Combination rule and last Fermat's theorem

Regla de combinación y último teorema de Fermat

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ABSTRACT

ENGLISH

The central objective of this document is to reason regarding the combined use of two or moremethods to solve a mathematical problem.

To facilitate understanding I present the topic with the help of a known problem. I have chosen the Fermat's Last Theorem because the polynomial that expresses it has few monomials and few variables.

ESPAÑOL

El objetivo central de este documento es razonar respecto al uso combinado de dos o más metodos para resolver un problema matemático.

Para facilitar la comprensión presento el tema con ayuda de un problema conocido. He escogido el último teorema de Fermat porque el polinomio que lo expresa posee pocos monomios y pocas variables.

Este documento está disponible en español con el título Regla de Combinación y Último teorema de Fermat.

Part 1 - Combination Rule

Let's think of a mathematical problem that can be stated in two or more ways. Let's think in three, for example.

Regarding a variable v , each statement delimits the interval of possible values and the three dimensions are not equally restrictive. This detail is irrelevant when we use the methods separately, without combining them.

What happens when our intention is to combine them? The two least restrictive dimensions allow values of v prohibited by the most restrictive bound. Is this irrelevant for the combination? Or is a rule necessary to ensure correspondence and coherence ?

Let's symbolize f_1, f_2, f_3 to the forms of statement. Suppose f_1 sets $v < 1$, f_2 sets $v < 2$, and f_3 sets $v < 3$. The form f_1 isinapplicable for $v \geq 1$, but the other two are applicable. We can combineall three nonchalantly or do we need to respect some norm of coherence and correspondence?

Let's mention something obvious. The combination is possible if we apply the forms less constraints within the interval $v < 1$ and we exclude the rest. That rest does not belong to the combination of the three forms, because it excludes f_1 . Applying the least restrictive forms within the interval allowed by f_1 is an essential criterion to have correspondence and coherence in the combination.

Is what has been explained valid in logical terms ? Assuming affirmative answer I state the rule explicitly.

COMBINATION RULE

↓

When we combine two or more ways of posing aproblem, the more restrictive way sets the validity interval of the combination.

Part 2 - Application Example

I show as an example the combination rule applied to Fermat's last theorem, which admits two elementary approaches, one arithmetic and the other geometric.

I write Fermat's equation.

$$a^n + b^n = c^n \tag{1}$$

o Geometric Approach

The triangular representation begins with $n \geq 2$, since $n = 1$ corresponds to the sum of segments of the same line.

$$\boxed{n = 2}$$

I repeat the original equation of the problem.

$$a^n + b^n = c^n$$

I factor

$$a\ a^{n-1} + b\ b^{n-1} = c\ c^{n-1}$$

I divide both members by c^{n-1} . Then I simplify.

$$a\ \left(\frac{a}{c}\right)^{n-1} + b\ \left(\frac{b}{c}\right)^{n-1} = c$$

Exchange members.

$$c = a\ \left(\frac{a}{c}\right)^{n-1} + b\ \left(\frac{b}{c}\right)^{n-1} \tag{2}$$

The following pair of relations is evident.

$$a < c\ \Rightarrow \frac{a}{c} < 1 \tag{3}$$

$$b < c\ \Rightarrow \frac{b}{c} < 1 \tag{4}$$

Relations (3) and (4) applied to (2) determine the following.

$$c < a + b$$

Exchange members.

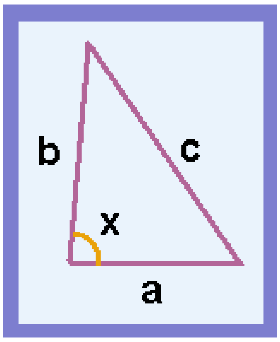
$$a + b > c \tag{5}$$

The following pair of relations is evident.

$$a + c > b \tag{6}$$

$$b + c > a \tag{7}$$

The inequalities (5, (6) and (7) are characteristic of a triangle.



By the cosine theorem we have the following.

$$a^2 + \ b^2 - 2\ a\ b\ \cos x = c^2 \tag{8}$$

Part 3 - Combined Arithmetic and Geometry

In the minimum triad a, b, c there is no common factor between numbers of the triad. So a and b cannot be the same. Symbolizing b to the greater we have the following.

$$\boxed{\frac{a}{b} < 1 \qquad \leftarrow \ \textit{arithmetic condition} \ \ (9)}$$

This condition can be expressed as an equation that satisfies a condition. I write the equation.

$$a = \frac{b}{q} \tag{10}$$

I write the condition.

$$q > 1 \tag{11}$$

In (8) replace a as indicated by (10) .

$$\left(\frac{b}{q}\right)^2 + \ b^2 - 2\ \frac{b}{q}\ b\ \cos x = c^2$$

group

$$\left(\frac{b}{q}\right)^2 + \ b^2 - 2\ \frac{b^2}{q}\ \cos x = c^2$$

I extract common factor b^2 .

$$\boxed{b^2 \left(\frac{1}{q^2} + 1 - \frac{2}{q} \cos x \right) = c^2 \quad (12)}$$

For $n \geq 2$ is $\cos x \geq 0$. I prove it starting at (8).

$$a^2 + b^2 - 2\ a\ b\ \cos x = c^2 \tag{8}$$

Clearance $\cos x$

$$\cos x = \frac{a^2 + b^2 - c^2}{2\ a\ b} \tag{13}$$

For $n \geq 2$, a procedure analogous to the procedure that allowed us to prove $a + b > c$ leads to the following.

$$a^2 + b^2 \geq c^2$$

So

$$a^2 + b^2 - c^2 \geq 0$$

I divide both members by $2\ a\ b$. Then I solve the second member.

$$\frac{a^2 + b^2 - c^2}{2\ a\ b} \geq 0 \tag{14}$$

I multiply M.A.M. (13) and (14) . Then I simplify.

$$\cos x \geq 0 \tag{15}$$

Exchange members in (12) .

$$c^2 = b^2 \left(\frac{1}{q^2} + 1 - \frac{2}{q} \cos x \right) \tag{16}$$

The inequality (15) implies in (16) the following.

$$c^2 \leq b^2 \left(\frac{1}{q^2} + 1 \right)$$

I divide both members by b^2

$$\frac{c^2}{b^2} \leq \frac{1}{q^2} + 1$$

Condition (11) expresses $q > 1$. So the following remains.

$$\frac{c^2}{b^2} < 2$$

I raise both sides to the power $\dfrac{n}{2}$.

$$\left(\frac{c}{b} \right)^n < 2^{\left(\frac{n}{2} \right)}$$

$$\left(\frac{c}{b}\right)^n < \left(\sqrt{2}\right)^n \tag{17}$$

I repeat equation (1) here.

$$a^n + b^n = c^n$$

I divide by b^n both members. Then I simplify.

$$\frac{a^n}{b^n} + 1 = \left(\frac{c}{b}\right)^n$$

exchange members

$$\left(\frac{c}{b}\right)^n = \frac{a^n}{b^n} + 1 \tag{18}$$

I replace the first member of (17) as indicated by (18) .

$$\frac{a^n}{b^n} + 1 < \left(\sqrt{2}\right)^n$$

$$\boxed{\frac{a^n}{b^n} < -1 + \left(\sqrt{2}\right)^n \quad (19)}$$

Inequation (9) expresses the following.

$$\frac{a}{b} < 1$$

I raise both members to the power n .

$$\frac{a^n}{b^n} < 1 \tag{20}$$

Condition (20) is the arithmetic condition, expressed to the n^{th} power.

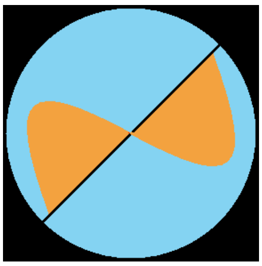
The arithmetic inequality (20) is more restrictive than the geometric inequality (19) . The combination rule explained in Part 1 of the document informs that we must use (19) within the interval bounded by (20) . This means excluding in (19) the values of n that imply $\frac{a}{b} > 1$. Consequently the region $n > 2$ is excluded.

Part 4 - Reflection

- The general conviction is to suppose that by elementary means a demonstration of the last Fermat's theorem is impossible.
- Our obligation is to find errors in the exposed development.
- We could attack the novel resource, which is the combination rule. Maybe it's a lie that we need to respect it to have correspondence and coherence in the combination of methods.
- Another possibility is to attack the geometric representation, in case it is inadequate.
- We could object to the use of inequalities, in case of generating uncertainty.

- Another attackable detail could be the artifice constituted by (10) and (11) , which translates the information from inequality (9) to a conditional equation.
- That translation leads to a quadratic equation, which is (12). We might object the second degree by invoking n^{th} degree of Fermat's equation.
- Surely there are more attackable details.
- A refuted detail would suffice to reject the proposition shown in this document and, consequently, discard the combination rule.
- The reader is invited to carry out this task.
- Personally I have not been able to do it.
- Until it is performed, I will assume that the combination rule is valid.

Pierre de Fermat (1601 - 1665)



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