The photon is imagined to be a quantized, transverse electromagnetic wave (TEM wave). Proceeding from that origin, novel and compact expressions for its voltage, current, charge, fields, flux, spin, and other parameters are developed herein a consilient system with few assumptions. This voltage plays a crucial role in the quantization criteria for the transverse **A**, **E**, and **B** fields of the photon, the foundational assumptions of Quantum Electrodynamics, and leads to a further question.

The Voltage of the Photon

A simple question led to this development [Bishop, 2012]. Since the photon has an electric field, **E**, measured in volts per meter in the direction of its field lines, what is the voltage of the photon? The answer turns out to be simple and elegant, with the assumption that the wavelength of the photon is the length of the photon. How long is a photon? How many times does it wiggle, if it wiggles at all?

From that starting point, many of its expected electromagnetic properties can be written out in closed form. Its energy is found to be divided evenly between its electric and magnetic fields, as with the classical Transverse ElectroMagnetic wave (TEM wave). The model is compatible with The Forbidden Equation, $i = qc$, [Bishop, 2016], as it has to be to stay within the confines of conventional electromagnetic theory. A frequency-invariant pair of derived quantum-electromagnetic properties, which are physical constants, can be associated with its intrinsic spin.

The photon began its conceptual career as a quantized TEM wave, confined with others of its kind in M. Planck's cavity oscillator [Planck, 1920a]. In order to derive analytic expressions for its electric and magnetic fields, another assumption is introduced, an analog of that analytic cavity. From there, the voltage of the photon is found to be an essential component of the foundational quantization rules.

Derivation of the Electromagnetic Parameters

The below are mostly average values for a photon one wavelength long regardless of the shape of its envelope. The frame of reference begins from a laboratory rest frame, then changes to a frame in which the photon is fixed, a frame moving at the speed of light. No regard is given to unobserved, special relativity effects such as rod-shortening. If the photon shortens up in flight, it would have to shorten to zero length, and its frequency would go to infinity.

First, a simplification of the constants. Using ϵ_0 or μ_0 together with c_0 in a single equation is algebraically, as well as geometrically, redundant. Each of these constants is associated with one or two dimensions in a three-dimensional, (x, y, z), Euclidean space. The speed of light, c_0, is in the direction of propagation of the light: $c_0 \rightarrow c_0\hat{\mathbf{z}}$, say, with $\hat{\mathbf{z}}$ the unit vector. ϵ_0 has two associated, orthogonal dimensions, one in the direction of propagation, $\hat{\mathbf{z}}$, and the other in the direction of the transverse electric field, $\hat{\mathbf{x}}$, say. Similarly, μ_0 has two associated, orthogonal dimensions, one in the direction of propagation, $\hat{\mathbf{z}}$, and the other in the direction of the transverse magnetic field, $\hat{\mathbf{y}}$, say. So the $\hat{\mathbf{z}}$ direction is being referenced, or 'overlaid' three times in a 'space' (ϵ_0, μ_0, c_0) that can be described as ((x, z), (y, z), z), two oriented planes and one oriented line. This adds unnecessary clutter while at the same time obfuscating the important role of the wave impedance.

With these two equations placed together side by side,

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad \text{and} \qquad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \tag{1}$$

it is a simple matter to cast the **electric permittivity, ϵ_0**, and **magnetic permeability, μ_0**, in terms of the **vacuum wave impedance, Z_0**, and the **vacuum speed of light, c_0**:

$$\epsilon_0 = \frac{1}{Z_0 c_0} \qquad \text{and} \qquad \mu_0 = \frac{Z_0}{c_0} \tag{2}$$

Z_0 is associated with the two perpendicular axes transverse to the direction of propagation, the plane of impedance. The Cartesian space that the set $\{Z_0, c_0\}$ refers to can be described as ((x, y), z). For one example of removing clutter, the **fine-structure constant** is often written out as $\alpha = e^2/(4\pi\epsilon_0 \hbar c)$, which can be condensed to $\alpha = Z_0 e^2/2h$ with this substitution [Bishop, 2007].

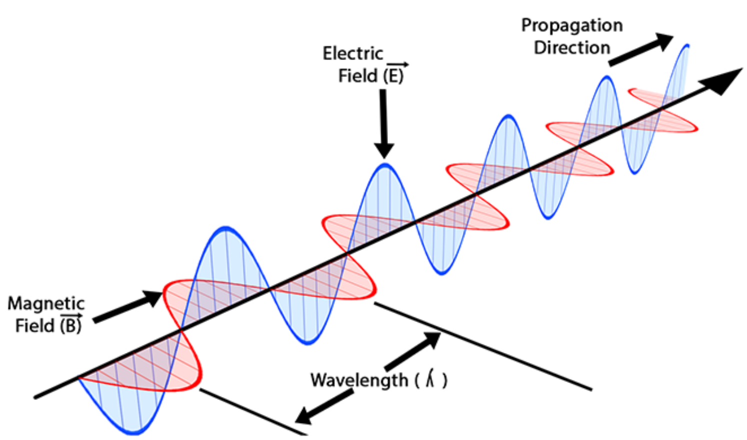

Figure 1. The fields of a classical TEM wave in free space

The set of independent physical constants $\{c_0, Z_0\}$ is used below to the exclusion of the set $\{\epsilon_0, \mu_0\}$ or the set $\{\epsilon_0, \mu_0, c_0\}$. Any occurrences of ϵ_0 and μ_0 in any imported equations are replaced using Equations (2). Z_0 plays an important role in all of this development.

With h as M. Planck's constant, c_0 the speed of light in vacuum, and f the frequency, the **energy**, E_P, of the quantum-mechanical photon is

$$E_P = hf \text{, or equivalently, } E_P = hc_0/\lambda \text{, with } f = c_0/\lambda \text{ and } \lambda = c_0/f \tag{3}$$

For this one-full-wavelength photon, its **time-of-passage**, T_p, through a fixed, transverse plane, is

$$T_p = 1/f , \tag{4}$$

which is the minimum amount of time required to recognize or register the frequency. The **average power,** P_P, of this photon is then

$$P_P = E_P/T_p \rightarrow P_P = (hc_0/\lambda)*(c_0/\lambda) \rightarrow P_P = hc_0^2/\lambda^2 \text{, or } P_P = hf^2 \tag{5}$$

Equation (5) can only hold if the wavelength of the photon is the length of the photon. For the TEM wave in free space, or guided by two conductors arranged to have an impedance of Z_0,

$$i = \sqrt{\frac{P}{Z_0}} \tag{6}$$

Therefore the **electric current**, i_P, of this photon is

$$i_P = \sqrt{\frac{P_P}{Z_0}} \tag{7}$$

Substituting Equation (5) in for P_P

$$i_P = f\sqrt{\frac{h}{Z_0}} \tag{8}$$

and, with $V = i/Z$, the **voltage of this photon** is

$$V_P = i_P Z_0 = f\sqrt{\frac{h}{Z_0}} Z_0 \tag{9}$$

or

$$V_P = f\sqrt{hZ_0} \tag{10}$$

Putting in numbers, the root term is 4.99624007*10^{-16} J-sec/coulomb. At 1 gHz, the voltage is then 5*10^{-7} V. The voltage of the 121.6nm H first ground transition photon (Lyman 1->2) is 1.23V. These values are not new; they, or perhaps some arithmetic multiplier of them, are inherent in the conventional description of both the classical light field and of the quantum-electrodynamical photon field, as shown below.

From Equation (8) the **electric charge on one side of the photon** is

$$\pm Q_P = \sqrt{\frac{h}{Z_0}} \qquad \text{and so} \qquad Q_P^2 = \frac{h}{Z_0} \tag{11}$$

The electric charge is the same for all photons. The value of $\pm Q_P$ is 1.32621132*10^{-18} coulomb. This is 8.27755100 times larger than the charge of the electron and 1.414… times larger than the Planck charge, q_P. commonly used in theoretical physics. $q_P = 4\pi\epsilon_0\hbar c_0$, is $q_P = \sqrt{2}Q_P$.

This Equation (11) establishes a link between the electric-charge concept, Planck's constant, and the wave impedance of the vacuum. The square of the electric charge of this photon is Planck's constant divided by the wave impedance. The plus/minus signifies that this Q_P is an analog of the charge on the plates of a capacitor/transmission line. The total electric charge is then $Q_P + -Q_P = 0$. The divergence is still $\nabla \cdot \boldsymbol{E} = 0$ outside of this photon, a condition required in the derivation of Maxwell's wave equations. In one view, "electric charge" is a name for the sides of the TEM wave where it stops, always with a positive side and a negative side as depicted in Figure 1., always moving at the speed of light according to the governing equation of electric current, $i = qc_0$.

The mysterious and celebrated **fine-structure constant**, α, can be expressed as the ratio of the squares of the invariant electronic and photonic charges:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c_0} \quad \leftrightarrow \quad \alpha = \frac{Z_0 e^2}{2h} \quad \leftrightarrow \quad \alpha = \frac{e^2}{2{Q_P}^2}\,, \tag{12}$$

a very different way of looking at it. Electric charge, Q, is a "square-root of reality" type of physics notion, as is voltage and many others. Bare Q is has never been measured all by itself, only its product with another such Q, such as the Q^2 in Coulomb's law. The squares in the last of Equations (12) are closer to a description of reality than the collection of terms in the conventional expression for α.

The units of electric permittivity, ϵ_0, are farad/meter, or capacitance per unit length. The length in question is in the direction of propagation of the TEM wave in vacuum, so the permittivity can be thought of as the scale-invariant "capacitance per unit length of space". The length in question here is the wavelength, λ, so the total **capacitance of the photon** is

$$C_P = \epsilon_0 \lambda \text{ , or, with Equations (2) and (3), } C_P = \frac{1}{fZ_0}\,. \tag{13}$$

Crosschecking against the definition of capacitance,

$$C = \frac{Q}{V} \rightarrow C_P = \frac{Q_P}{V_P} = \sqrt{\frac{h}{Z_0}}\frac{1}{f\sqrt{hZ_0}} \text{ which reduces to } C_P = \frac{1}{fZ_0}. \tag{14}$$

The units of magnetic permeability, μ_0, are henry/meter, or inductance per unit length. The length in question is in the direction of propagation of the TEM wave in vacuum, so the permeability can be thought of as the scale-invariant "inductance per unit length of free space". The length in question here is the wavelength, λ, so the total **inductance of the photon** is

$$L_P = \mu_0\lambda \text{ or, using Equ. (2) and (3), } L_P = \left(\frac{Z_0}{c_0}\right)\left(\frac{c_0}{f}\right)\text{, which reduces to } L_P = \frac{Z_0}{f}. \tag{15}$$

The **magnetic flux, φ_P**, can be found from the definition of inductance, with subscript L for "per unit length", a **geometric form factor in the plane of impedance, *GF***, of 1 for free space, and multiplying by the wavelength:

$$L_L = \frac{\varphi_L}{i} = GF\mu_0 \rightarrow L_P = \frac{\varphi_L\lambda}{i} = \mu_0 \rightarrow \varphi_P = \mu_0 i_P\lambda = \frac{Z_0}{c_0}f\sqrt{\frac{h}{Z_0}}\frac{c_0}{f} = \sqrt{hZ_0}. \tag{16}$$

For Maxwell et al's transverse electromagnetic wave (TEM wave), its energy content is split evenly between its electric field and its magnetic field. This criterion should continue to hold in the instant case, such that each has, from Equation (3), $E = hf$, $hf/2$ worth of the total energy. With W_E the **energy of the electric field** and W_M the **energy of the magnetic field**,

$$W_E = \frac{1}{2}VQ = \frac{1}{2}V_PQ_P\text{. With Equs. (10) and (11), } W_E = \frac{1}{2}f\sqrt{hZ_0}\sqrt{\frac{h}{Z_0}} \tag{17}$$

which reduces to

$$W_E = \frac{1}{2}hf \tag{18}$$

For the magnetic field energy,

$$W_M = \frac{1}{2}Li^2 = \frac{1}{2}L_Pi_P^2\text{. Substituting in Equations (8) and (15), } W_M = \frac{1}{2}\frac{Z_0}{f}f^2\frac{h}{Z_0} \tag{19}$$

which reduces to

$$W_M = \frac{1}{2}hf \tag{20}$$

The electric charge, Q, when invoked as an electric current, i, has been shown to only move at the speed of light according to its defining **equation of electric current, The Forbidden Equation, $i = qc_0$**

with q the charge per unit length. The common definition, $i = Q/t$, masks this equation. As a crosscheck on the development so far, with λ the length in question,

$$\boldsymbol{i} \;=\; \boldsymbol{q c_0} \quad \rightarrow \quad i_P \;=\; \frac{Q_P c_0}{\lambda} \text{. With Equ. (10) and (3), } \; i_P \;=\; \sqrt{\frac{h}{Z_0}}\frac{c_0}{\lambda} \text{, or } \; i_P \;=\; f\sqrt{\frac{h}{Z_0}} \qquad (21)$$

in agreement with Equation (8). The directions of c_0 and λ are the same, so their directions in the ratio cancel. The speed of light, the magnetic flux, the capacitance, the voltage, the electric current, and of course any length, are always in reference to one or two directions in space when used in a descriptive equation, unlike scalars such as temperature. This issue arises again below.

The expression for the voltage of the photon, Equation (10), can be crosschecked against the requisite **Companion Forbidden Equation** of voltage using Equation (16) divided by the wavelength,

$$\boldsymbol{V} \;=\; \boldsymbol{\phi_L c_o} \quad \rightarrow \quad V_p \;=\; \frac{\sqrt{h Z_0}}{\lambda} c_0 \;=\; \frac{\sqrt{h Z_0} f \lambda}{\lambda} \;=\; f\sqrt{h Z_0} \qquad (22)$$

in agreement with Equation (10).

Intrinsic Spin

The photon has a spin of either zero for linearly polarized light, or +-$h/(2\pi)$ for circularly polarized, an invariant property of all photons regardless of the frequency.

Both Equation (11) for the positive and negative electric charge Q_P, and Equation (16) for the total magnetic flux φ_P, show invariant properties, which do not depend on the frequency or wavelength of the photon:

$$\pm Q_P \;=\; \sqrt{\frac{h}{Z_0}} \qquad \text{and} \qquad \varphi_P \;=\; \sqrt{h Z_0} \qquad (23)$$

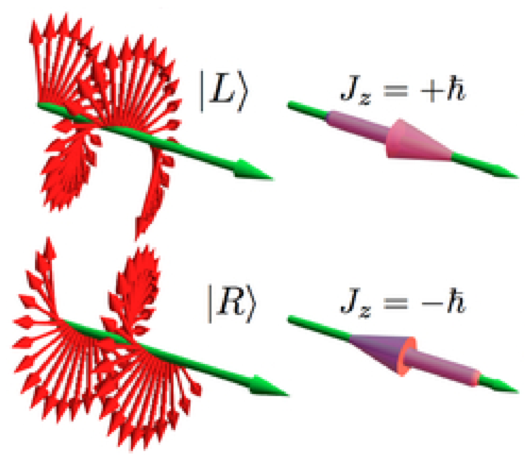

Figure 2. Circular polarization

Each of these quantities is a physical constant, being composed of other physical constants. The SI unit of Q_P is coulomb, and the units of the magnetic flux φ_P are kg*m*/amp, which can be thought of as angular momentum per coulomb, or as action per coulomb.

The value of +-Q_P is $1.32621132 \cdot 10^{-18}$ coulomb (or amp-second). The value of φ_P is $4.99624007 \cdot 10^{-16}$ kg*m^2/amp (or weber). Their ratio is the wave impedance:

$$\frac{\phi_P}{Q_P} = \sqrt{hZ_0}\sqrt{\frac{Z_0}{h}} = Z_0 . \tag{24}$$

Their product is Planck's constant:

$$Q_P \phi_P = \left(\sqrt{\frac{h}{Z_0}}\right)\left(\sqrt{hZ_0}\right) = h . \tag{25}$$

If a circularly-polarized, one-wavelength long photon makes one full rotation about its long axis, the associated electrical parameters above would also rotate through that angle of 2π. The product of charge and magnetic flux would rotate one Planck's constant worth of action per 2π radians, or $h/(2\pi)$. The intrinsic spin of the photon can then be expressed as

$$J_z = \pm\hbar = \pm Q_P \phi_P / 2\pi \tag{26}$$

Quantization of A, E, and B, the width of the photon

For the above Equations (4) through (26) only one spatial dimension was assumed, the wavelength in the propagation direction (*z* axis) of the photon. To find the expressions for the electric and magnetic fields and fluxes, an additional assumption about the size of its transverse dimensions (*x* and *y* axes) has to be introduced. How wide is a photon?

The cavity oscillator is a cubical box of arbitrary size, lined with perfectly-reflecting mirrors on its inside faces, and sitting on the laboratory bench. For the instant analytic volume, its length is the wavelength, λ and it moves with the photon. One choice for the transverse dimensions is the wavelength, to form a cubical box of volume λ^3. With that, the **electric field strength**, E_P, becomes

$$E_P = \frac{V_P}{\lambda} = \frac{f\sqrt{hZ_0}}{\lambda} = c_0\frac{\sqrt{hZ_0}}{\lambda^2} , \tag{27}$$

and the **magnetic flux density**, B_P, is

$$B_P = \frac{E_P}{c_0} = \frac{f}{\lambda c_0}\sqrt{hZ_0} = \frac{\sqrt{hZ_0}}{\lambda^2} . \tag{28}$$

As a crosscheck of Equations (27) and (28), with $B = \mu_0 H = H Z_0 / c_0$, the **magnetic field**, H_P, is $H_P = B_P/\mu_0 = c B_P / Z_0 = E_P / Z_0$, and

$$Z_0 = \frac{E}{H} = \frac{E_P}{H_P} = \frac{E_P Z_0}{c_0 B_P} = \frac{Z_0}{c_0}\left(\frac{f}{\lambda}\sqrt{hZ_0}\right)\left(\frac{\lambda c_0}{f\sqrt{hZ_0}}\right) = Z_0 \tag{29}$$

The **classical Hamiltonian**, ***H***, for an analytic volume, a box on the theoretical lab bench filled with electromagnetic radiation, is

$$\boldsymbol{H} = \frac{1}{2}\epsilon_0 \iiint_{Vol} |E(\boldsymbol{r},t)|^2 + c_0{}^2 |B(\boldsymbol{r},t)|^2 d^3\boldsymbol{r} \tag{30}$$

In this single-photon case, with the box attached to and moving with the photon, there are no additional components, modes, or photons to integrate over. As is done for the quantization of this Hamiltonian in quantum electrodynamics, substitute the squares of E_P and B_P into Equation (30). Dropping the unnecessary notation, and using Equation (2), the volume integral resolves to

$$\boldsymbol{H} = \frac{1}{2}\frac{1}{Z_0 c_0}\lambda^3\left(E_P^2 + c_0^2 B_P^2\right) \tag{31}$$

As above, Equations (17) and (19), the electric and magnetic energies should each be equal to $1/2hf$. For the **electric energy**, W_E, and using Equations (2) and the square of Equation (27),

$$W_E = \frac{1}{2}\frac{1}{Z_0 c_0}\lambda^3 E^2 = \frac{1}{2}\frac{1}{Z_0 c_0}\lambda^3 \frac{f^2}{\lambda^2} h Z_0 = \frac{1}{2}\frac{1}{c_0}\lambda f^2 h = \frac{1}{2}hf \tag{32}$$

For the **magnetic energy**, W_M, using Equations (2) and the square of (28),

$$W_M = \frac{1}{2}\frac{1}{Z_0 c_0}\lambda^3 B^2 = \frac{1}{2}\frac{1}{Z_0 c_0}\lambda^3 \frac{hZ_0}{\lambda^4}c_0^2 = \frac{1}{2}hf \text{ , so} \tag{33}$$

$$W_E = W_M = \frac{1}{2}hf \tag{34}$$

as before. Equations (32) and (33) appear to support the choice of the analytic volume, λ^3, and the expressions for E_P and B_P. It is consistent with the rest of this model. But it isn't necessary. The transverse dimensions needed for this derivation cancel out of the energy equations, so they are still indeterminate, with one constraint.

Let the transverse dimensions of the analytic box be some unknown X and Y, so the box volume is $XY\lambda$. For free space the geometric factor, GF, is the ratio between the height, X, and the width, Y, in the plane of impedance of the area the TEM wave moves through. It is a square with $GF = 1$, therefore $Y = X$.

With $Y = X$, the **electric field strength**, Equation (27), becomes an unknown function of X, the **width of the photon**:

$$E_P = \frac{V_P}{X} = \frac{f}{X}\sqrt{hZ_0} \rightarrow X = f\frac{\sqrt{hZ_0}}{E_P} . \tag{35}$$

And similarly for the **magnetic flux density**, Equation (28):

$$B_P = \frac{E_P}{c_0} = \frac{f}{Xc_0}\sqrt{hZ_0} = \frac{\sqrt{hZ_0}}{\lambda X} . \tag{36}$$

So no information about the width of the photon, its waveform, or its electric-field strength can be gleaned from the above analysis. These conclusions can be applied to the presumptions and equations of Classical Electromagnetism and of Quantum Electrodynamics to find the voltage and to search for X, the width of the photon, as follows.

With wave number $|\boldsymbol{k}| = 2\pi f_{\mathbf{k}}/c_0 = \omega_{\mathbf{k}}/c_0$, t for †, $\boldsymbol{e}^{\mu}$ the transverse polarization basis vectors, and $V_{OL} = L^3$ the cubical, macroscopic analytic volume, the foundational **quantized operator fields** are

$$\mathbf{A}(\mathbf{r}) = \sum_{\boldsymbol{k},\mu}\sqrt{\frac{\hbar}{2\omega_{\mathbf{k}}V_{OL}\epsilon_0}}\left\{\mathbf{e}^{(\mu)}a^{(\mu)}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}} + \bar{\mathbf{e}}^{(\mu)}a^{t(\mu)}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right\} \tag{37}$$

$$\mathbf{E}(\mathbf{r}) = i\sum_{\boldsymbol{k},\mu}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2V_{OL}\epsilon_0}}\left\{\mathbf{e}^{(\mu)}a^{(\mu)}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}} - \bar{\mathbf{e}}^{(\mu)}a^{t(\mu)}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right\} \tag{38}$$

$$\mathbf{B}(\mathbf{r}) = i\sum_{\boldsymbol{k},\mu}\sqrt{\frac{\hbar}{2\omega_{\mathbf{k}}V_{OL}\epsilon_0}}\left\{\left(\mathbf{k} \times \boldsymbol{e}^{(\mu)}\right)a^{(\mu)}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}} - \left(\mathbf{k} \times \bar{\mathbf{e}}^{(\mu)}\right)a^{t(\mu)}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\right\} \tag{39}$$

For the root terms in Equations (37) for **A**(**r**) and (39) for **B**(**r**), the use of Equations (2) reveal, again, something that is obscured by mixing together redundant physical constants. Substituting in $\epsilon_0 = 1/Z_0 c_0$, $\omega = 2\pi f$, and resolving $\hbar$ as $h/2\pi$, yields

$$\sqrt{\frac{\hbar}{2\omega_{\mathbf{k}}V_{OL}\epsilon_0}} = \sqrt{\frac{hZ_0c_0}{8\pi^2 f_{\mathbf{k}}V_{OL}}} \Rightarrow \frac{1}{2\pi}\sqrt{\frac{hZ_0c_0}{2f_{\mathbf{k}}L_X^2 L_Z}} = \frac{1}{2\pi}\frac{\phi_P}{L_X}\sqrt{\frac{\lambda_{\mathbf{k}}}{2L_Z}} \tag{40}$$

using the invariant magnetic flux density, φ_P, from Equation (15) for the latter expression, Two of the macroscopic lengths, the transverse L_X , have been labeled to emphasize their transverse directions and then brought out from under the root using the arguments leading to Equation (34). With $|L_x| = |L_y| = |L_z|$ the root term of Equation (38) for **E**(**r**) becomes

$$\sqrt{\frac{\hbar\,\omega_{\mathbf{k}}}{2\,V_{OL}\epsilon_0}} \;=\; \sqrt{\frac{h}{2\pi}\frac{\left(Z_0 c_0\right)\left(2\,\pi\,f_{\mathbf{k}}\right)}{2\,L^3}} \;=\; \sqrt{\frac{h f_{\mathbf{k}} Z_0 c_0}{2\,L_x L_y L_z}} \;\Rightarrow\; f_{\mathbf{k}}\frac{\phi_P}{L_x}\sqrt{\frac{\lambda_{\mathbf{k}}}{2\,L_z}} \;=\; c_0\frac{\phi_P}{\lambda_{\mathbf{k}} L_x}\sqrt{\frac{\lambda_{\mathbf{k}}}{2\,L_z}} \qquad (41)$$

With subscript j for summing over the three perpendicular axes in this quantum electrodynamic construct, **k** is commonly restricted to the countable

$$\left|\mathbf{k}_{\mathbf{j}}\right| \;=\; \frac{2\,\pi\,n_j}{L}, \quad n_j \;=\; \pm 1, \pm 2, \pm 3\ldots, \quad \text{for} \quad j \;=\; 1,2,3 \qquad (42)$$

For unit vectors ($\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$) along the (x, y, z) axes, **k** is in the direction |**k**| $\hat{\mathbf{z}}$, for example, and the solutions to Equations (37)-(39) are time-independent standing TEM waves propagating perpendicular to the faces of the mirrored box, still with an undetermined analytic width, X, of the quantized photon of wavenumber k. Considering propagation in the $\hat{\mathbf{z}}$ direction only, and so dropping the unneeded subscript j of Equation (43),

$$\left|\mathbf{k}\right| \;=\; \frac{2\,\pi}{\lambda_{\mathbf{k}}} \;=\; \frac{2\,\pi\,n}{L_z}, \quad n \;=\; \pm 1, \pm 2, \pm 3\ldots \text{ and so } \lambda_{\mathbf{k}} \;=\; \frac{L_z}{n} \qquad (43)$$

where L_z is the length of the box in the $\hat{\mathbf{z}}$ and **k** direction. With Equation (43), Equation (40) reduces to

$$\sqrt{\frac{\hbar}{2\,\omega_{\mathbf{k}} V_{OL}\epsilon_0}} \;\Rightarrow\; \frac{1}{2\pi}\frac{\phi_P}{L_x}\sqrt{\frac{\lambda_{\mathbf{k}}}{2\,L_z}} \;=\; \frac{1}{2\pi}\frac{\phi_P}{L_x}\sqrt{\frac{1}{2\,n}} \quad \text{[weber/meter]} \qquad (44)$$

and similarly for Equation (41), for the magnitude of the electric field, **E**(**r**), and for an amplitude of 1 of any component $\lambda_{\mathbf{k}}$:

$$\left|\boldsymbol{E}_{\mathbf{k}}(\boldsymbol{r})\right| \;=\; \sqrt{\frac{\hbar\,\omega_{\mathbf{k}}}{2\,V_{OL}\epsilon_0}} \;\Rightarrow\; f_{\mathbf{k}}\frac{\phi_P}{L_x}\sqrt{\frac{\lambda_{\mathbf{k}}}{2\,L_z}} \;=\; c_0\frac{\phi_P}{\lambda_{\mathbf{k}} L_x}\sqrt{\frac{1}{2\,n}} \quad \text{[volt/meter].} \qquad (45)$$

For **B**(**r**), Equation (39), the dimensions of **k** in the cross product, $2\pi/\lambda_{\mathbf{k}}$, are brought out and multiplied with Equation (45). For an amplitude of 1 of any component,

$$\left|\mathbf{B}_{\mathbf{k}}(\mathbf{r})\right| \;=\; \frac{2\pi}{\lambda_{\mathbf{k}}}\sqrt{\frac{\hbar}{2\,\omega_{\mathbf{k}} V_{OL}\epsilon_0}} \;\Rightarrow\; \frac{2\pi}{\lambda_{\mathbf{k}}}\frac{1}{2\pi}\frac{\phi_P}{L_x}\sqrt{\frac{1}{2n}} \;=\; \frac{\phi_P}{\lambda_{\mathbf{k}} L_x}\sqrt{\frac{1}{2\,n}} \quad \text{[weber/meter}^2\text{]} \qquad (46)$$

which crosschecks against $E \;=\; c_0 B$.

The **voltage, $V_{\mathbf{k}}$, of any component $\lambda_{\mathbf{k}}$** of the expansion of $\mathbf{E}_{\mathbf{k}}(\mathbf{r})$, Equations (45), (41), and (38), is seen to be

$$V_{\mathbf{k}} = f_{\mathbf{k}} \phi_P = f_{\mathbf{k}} \sqrt{h Z_0} \tag{47}$$

The voltage of the photon, and the TEM wave, was there in the conventional models *ab initio*.

The cubical cavity oscillator used for the foundational Quantum Electrodynamic Equations (37)-(39) has unique, geometric properties about it. Any cross-section of a cube, taken parallel to any face, is a square. Orienting this cube parallel to the (*x*, *y*, *z*) axes of a Cartesian coordinate system, the **k** vectors are taken as parallel to *x*, *y*, or *z*. The plane TEM waves are restricted to propagating through a square in the plane of their impedance. The geometric factor, *GF*, of a square is *GF* = 1, therefore the impedance, *Z*, of this confined space, in the specified $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ directions, is $Z = GF * Z_0 = Z_0$, the impedance of free space, a fact not evident in the literature.

The implied impedance can also be found by equating the electric and magnetic fields as follows:

$$Z_{\mathbf{k}} = \frac{E_{\mathbf{k}}}{H_{\mathbf{k}}} = \frac{|\boldsymbol{E}_{\mathbf{k}}(\boldsymbol{r})| Z_0}{|\boldsymbol{B}_{\mathbf{k}}(\boldsymbol{r})| c_0} = \frac{Z_0 c_0 \phi_P / \lambda_{\mathbf{k}} L_x \sqrt{1/2n}}{c_0 \phi_P / \lambda_{\mathbf{k}} L_x \sqrt{1/2n}} = Z_0 \tag{48}$$

With the wavelength, λ_K, in the direction of propagation of the photon, and **k** selected perpendicular to any pair of the walls of the box, the remainder dimensions of V_{OL} are in the transverse directions of **k** when applied to any particular photon that participates in the summation, the plane or subspace spanned by the $\mathbf{e}_\mu$ basis vectors. As shown above, these are undetermined dimensions *X* and *Y*, with the constraint that *Y* = *X* when $Z = Z_0$.

With the quantized photon's demonstrated impedance of $Z_{\mathbf{k}} = Z_0$, it is possible to equate any component of $|\mathbf{E}_{\mathbf{k}}|$ with the independently-derived E_P in an unknown *X*, identified above as the **width of the photon**, from Equation (35), with $\lambda_{\mathbf{k}} = \lambda$:

$$|\boldsymbol{E}_{\mathbf{k}}(\boldsymbol{r})| = c_0 \frac{\phi_P}{\lambda_{\mathbf{k}} L_X} \sqrt{\frac{1}{2n}} = c_0 \frac{\phi_P}{\lambda X} \tag{49}$$

and solve for *X*:

$$X = L\sqrt{2n} \tag{50}$$

If this were true, then the width of the photon would be larger than the box it is supposed to be confined in, for all *n*, an incredible proposition. Another interpretation is *X* = *L*, if the root term is ignored for some reason.

There are five different values for the width of the photon, *X*, considered so far: zero, infinity, *λ*, *L*, and $L\sqrt{2n}$. The *X* = 0 width of the point particle implies an infinite field strength. For $X \rightarrow$ infinity, $E \rightarrow 0$, also an untenable result. So *X* is still an undetermined, finite length, greater than zero and less than infinity.

Summary and Findings

The novel equations above were initially derived and crosschecked using the dimensional-analytic set {F, L, T, Q} (force, length, time, charge), then translated back into derived SI units.

Photon Property	Equation	SI Derived Units	{F, L, T, Q} Units
Electric permittivity	$\epsilon_0 = 1/(Z_0 c_0)$	farad/meter	$Q^2 L^{-2} F^{-1}$
Magnetic permeability	$\mu_0 = Z_0/c_0$	henry/meter	$F T^2 Q^{-2}$
Electric current	$i_P = f\sqrt{h/Z_0}$	ampere	QT^{-1}
Voltage	$V_P = f\sqrt{hZ_0}$	volt	$F L Q^{-1}$
Component voltage	$V_{\mathbf{k}} = f_{\mathbf{k}}\sqrt{hZ_0}$	volt	$F L Q^{-1}$
Electric charge	$\pm Q_P = \sqrt{h/Z_0}$	coulomb	Q
Capacitance	$C_P = 1/fZ_0$	farad	$Q^2 F^{-1} L^{-1}$
Inductance	$L_P = Z_0/f$	henry	$F L T^2 Q^{-2}$
Magnetic flux	$\varphi_P = \sqrt{hZ_0}$	weber	$F L T Q^{-1}$
Electric energy	$W_E = 1/2hf$	joule	F L
Magnetic energy	$W_M = 1/2hf$	joule	F L
Intrinsic spin	$J_z = \pm\hbar = \pm Q_P\phi_P/2\pi$	action quantum	F L T
Electric field strength	$E_P = f\phi_P/X$	volt/meter	$F Q^{-1}$
Electric field strength	$\lvert\mathbf{E_k}(\mathbf{r})\rvert = f_{\mathbf{k}}\phi_P/L_x\sqrt{1/2n}$	volt/meter	$F Q^{-1}$
Magnetic flux density	$B_P = \phi_P/(\lambda X)$	tesla	$F T Q^{-1} L^{-1}$
Magnetic flux density	$\lvert\mathbf{B_k}(\mathbf{r})\rvert = \phi_P/(\lambda_{\mathbf{k}} L_x)\sqrt{1/2n}$	tesla	$F T Q^{-1} L^{-1}$
Magnetic field strength	$H_P = (f/X)\sqrt{h/Z_0}$	ampere/meter	$Q L^{-1} T^{-1}$
Geometric factor, *GF*	varies	dimensionless	-
Fine-structure constant	$\alpha = e^2/(2Q_P^2)$	dimensionless	-

Findings:

- The discovery of the mainstream voltage of the photon, which has apparently not been addressed. Since the photon has an unsung voltage in their model, along with electric and magnetic fields, it also has a finite width, by their definition.
- A mathematical and logical unification of the TEM wave on the line with the TEM wave on the aether, which Maxwell and his followers had failed to accomplish. This may lead to entirely new lines of inquiry and possible application.
- Derivations of the apparently previously-unknown, closed expressions for the charge, capacitance, flux, etc. of the photon.
- An intriguing interpretation of the fine-structure constant, in which it is seen as the ratio of electronic and photonic charge. This constant only appears when photons and massive particles interact, and so its interpretation as their ratio of intrinsic electric charges is most direct.
- The identification of the occult wave impedance used in the QED analytic cavity, a figure not generally emphasized.
- The identification of the variable, X, which represents the width of the photon, as well as the TEM wave on the line or in free space, in conventional electromagnetics.
- An expression for the intrinsic spin in terms of the photon's charge and magnetic flux density.

Several useful mathematical relations and techniques were employed in the above, not all of which were called out. There are the crucial substitutions of Equations (2), which condense the number of physical constants employed. There is The Forbidden Equation: $i = qc$, the governing equation of electric current, which is found to be compatible with this model. The equal partition of the electric and magnetic field energies of the electromagnetic wave was used twice to verify the derivations. The wave impedance of free space, or the aether, Z_0, along with its geometric form factor, *GF*, [Catt, et al, 1978] is shown to be a crucial concept for any photon model, being as fundamental as the speed of light.

The initial assumption, that the wavelength of the photon is the length of the photon, is supported by the derived quantities. Any other length assumption, from a point-like zero to an infinite extent, either complicates or renders absurd the expressions for electric field, current, voltage, and the other equations that involve frequency or wavelength.

A second assumption, famously proposed by Maxwell and the others, is that light is an electromagnetic wave, similar to the transverse electromagnetic waves of an electric circuit/transmission line. That hypothesis was used directly to apply the electrical parameters of the TEM wave on the transmission line to this photon model with and including the quantization conditions.

M. Planck, in his June, 1920 Nobel Prize acceptance speech, titled in translation "The Genesis and Present State of Development of the Quantum Theory" [Planck, 1920b] addresses the problem of the volume of the photon in his closing remarks:

"There is one particular question the answer to which will, in my opinion, lead to an extensive elucidation of the entire problem. What happens to the energy of a light quantum after its emission? Does it pass outwards in all directions, according to Huygens's wave theory, continually increasing in volume and tending towards infinite dilution? Or does it, as in Newton's emanation theory, fly like a projectile in one direction only?

"In the former case the quantum would never again be in a position to concentrate its energy at a spot strongly enough to detach an electron from its atom; while in the latter case it would be necessary to sacrifice the chief triumph of Maxwell's theory- the continuity between the static and the dynamic fields- and with it the classical theory of the interference phenomena which accounted for all their details, both alternatives leading to consequences very disagreeable to the modern theoretical physicist. Whatever the answer to this question, there can be no doubt that science will some day master the dilemma, and what may now appear to us unsatisfactory will appear from a higher standpoint as endowed with a particular harmony and simplicity. But until this goal is reached the problem of the quantum of action will not cease to stimulate research, and the greater the difficulties encountered in its solution the greater will be its significance for the broadening and deepening of all our physical knowledge."

Science has not yet mastered this dilemma, as the famous metaphysics of the wave-particle duality and the results of the double-slit experiment highlight. Nor has Maxwell triumphed in establishing "the continuity between the static and the dynamic fields", for it has not be proven that the static field exists in the first instance, nor that it can be instantly accelerated and transformed into a field that moves at the speed of light. The math works out to some degree, but that cannot make it govern anything, as the case history of Quantum Electrodynamics shows [Consa, 2021].

There are several different conceptual models for the photon in addition to the quantum electrodynamic "packet of energy" idea. But all of them have to have an electric field, **E**, and its attendant voltage, V_P, and electric charge, Q_P, or whatever may come to replace those ideas, in order to conform to observation. No such model can be taken seriously without either an explicit accounting of all of the electromagnetic parameters of the TEM wave in free space or, perhaps, by replacing large swaths of electrical engineering and physics with something better. Short of the latter, and following M. Planck, they must address this question about its electric field strength, **E**, directly and without subterfuge:

How wide is a photon?

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