Undecidability of the Riemann Hypothesis

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Abstract: The Riemann zeta function in the Riemann hypothesis equals zero for both all negative even integers and an infinite number of complex numbers with real part 1/2. We can conclude that Riemann hypothesis is undecidable.

Keywords: Riemann hypothesis, Riemann zeta function, nontrivial zero, infinity, undecidability.

1. Introduction

The Riemann hypothesis was proposed by Bernhard Riemann (Riemann, 1859).

When studying the distribution of prime numbers Riemann extended Euler's zeta function (defined just for s with real part greater than one)

$\zeta(s) = 1 + 1/2^s + 1/3^s + \ldots$

to the entire complex plane (except the simple pole at s=1 with residue one). Riemann noted that his zeta function had trivial zeros at -2, -4, -6, ...; that all nontrivial zeros were in the critical strip of non-real complex numbers with $0 \leq \text{Re}(s) \leq 1$, and that they were symmetric about the critical line $\text{Re}(s) = 1/2$; and that the few he calculated were on that line. Riemann conjectured that all of the nontrivial zeros are on the critical line, a conjecture that subsequently became known as the Riemann hypothesis.

Hardy proved in 1914 that an infinite number of the zeros of the Riemann zeta function do occur on the critical line (Hardy, 1914) and in 1989 Conrey showed that over 40% of the zeros in the critical strip are on the critical line (Conrey, 1989).

In this paper, the Riemann hypothesis is undecidable based on the fact that the Riemann zeta function in the Riemann hypothesis equals zero for both all negative even integers and an infinite number of complex numbers with real part 1/2.

2. Undecidability of the Riemann Hypothesis

The Riemann zeta function in the Riemann hypothesis is a complex function, and the complex field arises because of the square root of -1.

Does the square root of -1 exist or not? From the perspective of the real axis or the real number field, the square root of -1 can be determined as non-existent, although the state at infinity is undecidable for the real number field. Because if the square root of -1 exists, then we can get the contradiction that -1 is equal to 1, but maybe we can avoid this contradiction by specifying that the algebraic identity

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

does not apply to the case when both a and b are negative real numbers. However, if the square root of -1 exists, it also means that the square root of the discriminant of a quadratic equation exists when the discriminant is less than 0, which must mean that the conic has an intersection with the real number line, but if the intersection occurs on the real number line it must mean that the discriminant is not less than 0, which also leads to a contradiction.

Therefore, when the discriminant of a quadratic equation is less than zero, it has no intersection with the real axis. But at this time, by introducing imaginary numbers, we can make them intersect at infinity. However, at this time, the existence of imaginary numbers must theoretically assume that there is an intersection at infinity, which is an undecidable proposition.

The Riemann zeta function in the Riemann hypothesis equals zero for both all negative even integers and an infinite number of complex numbers with real part 1/2.

When the zero root is only on the axis where the real part is 1/2, and not on the real axis, the existence of this imaginary root implies that the curve represented by the Riemann zeta function has an intersection with the real axis, and that this intersection is thought to occur at infinity, or assumed to occur at infinity, and then expressed in the form of the square root of -1 on the complex plane.

The state at infinity is undecidable for the real number field. However, the zeros of the Riemann zeta function in the Riemann hypothesis occur on both the real axis and the axis whose real part is 1/2 in the complex plane, and there are infinitely many of them. So the question about the state at infinity is unavoidable, and this is an undecidable proposition.

Therefore, the Riemann hypothesis is undecidable.

References

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