Poynting's Theorem and Undecidability of The Logic of Causality in Light of EPR Completeness Condition

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Abstract

The most elementary empirical truth associated with any experiment involving light (electromagnetic radiation) propagation is the distinction between the source (region of cause) and the detector (region of effect), i.e. "cause/effect" distinction, based on which one can speak of "distance between source and detector", "propagation from source to detector" and, therefore, "action at a distance", "velocity of propagation". According to EPR's completeness condition, "cause/effect" distinction should be taken into account in a theory that is supposed to provide explanations for such an experiment, the simplest one being the Hertz experiment. Then, in principle, one can decide whether "cause before effect" or "cause after effect" i.e. the logic of causality remains decidable. I show that, working with Maxwell's equations and "cause/effect" distinction to explain Hertz experiment, Poynting's theorem is unprovable. It is provable if and only if "cause/effect" distinction is erased by choice through an act of free will, but the logic of causality becomes undecidable. The current theoretical foundation behind the hypothesis of 'light propagation' comes into question as theoretical optics is founded upon Maxwell's equations and Poynting's theorem. A revisit to the foundations of electrodynamics, with an emphasis on the interplay among logic, language and operation, seems necessary and motivated.

Keywords: Poynting's theorem; light propagation; Hertz experiment; EPR completeness; logic of causality; choice and free will; decision problem.

1 Introduction

Light can not be seen either in wave or particle (photon) form; rather both of them are theoretical hypotheses that let us explain physical phenomena and experimental observations, albeit with never ending debate and discussions among scientists regarding the nature of light [1]. The hypothesis of light propagation as electromagnetic wave, carrying electromagnetic energy, is rooted to Maxwell's equations [2,3] and Poynting's theorem [4], which is considered to have been experimentally verified by Hertz experiment [5]. Today, such knowledge is part of standard textbooks [6,7] and forms the basis of classical optics [8]. On the other hand, the hypothesis of photon propagation that was put forward by Einstein [9,10], is yet to have a well accepted description in terms of wave function so as to justify the intuition of "a propagating particle/quantum of light", although there have been attempts by some authors [11–13] (see also ref. [14] for a detailed analysis and history of such ideas). The basis of quantum optics relies on the quantization of Maxwell's equations and the conservation principles dictated through Poynting's theorem, irrespective of whether one builds on a wave function description of photon [14] or chooses to proceed without such ideas [15–19] which are unavoidably associated conceptual issues rooted to the concept of 'wave function' itself [20–23]. Based on such hypothesis, that *photons propagate*, one seeks to know the path of propagation of a photon in double slit experiments [24–28], does experiment with entangled photons [28–30] and hidden variable theories [31,32], demonstrates quantum teleportation [33–38], processes quantum information [39,40], etc.

It is important to note that the most basic element of reason, which is common to all investigations concerning the propagation of light, is a distinction between the region of cause and the region of effect. Let me call it "cause/effect"

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distinction. In Hertz experiment these are the source coil and the detector coil [5]. In photon related experiments these are the photon source and the photon detector [25–41]. It is only such prior distinction that lets one speak of "distance" or "spatial separation" between those two regions, irrespective of whether the experiment is about classical physics or quantum physics [42–44]. Without such premises it is meaningless to speak of "propagation (from source to detector)", "velocity of propagation (distance from source to detector/time)" and "action at a distance (from source to detector)" [45–47], that let us speak about the logic of causality [48, 49]. Essentially, without "cause/effect" distinction in a theory, it is impossible to decide whether "cause before effect" or "cause after effect" i.e. the logic of causality becomes undecidable.

Here, I show that if one works with the standard Maxwell's equations to explain the physical reality of the Hertz's experiment, albeit with honest symbolic expressions to express "cause/effect" distinction and abides by EPR's completeness condition of a physical theory (ECC) [21], then Poynting's theorem is unprovable. Only a choice of erasure of "cause/effect" distinction renders the Poynting's theorem provable. However, as a consequence of this choice, the theory becomes incapable of deciding "cause before effect" or "cause after effect" i.e. the logic of causality becomes undecidable. The present work have the following different aspects which are worth noting. Firstly, from historical, as well as engineering point of views, this work (along with ref. [50]) provides an independent theoretical support to Tesla's experimental objections against the interpretations of Hertz experiment [51]. Secondly, this work provides an example of how choice and free will have played a part in legitimizing a *classical* theory which, therefore, exemplifies a deviation from the usual practice of associating choice and free will exclusively to *quantum theory* [52–56] following Schroedinger [20]. Thirdly, this work signifies the essence of investigation regarding logic, language and operation, involving a subtle interplay between Hilbert's formalism, Brouwer's intuitionism and Einstein's operationalism, that has been demonstrated in a recent string of articles which may potentially affect the foundations of physics in a radical fashion [50, 57–60]. In a nutshell, the present work, along with ref. [50], provides strong motivations to rethink about the foundations of electrodynamics from an elementary standpoint.

2 Logic, language and operation

Language, i.e. expression through symbols [61], is the only mean of our communication in the pursuit of "truth" [62–65], that is perceived through our sense experiences and experimental observations as far as physics is concerned [21, 66]. Such message is explicitly manifest from the following words of Bohr on p 3 of ref. [67]: "*... the description of the experimental arrangement and the recording of observations must be given in plain language, suitably refined by the usual physical terminology. This is a simple logical demand, since by the word "experiment" we can only mean a procedure regarding which we are able to communicate to others what we have done and what we have learnt.*". Therefore, refinement of language and logic, in connection to operation (experiment and measurement) [68, 69], is an indispensable part of an investigation concerning the foundations of physics [57, 58]. This also becomes evident from Einstein's view of physics [66]: "*Physics constitutes a logical system of thought.... The justification (truth content) of the system rests in the proof of usefulness of the resulting theorems on the basis of sense experiences, where the relations of the latter to the former can only be comprehended intuitively.*", which signifies both Hilbert's emphasis on logical treatment of the statements of physics [70](also see refs. [71, 72]) and Brouwer's emphasis on the role of intuition in science [73–75]. It is to be noted, therefore, that the truthfulness, or faithfulness, of the correspondence between expressions and experiences (experiment and measurement) is essential when the truth content of a theory is being judged on the basis of experiment and observation perceived through sense experiences [57]. Indeed such is the essence of EPR's completeness condition (ECC) of a theory that states, "*every element of physical reality must have a counter part in the physical theory*" where physical reality "*takes the form of experiment and measurement*" [21].

In the discussion which follows, I write a truthful symbolic correspondence between experiment and theory, in light of ECC, so as to analyze Hertz experiment in terms of Maxwell's equations. To mention, I shall ignore the uprovability of the first Maxwell's equation in light of ECC, that has been recently reported in ref. [50].

3 Elements of Reason – Propositions and Refinement of Symbols

Let me begin with stating the elements of reason in clear terms, based on which I shall discuss Poynting's theorem [4], using the modern vector notations available in the standard textbooks [6, 7]. The following propositions constitute an act of intuitive comprehension of physical reality which will result in a consequent refinement of the symbols. The analysis, based on these propositions, is the logical treatment of the statements of physics. Therefore, on the whole, it is an investigation concerning logic and intuition so as to provide a truthful interpretation of experimental operations with an aim to satisfy ECC.

3.1 The Logical Connectives

I am going to use the following logical connectives in the respective symbolic forms in course of this analysis – "$\wedge$", "$\neg$", "$\rightarrow$", "$\equiv$" stand for "logical conjunction" (AND), "logical negation" (NOT), "logical implication" (IF....THEN..), "logical equivalence" (IF and only IF) respectively. The essence of the logical connectives can be understood from the following truth tables, where A and B are arbitrary propositions:

A	B	$A \wedge B$	$A \rightarrow B$	$A \equiv B$
F	F	F	T	T
T	F	F	F	F
F	T	F	T	F
T	T	T	T	T

A	$\neg A$
T	F
F	T

I note that in case of "$A \rightarrow B$", B is the premise and A is the conclusion because the falsehood of B necessarily leads to the falsehood of A, but the falsehood of A does not necessarily lead to the falsehood of B.

With such basic clarifications I proceed with the following propositional analysis. For more details on mathematical logic, the reader may consult, for example, refs. [76–78] among many others.

3.2 The Propositions

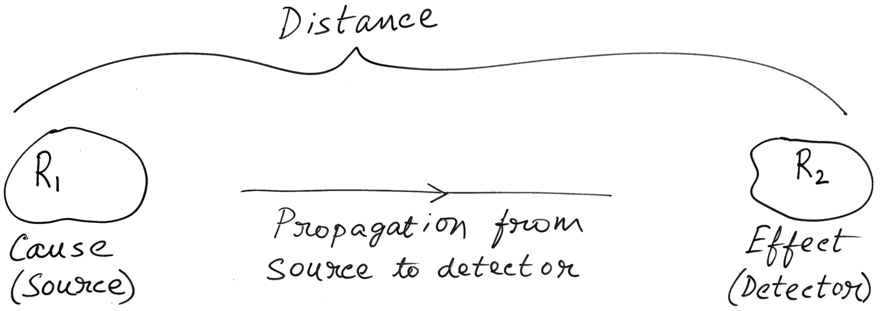

Figure 1: The source lies in the region of cause R_1 and the detector lies in the region of effect R_2.

Considering the crude, but sufficiently demonstrative for the present purpose, schematic representation of Hertz experiment depicted in fig.(1), I consider the following propositions.

P_1: There is a distinction between the source, i.e. a region R_1 where the cause arises, and the detector, i.e. the region R_2 where the effect occurs.

[*Significance*: The words "cause" and "effect" acquire meanings which are distinct from each other i.e. assertion of "cause/effect" distinction. This distinction is encoded in the subscripts "1" and "2", whose erasure is tantamount to the symbolic manifestation of the denial of "cause/effect" distinction.]

P_2: There is a clear notion of "distance" between R_1 and R_2.

[*Significance*: The word "distance" expresses the realization of a spatial separation between R_1 and R_2 so that the terminologies like "propagation", "velocity", "action at a distance" can acquire meaning. $P_2 \rightarrow P_1$ i.e. the latter is the premise based on which the former can be asserted.]

P_3: There is a common notion of "time" for R_1 and R_2.

[*Significance*: If and only if a common notion of "time" is considered, then only a comparison of the "time" marks, which label events at R_1 and R_2, become meaningful (this is part of the motivating arguments of

Einstein's special relativity [79]). Without such comparison, a notion of "time gap between two events at R_1 and R_2", become inexplicable, which however is necessary for judging whether "cause before effect" or "cause after effect". $P_3 \to P_1$ i.e. the latter is the premise based on which the former can be asserted.]

Now, I may note the following *implications*:

1. "A time ordering of events occurring in R_1 and R_2" $\to (P_1 \wedge P_3)$.
2. "Action at a distance" $\to (P_1 \wedge P_2)$.
3. "Finite velocity of propagation of action from R_1 to R_2 [velocity = distance/time]" $\to (P_1 \wedge P_2 \wedge P_3)$.

Thus, P_1 is the most elementary truth that must be accounted for in a theory which is meant for an explanation of the physical reality of Hertz experiment. ECC demands that too.

The falsehood/negation of P_1 can be written as follows:

$\neg P_1$: There is NO distinction between the source, i.e. a region R_1 where the cause arises, and the detector, i.e. the region R_2 where the effect occurs.

[*Significance*: The words "cause" and "effect" have NO computational content/value, or, are NOT manifested in the equations of the theory. Therefore, all the three implications listed above become false.]

Therefore, I observe that a theory allowing $\neg P_1$ is

- incapable of deciding whether the logic of causality holds ("cause before effect") or not ("cause after effect"),
- trivially, an EPR incomplete description of an experiment with P_1 like what has been depicted in fig.(1).

3.3 Refined Symbols

Maxwell's equations hold for localized charge and current distributions. Here, there are two such regions R_1 and R_2 which have been considered to be distinct by P_1. Therefore, I consider two sets of Maxwell's equations corresponding to R_1 and R_2 as follows.

3.3.1 Equations For R_1

The relevant quantities and equations for R_1 are as follows. $\rho_1, \vec{J}_1$ are the charge density and the current density respectively, $\vec{E}_1, \vec{B}_1$ are the electric field and the magnetic field, respectively, produced due to $\rho_1, \vec{J}_1$. The equations relating these quantities are as follows:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E}_1 &= \frac{\rho_1}{\epsilon_0}, \qquad \vec{\nabla} \times \vec{E}_1 = -\frac{\partial \vec{B}_1}{\partial t}, \\ \vec{\nabla} \cdot \vec{B}_1 &= 0, \qquad \vec{\nabla} \times \vec{B}_1 = \mu_0 \vec{J}_1 + \mu_0 \epsilon_0 \frac{\partial \vec{E}_1}{\partial t}. \end{aligned}$$

Remark: Here, ρ_1 and $\vec{J}_1$ are the source charge density and the source current density which produce the fields $\vec{E}_1$ and $\vec{B}_1$. $\vec{E}_1$ and $\vec{B}_1$ can NOT act upon ρ_1 and $\vec{J}_1$, but only on charge and current localized in any location that is remote from R_1 e.g. R_2 in the present case. Indeed such intuition is already evident from Maxwell's statement of Coulomb's hypothesis, clearly stated in his book [3], but computationally accounted for only recently in ref. [50]. $\rho_1 = 0, \vec{J}_1 = \vec{0}$ outside R_1, which can be termed as vacuum with respect to R_1 because other localized charge and current may be present outside R_1 which can interact with $\vec{E}_1, \vec{B}_1$, but such charge and current are not $\rho_1, \vec{J}_1$ that appear in the above equations.

3.3.2 Equations For R_2

The relevant quantities and equations for R_2 are as follows. $\rho_2, \vec{J}_2$ are the charge density and the current density respectively, $\vec{E}_2, \vec{B}_2$ are the electric field and the magnetic field, respectively, produced due to $\rho_2, \vec{J}_2$. The equations relating these quantities are as follows:

$$\begin{aligned} \vec{\nabla} \cdot \vec{E}_2 &= \frac{\rho_2}{\epsilon_0}, \qquad \vec{\nabla} \times \vec{E}_2 = -\frac{\partial \vec{B}_2}{\partial t}, \\ \vec{\nabla} \cdot \vec{B}_2 &= 0, \qquad \vec{\nabla} \times \vec{B}_2 = \mu_0 \vec{J}_2 + \mu_0 \epsilon_0 \frac{\partial \vec{E}_2}{\partial t}. \end{aligned}$$

ρ_2 and $\vec{J}_2$ are generally called test charge density and test current density on which some external fields act, which are here $\vec{E}_1$ and $\vec{B}_1$ – NOT $\vec{E}_2$ and $\vec{B}_2$ which are rather produced due to, or sourced by, ρ_2 and $\vec{J}_2$.

Remark: At this point one may argue that consideration of ρ_2 (charge acted upon) and $\vec{J}_2$ (current acted upon) should have sufficed for the calculation to be performed. However, as I shall point out shortly, it is necessary to consider a whole new set of Maxwell's equations for R_2 so as to perform calculations.

4 The electrostatic part – A hint of unprovability of Poynting's theorem

Hertz writes, "*The total force may be split up into the electrostatic part and the electromagnetic part;...*", and explains the observations accordingly [5]. So, let me calculate the electrostatic work done by $\vec{E}_1$ upon the total charge in R_2 and verify whether it matches the standard results. The equations for R_1, which are relevant for the present calculation:

$$\vec{\nabla}\cdot\vec{E}_1=\rho_1/\epsilon_0,\qquad \vec{\nabla}\times\vec{E}_1=0\quad\equiv\quad\vec{E}_1=-\vec{\nabla}\phi_1, \tag{1}$$

where ϕ_1 is the scalar potential. Then, the amount of work done, due to $\vec{E}_1$, on an infinitesimal charge $dq_2=\rho_2 d\tau$, contained in an infinitesimal volume $d\tau$ at a point P in R_2, is given by

$$\begin{aligned} W_{21}[\rho_2 d\tau] &= -\int_\infty^P(\rho_2 d\tau)\vec{E}_1\cdot d\vec{\ell}\qquad[\text{such that } P\in R_2]\\ &= \int_\infty^P(\rho_2 d\tau)(\vec{\nabla}\phi_1)\cdot d\vec{\ell}\\ &= (\rho_2 d\tau)\phi_1[P]\qquad[\because \phi_1[\infty]=0 \text{ by boundary condition}]. \end{aligned}\tag{2}$$

Therefore, the total work done on all the charge in R_2 is given by

$$\begin{aligned} W_{21}[R_2] &= \int_{R_2}\rho_2\phi_1\;d\tau\;=\epsilon_0\int_{R_2}(\vec{\nabla}\cdot\vec{E}_2)\phi_1\;d\tau\qquad[\text{using } \vec{\nabla}\cdot\vec{E}_2=\rho_2/\epsilon_0]\\ &= \epsilon_0\int_{R_2}[\vec{\nabla}\cdot(\phi_1\vec{E}_2)-\vec{E}_2\cdot(\vec{\nabla}\phi_1)]\;d\tau\\ &= \epsilon_0\int_{R_2}\vec{E}_1\cdot\vec{E}_2\;d\tau+\epsilon_0\oint\!\!\!\oint_{\partial R_2}(\phi_1\vec{E}_2)\cdot d\vec{S}_2 \end{aligned}\tag{3}$$

Here, even if I ignore the second term (i.e. the surface integral), the first term does not lead to the familiar result "$\frac{\epsilon_0}{2}\int E^2\;d\tau$". The "1/2" is missing, which actually appears in the textbook calculation because of the exclusion of the self-energy of point-charges while calculating the self-energy of a configuration of point charges and the corresponding extrapolation to continuous charge distribution [6, 7]. The most crucial point to note here is that I can get the resemblance with the familiar "E^2" if and only if I erase the distinction between R_1 and R_2 to write $\vec{E}_1=\vec{E}_2=\vec{E}$ (by erasing the subscripts "1" and "2"). That is, I choose to invoke $\neg P_1$, which is a choice of erasure of "cause/effect" distinction, to obtain a desired result. However, due to this choice, the logic of causality becomes undecidable. This hints towards the fact that either Poynting's theorem is unprovable or logic of causality is undecidable. This should be explicitly manifest from what follows next.

5 Revisiting Poynting's theorem

Nowadays available in any textbook on electrodynamics, the proof of Poynting's theorem starts with the application of the Lorentz force to calculate the rate of work done, by external fields, on a test (point) charge [6, 7]. Here, I redo the calculations within the present context of Hertz experiment.

The Lorentz force, due to $\vec{E}_1$ and $\vec{B}_1$, on a point charge q_2 in R_2 is given by

$$\vec{F}_{21}[q_2]=q_2(\vec{E}_1+\vec{v}\times\vec{B}_1). \tag{4}$$

The work done by $\vec{E}_1$ and $\vec{B}_1$, in time dt, to move a point charge q_2 in R_2, with velocity $\vec{v}=d\vec{\ell}/dt$, is given by

$$\begin{aligned} dW_{21}[q_2]=\vec{F}_{21}[q_2]\cdot d\vec{\ell} &= q_2(\vec{E}_1+\vec{v}\times\vec{B}_1)\cdot\vec{v}dt\quad[\because \vec{v}=\frac{d\vec{\ell}}{dt}\;\equiv\;d\vec{\ell}=\vec{v}dt]\\ &= q_2\vec{E}_1\cdot\vec{v}dt\qquad[\because(\vec{v}\times\vec{B}_1)\cdot\vec{v}=0]. \end{aligned}\tag{5}$$

$$\tag{6}$$

Therefore, the rate of work done due to $\vec{E}_1$ and $\vec{B}_1$, on a point charge q_2 in R_2 is given by

$$\frac{dW_{21}[q_2]}{dt} = q_2 \vec{v} \cdot \vec{E}_1. \tag{7}$$

Now, to carry forward the computation for a charge distribution, the general practice is to replace the point charge q_2 with an infinitesimal charge dq_2, followed by an integration over the relevant volume. So, we proceed as follows. The rate of work done due to $\vec{E}_1$ and $\vec{B}_1$, on an infinitesimal charge $dq_2 = \rho_2 d\tau$, contained in an infinitesimal volume $d\tau$ in R_2, is given by

$$\frac{dW_{21}[dq_2]}{dt} = dq_2\ \vec{v} \cdot \vec{E}_1 = (\rho_2 d\tau)\ \vec{v} \cdot \vec{E}_1 = (\vec{J}_2 \cdot \vec{E}_1)\ d\tau \qquad [\because \vec{J}_2 = \rho_2 \vec{v}]. \tag{8}$$

Therefore, the rate of work done due to $\vec{E}_1$ and $\vec{B}_1$, on the total charge contained in R_2, is given by

$$\therefore \qquad \frac{dW_{21}[R_2]}{dt} = \int_{R_2} (\vec{E}_1 \cdot \vec{J}_2)\ d\tau. \tag{9}$$

Computation would have stopped/halted here if I would have just considered only ρ_2 and $\vec{J}_2$, and not Maxwell's equations for R_2.

Now, in (9), I write $\vec{J}_2$ in terms of $\vec{E}_2, \vec{B}_2$ by using Maxwell's equations for R_2 and proceed with the calculations as follows:

$$\begin{aligned}
\frac{dW_{21}[R_2]}{dt} &= \frac{1}{\mu_0} \int_{R_2} \left[\vec{E}_1 \cdot (\vec{\nabla} \times \vec{B}_2) - \mu_0 \epsilon_0 \frac{\partial \vec{E}_2}{\partial t} \cdot \vec{E}_1 \right] d\tau \\
&= \frac{1}{\mu_0} \int_{R_2} \left[\vec{B}_2 \cdot (\vec{\nabla} \times \vec{E}_1) - \vec{\nabla} \cdot (\vec{E}_1 \times \vec{B}_2) - \mu_0 \epsilon_0 \frac{\partial \vec{E}_2}{\partial t} \cdot \vec{E}_1 \right] d\tau \\
&\qquad [\text{using } \vec{\nabla} \cdot (\vec{E}_1 \times \vec{B}_2) = \vec{B}_2 \cdot (\vec{\nabla} \times \vec{E}_1) - \vec{E}_1 \cdot (\vec{\nabla} \times \vec{B}_2)] \\
&= \frac{1}{\mu_0} \int_{R_2} \left[-\vec{B}_2 \cdot \frac{\partial \vec{B}_1}{\partial t} - \mu_0 \epsilon_0 \frac{\partial \vec{E}_2}{\partial t} \cdot \vec{E}_1 - \vec{\nabla} \cdot (\vec{E}_1 \times \vec{B}_2) \right] d\tau \qquad (10) \\
&\qquad [\text{using } \vec{\nabla} \times \vec{E}_1 = -\partial \vec{B}_1 / \partial t \text{ in the first term}] \\
&= \frac{1}{\mu_0} \int_{R_2} \left[-\vec{B}_2 \cdot \frac{\partial \vec{B}_1}{\partial t} - \mu_0 \epsilon_0 \frac{\partial \vec{E}_2}{\partial t} \cdot \vec{E}_1 \right] d\tau - \frac{1}{\mu_0} \oiint_{\partial R_2} (\vec{E}_1 \times \vec{B}_2) \cdot d\vec{a}_2. \qquad (11)
\end{aligned}$$

The above expression manifests the distinction between R_1 and R_2 (i.e. P_1) through the subscripts "1" and "2" to denote the fields originating in the respective regions. The logic of causality is decidable in principle. However, Poynting's theorem is unprovable.

Now, it is trivial to observe that if and only if I erase the distinction between R_1 and R_2 (i.e. $\neg P_1$) to write $\vec{E}_1 = \vec{E}_2 = \vec{E}$ and $\vec{B}_1 = \vec{B}_2 = \vec{B}$, then only the expression (10) can be written, through a deliberate loss of symbolic clarity by erasing all the "1" and "2" in the subscripts, as follows:

$$\frac{dW}{dt} = \frac{1}{\mu_0} \int_{\tau} \left[-\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot \vec{E} \right] d\tau - \frac{1}{\mu_0} \oiint_{\partial \tau} (\vec{E} \times \vec{B}) \cdot d\vec{a}, \tag{12}$$

where one can now introduce the Poynting vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$ in the third term.

Indeed, in standard practice [6,7], one writes "$(\vec{E} \cdot \vec{J})$" as the integrand of (8) by erasing the subscripts "1", "2" of "$(\vec{E}_1 \cdot \vec{J}_2)$" and consequently looses the information regarding whether "$\vec{J}$" is the current in the source or the detector, which allows one to recklessly use Maxwell's equations to achieve a desired result. The rest of the steps of calculation, which lead to the Poynting's theorem from (12), are trivial and available in standard textbooks [6,7]. Thus, $\neg P_1$ renders Poynting's theorem to be provable, but the logic of causality becomes undecidable in principle.

Therefore, I may assert that only one of the following is true in the context of an explanation of Hertz experiment on the basis of standard Maxwell's equations.

- P_1 holds. Poynting's theorem is unprovable, but the logic of causality is decidable. The theory manifests ECC by taking into account the distinction between R_1 and R_2 that is schematically depicted in fig.(1).

- $\neg P_1$ holds. Poynting's theorem is provable, but the logic of causality is undecidable. The theory violates ECC by erasing the distinction between R_1 and R_2 denying truth manifested through the schematics of fig.(1).

My assertion points towards the following aspects of the current status of theoretical physics as far as the understanding of light is concerned.

1. ECC is not discussed in the context of Maxwell's equations (excluding ref. [50]) and Poynting's theorem, considering the physical reality of Hertz experiment. Therefore, in the standard literature, discussion of ECC appears to be exclusively relevant in quantum physics, which is quite misleading. This is significant because the hypothesis of *photon propagation* forms the basis of experiments with photons and such a hypothesis is just an extrapolation of the classical concept of light propagation as wave founded upon Maxwell's equations, Poynting's theorem and Hertz experiment.

2. Both P_1 and $\neg P_1$ are considered to be simultaneously true, signifying a decision problem at the root of the current understanding of electromagnetic phenomena in terms of Maxwell's equations and Hertz experiment. Meaning, borrowing words of Hilbert and Ackermann [78], there is a "*problem of universal validity*" of P_1 that plagues the currently accepted proof of Poynting's theorem whose consistency in the backdrop of Hertz experiment has remained hitherto unquestioned (prior to this present work).

5.1 "Necessity", Choice and Free Will

In the context of this work, I find it interesting to mention Poincare's statement from page no. 6 of ref. [80], regarding mathematical reasoning: "*It is therefore at the beginning of Arithmetic that we must expect to find the explanation we seek; but it happens that it is precisely in the proofs of the most elementary theorems that the authors of classic treatises have displayed the least precision and rigour. We may not impute this to them as a crime; they have obeyed a necessity.*"

Poincare does not explain what the "necessity" is, which leaves me the room for the following speculation that I find appropriate in the present context. This necessity is to achieve a result that has been preconceived in mind by the respective author, who invokes choice by his/her free will as per requirement when such an achievement seems impossible through a logical discourse. Now, it is essential to distinguish the steps of reasoning that follow a logical discourse, from the steps of reasoning invoked through choice. This can only be done by being precise and rigorous in reasoning (alongside calculation) and, therefore, in language. It is to search for such precision and rigour in reasoning in physics that Hilbert formulated his sixth problem [70–72]. This has served as the motivation for the propositional analysis (arithmetization of language) in this work to begin with, so as to make the pivotal role of "cause/effect" distinction as clear as possible. This consequently has led to the identification of the choice of erasure of "cause/effect" distinction invoked by free will to achieve a result theoretically.

In view of this I may make the following remarks. Schroedinger discussed, in the context of quantum theory, how the experimenter creates physical reality through choice and free will concerning measurement [20]. Such discussion has led to the conception of even theorems in modern days [52–56]. However, choice seems to constitute a part of human reasoning [81–86] and not necessarily related to any particular theory. This can be immediately identified from Turing's distinction between "choice machines" and "automatic machines", where he worked with the latter only to implement arithmetization of language into practice [87] and thereby showing an application of Hilbert's decision problem [78]. The present work, being concerned with classical theory of electrodynamics, firmly establishes that choice and free will are part of human reasoning and the theoretician can create a result through choice and free will, which may be used by the experimenter as a language to speak about physical reality even in the simplest of contexts like Hertz experiment.

5.1.1 Foundations of Optics and a Hidden Choice Unraveled

Now, such observations do affect the modern day status of physics where much more complex experiments than Hertz experiment are performed, but the theoretical premise remains the same, namely, Maxwell's equations and Poynting's theorem. Optics [8], especially quantum optics [14, 15, 17, 18], holds the key for the modern day renaissance in quantum information science [30] and forms the basis of all our understandings concerning any photon related experiment. Explanations of photon propagation, in terms of wave function [11, 14] and in terms of quantum field theoretic techniques [15], both refer to Poynting's theorem when necessary. Therefore, the theoretical premise based on which one analyses experiments involving

- "choice" and "erasure" [88, 89], is itself constructed through a choice of erasure,

- involving hidden variables [31,32], is itself plagued by a hidden choice - now unraveled.

Certainly such mentions are far from any concrete technical assertions and they need to be analyzed through further investigation concerning precision of language in which physics is written [50,57,58]. Nevertheless, a discomfort lingers from the logico-linguistic point of view whose significance is manifested through Hilbert's sixth problem [70–72] and other recent analyses concerning the logico-linguistic foundations of physics [50,57,58,60]. To be more specific I may assert that an understanding of quantum phenomena through photon related experiments (e.g. ref. [24] introduces quantum mechanics through double slit experiment with photons), demands the declaration of the postulates of electrodynamics, if any, alongside the postulates of quantum mechanics, where the "cause/effect" distinction must be clearly asserted or negated so as to make the foundations logical or mathematically treated [70–72]. In view of this, definitely the question arises that how to take into account the empirical truth of causal light propagation in the theory, in light of ECC. This is why the present analysis motivates the need to revisit the foundations of electrodynamics with a renewed focus on logic, language and operations, where the intent should be to write the theory so as to explain the observations i.e. the theory should be on "trial" alongside the presently accepted ones – an attitude well manifested through the words of Jaynes [90] that led to Jaynes-Cummings model [91].

6 Conclusion

I conclude with the following remarks regarding what one may learn from the analysis presented in this work.

If a distinction is made, and maintained, between cause (source) and effect (detector/test) in theory, so as to provide a EPR-complete description of Hertz experiment, then Poynting's theorem is unprovable. So, a theoretical proof of electromagnetic energy propagation over a distance, based on the standard Maxwell's equations, is impossible in light of EPR completeness condition. However, the logic of causality, or the law of causation, remains decidable. In this case, Hertz's conclusion regarding "propagation of electromagnetic actions" can not be theoretically interpreted in terms of (the unprovable) Poynting's theorem.

On the other hand, a choice of erasure of "cause/effect" distinction, in theory, renders the Poynting's theorem to be provable. However, this choice of erasure makes the theory provide an incomplete description of physical reality of the Hertz experiment, in EPR's sense. Also, the logic of causality becomes undecidable. That is, using such a theory one can not decide whether "cause before effect" or "cause after effect" because there is no distinction between "cause" and "effect" in the theory. The phrase "propagation of electromagnetic action" loses its essence because "propagation" from "where to where" can not be understood from such a theory.

Therefore, either way, Hertz experiment loses the theoretical foundation based on standard Maxwell's equations. From the historical point of view this work provides an independent theoretical support to Tesla's experimental objections against the usual interpretation of electromagnetic wave propagation in terms of Maxwell's equations [51]. From Hilbert's logical point of view [70], this work provides an important example of propositional analysis of the statements of physics to gain clarity in reasoning with an attachment to experimental realizations i.e. the connection among logic, language and operation is signified. Also, this work showcases the role of choice and free will in writing a classical theoretical result, thus establishing an example outside the usual regime and practice of discussing choice and free will exclusively in the context of quantum theory [20,52–56]. It is interesting to observe that what appears to be a symbolically true expression of the practical experience of Hertz's experimental set up, only provides the sense of a correlation between the source (cause) and the detector (effect) — a trait that is spoken about and discussed exclusively in the context of quantum entanglement and related experiments. This observation is important because the reasoning associated with such scenarios, involving photons, give rise to paradoxical obstacles in our understanding and interpretations in terms of quantum optics [14–18]. Such literature, ironically, is based on an unquestioned acceptance of Maxwell's equations and Poynting's theorem as the theoretical basis of light propagation (namely, classical optics [8]), which is extrapolated to quantum regime, with little doubt on such classical premises, to result in the hypothesis – *photon propagates*, supported through further theoretical constructions based on quantization methods [14]. Based on such hypothesis, one seeks information regarding the path of propagation of a photon, from the source to the detector, in a double slit experiment. It seems, alongside the discussions and debates concerning "delayed choice" [25,88] and "quantum erasure" [89] in photon related experiments, and the role of choice and free will in quantum theory and measurement [20], it is now equally important of being aware of the choice of erasure of "cause/effect" distinction in the proof of Poynting's theorem in the backdrop of Hertz experiment. This is because, now it can be doubted whether we, at all, have a theoretical proof of electromagnetic energy propagation and, if not, whether we have a theoretical proof for light propagation. It seems that a revisit to the foundations of electrodynamics seems more essential than ever, so as to have a more refined understanding of light 'propagation', especially in light of the Nobel Prize 2022 in physics [30].

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