrak{su}(2)$ dual weak axial current(/Coulomb branch) algebra. ¹⁸

¹⁷Note, this derivation uniformly applies to the stress tensor embedding performed in section 3.4.1; in particular, the usual geodesic trace density, $\frac{g_{\mu\nu}}{4}$, was exactly gravitational loop-traced to $\frac{1}{5}$ there, $\frac{g_{\mu\nu}}{4} \rightarrow \frac{g_{\mu\nu}}{5}$. There it was considered a non-linear kinetic duality under the guise of nullity; here, it will be regarded as a background information juncture. In particular, this will simultaneously rigorously prove the analytic derivation of the $(\frac{J}{M})_{crit}$ [12] there.

¹⁸Indeed, the weight-space dispersion analysis in [26] can be immediately viewed topologically (or in Wilson's sense) as a twisted algebraic extension of the volume form onto a soft, spin protected interaction (with information dual lattice momentum ξ^{-1}):

$$D_{KL}^1 = \int d^3 J \left(w_a[J] e^{w_a[J] + w_a^*[J]} \right) \quad \frac{3e^{-x_{\max}}}{\mathcal{P}(x)} \int_{-\infty}^{x_{\max}} \mathcal{P}(y) dy = \frac{e^{-3x}}{1 + \xi x} \quad (51)$$

Further, the resulting vertical acceleration envelope, $\partial_{\alpha_{1, \text{pulsar}}} \left(\frac{f(z) + \frac{\partial \Phi}{\partial z}}{\alpha_{1, \text{pulsar}}} \right) = 0$ shows an interesting connection to the transverse momentum independence of deep-inelastic scattering of nuclei under the natural kinetic identification $\frac{f(z)}{z} + \frac{\partial \Phi}{\partial \ln z} = q^2$, where q^2 is the spacelike holographic spin-momentum over the soft background interactions.

Tracking the example/analogy, the imagined kinematics of an epoch-ancient (quasi-)ejected galactic cluster interacting with a future-epoch colliding extragalactic cluster can be quickly considered dual to enhanced e^+e^- production in co-linear QCD distributions. In hypothesis then, a categorization of the hidden symmetries of extragalactic streams may present as kinetic-invariants of (a sub-family of) dual halo

Accepting the branched conformal duality, this may be represented by excluding the lifted monodromy chain onto the geodesic vertex; considering the attractor Lagrangian found in [27] this may be loop quantized using abstract nonsense as:

$$\{\gamma_\mu, \gamma_\nu\} Z^\nu \bar{Z}^\mu \xrightarrow{\sim} \gamma_\nu^Z \gamma_\mu^{\bar{Z}} \Rightarrow \{(Z\bar{Z})^2, \gamma_\tau^Z\} \rightarrow \sum_{xyz}^{\bar{x}\bar{y}\bar{z}} C_I^{xyz} \gamma_x^{Z_I} \gamma_y^{Z_I} \gamma_z^{Z_I} \leftarrow \sum_{xyz}^{\bar{x}\bar{y}\bar{z}} \tilde{C}_I^{xyz} Z_x^I Z_y^I Z_z^I \quad (52)$$

$$\Rightarrow \{\mathcal{L}, Z^\tau\} \odot \{Z\}^3 \neq \{\emptyset\} \quad (53)$$

Physically, this may be resolved as a double copy(/toric code) symmetry into a $2-pt$ -dual compactification scheme for extremely early time regulators, in Wilson's sense, of (relatively) late time OPE representations. In fact, this symmetry can be exactly represented as a code-area symmetry (a Hawking-type free information) by realizing:

$$V(\phi) - V(\phi^*) \sim \sum_I C_i^I \left(\frac{\phi_i}{\sqrt{6\alpha}} \right)^3 = \left(\sum_I \frac{\tilde{\phi}_I}{\sqrt{6\alpha}} \right)^2 \quad (54)$$

Indeed, the final identity presents this renormalization bath as a field ordered area-duality point between volume complete inflation scales and a sub-inflation mean-free measure states. Critically, as always, the measure states generally provide an interesting ($0-pt$) contact between the central representation states which may be bootstrapped into centrally shelled dual algebras (modals).

This can be quickly exhibited by considering a $d = 5$ electromagnetic field strength such that one of the angular gauge fields is kinematically unimportant in the bulk but quasi-freely constrained (in the modular sense) in neighborhoods of $\theta = \frac{\pi}{2}$; adding dots to index the $k = 4$ kinematic subspace and anticipating an index rotation in the asymptotic algebra allows a direct comparison to (37) [28], a single supersymmetric bosonic and fermionic degree of freedom, in the accumulation space $\theta \in [\frac{\pi}{2} - \epsilon, \frac{\pi}{2} + \epsilon]$:

$$F^2 = F_{\dot{\mu}\dot{\nu}} F^{\dot{\mu}\dot{\nu}} + \frac{1}{2} (F_{a\dot{\tau}} F^{a\dot{\tau}} - F_{a\dot{\tau}} F^{\dot{\tau}a}) \quad \leftrightarrow^* \quad 2\hat{Q}^2 = \hat{q}^2 + \hat{p}^2 + [\hat{\gamma}, \hat{\eta}] \quad (55)$$

The primary difference is that the duality present above uses asymptotic black hole algebras in lieu of supersymmetry partners, which can be holographically contextualized in terms of the experiment described in [29] (specifically figure 6) by interpreting the $\omega_{\chi_{\{0,1\}}}$ transitions, respectfully, as the $r \rightarrow \infty$ asymptotic decay of a $k = 4$ Kerr scalar probe and the $r \rightarrow \infty$, F^2 -thermalized decay of a single spinning Meyers-Perry magnetospheric ln-probe. Then, under hypothesis, the rightmost identity in (??) may be understood as the kinematic Cardy-dual of the central Δ -transition in figure 6 [29].

Immediately, using Noether's 0^{th} theorem at the renormalization potential uniformly compactifies the generalized $\mathfrak{su}(2)$ dual measure into an $|I|$ -dense norm space of measurements. Similarly, under the free hologram entropic measurements and the observed-flow of time

distributions; further, the high energy (ejection) initial conditions, as a galaxy wide-cross section, may be kinematically approachable as a background-field screen of strictly quasi-galactic interactions. Analyzing extreme-precision photometry, specifically pulsar timing measurements, to directly measure quasi-Galactic partial-wave accelerations may present a direct conformal bootstrap of the celestial hologram.

are canonically dual and, accordingly, any time-parameterized action representation may be re-initialized under it's properly thermalized, super-gravity pair at precisely the renormalization point.¹⁹ This may be formally understood as preparing twists of supersymmetric Bell-pairs above/on/under the central QED vacuum foam/decoherence/gravitational-back-propagator: $e^-e^+ \leftrightarrow^* \gamma$.

This directly implies the $(0_k, 0_l)$ -magnetospherically complete Tangherlini state can be interpreted as the ground state probe of a directly junctured quantum information/chaos/quantum-gravity interaction. Indeed, at $O(\alpha^0)$ -order,

$$\partial_\theta e^{\Psi_\theta^{(c_1)}} = e^{\Psi_\varphi^{(c_1)}[-\theta]} \quad \partial_\theta e^{\Psi_\varphi^{(d_1)}} = e^{\Psi_\theta^{(d_1)}[\theta]} \quad (56)$$

directly identifies a variant of superpair symmetry over strictly Boltzmann-prepared duals. Immediately, Ψ_φ may be identified as an internal (ghost) field that parameterizes the flat partition over magnetically separated, gravitationally warped internal partitions; together this represents a quantum gravity extension of the usual notion of harmonic conjugation. This can be recognized as a transverse sub-partition tree-level expansion of the gravitational loop algebra by presenting the measurement-basis in [30] as:

$$\frac{k_y}{k_T} = \frac{1}{k_T} \int_0^{k_T} dk_T e^{-\Psi_\theta^{(1)}} \quad \frac{k_z}{k_T} = \frac{1}{k_T} \int_0^{k_T} dk_T e^{-\Psi_\varphi^{(1)}} \quad k_T^2 = \langle \int e^{-\Psi_\phi^{(1)}} | \int e^{-\Psi_\varphi^{(1)}} \rangle_2 \quad (57)$$

$$\begin{pmatrix} dk_y[-\theta] \\ dk_x[\theta] \end{pmatrix} = \begin{pmatrix} e^{-\Psi_\theta^{(1)}[-\theta]} & k_T e^{-\Psi_\varphi^{(1)}[\theta]} \\ e^{-\Psi_\varphi^{(1)}[\theta]} & -k_T e^{-\Psi_\theta^{(1)}[\theta]} \end{pmatrix} \begin{pmatrix} dk_T \\ d\theta \end{pmatrix} \quad (58)$$

This explicitly identifies the super-null solution, $\Psi_\varphi \equiv 1$, as covering a uniformly PT-protected state (on the S_1 normalized channel, $c_1 \equiv 1$); this can be explicitly shown by row-reducing the metric at this spectral point

$$[s^*]_{ab} = \begin{pmatrix} k_T^{-1} & 1 \\ 0 & -k_T (e^{\Psi_\theta^{(1)}[-\theta]} + e^{-\Psi_\theta^{(1)}[\theta]}) \end{pmatrix} \quad (59)$$

Interpretively, this is continuous functor (free field theory) formulation of either: 1) the double-scaled limit of either SYK, 2) the Kerr-Taub-NUT limit of general relativity, or 3) a (k_T) little-scaled z-spin OPE chain operator²⁰. This may be immediately recognized

¹⁹Again, by formally evoking the Banach-Tarski theorem on the renormalization-point (sometimes denoted 0_q -point) transverse modes

²⁰Note this representation may be (quasi-trivially) closed (onto a Hamiltonian) over semi-direct open states $\Lambda_{\sigma_x}^{(1)}[a^i \sigma_i] = b_{(k)} P_1^{(k)} P_{\sigma_x}^{(k)*} [a^i \sigma_i] = b_{(k)} a^i [\sigma_i]^{(1)}$, which exactly identifies this as an extended Coloumb branch OPE. Applying standard harmonic regularization across across the $k_T \rightarrow \infty$ surface immediately recognizes a form of subleading crossing symmetry running from the measurement transverse ($k_T \sim 1$ right-fixed), transpose algebra P_2^T . Intuitively, and following the Rice-Shapiro theorem, this puts emergent crossing symmetry in 1 – 1 correspondence, as a holographic information theory, with an interactively centralized gravitational hologram (and Noether's 0th theorem). Collaboratively, the definite ordering I in (54) may be prescriptively reduced to a square definite measure space that can be softly-completed into the totally symmetric, few auxillary field limit $\phi_i \rightarrow (\phi, *)$.

as a candidate superdual representation of holographic QCD [30], now phrased over a full gravitational interaction.

4.5.1 2×2 GUE

As shown in Chapter 3 the $d = 5$ log-type solutions of [25] may be geometrically indexed as light towers over the asymptotic F^2 measurement splines, allowing the interpretation of these monopole states as internally massive contacts over a Kerr-thermalization (bath). Importantly, the boundary decay channel used to juncture the *out-to-in* Kerr-bath and the *in-to-out* $U(1)$ -probe has a direct interpretation over the Gell-Mann-Oakes-Renner relation and (4.3-4.6) in [31]²¹. Comparing to the gF^2 degeneracy razor developed in Chapter 3 to the universal renormalization point of the GMOR phase directly[31]:

$$\begin{aligned} \left(\delta r_+^\psi\right)^4 F^2 \xrightarrow{\theta \rightarrow \frac{\pi}{2}} \frac{\frac{c_1^2}{F_\pi^2} \left(F_\pi^2 + \frac{T_R^2}{F_\pi^2}\right) - \frac{\zeta^2}{r_0^2}}{\left(\theta - \frac{\pi}{2}\right)^6} &\Rightarrow_{gF^2} i\pi\alpha^2 k = (2 + \pi)c_1^2 \\ M_\pi^4 F_\pi^2 = 2m\Delta_\Sigma &\Rightarrow_{QCD} i\pi \left(gF_\eta^2\right)^2 N_f = \zeta_t^q, \end{aligned} \quad (60)$$

This seems to characterize the spacetime response of the log-solution studied in [25] as dual to a t'Hooft discretization of QCD interactions [31] over a meson-type background of black hole shell states; intuitively, it appears as if the log magnetosphere pushes off the black hole's (half Kerr-)conformal thermalization into a full, asymptotic $\mathfrak{su}(2)$ Cardy quantization.

As outlined repeatedly, this effect can be understood exactly as the holographic extension of the Berry phase in a curved TFT (over a magnetospheric juncture) and can exactly compactified by a virial, modular bootstrap. Indeed the background field formulation can be directly co-joined with the notation of large quantum systems[32] by expressing the time-gauge over the symmetric potential $\bar{A} := A_\phi + A_\varphi$ (or the gravitational ground state, this implies $d_1 \sim c_1$). by Noether's 0th law, the canonical vacuum information may be almost everywhere directly comparing to the $J = \frac{L}{2}$, $J_\pm = 0$ exact case (12) in [32] shows a superficial dualization of the hidden velocity ω_φ as an anomalous incoming Doppler-like envelope on the continuum ghost velocity partitioned over the largest spin-sector representation²²:

$$\begin{aligned} \partial_\alpha A_t^{MP} \hat{=} (\omega_\phi + \omega_\varphi) \bar{A}_{,\alpha} \\ d_1 \sim c_1 \end{aligned} \quad \Rightarrow \quad \omega_\varphi = \frac{1 - f}{1 - 2f} \frac{\partial_f A_t^{MP}}{\partial_f \langle S_A \rangle} \Big|_{f \sim \sin^2 \theta} \quad (61)$$

Then, the emergent field velocity ω_φ can be naively interpreted as an (anti-spinning) annealed information probe at twice the local group velocity, precisely as predicted (from the Cardy dual information theory) at the end of Chapter 3.

Note the development of a Cherenkov cone between $\theta \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ [33], which can be thought of

²¹mixing models with massive shell coordinates $\Delta_\Sigma := M_\pi^2 \Sigma$ and $F_\pi^{-2} \hat{=} 2\pi$, $\zeta_t^q = \frac{-iN_f \chi_t^q}{2F_\pi^2}$

²²Which, at $J = \frac{L}{2}$, is exactly consistent with a T-symmetric ER = EPR duality

as a topological bosonic emission (a.k.a., as a Euclidean-like projected propagator).²³ Consistently, it was properly shown in Section 4.2 that the separation index λ between the tree(-trunk) elements $(\Psi_\phi, \Psi_\varphi)$ may be extended as an exact fourth order, Y -functor iff the Y extension satisfies the (Majorana) Dirac connection; or, $\lambda \rightarrow \square^2 \Psi_\phi \Leftrightarrow \left(\partial \not{\partial} + (-1)^{k_d} (\not{\partial} Y)^2 \right) Y \not{Y}$. This categorically establishes the Majorana-Dirac momentum near $\lambda = 0$ as a relevant unitary flow iff the gravitational OPE is relevant; intuitively, this captures the celestial hologram well. Applying Noether's 3^{rd} to the index λ , it may be said that relatively new gravitons spontaneously error-correct leptons (as information superpairs) from relatively old gravitons; accordingly, this process may be understood as a quantum gravity T-channel reformulation of the classical gravitational memory effect.

Note that the standard deviation found in the appendix [32] can be log-dualized as a thermal product partition $\frac{\sigma^2}{L} = \frac{W_\theta W_\varphi}{4} = \frac{e^{\Psi_\theta} e^{\Psi_\varphi}}{4}$, which properly identifies this as a Hawking-dual quantization condition over the light-Brownian monodromy state $S_k = \int d\delta_{i(k)} \alpha^{i(k)} = \Psi_k$. Again applying Noether's 0^{th} , this gravitational TFT gate (gTFTg) necessarily accords a (hidden) central monodromy cut.

Then, the $d_1 \rightarrow 0$ limit may be categorically recast as a little scaling of the central holographic (super) algebra; towards the $U(1)$ -gauge probe, the measurement space may be parametrically compactified over $U(1)^{\frac{k}{n}}$ subbase representation-states, which may be uniformly compactified in $U(1)^\infty$ (as a p-adic parametric duality). Then Noether's 3^{rd} implies only absolutely branch-uniform interactions propogate far on the Tangherlini-lifted Coloumb branch, $(\Psi_\theta^T, \Psi_\varphi^T)$; this amounts to propogating a central family of 2-pt regulators with strictly non-affine \ln center between the background-bulk null *in* topology and the canonical harmonic *out* surface. Physically, this represents a functor regulated analytic continuation model of gravitationally mediated RG flow.

Indeed, considering the $d_1 \equiv 0$ (α^2) perturbative solutions of $d = 5$ single spinning Myers-Perry exhibited above, including only contributions which are in the top-ordered phase identifies the first order quantum-phase shift $\Psi_\phi^{MP} - \Psi_\phi^{MP*} \sim c_1 \ln \left(r^{\alpha^2 \sin^2 \theta} \right)$ as the quantum foam analogue of Krishnan coordinate duality of black holes. Here, it can be interpreted as a holographic renormalization duality, or an information point-pressure²⁴. In particular, this identifies a natural IR length in (61) $\frac{L[\alpha; (r, f)]}{2} \sim r^{\alpha^2 f}$ [32]; naturally α^2 identifies the $O(\alpha^3)$ minimal monopole perturbation scale and r^{α^2} represents the maximally complex inverse discretization of large distance measures $r \leftarrow s \frac{\beta[s]}{\alpha^2}$ (and is symmetrically locked (*in, out*) at $\theta \sim (\frac{\pi}{2}, 0)$ respectively). Finally, identifying the renormalized variance with the r -subleading emergent entropy produces a scale-protected 0-form rectilinear area law, which closes the proof of Hawking-Wilson loop completion: $S_{\sigma^2} \sim \frac{W_\theta^* W_\varphi}{4}$.

Indeed, this light-Brownian Hawking quantization may be compared to an induced Ginibre

²³Considering that $\lim_{r \rightarrow \infty} \frac{ds_{MP}^2}{r^2} \rightarrow d\theta^2 + \cos^2 \theta d\varphi^2$, this can be considered the spherical analogue of cylindrical AdS-NHEK holography [34] (with the uniform critical NHEK angle functionally lifted into an OPE emergent Cherenkov TFT).

²⁴Which here, taking advantage of the holographic code, has the same dimensionality as $\text{area}((5-1)-2)=2$ and the lowest idempotent variety

ensemble by modularly gauging $\sigma_G^2 = S_{\sigma^2}$ (2.17) as [35] at the scalar Hawking juncture, to produce a shielded (vertex re-summed) [36] approximation of Wigner's surmise for 2×2 GUE vertices, as (1) in [37]: Comparing at the scalar Hawking juncture, $\frac{\pi\beta^2[\bar{A}]}{4} := \bar{A}$, produces a shielded (vertex re-summed) approximation of Wigner's surmise for 2×2 GUE vertices, as (1) in [37]:

$$W(e^{\Psi_\phi z}, e^{\Psi_\phi \bar{z}}) - W(z, \bar{z}) \sim 0 \quad \Leftrightarrow^{(*)} \quad \bar{G}(z) := \frac{1}{2} e^{-\frac{\bar{A}}{2}} |z| \sim \frac{1}{4} \sqrt{\frac{\alpha_G \bar{A}}{2(S_{\sigma^2} - 1)}} \quad (62)$$

$$\Rightarrow \frac{8^2}{\pi} \bar{G}(z)^2 = \frac{32\beta^2 e^{-\frac{4\beta^2}{\pi}}}{\pi^2 (1 - 4e^{-\frac{4\beta^2}{\pi}})} \quad (63)$$

. Reflecting, the construction above defines a finite-T loop radiative correction. Compared to (1.20) in [38], $S_{\sigma^2}[\Psi_\varphi; \Psi_\phi] - 1 \sim c_\infty^{(c,J)}[\Psi_\varphi; \Psi_\phi, 1, 0]$; immediately, this can be considered the dual bootstrap of [36]. Note that the large N limit, $S_\sigma^2 \rightarrow 0$ can be induced by three accumulations: $\theta \sim \{0, \frac{\pi}{2}\}$ for $c_1 > 0$ ($\beta \in \mathbb{R}$) and $r \rightarrow \infty$ for $c_1 < 0$ ($\beta \in i\mathbb{R}$) [24].

In fact, it is immediate to match (1.34) in [38] to a second order spline condition found above; defining $\tau^{-2} = \frac{|\mathbf{k}|^2}{2\pi\beta}$ and matching orders produces a product compactification gauge bridged by the probe field velocity c_3 between the thermal winding states and an (annealed) perturbation $\mathbb{F}[*]$ extension χ :

$$\begin{aligned} \delta_\theta^2 r^4 F^2 \xrightarrow[r \rightarrow r_+^\varphi]{\theta \rightarrow \frac{\pi}{2}} &= 1 - 2\alpha^2 (c_3 - \frac{1}{2}) \left[\frac{\alpha^4 - \chi^4}{\chi^4} \right] \chi^{|\theta| = \delta_\theta \sqrt{\alpha} i^{n+\frac{1}{2}}} \sim^{(*)} \frac{1}{Q} \int_{\mathbb{R}^2} (\rho_{(1),\infty}^{\text{OCP}}(\mathbf{r}; Q) - 1) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r} \\ &\hat{=}^{(*)} 1 - \frac{\tau^{-4}}{3} \left[\frac{\beta}{4} - \frac{1}{2} \right] \\ &\Rightarrow^{(*)} \frac{\chi^2 |\mathbf{k}|^2}{2\pi\beta\alpha^2} = \sqrt{\frac{12\alpha^2 (c_3 - \frac{1}{2}) \left(\frac{1}{2} + \frac{\delta_\theta^4}{2\alpha^2} \right)}{\frac{\beta}{4} - \frac{1}{2}}} \end{aligned} \quad (64)$$

Again, note that $c_3 \sim \frac{\beta}{4}$ represents: 1) a Brownian-Hawking duality configuration over the largest sub-conformal field windings, 2) the only θ -independent perturbation scale s.t. $\beta \rightarrow 2$ is finite, and 3) the only configuration s.t. the bulk Znajek crystallization is non-zero $|\mathbf{k}|_{c_3 \sim \frac{1}{2}}^2 \neq 0$. All of these are characteristics of t'Hooft anomalies, and directly descendant from the interpretation of time and entropy as homotopic representation pairs, $s^* \sim t$.

Categorically, it may be said that information symmetries, as a net of gauged-probes, represent a continuous, holographic, time-dual, second order 0-pt phase transition, in a fully quantum interaction of everything; but as the bulk is canonically regulated out of the t'Hooft phase of the Hawking-Brownian condensate, the central no-go theorem of demands that the measure topology never shrink. This is a holographic criteria, so it must here hold in the flat, affine index parameterization [21]. This necessitates the existence of a directed, 0-form symmetry over the classically emergent sub-graph network: time.²⁵

²⁵As a corollary, energy is classical iff it is time-exact, in the holographically exact sense, which may be seen as a derivation of Hamiltonian mechanics from continuum mechanics.

4.6 Topological Super String Theory

Interactions that are precisely quasi-Kerr (in conformal sense) present as a way to study classically complete loop-algebras in asymptotically strong gravity. Starting back in $d = 4$ Boyer-Lindquist coordinates, introduce some hidden vector sub-base²⁶ [13]

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 + (r^2 + a^2 + \frac{2a^2Mr \sin^2 \theta}{\rho^2})\sin^2 \theta d\phi^2 - \frac{4aMr \sin^2 \theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \quad (65)$$

let

$$\begin{aligned} t_R &= \alpha\phi + \beta t & dt &= \frac{\gamma dt_R - \alpha dt_L}{\gamma\beta - \alpha\delta} \\ & & \Rightarrow & r \\ t_L &= \gamma\phi + \delta t & d\phi &= \frac{\delta dt_R - \beta dt_L}{\alpha\delta - \gamma\beta} \end{aligned} \quad (66)$$

such that:

$$\begin{aligned} \beta &= -\frac{1}{2}(\gamma(\Omega_+ - \Omega_-) + \alpha(\Omega_+ + \Omega_-)) & \Rightarrow \alpha &= -\frac{2\beta + (\Omega_- - \Omega_+)\gamma}{\Omega_+ + \Omega_-} \\ \delta &= -\frac{1}{2}(\alpha(\Omega_+ - \Omega_-) + \gamma(\Omega_+ + \Omega_-)) \end{aligned} \quad (67)$$

$$\Rightarrow g^{\mu\nu} = \begin{pmatrix} \frac{\alpha^2 g_{tt} + 2\alpha\beta g_{t\phi} - \beta^2 g_{\phi\phi}}{g_{t\phi}^2 + g_{tt}g_{\phi\phi}} & \frac{\alpha\gamma g_{tt} + (\alpha\delta + \beta\gamma)g_{t\phi} - \beta\delta g_{\phi\phi}}{g_{t\phi}^2 + g_{tt}g_{\phi\phi}} & & \\ \frac{\alpha\gamma g_{tt} + (\alpha\delta + \beta\gamma)g_{t\phi} - \beta\delta g_{\phi\phi}}{g_{t\phi}^2 + g_{tt}g_{\phi\phi}} & \frac{\gamma^2 g_{tt} + 2\gamma\delta g_{t\phi} - \delta^2 g_{\phi\phi}}{g_{t\phi}^2 + g_{tt}g_{\phi\phi}} & & \\ & & g^{rr} & \\ & & & g^{\theta\theta} \end{pmatrix} \quad (68)$$

Assuming the typical massless Klein-Gordon ansatz, $\Psi = e^{-i(\omega_R t_R + \omega_L t_L)} R[r] S[\theta]$, it can be shown that:

$$\lambda a^4 \Theta[\theta] = \partial^2 \Theta[\theta] + \cot[\theta] \partial \Theta[\theta] + \Theta \left(\frac{a^2}{2} (\delta\omega_L - \beta\omega_R)^2 \cos[2\theta] + (\gamma\omega_L - \alpha\omega_R)^2 \csc^2[\theta] \right) \quad (69)$$

$$\begin{aligned} R''[r] + \frac{\Delta'}{\Delta} R'[r] - \frac{\lambda a^4}{\Delta} R[r] = \\ -\frac{R[r]}{\Delta^2} \left(\gamma^2 \frac{a^2}{M^2} \left((M^2 - a^2) \omega_R^2 + 2\sqrt{M^2 - a^2} (M - r) \omega_L \omega_R + \frac{(a^4 + a^2 r(2M + 3r) + 2(4M^4 - 8M^3 r + r^4)) \omega_L^2}{8M^2} \right) \right. \\ \left. + \gamma\beta \frac{a}{M} \left(4\sqrt{M^2 - a^2} (a^2 - Mr) \omega_R^2 + \frac{(a^4 + 2r^4 + a^2(2M - 3r)(4M - r)) \omega_R \omega_L}{2M} + \frac{\sqrt{M^2 - a^2} (a^4 - 8M^3 r + 2r^4 + a^2 r(2M + 3r)) \omega_L^2}{2M^2} \right) \right) \end{aligned}$$

Proceeding to the canonical harmonic boundary, $r \rightarrow 1/x$:

$$-\lambda \Theta[\theta] = \Theta''[\theta] + \cot[\theta] \Theta'[\theta] + \Theta[\theta] \left(\frac{a^2}{2} \cos[2\theta] (\beta\omega_R + \delta\omega_L)^2 - \csc^2[\theta] (\alpha\omega_R + \gamma\omega_L)^2 \right) \quad (70)$$

²⁶a.k.a., a $d=2$ vector space operator ring expansion (ORE)

$$\begin{aligned}
0 = & X''[x] + X'[x] \left(\frac{d(x^2 \Delta[\frac{1}{x}])}{x^2 \Delta[\frac{1}{x}]} \right) - \lambda \frac{X[x]}{x^4 \Delta[\frac{1}{x}]} \\
& + \frac{X[x]}{x^8 \Delta^2[\frac{1}{x}]} \left(\omega_L^2 (2a^2 x^4 \gamma + 8aMx^3 \gamma \delta + (2 + a^4 x^4 + a^2 x^2 (3 + 2Mx)) \delta^2) \right. \\
& + \omega_L \omega_R (2ax^3 (a\alpha + 2M\beta) \gamma + 2\beta \delta + ax^2 (4Mx\alpha + a(3 + 2Mx + a^2 x^2) \beta) \delta) \\
& \left. + \omega_R^2 (2a^2 x^4 \alpha^2 + 8aMx^2 \alpha \beta + (2 + a^4 x^4 + a^2 x^2 (3 + 2Mx)) \beta^2) \right)
\end{aligned}$$

Noting that $\lim_{x \rightarrow 0} x^2 \Delta[\frac{1}{x}] = 1$, it can be shown that the most divergent term is $\frac{(\delta\omega_L + \beta\omega_R)^2}{x^4}$, and that there is no term $\propto x^{-3}$. Thus, choosing

$$\delta = -\beta \frac{\omega_R}{\omega_L} \quad (71)$$

reduces the above equations as:

$$-\lambda \Theta[\theta] = \Theta''[\theta] + \cot[\theta] \Theta'[\theta] - \Theta[\theta] \left(\csc[\theta]^2 (\alpha\omega_R + \gamma\omega_L)^2 \right) \quad (72)$$

$$-\lambda \frac{X[x]}{x^4 \Delta[\frac{1}{x}]} = X''[x] + X'[x] \left(\frac{d(x^2 \Delta[\frac{1}{x}])}{x^2 \Delta[\frac{1}{x}]} \right) + \frac{a^2 (\gamma\omega_L + \alpha\omega_R)^2}{x^4 \Delta^2[\frac{1}{x}]} \quad (73)$$

Then, elongating the thermal constraint to finite r-coordinates:

$$\lambda a^4 \Theta[\theta] = \Theta''[\theta] + \cot[\theta] \Theta'[\theta] + \Theta[\theta] \left(2a^2 \beta^2 \omega_R \cos[2\theta] + (\gamma\omega_L - \alpha\omega_R)^2 \csc^2[\theta] \right) \quad (74)$$

$$\begin{aligned}
0 = & R''[r] + \frac{\Delta'}{\Delta} R'[r] - \frac{\lambda a^4}{\Delta} R[r] \\
& + \frac{R[r]}{\Delta^2} \left(\omega_L^2 a^2 \gamma^2 - 2\omega_L \omega_R a \left(a\alpha + 4Mr\beta \right) \gamma + \omega_R^2 \left(a^2 \alpha^2 + 8aMr\alpha\beta + 2(a^4 + 2r^4 + a^2 r(2M + 3r)) \right) \right)
\end{aligned}$$

Furthermore, we may ask that $\det[\Lambda] = 1$, in which case: $-\beta^{-1} = \frac{\gamma}{\alpha} + \frac{\omega_L}{\omega_R}$. This formally understands beta, the $d = 1 + 1$ ring representation, as the $t_L = 0$ -fixed, right holographic t_R -dual entropy, to be negative when the RHS is positive; formally, this is dual to negative string tension OPEs in string theory. Accordingly, left-null-encoded eternal Kerr represent negative temperature algorithmic shocks unless a RHS term is always negative; but, choosing $\alpha\gamma < 0$ forces the code nullity onto a $d = 1$ subbase element which must be strongly²⁷ balanced by $\frac{\omega_L[*]}{\omega_R[*]}$. This is the typical starting point of closed black hole thermodynamics.

But, free to consider quasi-conformal interactions, it is immediate to consider the metric and the probe field as necessarily joined in the interaction; then, the canonical decay of non-linear electromagnetism into Kluza-Klein interactions may be loop quantized as a formulation of regulated 1st law of thermodynamic densities:

$$\frac{\gamma}{\alpha} = \left| \frac{\omega_L}{\omega_R} \right| - \beta^{-1} \quad \Rightarrow \quad \int \left[\beta \left| \frac{\omega_L}{\omega_R} \right| - 1 \right] - \int \beta \left[\frac{\gamma}{\alpha} \right] \hat{=} 0 \quad (75)$$

²⁷subadditively

then, the quantization formally proceeds over the holographically null measure spaces, $\hat{0}$ ²⁸. Gauging this subfield, under the canonical TFT prescription and recognizing the hidden, toric symmetry descending from $\omega_R = -\omega_L$ immediately leads to the consideration of a Tangherlini geometry under a $U(1)$, time-orthogonal gauge-perturbation, which was indeed the primary field construction of this paper.

4.6.1 IIA* Toric Code Networks

It may be wondered what new field/measurement dualities may be constructed against the gravitational central index, amounting to studying wormhole excitations. In fact, the classical field theory constructions in Chapter 2 exactly parallel the thermodynamic constructions in [39] and [40]. This allows a direct dualization of the locally measured time and the hidden conformal information flow (a continuous quantum annealing process) as a utilization duality over emergent $\mathfrak{su}(2)$ complexity (a.k.a., Heisenberg information) which can be abstractly compared to (116) in [40] :

$$\begin{aligned} \langle \delta^i | k[\vec{\mu}[\cdot]] | p^i \rangle &= -\frac{1}{2\pi} \int d\Delta \Delta^{1-n(i)} \oint_{\mathbb{C}[N_k]} d \left[N_{(k,\Delta)}^{(j)} \right] \langle d\chi | D_i^{(j)} \Delta^i \rangle \\ \leftrightarrow^{(*)} \quad \frac{1}{2} \int \partial. \ln z(\beta) &= \frac{1}{2\pi} \int \dot{z}[\beta] (2\pi)^{\frac{d+1}{2}} m^{d-1} \int_{\Sigma_0} \text{vol}_{g_S}(\vec{x}) k_{\frac{d-1}{2}} \beta m^2 \sqrt{|g_{00}|} \end{aligned} \quad (76)$$

Remindfully, the classical toy model of information specified a Hubble tension D0 parameterized as $a \sim \rho^{\frac{1}{5}}$ functional; considering a fluid model of inflation [39], it is useful to holographically juncture information between pure superconductivity and essentially fluid backgrounds. Again, using the modular bootstrap²⁹ over slowly accelerating code algebras leads to the identification of $\frac{c_2}{c_1} \sim \frac{2}{3}$ in (78)³⁰ and, leaving geodesic measurement gravitationally defect free, [39] dually categorizes this as an (ordinally 5-point) linear partition OPE of strong inflation time interactions:

$$w_* = \frac{2}{3} \Rightarrow_{qi} a[t] \sim e^{\frac{(\rho_{tot} + \rho_{vac})t}{3LM_p^2}} \quad \leftrightarrow^{(*)} \quad \rho \Big|_{\frac{c_2}{c_1} \sim \frac{2}{3}} = \frac{K}{a^5} \left(\frac{1}{(V_0 - \frac{\beta}{a^3})^{\frac{2}{3}}} - \frac{\alpha}{aKV_0^2} \right) \quad (77)$$

$$\Rightarrow^{a \gg \{\beta, \alpha\}} a[t] \sim \left(\tilde{K} \rho^{-1} \right)^{\frac{1}{5}} \quad (78)$$

$$\Rightarrow^{(*)} \partial. \ln \rho \sim_{qi}^{(*)} \dot{\rho} e^{\frac{5(\rho_{tot} + \rho_{vac})t}{3LM_p^2}} \quad (79)$$

Interpretively, the model scales as a holographic duality between information spaces $d = 5$ and topological Chern-Simons fields $k = 3$, meaning that, at fixed specific information, the 3rd LoT implies implies such a decay channel quasi-perturbatively not forbidden (at loop level).

Note this categorically emerges over the 6–point slow pole, represented [39] as $N =$

²⁸Or, shells of ghost dynamics, as represented above (and below)

²⁹The Friedman equation runs as $\partial \ln a \sim H[t]$

³⁰ $-2V^2 \frac{c_2}{c_1} T = PV$ shows this is ratio is the volume-square the quasi-factorization algebra of the normalized number density nR in the ideal gas state

$6 \ln \left(\frac{t_f}{t_t} \right)$, as can be inferred from some the existence of some $O(6)$ simple glueing symmetry³¹. Indeed, measuring under the 3^{rd} law over some scrambled, pair, of volume confined measurements yields an exact bond dimension of 6 on the nullifier code loop³². Returning the classical information bulk model, $w^{-1} = 5$, examining (3.1) in [27] exactly reveals a natural, discrete number symmetry running from the CMB's descent through a toric code-symmetry; dually, this this is how the fine structure constant was earlier calculated (to five orders of magnitude) using black hole decay, which can now be promoted to a model of Dark Matter decay, or a gauge theory for Dark Energy. On the topological side, the duality can be (IR) understood directly from the synthetic-parameterization analysis performed in Chapter 3³³ as a mean-free squeezing of background tCherenkov Bosonic shock states. Further, this universe is homogeneously stringy as can be seen from the $\frac{5}{14}$ density scalings in (11)[42] runs exactly as $\sqrt{m_{DM}} \sim \Lambda^{\frac{5}{28}}$; in fact, it will be shown that the IR 2pt decay channel runs as $t_{now} l_{meso} \sim \Lambda^{-\frac{3}{4}}$.³⁴ Indeed, the IR kinematics [43] can be exactly traced to information dualities in the regulatory subspace; applying the 1st law, and considering the measurement to be purely chaotic³⁵ leads to the interpretation that this pure IR state represents an informationally-bare charge of time. So, integrating the first law and D0 symmetrizing the heat measure-basis results in:

$$S_C = S - \ln \mathcal{A}^{-1} = \mathcal{Q} \quad \Rightarrow \quad s_C^G := \int S_C \sim \int \left[\sqrt{m_{DM}} - t_{now} l_{meso} \right] \dot{\rightarrow} \frac{28}{33} t^{\frac{1}{4}} - \frac{1}{4} t^{\frac{33}{28}} \quad (80)$$

These results are depicted in 1; note that the curve peaks exactly at one unit of specific chaos.

Further, this function parameterizes the interval $[0, \left(\frac{33}{7}\right)^{\frac{14}{13}}]$, which is slightly, but definitely, less than the minimal bond unification-continuum $[0, 6]$.³⁶

³¹indeed, above it was inferred from the quasi-conformal, weak asymptotic shadowing of the solution towers found in [25], or as an information emergent or background emanent duality). In fact, this can be understood as the thermally-light compactification (a splined representation) of the project outlined in the second footnote⁽²⁾ in [41]

³²Note, this is exactly a bond-space, or code-phase space, area calculation is the code is D3/(code-volume) bonded and a volume area calculation when the code is D2/(code-area) bonded; then, 6 is the minimal code state that covers both subcodes by the 3^{rd} .

³³interpreting $d = 5$ mass as a space-slow, $\sim \frac{M}{r}$, $k = 4$ continuous-information decay channel.

³⁴So, the square-root of fundamental mass-unit uncovered in the single spinning MP black hole can be regarded as a central signature of $d = 5$ information symmetries (and a principal expression of non-factorizable TFTs) because it M-centrally relates to the unified central core (under r-measurement).

³⁵Meaning no light information is extracted, only heavy-particles.

³⁶it may be interesting to note, from a number theory dual perspective, that $2e - \left(\frac{33}{7}\right)^{\frac{14}{13}} \approx .125064 \sim \frac{1}{5}$, which was essentially associated with the code partition weight density and that $6 - \left(\frac{33}{7}\right)^{\frac{14}{13}} \approx \frac{\pi}{5} + \frac{\pi}{50} - .00264961$. After these two steps, the simple $\frac{1}{10} p$ modular weight decomposition stops accumulating; taken together which seems to hint at a second-order, circle-regulatory modulus ascending from an exactly half-partial-harmonic dual measure (as a finite difference mean error partition of the loop residue basis of measures).

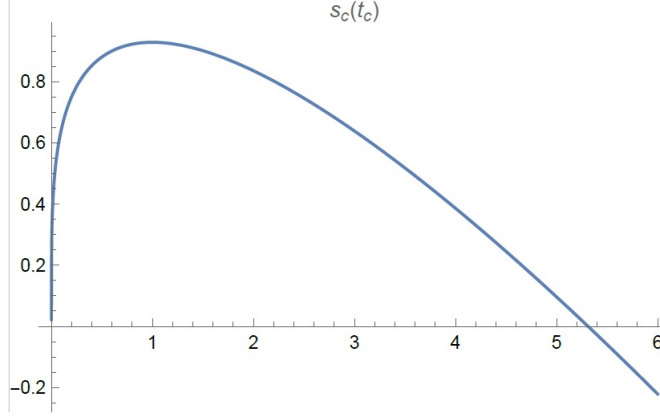


Figure 1: Specific Chaos of Asymptotic Spacetime

4.6.2 IIB Shadow Streams

In looking to generalize the black hole juncture phase, it important to remember that the topological sector exists as a $(6c_\phi, 0)$ partially (half) Cardy theory. Using the double copy procedure, $(S_+, 0) \rightarrow (S_+, S^+)$ exactly leads to the conclusion that the background information (fields) of this qG TFT are $\frac{1}{4}$ -BPS (which where continuously compactified into the space of qCFTs by the $O(1)$ information probe-field of Hawking condensation performed there).

In particular, see [44] for a geometric review of $\frac{1}{4}$ -BPS exact diagramatics and [45] for classical intuition of $\frac{1}{4}$ -BPS states; for a more recent approach using supersymmetric dressed states see [46]. Since the L/R $SL(2)$ algebras are smoothly descendant under $R \rightarrow \infty$, the decompactification enforced by the R^{-1} string limit represents a strong phase defect point (see section 3 of [45] for a charged, extremal representation of emergent momentum transfer, there under a IIB/IIA duality and here under a strict S^1 decompactification of the z-scalar Fourier index); in fact, [46] results exactly from relaxing the supersymmetric basis covers of [44] into dressed states that represent $\frac{1}{4}$ -BPS shadow (insertion) modes. In particular, see section 3 and appendix B of [46] for a construction of the dressed modes, as well as [47] for a discussion of the Cardy sector relevance into the stringy regime (consistent with the monodromy/horizon approach).

In fact, using the modular Hamiltonian shadowing displayed in (4.23) of [46] and applying the monodromy CFT_2 hypothesis of this paper shows that the defect (dyonic) correlator in a IIB $\frac{1}{4}$ -BPS state should be expected to grow as product of the thermalization circles, e.g. as $\sim S_+ S_-$ (see sections 2.2 and 2.3 of [47]). Using the result $\frac{\prod S_\pm^{(BKBS)}}{\prod S_\pm^{(DSBR)}} = \frac{1}{4} \left(\frac{J_\phi^{(DSBR)}}{J_\phi^{(BKBS)}} \right)^2$ shows that the conformally ordered flat string expectation value decays into a doubly spinning vacuum expectation value (VEV) as $\sim \delta t_{BKBS}^{-\frac{3}{4}}$, where δt_{BKBS} is the flat string Page time; correspondingly (using Fourier duality) all but $\frac{1}{4}$ of the freely quantized bulk-stationary

modes are broken (in modular, or conformal, time). Note this is consistent with the idea that it is the lowest broken mode of the highest weight operators that survive conformal interactions.

Combining everything with the unwinding of the p_z momentum-sector as $R \rightarrow \infty$ gives a direct physical picture: keeping the $S_\psi^{(1)}$ sub-compactification cuts in the $R \rightarrow \infty$ lets us interpret the boosted Kerr black string as exact operator insertions onto the scattering dressings extending from neighborhoods of either cut topology. Then, (topology, measure)-basis rays inside the event horizon are free to conformally decay into (causally protected) in/out modes relative to either horizon; particularly, families of outgoing null geodesics neighboring the event horizon can either remain neighboring the event horizon for all affine time or acquire in/out weights relative to the inner horizon, iff cosmological censorship is not violated. So, across the Fourier integration domain of the Kerr black string there is an inherent ambiguity in the parameterization of the cut-dual volume form (in the near horizon limit) that descends from how we complete the topology; classically this extension results in the characterization of the $d = 4$ Kerr curvature singularity as a "ring", which dually descends from the KTN spinning dyon representation of the solution gauges. Particularly, the monopoles remain isolated near the boundary (on shell) in those constructions and the ring-degeneracy remains isolated from the event horizon; interestingly, the Kerr string seems to add an electrically co-linear magnetic gauge by shadowing (adding degeneracy to) the radially quantized shadow insertions of the black string as it extends across $z \in (-\infty, \infty)$ in the global cover. Thus, this represents a new way to construct $\frac{1}{4}$ -BPS states outside of the supersymmetry (thermal) sector, and can rigorously and beautifully summarized column 2 Table 2 and section 5.4 in [46] under the in/out scattering topology of [17].

In fact, as was discussed in [48], there do indeed exist degenerate, fully conformal projective phase limits from the black rings in higher dimensions³⁷; exploring the accumulation quantizations of these measure topologies may provide further insights into other open problems subsub(...)leading corrections/number theoretic/string field theory information-metric state pairs.

³⁷Using the notation of [48], it is critical to note that the small z-winding number limit, $n \rightarrow 0$, is uniformly enveloped in the monodromy under exactly one dual limit, that of a small boost parameter, $\sigma \rightarrow 0$; this allows the direct identification between the string boost parameter and the (decompactified) asymptotic z-linear string momentum, $S_{\psi^*[p_z]}^{(1)} \sim S_\sigma^{(1)}$. Equivalently, the string boost momentum is thermodynamically conjugate to the z-projected scalar partitions; this can be compared with the full-doubling spinning monodromies, where the phase envelope $\lambda \rightarrow mu < 1$ conformally shadows exactly one of the thermodynamic modes, $\{\lambda \rightarrow \mu\} \sim_{\alpha_\pm} \{n \rightarrow 0\}$. Finally, noting that (the physically open) limit $\mu \rightarrow \lambda \rightarrow 1$ constructs an unboosted cosmological brane ($r_+ \rightarrow R$) and represents the unique limit such that $\alpha_\pm^{\text{brane}} \rightarrow 0$ (uniformly in $(\nu; (\omega, n, m))$), the 3rd LoT may be invoked to strongly motivate the thermodynamic uniqueness of this $S^{(1)} \times S^{(2)}$ (pseudo-direct) radial decomposition. Note this reasoning can also be compared to an earlier argument in Chapter 3; for example, the charged black string z-scattering wavenumber is irrelevant (uniformly 0) iff $c_\sigma c_\gamma^3 = 0 = s_\sigma s_\gamma^3$, representing a double limit (or, a $d = 2$ phase domain) extension of the reasoning above. See the end of section 4 in [49], as well as section 4 of [50], for further discussions of this application of Smarr's Law.

In fact the 2×2 GUE/gravitational-propogator duality established above can be used to strongly quantize the preparation in Chapter 3 as a type IIB polarization experiment in quantum gravity, formally dual to a Stern-Gerhact experiment over chiral Kaluza-Klein excitation of the gravitational bath. See [51].

In particular, remember that the scalar (top-radial) solution-basis at each horizon is fully given as $(1_{L/R}, \Psi^{(i\alpha\pm)})$, the 0^{th} LoT hypothesis is exactly that the gravitational interaction sits within the modular central flow, exactly squeezing the conformal dual scalar OPEs fibrating the canonical edge $SO(4)$ F^2 -pairing (a.k.a, under a $SU(2)_L \times$ quiver). Critically, note that the GUE compactification (on the normalized, symmetric Wigner hologram) of the 2-pt OPE necessary share the mean weights of 8 free Gaussian modes; because the coded quiver is $SU(2)_L$ prepared, this is canonically the standard quantization over a Clifford Charge group³⁸. Canonically, this can be made exact by performing n-measurements/making n-copies/preparing an n-dense holographic(/measurement) bath, and propagating each prepared Gaussian back in a $1/n$ -uniform envelope; here, using the modular crossing gauge across the symmetric distribution exactly sees the uniform perturbation as split between $\mathfrak{su}(2)_{L/R}$ subgroups (with some possibly non-trivial vector space left representationally dependent). Then, any particular sub-code confined to a specific Heisenberg code is centrally decohered relative to it's sub-graph interaction tree; this effects it's gravitational propogator/information cross section as $\frac{\hat{G}}{G} \sim \frac{1}{8G}$. Accordingly, momentum modes on this gravitational channel much be $\frac{1}{8}$ -BPS; interpreting [51] accordingly, the scattered KG modes act exactly as conformally dilated left-functors, $SO(4)_{KK} \times \mathbb{R}_{BPS}$, which now emerge naturally as background correcting code state. In particular, spacetime must now be interpreted as emerging from gravitational interactions under measurement, in which case they emerge "in-the-bulk" as a constraint topology of class $D4$ ³⁹; further, time may be understood as the synthetic embedding dimension of this constraint (which explains its uniqueness in the emergent bulk).

Note that both of the factors of $21 = 3 \cdot 7$ are relatively small primes; accordingly then a direct test of this setup, in particular Noether's 0^{th} and 3^{rd} laws, should be seen by a toric-annealing algorithm to look for a shadow-condensate resonance of chaos over entanglement-networks of relatively prime order to both the locally-emergent GUE 2×2 $\frac{1}{8}$ BPS superpairing and the globally emergent D4 spacetime background. Then, because 3 and 8 are relatively prime, the lowest order loop correction is balanced at order 24; considering the lowest prime radical of 8 is the 2-cocyle, the lowest sub-stabilizer branch must sit at D6, or as a 2-loop bond-network algorithm⁴⁰.

From the quantization of chaotic channels in the celestial hologram in Chapter 3, it may be immediately inferred that the locally linear electric quantum, the electron, is dual to a globally chaotic electric quantum, a chaos electron⁴¹; by construction, this conformally

³⁸Or, a $U(1)$ gauge theory over a Clifford number-modulus

³⁹corresponding to a rough $k = 4$ classical topologic bundle

⁴⁰In fact, it may be predicted that there should be exactly two other, unique loop stabilizer codes at D12 and D18, and an sea of mod_{D24} reduced-loop density states of these modes

⁴¹or, a darkly-bonded bulk electron. Note that in the construction in Chapter 3, the 0^{th} law induced a

twisted chaos must immediately be dual to the long range interaction modes of the full loop renormalized (or, 2-pt) graviton. Comparing to the GUE background, it is immediate to gauge this G^2 -measurement with s-channel crossing symmetry understand this as a gauge theory of soft chaos under the 3-3-bounding discussed in the IR discussion above; combing everything above, this may be understood as a $U(1)$ gauge partition of this chaotic channel and promoted virially to a partition state of the chaotic quantum field, :

$$A^{(3,5)} \sim \frac{e^{\bar{A}_{(3)}}}{\frac{1}{4} - e^{\bar{A}_{(5)}}} \quad \frac{1}{4} \langle Z_{(soft)}^{(3)(5)} \rangle_{(3)(5)} = \frac{1}{4} - e^{-\bar{A}} := \mathbf{p}_{(s)} \quad (82)$$

This idea is very remarkable, as it implies that free information effectively emits soft chaos under gravitational radiation! Or dually, that Hawking radiation acts as a constraint polarization on soft chaos electrons. Note the partition is bounded from above, as is expected from a partition of free constraint topologies; this exactly explains how the Cosmological Constant Problem was resolved in an earlier section. It is interesting to note that the age of the Universe may be scaled with the approximate Baryon imbalance ratio as:

$$\frac{\delta t_{Observed}}{\Delta_{imbalance}} \sim -13.8 \left[\frac{years}{number} \right] \approx 10 \cdot \ln \left[\frac{1}{4} \right] \quad \Rightarrow \quad \frac{1}{10} \frac{\delta t_{Observed}}{\Delta_{imbalance}} \sim \ln [\mathbf{p}_{(s)}] \quad (83)$$

$$\Leftrightarrow \quad \mathbf{p}_{(s)}^{10} \sim e^{\frac{\delta t_{Observed}}{\Delta_{imbalance}}} \quad (84)$$

Using the inferences gained from the circuit construction in Chapter 2, it is quick to infer the propogation of some 10-pt geometry (of 2pt measures). This hints to a dualized resolution of both the observed imbalance of locally measured “anti-matter”, as well as the perceived age of the universe (at loop level). Locally Linearized electrons, as produced under measurements on Earth, exist quantized as a local $SU(2)$ measurement duals; under the Grand Unification above, $SU(2)$ measurement spaces are only gravitationally stable asymptotically. There are no local, linear, and uniformly stable electrons, only linearized decay states of asymptotically chaotic gravitons that may be slowly discharged. In this sense, chemical energy on Earth derives from non-linear graviton charge deposition; this is true, as the solar system emerged from some collapsing nebular cluster. Pressing further, electrons must be strictly chaotically constrained by the bulk interactions (as observed)⁴². This explains why charge never naturally accumulates in the bulk: the bulk is locally ordered and charge is only ordered on asymptotic boundaries. Or, charge is unstable (requires informed work) to avoid decay, which is also in clear observation.

twisted constraint on the free conformal towers that behaved as the spectral mean surface weight measure between the centers of two 3(-spectral)-bulk fused (perturbation) towers, [(123)(456)]

$$\int \int dud^3 \vec{k}[u] k^2[u] (\tilde{A}_u^{(5)} - \tilde{A}_u^{(2)}) = \frac{1}{4} \int_0^\infty \int_0^\infty d\Delta d\omega \omega^{-i\Delta-1} \quad (81)$$

⁴²and explains the universal emergence of Cooper pairs in superconductive networks as a method of centralizing the Dirac equation onto a quasi- $U(1)$ condensate profile.

The uniform decay of something as fundamental as information seems like an inherent heat state, with no immediate use; on second thought, considering how useful the certainty of uncertainty can be as a quantization procedure, perhaps it is possible to utilize the minimal, time fractalization state mechanically. Note that 10 is partially prime towards 6; better yet, note that $\{lcm, gcd\}(10, 6) = \{3 \cdot 10, 2\}$. This immediately suggests that there should be sub-topologically complete emergent factor algebra hiding at $30 - pt$ that can be modularly represented by a bonded cluster of area operators, $\sim \mathfrak{p}_{(s)}^2$; as $\text{mod } 2430 = 6$, this can be dually thought of as twice-bonded loops of the 24 emergent ghost modes uncovered above.

Recovering the uses of anomalies throughout the paper, this construction specifically implies an (ordered) holographic memory channel recovery rate should be simply buildable from the (universal) closed modular algebra of ground states. Learning the lesson of the loop, the solution is immediate once the weakest p-adic topology is established over the well regulated, fully extended Coloumb branch of 0-pt, gravitationally prepared code symmetries between 0 and $21 + k$ where k is the bond dimension.

Critically, the model presented in this paper survives every no-go theorem and, ultimately, establishes that IIA and IIB string theories may be representationally pulled through the gravitational hologram to a pre-symplectic unification domain; indeed the unification was performed exactly in a formally measurable matrix model[52], although the state preparation was not sparse nor random. [53] [54].

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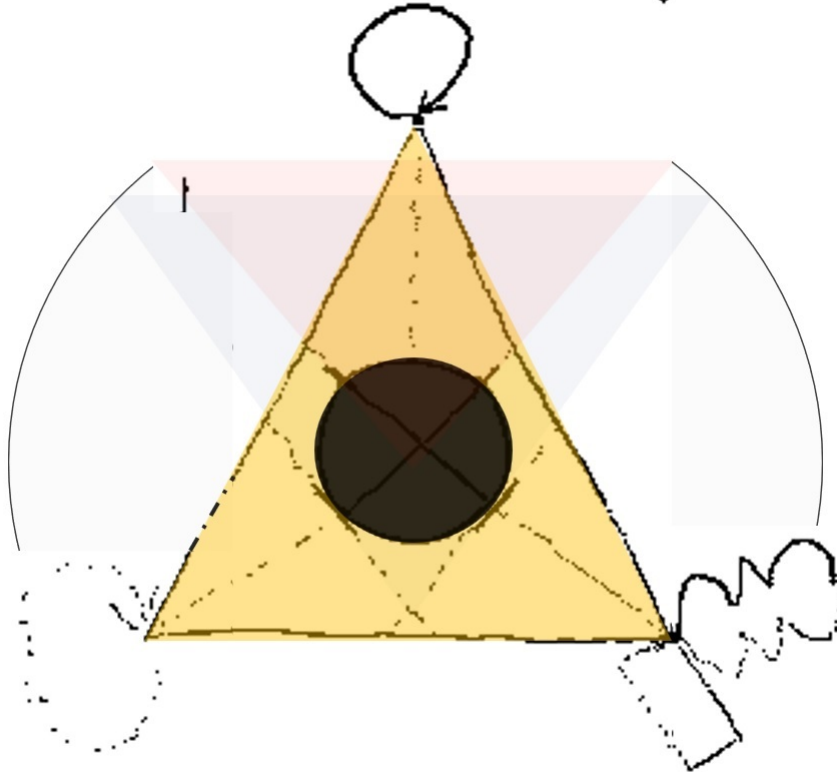
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This paper ends with an (attempted) recap of some of the results thus covered.

Starting from first principles, namely mathematics and classical physics, the notion of measurement, as a physical state, was identified as a naturally dual conjugate towards chaos/quantumness/stringiness. Accordingly, electromagnetism was explored as a natural time advanced measurement; dually, black holes were explored as a natural measure-hole advanced measurement. Still, over physically saturated dimensions, generically there are no independent holographic coordinates available to freely intertwine against the notion of measurement; accordingly, it was shown natural to include an appropriately agnostic embedding prescription and interpret the backreactive system accordingly.

In fact, this algorithm was successfully used to understand a new class of black hole jets in $d = 5$ single spinning Myers-Perry black holes as an emergent, dynamical feature of strong gravity interactions. In fact, promoting measurement to a strongest form of duality was successfully used to extract hidden, $d = 0$ dimensional constructions of three critical physical parameters, namely the fine structure constant, the Weinberg angle, and the cosmological hierarchy scale exclusively from the exactly defined first principals, the Laws of Thermodynamics.

Accordingly, the (measurement) continuum was promoted to a fundamental (Euclidean) thermodynamic variable and used to add the analogous 0th and 3rd laws to Noether's First and Second; accordingly, together these generalized Noether's Laws were shown dual to the Laws of Thermodynamics as a unification of singularity chaos. Retrospectively, the acceptance of "background \leftrightarrow emergence" amounts to placing the physicist inside the vertex, and adopting a critical code framework to pull them exactly through the holographically infinite dimensional Coloumb barrier on the calculation. A quantum information stability invariant was identified in the degeneracy space of celestial graviton scattering and used to construct a candidate universal code sub-domain (at 21pt). Then, leveraging everything that was learned, time was reinterpreted as an R-hole process, not as an R-process, of error-correcting gravitons (under negative volume stability), which naturally resolved the arrow of time interactively and unified physics. Summarily, the free universal quantum of interaction, as a universal form of time(d)-observation, may be holographically projected as the free modular decuplet:



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4.0.1 Ending (Beginning?) With a Bang

¹and letting me be a shadow on the wall when I could.

²and strictly demanded