Some notes on shadows

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Abstract: These notes were taken whilst thinking about the monotone-light factorization, which lead to the productive idea of a shadow category, or a certain kind of category with a preordered structure.

§0 Preamble

Conventions

By an isomorphism we will always mean a bijective n-categorical isomorphism. By a pushout or pullback, we will mean an n-pushout or n-pullback. We avoid working with the finer details of n-categories, but appreciate their relationship to one-categories for the purposes of localization.

Let $K=\partial K^2$ be a boundary-forming and simply connected space. Let the convex portion of space about K have an inner product

$$\begin{array}{c}\mathbf{Sets} \times \mathbf{Top} \to \mathbf{DispMfld} \\ \downarrow \\ \mathbf{SSets}\end{array}$$

so that there is a projection onto a curve within $\mathscr{U}(K)$.

We will call K an (ε,δ)-chain, and we will call its pushout into SSets a "shadow." We will operate using the commutative fusion rule

$$\varepsilon \bigstar \delta = \delta \bigstar \varepsilon = \mathrm{Hom}(\mathbf{Sets} \times \mathbf{Top}, -)$$

We characterize each unique geometric fibration $\theta \to \{-\}$ according to a "length spectrum," which "records" information about the number of objects with maps into identical tangent categories.

Definition 0.0.1 A δ-transitive connection is a first-degree connection on an (ε,δ)-chain.[1]

Let A be a geometric series with a least element α. Then, there is a relationship

$$\sup(A)\mathbf{R}\alpha$$

of rank p, which accords with the p-weight[2] of an associated module in A.

§1 Chain Transitivity

A sequence Q of weakly chained ind-spaces give a precise Fourier projection onto the interior of a topological space as determined by the display maps which foliate Q. We define Q to be a chain-connected space; that is, for any two elements {p,q} in Q, they may be compared by $p\mathbf{R}q=(q\mathbf{R}p)^{-1}$.

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1 See [ChainTr]; we must cherish the authors of this paper for their marvelous account of the succession function, which had yet to make its more operadic appearance. [GSP] gives a similar indispensable account.

2 See [Calc], pg. 13

We say a chain-connected space has the property of *chain transitivity* if, for any two sections

$$\{p_1 \to q_1, p_1 \to p_1\} \simeq \mathrm{fib}(Q),$$

there are the retracts

$$\{q_1 \to p_1, p_1 \to p_1\} \simeq \mathrm{cofib}(Q^{op})$$

This definition has been precipitated slightly beforehand, but now receives its precise incarnation.

Definition 1.0.1 . A *display map*

$$\phi_i : ((p \bigstar q) \vee (q \bigstar p)) \to ((p \vee q) \vee (q \vee p))$$

is a fusion-distributive logical connective.

Example

Let $\phi_i(\mathscr{P}) \to \phi_i^{-1}$ be a "twist" on an algebra $\mathscr{A}$. We call this map of displays $\phi_i(-) \to \phi_i(-')$ a *display block* if it covers a frame $\mathscr{f}$ in the π_0-cocycle of an arbitrary system of involutions.

Here, an involution means a coefficient q attached to a function field f

$$\pi_0: q(f) \to ((q^{-1}(f)) \vee (q(f^{-1}))$$

Following [Beardsley], we will write $\mathbf{L_n}$ for the appropriate localization functor $\pi_0\zeta \to \zeta$, where

$$\zeta = q(f) \to (-)$$

Let $\mathfrak{g}$ be an arbitrary geodesic in some presentable category. Then, we have a restriction from the *higher bundle* of $\mathfrak{g}$ to the $\mathfrak{g}$-action *on objects*. This is a localization from the ∞-Cats form of a representation to the geometric form.

Definition 1.1.0 A **Jordan form** is the kernel of some ζ-object. An object with a Jordan form satisfies the idempotent criterium on pg. 4 of [Jord].

Definition 1.1.1 Let S: X → X be a continuous map. A sequence $\{x_n\}$ in X is an asymptotic pseudo-orbit of S if[3]

$$\lim_{n \to \infty} d(S(x_n), x_n + 1) = 0$$

Remark It would be interesting, at least to the author, if every ζ-object object had an asymptotic pseudo-orbit.

For two rank one isomorphisms x**R**y and x'**R**y', we take the difference

$$\{x \cup y\} \backslash \{x' \cup y'\} = \omega$$

[3] [Strong], definition 2.3

to be a pseudo-orbit of an **R**-algebra $\mathscr{A}$. If we let {x≃x'},{y≃y'} be distinct equivalence classes, then we can take the immersion

$$\gamma\omega \hookrightarrow \omega$$

from the outer sum of a pair and the inner hom of the other.

§2 Shadows

Definition 2.0.1 A *po-category* is a category $\mathfrak{PO}$ with a 2-isomorphism into the bi-category[4] SSets × $\mathbb{I}$.
Definition 2.0.2 A *shadow category* is a po-category

$$\mathfrak{shad}=\mathrm{Push}(\mathfrak{PO})$$

which is locally a pushout.

A shadow is a compactly generated object in $\mathfrak{shad}$, which is globally presentable.
Proposition 2.1.0 $\mathfrak{shad}$ is Cartesian-closed.
Proof Let ω_1,ω_2 be two real cones, with the inclusions

$$\omega_1 \in \mathfrak{PO}$$
$$\omega_2 \in \mathrm{Push}(\mathfrak{PO})$$

The bijection

$$\omega_1 \leftrightarrow \omega_2$$

automatically makes $\mathfrak{shad}$ symmetric monoidal, which means it is Cartesian closed.

Let **L** be a diagram with a unique factorization f: y' → y, where proj{y,y'}=z. Assuming each object is a po-category, we have an isomorphism

$$\mathfrak{shad} \simeq z$$

Definition 2.1.1 An E-chain is a strictly modellable set of integrands with morphisms into $\mathfrak{shad}$.
Here, we have explicitly defined the boundary-forming parts of an orbifold to be those series of fibrations which admit connections into the shadow category.
Proposition Any map $E \rightarrow \mathfrak{shad}$ is at least a $\underline{d}$-shadowing.[5]

A "shadow" is, in some sense, any continued fraction which has stable approximation as an algebraic symbol σ.

$$\mathrm{Map}(E,\mathfrak{shad}) \simeq \partial\sigma_k \rightarrow \sigma$$

Here, the left adjoint is a totally disconnected free variable.
We can compute this more simply by putting

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[4] Where $\mathbb{I}$ is the category of intervals
[5] See [ChainTr], pg. 3.

$$\sigma_{\text{DISC}} \to \sigma_{\text{FIN}} = \sum_{i=0}^{\infty} (d(\partial i, i)) = \pi_0(\tau).$$

Axiom 2.1.2 *Adjointness*
For two stable objects $\pi_0(\tau_0) \vdash\dashv \pi_0(\tau_{k+1})$, with adjoint morphisms f and f', these morphisms shall be called *mutually orthogonal* and *metric-forming*.

We take a simple comparison triangle, $\widetilde{\Delta} = \angle abc = \sum_{\angle a}^{c} p^{\circ}$, and calculate, say, the standard deviation of each angle from the arithmetic mean. Then we obtain a measure of the hyperbolicity (as measured by the Cat(k)-number) of a space, specifically a positive or negative sectional curvature.

References

[Beardsley] J. Beardsley *Some notes on the category of p-local harmonic spectra* (2013)
[ChainTr] W.R. Brian, J. Meddaugh, B.E. Raines *Chain transitivity and variations of the shadowing property*
[GSP] A. Barzanouni, E. Shah *Chain Transitivity for maps on G-spaces* (2019)
[Strong] M. Hirsch, H. Smith *Chain Transitivity, Attractivity, and Strong Repellors for Dynamical Systems* (2001) (Journal of Dynamics and Differential Equations)