GRAVITY EXTENSIONS EQUATION AND COMPLEX SPACETIME

TOMASZ KOBIERZYCKI

Abstract. This hypothesis will present a possible extension of Einstein's field equations [1][2][3][4] which reduces to his field equation by contractions. Basic concepts such as the description of the inertial system or the definition of a physical observer are discussed. The field equation predicts the existence of exactly four-dimensional space-time, as only four-dimensional space-time has an equal number of unknowns for each term of the equation. The equation itself can be written in two mixed and fully covariant forms:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{0.1}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2} R_{\sigma\phi} g_{\mu\nu} = \kappa T_{\mu\phi} g_{\sigma\nu} \tag{0.2}$$

This model relates the field of matter to the curvature of space-time in a direct way, if matter is not present at a given point in space, it is simply flat space-time, which makes it a requirement that the momentum energy tensor does not zero in the presence of space-time curvature. In this work, he does not give the exact solutions of the equations, only their derivation and their form in a particular case. I will also present a possible way to quantize field equations using complex space-time. This gives the quantum field equation which I can write as

$$\left(R_{\phi\mu\sigma\nu}\right)^{\dagger} R_{\phi\mu\sigma\nu} = \kappa \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{0.3}$$

This equation uses a special complex space-time and generates real invariants by using complex conjugates and transpositions. This removes it from singularity field theory as only normalizable fields that do not possess any kind of infinity are a consequence of this normalization.

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1. Classic section

1.1. Description of the inertial system.

According to Newton's laws of motion, motion with constant velocity or lack of it gives an inertial frame of reference. This definition can be simplified even more that it is a system in which there are no forces associated with the motion of this system. Such a system does not feel the forces associated with its movement. The key question is whether an observer under the influence of a gravitational field can be treated as an inertial observer?

According to the equivalence principle, the gravitational field cannot be locally distinguished from the acceleration, on the other hand, omitting the tidal forces, one can look at the inertial system as the system of any falling observer in the gravitational field. This observer locally has no weight, no force acts on him. There are two possibilities, either the observer is in uniform motion or the observer is at rest. The first possibility can be ruled out for obvious reasons only the second possibility remains, the observer in the gravitational field is motionless. This means that every observer in the gravitational field that is not subjected to any kind of apparent force related to e.g. standing on the surface of the gravitational field source or any other force is treated as an inertial system. And according to this observer's perspective, this gravitational field needs to be described. This means that any non-inertial observer cannot see the true cause of motion because there are forces in his system, so they exclude him from being an inertial system. The description of the laws of physics and thus motion must always be seen from the perspective of an inertial observer as only he perceives the true cause of motion, which also applies to the gravitational field.

An inertial frame of reference is defined as one which, under the influence of a physical field, remains completely motionless from its own perspective, where this motionlessness is only locally defined. This locality makes us ignore the tidal forces that will naturally accompany the gravitational field, and thus the physical field that is the source of motion. However, the definition of the physical field itself, i.e. the gravitational field, the field that causes motion, is more delicate. It results from the definition of the inertial system itself, physical fields are a field from the perspective of which no inertial observer can be described as an inertial observer, so he must be in motion relative to this field. This means that the field itself cannot give us the whole picture of how motion physically occurs. Only the perspective of a field that is the source of motion and an inertial observer that is able to detect true non-relative motion as a combination gives us a description of physical reality. Any motion that is relative depends on the system in which it is measured, the physical field or simply the gravitational field cannot be dependent on the system in which it is measured, it must be a source of motion for every inertial observer.

1.2. **Light signals and their interpretation.** The basis of the Theory of Relativity (Special) is the constancy of the speed of light for each inertial system. This creates a transformation of the frame of reference so that the speed of light is conserved for each observer. For two observers observing an event of length ds, you can write this transformation as a requirement $ds^2 = \eta_{\mu\nu}dx^{\mu}dx^{\nu} = \eta_{\mu\nu}dx'^{\mu}dx'^{\nu}$ where prime coordinates are the second observer. This means that the light signal plays a key role in building the concept of distance in space-time. Consider a light signal propagating from the point $\mathbf{x}$ then the space-time distance (interval) can be written as:

$$ds^2 = \eta_{\mu\nu}dx^{\mu}\left(\mathbf{x}\right)dx^{\nu}\left(\mathbf{x}\right) \tag{1.1}$$

This means that distance is fundamentally linked to the ability to send a light signal from a given point in spacetime. How a given observer measures his/her axis of time and space is not absolute, but the magnitude is. So what is its physical meaning? A light signal sent from a given point in space-time determines that event happening at that point in space-time. This means events are understood as light signals propagating from every existing point in space at every possible point in time. Of course, these signals do not have to be physically sent, it's just a geometric fact, so for a light signal, the interval is always zero $ds^2 = 0$. An event for itself has no distance from its beginning. Despite this, different observers will perceive differently how these events occur, the light signal does not "perceive" the distance between itself and every other event. The light signal is immediately located in that place and time in which the event is present from the perspective of the given observer. This gives a fairly obvious interpretation of how an inertial observer defines his laws of motion and how he perceives time and space.

Since the observer is always stationary relative to the light signal emitted from any point in time and space, this gives an additional important rule in determining motion, the observer is always stationary from the perspective of any light interval, i.e. an interval with zero distance in time and space. The inertia of this observer is always defined with respect to the event itself, or more precisely with respect to space-time. The observer itself is always motionless relative to any event that happens in space-time. So any truly inertial frame of reference is defined by the impossibility of motion with respect to the event. Thus, space-time consists of inertial observers and this inertia results from the invariance of the speed of light for each observer, i.e. events in space-time.

1.3. **Observer definition.** The observer is understood as a frame of reference capable of measuring time (clock) and distance (ruler). The units of measurement must always be chosen to express distance in time or space. This means that if I measure distance in meters and seconds I have to express both units in meters or seconds, which is achieved by multiplying time by the speed of light (meters) or dividing distance by the speed of light (seconds). This is a fairly basic assumption in the Theory of Relativity.

What is crucial for extending the field equations is the exact physical definition of the phenomenon for a given observer. A physical phenomenon is simply such a phenomenon that meets the previous assumption, the observer is completely at rest relative to the event, which means that it is defined as an inertial frame of reference. The previous definition, of course, only makes sense in the case of flat space-time, so it is not a general case. To go to the general case, it is necessary to define an observer in a gravitational field as still an observer motionless relative to an event in which a gravitational field is present. Before discussing free fall from the perspective of an inertial observer, one key point needs to be addressed.

Space-time in the mathematical description must adhere to the principle according to which the observer remains inertial to the event, this means that locally, as in the Theory of Relativity, the observer locally measures flat space-time, which is not true globally. The whole point of this paper is to show that there are other field equations that reproduce this principle but with an additional condition. This condition is that the gravitational field is fully dependent on the existence of a field of matter and/or energy at every point in space-time. In the absence of matter at any point in space-time, these equations become equations for flat space-time, which means that literally the source of deviations from flat space-time must be the presence of matter at the point where this space-time deviates from it, otherwise we get flat Minkowski space. There is an additional principle that is central to this whole model and its assumptions, the equivalence of gravity and the field of matter. Which I will discuss in more detail later in the section on the non-zero momentum energy tensor requirement.

1.4. **Light clock.** In the Theory of Relativity, time is measured using a theoretical light clock that measures the time between successive reflections of a light ray. Such a hypothetical clock has quite important implications for how distance and the passage of time are defined. Since the speed of light is constant in a vacuum for any observer, it is a contentious but important question as to what the speed of light really remains constant. Let's consider three possible scenarios:

1. Relative to some field - a medium once called ether.
2. Relative to an inertial observer.
3. It's just a constant relative to nothing.

Thanks to the light clock, you can answer the question. If there is a field with respect to which light always has a constant speed - that field is an event, the inertial observer perceives that for each event the speed of light is constant, so this does not contradict the second point of this assumption. This also explains why the speed of light is the speed of causality in a simple way, events propagate at the speed of light and any observer who is not moving at the speed of light and therefore not an event stays still relative to the event which makes the speed of light remain constant and the information moves exactly at the speed of light (only in the sense of a physical event).
Now a thought experiment using a light clock that can rule out the third option is as follows, I have two observers A and B both of whom use a light clock to measure time, but you don't know which is moving and which is not. These observers at some point in time t meet and compare their clocks, always the clock of the moving observer will have fewer ticks. It is impossible to define whether the observer A was in motion or the observer B was in motion, because without the top-down assumption that this particular observer is in motion and the other one, it is not possible to perform an experiment using light signals that will distinguish which one is truly in motion. However, what makes it possible to distinguish which is in motion is the indication of the first clock relative to the second, if the observer A was in motion, his light clock has a smaller clock indication. This means that both the clock and the observer A were physically in motion relative to an event that lasted a certain amount of time. The difference is that the light in the clock seems to tick normally for observer A and slower for observer B, but observer B will say the same about observer A. This means that since A's clock is in true motion, observer B is right and observer A's perspective is wrong. A's clock is ticking slower objectively because fewer events occur physically for this observer - and since the speed of light is constant relative to events as their number decreases relative to observer B for whom more events occur, it will find that fewer events occur for observer A which is a physical fact. So this paradox clearly shows that the speed of light is constant only and exclusively with respect to the event.

1.5. **Lorentz transformations.** Lorentz transformations [5][6][7][8][9] are the basis of the Special Theory of Relativity, they transform the coordinates of one frame of reference to match what that observer perceives while maintaining constant the speed of light. These transformations can be written for the space-time interval of two observers as:

$$\eta_{\mu'\nu'}dx^{\mu'}(\mathbf{x})\,dx^{\nu'}(\mathbf{x}) = \eta_{\mu'\nu'}\Lambda^{\mu'}_{\alpha}(\mathbf{x})\,dx^{\alpha}(\mathbf{x})\,\Lambda^{\nu'}_{\beta}(\mathbf{x})\,dx^{\beta}(\mathbf{x}) \tag{1.2}$$

However, more generally, these transformations can be approached in curved spacetime using tetrad fields, then in general the metric tensor can be written as:

$$g_{\mu\nu}(\mathbf{x}) = e^{a}_{\mu}(\mathbf{x})\,e^{b}_{\nu}(\mathbf{x})\,\eta_{ab} \tag{1.3}$$

The vensors can be written using the field of tetrads [10] as a combination of them with the hexagon of a Minkowski space as:

$$\hat{e}^{\mu} = e^{a}_{\mu}(\mathbf{x})\,\hat{e}_{a} \tag{1.4}$$

Where the sensor with index μ denotes any curved space-time, and the sensor with index a denotes Minkowski space sensors. Lorentz transformations can be performed locally on these vensors:

$$\hat{e}_{b'} = \Lambda^{a}_{b'}(\mathbf{x})\,\hat{e}_{a} \tag{1.5}$$

Eigentime, i.e. the time measured by a clock moving along a given trajectory, can be written for flat space-time as:

$$d\tau = \int_{P}\frac{ds}{c} \tag{1.6}$$

Writing it out, I get the expression:

$$\tau = \sqrt{1-\frac{v^{2}(t)}{c^{2}}}dt \tag{1.7}$$

The interpretation of this expression is central to the whole of this work, if events are moving exactly at the speed of light, this event tells you how many events occur for a given observer. For a massless particle, of course, this value will be zero, it means that events do not perceive time, or more precisely, the speed of light remains constant relative to the given event because it is the speed of propagation of that event in space. In the general case, for any space-time, the eigentime value is equal to the expression 5.5 more precisely:

$$d\tau = \frac{1}{c}\int_{P}\sqrt{g_{\mu\nu}(\mathbf{x})\,dx^{\mu}(\mathbf{x})\,dx^{\nu}(\mathbf{x})} \tag{1.8}$$

1.6. **Free fall.** Fall of freedom is a basic gravitational phenomenon, the assumption is that during free fall the falling observer remains motionless in relation to the gravitational field. Additionally, the gravitational field is equivalent to the field of matter or energy. From these two assumptions, only one interpretation of what happens in free fall can be gained that is consistent with experience.

Let's consider a thought experiment, I have an inertial frame U and a gravitational field source U', the mass of the frame U' is much greater than the mass of the inertial frame U so that the gravitational influence of the first frame is negligible. Since the system U is at rest, it means that the system U' is in motion. The first frame is approximately point-like and the U' frame is a spherical mass expressed by the energy density function $\rho = \frac{\int_0^R m(r)drc^2}{\frac{4}{3}\pi r^3}$ such that the integral of $m(r)$ is equal to the initial mass or rest mass of the system $\int_0^R m(r)dr = m_0$. Where R is the surface radius of this mass. An inertial observer perceives that the motion of the U' system is directed spherically in all directions as the relative size of this object "increases", the U' system expands in all directions, but when this system expands enough for the U system to hit it with surfaces the size of both will not change himself. This means that both objects must have experienced exactly the same expansion in space. This can be described by the Ricci tensor and the equivalence of this tensor to the field of matter. However, the units of the energy tensor and the Ricci tensor are not the same, it is necessary to use Einstein's constant, the whole thing can be written as:

$$R_{00} = n\kappa T_{00} \tag{1.9}$$

Where the numeric constant n is some number. The key here is that only the time-time component of the Ricci tensor and the momentum energy tensor are taken into account. Due to the fact that the geodesic lines move away from each other or the volume form increases over time, this constant must have a negative sign, so $n = -a$ where a is a certain number. This means that the observer U is stationary but time is expanding with the gravitational field source U'. Objectively, both observers remain at rest, while time expands with the gravitational field. Writing the whole thing as an equation:

$$R_{00} = -a\kappa \frac{\int_0^R m(r)drc^2}{\frac{4}{3}\pi r^3} \tag{1.10}$$

This equation is crucial for the whole of this work, it shows the equivalence between the material field and the expansion of space-time and thus the inertia of both systems U and U' as systems that are physically stationary.

1.7. **Spacetime.** According to the principle written in the previous chapter, gravitational systems remain inertial. It should be remembered, however, that these systems are truly inertial, they must meet not only the immobility in the gravitational field in the classical sense, but also in the sense of the necessity of immobility in relation to the light ray sent from a given point in space-time. According to the Theory of Relativity, this requirement can be written as a transition from flat to curved spacetime:

$$ds^2 = g_{\mu\nu}\left(\mathbf{x}\right) dx^{\mu}\left(\mathbf{x}\right) dx^{\nu}\left(\mathbf{x}\right) \tag{1.11}$$

$$g_{\mu\nu} = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} \tag{1.12}$$

On the other hand, the space-time metric itself must be not so much arbitrary as fulfilling the principle described in the previous chapter. To obtain this, we need a gravitational field equation whose solution is a space-time metric satisfying this equation. Before proceeding to the derivation of this equation, it is important to introduce all the rules that must be met so that such a space-time satisfies the principle of immobility of any system defined in this space-time.

This means that each system must be defined as inertial, while the structure of space-time itself depends on the field of matter or energy at a given point. From the previous chapter it can be deduced that the momentum energy tensor does not disappear at any point in spacetime, it is necessary that the Ricci tensor does not disappear with it. This is quite a simple rule, the farther from the center of mass, the lower the density of matter, the closer it is, the greater, the surface of this matter is only conventionally understood as the limit beyond which the mass does not increase because the amount of matter, and more precisely the rest mass, does not increase. However, the density of matter will continue to decrease indefinitely no matter how far we move away from the central mass, which means that the gravitational field does not disappear and the field of matter does not disappear with distance.

Since it follows from the assumptions that we are studying the deviation of space-time from Minkowski space, the principle that the inertial observer should be motionless relative to the hypothetical light signal sent from a given point still applies. Combining all these principles into one principle, we get a space-time consistent with this general idea of inertial systems. The last step to success in this reasoning is to find field equations that satisfy this principle, fortunately there is only one way to derive such an equation and doing so mathematically complicates Einstein's field equations. However, the assumption is that it gives a better description of the gravitational field and reduces to the field equations in a suitable way.

1.8. **Finding the field equation.** The field equation must reduce to the Einstein field equations but this is only one requirement, the second requirement is the immobility of all observers relative to the light signal sent from a given point in space-time, the last requirement is that the momentum energy tensor must not decay at each point otherwise we get Minkowski space. An equation that satisfies all these conditions is very difficult to find without any mathematical clue, it turns out that all assumptions are satisfied if we take only the first law. The field equations must be reducible to Einstein's field equations.

The Einstein field equations have ten unknowns, they consist of the Ricci tensor, the Ricci scalar, the metric tensor, and the momentum energy tensor. I'll start by writing these equations without the momentum energy tensor, i.e. I'll write the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{1.13}$$

The key question is can there be a tensor that reduces to the Einstein tensor? Well, yes, the Einstein tensor can also be written as two contractions of the Riemann tensor and contractions of the Ricci tensor with the metric tensor:

$$G^{\rho}_{\mu\rho\nu} = R^{\rho}_{\mu\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.14}$$

Such writing of the Einstein tensor automatically yields a tensor whose contraction leads to the Einstein tensor. Now to get the tensor that I am looking for I only need to change the index ρ to another index, in this case I will use the index σ which will give me a new tensor:

$$G^{\rho}_{\mu\sigma\nu} = R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.15}$$

For obvious reasons, this tensor reduces to the Einstein tensor, the problem is that this tensor has 256 components, of which only 20 are independent in four-dimensional space-time. Interestingly, this tensor has only the same number of unknowns in four-dimensional spacetime, which can be written as an equality, where on one side there are the number of independent components of the Riemann tensor on the other side twice the number of components for a symmetric second-order tensor:

$$\frac{n^2\left(n^2-1\right)}{12} = n\left(n+1\right) \tag{1.16}$$

The solution to this equation is four-dimensional space-time, or zero-dimensional space-time, I omit solutions with a negative number for obvious reasons. The last step to complete the field equation is the other side of the equation, namely the momentum energy tensor.

1.9. **Field equation.** The last element of the full field equation is the momentum energy tensor, in this case I need to write a tensor whose contraction leads to the momentum energy tensor itself. I need to use the ρ and σ indices the same way as in the Einstein tensor except that I want to get the momentum energy tensor not its trace multiplied by the metric tensor. So a contraction is needed that leads to the Knocker delta, I can write this part as:

$$T^{\rho}_{\mu\sigma\nu} = T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{1.17}$$

It can be easily verified that in fact the contraction of the index ρ and σ leads to the Knocker delta $g^{\kappa\rho} g_{\rho\nu} = \delta^{\kappa}_{\nu}$ acting on the momentum energy tensor will change its indices to be consistent with the rest of the equation, so I finally get the field equation:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{1.18}$$

It can be checked again that this equation reduces to Einstein's equations with the difference that there is not only one possible contraction, it means that the field equation can not only be reduced to one equation but to several different equations. Their common feature is that for a vacuum they all reduce to one equation, the Ricci tensor is equal to zero. This results in their common feature, returning to the extended equation, as will be shown later in this work, this equation meets all the requirements that are necessary for the observer to always be motionless in relation to the event, i.e. simply space-time, which is equivalent to a light signal sent from a given point of it and additionally satisfy this principle for the gravitational field.

This equation, however, is quite a complicated equation and is at the cost of being mathematically more complicated as a whole than Einstein's field equations on the other hand, it is possible that this equation solves problems that are observed in cosmology (dark matter and energy) and does not require addition to the gravitational field no additional ingredients to make certain predictions match them. The cost of this is the mathematical complication of the gravitational field equations, where already difficult field equations become even more difficult to solve. The principle on which these equations are based can also be highly controversial as gravity acts as an attraction here this attraction is a kind of illusion in fact we are talking about the expansion of time which looks like an attraction from a certain perspective. However, it gives a theoretically reasonable description of every observer as inertial, which also applies to the gravitational field.

1.10. **Requirement for a non-zero momentum energy tensor.** The extended Einstein tensor for a vacuum will always give a Minkowski space. This can easily be proved if I zero the components of the Riemann tensor by setting all indices equal to each other, I get:

$$-\frac{1}{2}R_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma}=0 \tag{1.19}$$

This equation will end up with a Ricci tensor equal to zero, I can rearrange it in two ways, the first way is to write the equation as a Ricci scalar times the metric tensor is equal to zero but remember that there are components of the Ricci tensor in the equation, so I can use the metric tensor identity which will give E equality of diagonal elements Ricci tensor of zero:

$$R_{\sigma\sigma}=0 \tag{1.20}$$

So I used the fact that the metric tensor gives the Knocker delta δ^{κ}_{σ}, that's part of the equation, but since the Ricci tensor is equal to zero (its diagonal elements) then whenever there are diagonal elements of the Ricci tensor, the Riemann tensor will also be equal to zero. You can prove from this equation that if all the diagonal elements of the Ricci tensor are equal to zero, the Riemann tensor will also be equal to zero. I'll write the field equation again only this time for the diagonal components of the Ricci tensor, where I take into account that if the Ricci tensor has non-diagonal components it gives zero so only the diagonal components of the metric tensor are allowed:

$$R^{\sigma}_{\mu\sigma\nu}-\frac{1}{2}R_{\sigma\sigma}g^{\sigma\sigma}g_{\mu\nu}=0 \tag{1.21}$$

$$R^{\sigma}_{\mu\sigma\nu}=0 \tag{1.22}$$

$$R_{\mu\nu}=0 \tag{1.23}$$

Which ultimately gives the Ricci tensor, which is always zero, and therefore the Riemann tensor, which is always zero:

$$R^{\rho}_{\mu\sigma\nu}=0 \tag{1.24}$$

Which proves that the momentum energy tensor must be nonzero at every point in space for the equation not to result in flat spacetime. The extended Einstein tensor is zero, so for vacuum it just gives Minkowski space. Here these two equations are completely different from each other in relativity there are solutions for vacuum here there are no solutions for vacuum other than flat space-time, so another requirement that was assumed is satisfied by the field equation.

1.11. **Momentum energy tensor.** The momentum energy tensor [11] is one of the fundamental mathematical objects in field equations. In the Theory of Relativity, the only source of the gravitational field is the momentum energy tensor, in this hypothesis it is not entirely so. Write the field equation again, but so that the Riemann curvature tensor is on the left side of the equation:

$$R^{\rho}_{\mu\sigma\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} + \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.25}$$

As you can see, the source of space-time curvature will be the momentum energy tensor, but also its combination with metric tensors, and the Ricce tensor and its combination with metric tensors. This means that, contrary to the theory of relativity, the source of the gravitational field is not only the momentum energy tensor but also the metric tensor because, as I showed in the previous chapter, the extended Einstein tensor is nullified in the absence of a matter field, so the influence of the Ricci tensor is meaningless without the presence of energy. However, when energy is present, things get complicated, the curvature consists of the momentum energy tensor and combinations with metric tensors, but also the Ricci tensor and its combinations with metric tensors. To better understand this fact, let's write the Ricci tensor in the previous equation:

$$R^{\rho}_{\mu\sigma\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} + \frac{1}{2} R^{\rho}_{\sigma\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.26}$$

This leads to an apparent contradiction in the reasoning about curvature, to calculate the curvature of spacetime you need to use its contraction which seems to be meaningless, now I will write the field equation so that $\sigma = \mu, \rho = \nu$ which finally gives me the equality:

$$\kappa T_{\mu\mu} = -\frac{1}{2} R_{\mu\mu} \tag{1.27}$$

This means that the diagonal elements of the momentum energy tensor are equal to the diagonal elements of the Ricci tensor. Now for non-diagonal elements of the momentum energy tensor I get , where $\sigma = \nu$ and $\rho = \nu$:

$$\kappa T_{\mu\nu} = -\frac{1}{2} R_{\mu\nu} \tag{1.28}$$

Which shows that the momentum energy tensor is fully defined by the Ricce tensor and vice versa, the Ricce tensor is defined by the momentum energy tensor. Of course, when solving the field equation, additional terms from the equation itself appear in the equations.

1.12. **Conservation laws and covariant derivative.** With respect to the covariant derivative, the Einstein tensor and the momentum energy tensor are zero, which is a key conservation law in relativity. It can be easily shown that from the Einstein field equation it is possible to derive an extended field equation using only metric tensors whose covariant derivative is equal to zero, so they can be treated as constants, instead of deriving the formula from the Banach equation, it will be easier to do it from the Einstein equations. The trick is to choose the right summation indices of the Riemann tensor that yields the Ricci tensor. I can write this whole process except that it will use the covariant derivative altered by the metric tensor to have a superscript $\nabla^{\nu} = g^{\nu\alpha}\nabla_{\alpha}$ by writing the whole equation starting with the Einstein tensor [12]:

$$\nabla^{\nu}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = 0 \tag{1.29}$$

$$\nabla^{\nu}\left(R^{\rho}_{\mu\rho\nu} - \frac{1}{2}R_{\rho\kappa}g^{\kappa\rho}g_{\mu\nu}\right) = 0 \tag{1.30}$$

$$\nabla^{\nu}g_{\sigma\alpha}g^{\alpha\rho}\left(R^{\rho}_{\mu\rho\nu} - \frac{1}{2}R_{\rho\kappa}g^{\kappa\rho}g_{\mu\nu}\right) = 0 \tag{1.31}$$

$$\nabla^{\nu}\delta^{\rho}_{\sigma}\left(R^{\rho}_{\mu\rho\nu} - \frac{1}{2}R_{\rho\kappa}g^{\kappa\rho}g_{\mu\nu}\right) = 0 \tag{1.32}$$

$$\nabla^{\nu}\left(R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu}\right) = 0 \tag{1.33}$$

The same reasoning can be applied to the momentum energy tensor, except that I have to rewrite this tensor as a product of the Knocker delta, then this delta as a product of two metric tensors, which doesn't change the result but allows us to get the correct tensor. Writing the entire mathematical transformation:

$$\nabla^{\nu}T_{\mu\nu} = 0 \tag{1.34}$$

$$\nabla^{\nu}T_{\mu\kappa}\delta^{\kappa}_{\nu} = 0 \tag{1.35}$$

$$\nabla^{\nu}T_{\mu\kappa}g^{\kappa\rho}g_{\rho\nu} = 0 \tag{1.36}$$

$$\nabla^{\nu}g_{\sigma\alpha}g^{\alpha\rho}T_{\mu\kappa}g^{\kappa\rho}g_{\rho\nu} = 0 \tag{1.37}$$

$$\nabla^{\nu}\delta^{\rho}_{\sigma}T_{\mu\kappa}g^{\kappa\rho}g_{\rho\nu} = 0 \tag{1.38}$$

$$\nabla^{\nu}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} = 0 \tag{1.39}$$

So this field equation, just like Einstein's equation, satisfies the conservation principle, which is a very important feature. Otherwise, the model would be mathematically inconsistent.

1.13. **Free Fall Field Equation.** In the chapter on free fall, I wrote down an equation that I can now fully derive from the field equation. A special case of the field equations is one where the components of the Riemann tensor are zero and I get only the diagonal elements of the metric tensor, the momentum energy and the Ricci tensor. By writing this equation:

$$R^{\mu}_{\mu\mu\mu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} = \kappa T_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} \tag{1.40}$$

$$-\frac{1}{2} R_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} = \kappa T_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} \tag{1.41}$$

$$-\frac{1}{2} R_{\mu\kappa} \delta^{\kappa}_{\mu} = \kappa T_{\mu\kappa} \delta^{\kappa}_{\mu} \tag{1.42}$$

$$-\frac{1}{2} R_{\mu\mu} = \kappa T_{\mu\mu} \tag{1.43}$$

$$R_{\mu\mu} = -2\kappa T_{\mu\mu} \tag{1.44}$$

Now I can go back to free fall, and from the perspective of an observer moving with the frame of reference, I will have four equations to solve for dust:

$$R_{00} = -2\kappa\rho \tag{1.45}$$

$$R_{11} = R_{22} = R_{33} = 0 \tag{1.46}$$

So the constant we're looking for is two, which was discussed in the chapter on free fall, and according to that chapter, the field equation gives exactly the expected result. Going back to the equation itself, this is the only way to write the field equation where the elements of all tensors are only diagonal without getting a flat space-time. The unknown in this equation is the metric tensor and the equation itself is quite complicated despite the relatively simple notation, one component of the Ricci tensor differs from the Theory of Relativity. The interpretation of this equation is the same as for free fall. The minus sign means that the volumetric form, as the radius coordinate decreases, increases in proportion to the density, which becomes larger and larger. Of course, real mass is described by mass functions from the radius, not from the point concentrated mass - the point mass will be a singularity for a zero radius equal to zero, but if the mass is a function that vanishes at zero then you can get rid of the singularity. This model is not free of singularities, which inherits in a sense from the Theory of Relativity, on the other hand, however, these singularities are completely dependent on the field of matter.

1.14. **The universe and its expansion as a result of the field of matter.** From the previous chapter, you can assume that the universe behaves like dust and calculate the value of the Ricci tensor for the present conditions. These calculations result in a result very close to the cosmological constant, with the difference that here it is not a constant but a value dependent on the density of the matter field, where dark matter must be taken into account in mass calculations. For dust as in the previous chapter, in the frame of reference moving with the dust, I consider only one non-zero component of the Ricci tensor. This is a time-time component, the direct effect of the time-time component is to increase the distance between geodesics in time, which results from the negative sign of this component. I will now write the value of the Ricci tensor where I multiply the mass of the universe by $1 + \frac{85}{15}$ where the fraction represents the dark matter of which there is about 85% in the universe:

$$R_{00} = -2\kappa\rho \tag{1.47}$$

$$\rho_0 = \rho_U \left(1 + \frac{85}{15}\right) \tag{1.48}$$

$$\rho_0 \approx 3 \cdot 10^{-27} [kg/m^3] \tag{1.49}$$

$$R_{00} \approx -1 \cdot 10^{-52} [m^{-2}] \tag{1.50}$$

Which is a result practically equal to the observed cosmological constant [13], except that it is not a constant but a value depending on the field of matter. This means that, according to this model, the cosmological constant is not constant but variable depending on the density of matter in the universe. The lower the density, the lower the value, the higher the higher the value. On the local scale, this magnitude is so small that other gravitational fields dominate, while on the global scale of the universe, this magnitude will be visible as it is the effect of the global gravity of the universe. This means that according to this model the universe expands in the same way as any other gravitational field. Just like in free fall, the earth gets bigger and bigger as it expands, so time expands as the earth moves, which makes it appear stationary to an observer on its surface, but to any other observer in free fall it is. he is motionless and the earth is expanding with time all around, the same applies to the universe, we are in free fall relative to the universe, therefore we perceive its expansion in time as the expansion of space, where according to this model it is not space that expands but time what it gives the impression of matter moving away from each other inside the universe.

1.15. **Dust field equation.** For example, I can write the field equation for dust [14][15]:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} = \kappa\rho U_{\mu}U_{\kappa}g^{\kappa\rho}g_{\sigma\nu} \tag{1.51}$$

In a system co-moving with dust, this equation reduces to:

$$R^{0}_{0\sigma\nu} - \frac{1}{2}R_{\sigma 0}g^{00}g_{0\nu} = \kappa\rho U_{0}U_{0}g^{00}g_{\sigma\nu} \tag{1.52}$$

$$R^{0}_{0\sigma\nu} - \frac{1}{2}R_{\sigma 0}g^{00}g_{0\nu} = \kappa\rho U_{0}U_{0}g^{00}g_{\sigma\nu} \tag{1.53}$$

$$-\frac{1}{2}R_{\sigma 0}g^{00}g_{0\nu} = \kappa\rho U_{0}U_{0}g^{00}g_{\sigma\nu} \tag{1.54}$$

$$R_{\sigma 0}g_{0\nu} = -2\kappa\rho U_{0}U_{0}g_{\sigma\nu} \tag{1.55}$$

$$R_{\sigma 0}g_{0\nu} = -2\kappa\rho g_{\sigma\nu} \tag{1.56}$$

Which is the solution of the field equations for an observer co-moving with the dust. From the equation written above, the zero component of the Ricci tensor is equal to:

$$R_{00}g_{0\nu} = -2\kappa\rho g_{0\nu} \tag{1.57}$$

$$R_{00} = -2\kappa\rho \tag{1.58}$$

Which is consistent with the previous calculations for only diagonal indices of all components. The equation itself, however, is more complicated and has as many as ten unknown components of the metric tensor, i.e. all its components, plus four unknowns of the Ricci tensor, which are all components having as one index, the time index. The sign of the Ricci tensor is always negative with respect to the field of matter. This equation does not specify the residual components of the Ricci tensor, so the four components of the Ricci tensor and the ten components of the metric tensor are crucial.

1.16. **Non-vanishing matter fields and dark matter.** The requirement that the momentum energy tensor is non-vanishing at every point in space so that the curvature is non-zero automatically generates seemingly extra matter in the universe. According to this model, it is not extra matter but matter itself is treated as a continuous field. This means that every physical object extends to infinity in space. Depending on the adopted model of matter, this will give different geometries of space-time. This means that the geodesic equation no longer describes the trajectory of a single object, but must be treated as the motion of a continuous field. This means that I have to consider not the trajectories but the motion of the whole region of space or the whole field in general. This means that matter can only be locally gravitationally defined as a constant amount of mass or energy.

Exactly the same requirement works for the momentum energy tensor, it must be defined over the whole manifold and in addition if the manifold is not a Minkowski space it must be non-zero over the whole manifold. The requirement for rest mass as total mass in the field is no longer met, rest mass is only locally conserved, global mass not equal to rest mass, rest mass is only the field of greatest concentration of the field if the field has one. This means that the field is treated as a continuous field of matter defined over the whole manifold, not just one field or point. Therefore, the total field mass or energy has no interpretation, only the local field energy or mass can have an interpretation. It locally causes gravitational effects, so it is responsible for the modifications of space-time from Minkowski space. This is a possible explanation for dark matter as missing mass, the amount of total mass not equaling the rest mass.

This means that the momentum energy tensor must be defined in such a way that its value always agrees with the local and global curvature of space-time. This only says that the transition from the definition of matter to the definition of the field of matter is necessary for this model to make sense. The definition of rest mass must therefore be globally replaced by a field which only locally has a rest mass, and only locally this mass remains constant. Since the field is equivalent to the gravitational field, the law of conservation of the field requires that this tensor gives zero covariant derivative. This requirement is crucial to the whole field equation and is described in the chapter on conservation laws.

1.17. **Model and Principle Problems.** The biggest problem with this model is that it is not a quantum model, it is still a classical theory of gravity which is in opposition to the quantum theory. The second problem of the model is the occurrence of singularities. It is impossible to remove singularities from this model if any mass is concentrated in a point, it leads to infinite curvature of space-time, in this case negatively infinite. The ideal model would be devoid of singularities, however, this model, through the mathematical description used, generates singularities depending on the field of matter, the key question is whether these singularities can be removed. Alternatively, whether these singularities can be surpassed. The key question is also whether any non-quantum theory of gravity will lead to singularities, so it is necessary to look for quantum theory or singularities in the real world that exist and have physical significance.

If singularities do not occur in the real world (I mean the points where the curvature of space-time becomes infinite) eventually quantum gravity is needed to get around this problem, but if singularities are real and occur in the physical world there must be accurate ways to understand the geometry of such objects. Singularities must be surpassable for the field of matter, geodesics cannot end at singularities, in relativity geodesics end at singularities the key question is whether geodesics end in this model the same as in relativity, unfortunately the exact answer to this question requires, firstly, exact solutions of the field equations which are not presented here, secondly, their analysis. A good clue to this is that singularities only appear under the influence of the point mass, assuming that the mass is a continuous field that only concentrates in a given region of space and this concentration is equal to the rest mass, and for zero distance from the mass, the singularity disappears for the zero radius disappears with the mass, it means that in purely theoretical imprecise considerations, singularities can be avoided.

The mere fact that for a positive momentum energy tensor, the Ricci tensor is negative in the simple case of writing the field equation gives hope that the geodesic lines, instead of ending at the singularity, start there. Because if you reverse the motion away from the singularity, you get an expansion of space-time that slows down until you get to infinite distance from the source of mass, then you'll just be left with flat space-time. Of course, a formal mathematical proof of this reasoning is necessary to be sure of this, but the reasoning makes sense. Since the momentum energy tensor cannot be fully defined negatively, we also avoid the problem of the infinitely positive Ricci tensor which gives exactly the singularity where every geodesic ends. This model, however, conflicts with quantum physics, and only time and experimentation can show which approach is true.

1.18. **Fully covariant field equation.** The field equation can be converted to a fully covariant form. To do this, just use the metric tensor to lower the index ρ, such an equation can be written as:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} = \kappa T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} \tag{1.59}$$

$$g_{\phi\rho}R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}g_{\phi\rho}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} = \kappa g_{\phi\rho}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} \tag{1.60}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}\delta^{\kappa}_{\phi}g_{\mu\nu} = \kappa T_{\mu\kappa}\delta^{\kappa}_{\phi}g_{\sigma\nu} \tag{1.61}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2}R_{\sigma\phi}g_{\mu\nu} = \kappa T_{\mu\phi}g_{\sigma\nu} \tag{1.62}$$

This form of the field equation may in many cases be more useful than the previous one, it may simplify calculations. Of course, since this is a tensor equation, the form of this equation doesn't matter. For example, I can derive the field equations for all diagonal elements simply by zeroing the Riemann tensor, making all diagonal elements zero:

$$-\frac{1}{2}R_{\phi\phi}g_{\phi\phi} = \kappa T_{\phi\phi}g_{\phi\phi} \tag{1.63}$$

I omit the metric tensor because its values are identical on both sides of the equation, which gives exactly the same result as before:

$$R_{\phi\phi} = -2\kappa T_{\phi\phi} \tag{1.64}$$

Going further, we can compute the whole equality for non-diagonal elements:

$$R_{\phi\mu\nu\nu} - \frac{1}{2}R_{\nu\phi}g_{\mu\nu} = \kappa T_{\mu\phi}g_{\nu\nu} \tag{1.65}$$

$$-\frac{1}{2}R_{\nu\phi}g_{\mu\nu}g^{\nu\phi} = \kappa T_{\mu\phi}g_{\nu\nu}g^{\nu\phi} \tag{1.66}$$

$$-\frac{1}{2}R_{\nu\phi}\delta^{\phi}_{\mu} = \kappa T_{\mu\phi}\delta^{\phi}_{\nu} \tag{1.67}$$

$$-\frac{1}{2}R_{\mu\nu} = \kappa T_{\mu\nu} \tag{1.68}$$

Where in the last line of equality I used the symmetry of the Ricci tensor.

1.19. **Contractions of field equations.** There are exactly six contractions of the field equations, write them all down. Starting from writing the field equation:

$$R_{\phi\mu\sigma\nu} - \frac{1}{2}R_{\sigma\phi}g_{\mu\nu} = \kappa T_{\mu\phi}g_{\sigma\nu} \tag{1.69}$$

Now it will save all contractions:

$$g^{\mu\phi}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\mu\phi}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\mu\phi}T_{\mu\phi}g_{\sigma\nu} \tag{1.70}$$

$$R_{\sigma\nu} = -2\kappa T g_{\sigma\nu} \tag{1.71}$$

$$g^{\sigma\phi}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\sigma\phi}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\sigma\phi}T_{\mu\phi}g_{\sigma\nu} \tag{1.72}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} \tag{1.73}$$

$$g^{\nu\phi}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\nu\phi}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\nu\phi}T_{\mu\phi}g_{\sigma\nu} \tag{1.74}$$

$$R_{\mu\sigma} = -\frac{2}{3}\kappa T_{\mu\sigma} \tag{1.75}$$

$$g^{\mu\sigma}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\mu\sigma}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\mu\sigma}T_{\mu\phi}g_{\sigma\nu} \tag{1.76}$$

$$R_{\phi\nu} = -\frac{2}{3}\kappa T_{\phi\nu} \tag{1.77}$$

$$g^{\mu\nu}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\mu\nu}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\mu\nu}T_{\mu\phi}g_{\sigma\nu} \tag{1.78}$$

$$R_{\phi\sigma} = -\kappa T_{\phi\sigma} \tag{1.79}$$

$$g^{\sigma\nu}R_{\phi\mu\sigma\nu} - \frac{1}{2}g^{\sigma\nu}R_{\sigma\phi}g_{\mu\nu} = \kappa g^{\sigma\nu}T_{\mu\phi}g_{\sigma\nu} \tag{1.80}$$

$$R_{\phi\mu} = -8\kappa T_{\mu\phi} \tag{1.81}$$

All these contractions have one thing in common, for a zero momentum energy tensor they reduce to one equation, the Ricci tensor is equal to zero $R_{\mu\nu} = 0$, where the indices are different for each of these contractions. Only Einstein's field equations, on the other hand, satisfy the rule that the covariate derivative is zero for them, all other contractions fail on this point.

1.20. **The field equation for an ideal liquid.** The field equation for the simplest case, which is dust, consists of sixteen equations to be solved, while the equation for an ideal liquid has as many as sixty-four equations to be solved, of which ten metric tensor functions are sought. Since the momentum energy tensor for a liquid in a system moving with the liquid has only four components, I can write the field equation as:

$$R_{\mu\mu\sigma\nu} - \frac{1}{2} R_{\sigma\mu} g_{\mu\nu} = \kappa T_{\mu\mu} g_{\sigma\nu} \tag{1.82}$$

The Riemann tensor will zero and I get the final equation:

$$-\frac{1}{2} R_{\sigma\mu} g_{\mu\nu} = \kappa T_{\mu\mu} g_{\sigma\nu} \tag{1.83}$$

$$R_{\sigma\mu} g_{\mu\nu} = -2\kappa T_{\mu\mu} g_{\sigma\nu} \tag{1.84}$$

Now I can write down all the components of this equation:

$$R_{\sigma 0} g_{0\nu} = -2\kappa \rho g_{\sigma\nu} \tag{1.85}$$

$$R_{\sigma a} g_{a\nu} = -2\kappa P g_{\sigma\nu} \tag{1.86}$$

In particular, he will write down sixteen of the sixty-four equations that determine the diagonal elements of the metric tensor:

$$R_{\sigma\mu} g_{\mu\sigma} = -2\kappa T_{\mu\mu} g_{\sigma\sigma} \tag{1.87}$$

$$R_{0\mu} g_{\mu 0} = -2\kappa T_{\mu\mu} g_{00} \tag{1.88}$$

$$R_{a\mu} g_{\mu a} = -2\kappa T_{\mu\mu} g_{aa} \tag{1.89}$$

Now that's quite a lot of equations, and they're not simple equations to solve.

1.21. **Physical Field - Interpetation.** The field equations are very complicated, if there are solutions to them, they are not easy to find, as can be seen from the example from the chapter on the perspective of a falling observer. The momentum energy tensor does not vanish at any point. However, it is also possible to determine geometrically what the field equation means, I will write it again in a fully covariate form:

$$R_{\phi\mu\sigma\nu} - \frac{1}{2} R_{\sigma\phi} g_{\mu\nu} = \kappa T_{\mu\phi} g_{\sigma\nu} \tag{1.90}$$

The Riemann tensor is responsible for the total curvature of the manifold, while the Ricci tensor will be responsible for the volume changes of the manifold, the difference between the two is equivalent to the field of matter. It is obvious that both the momentum energy tensor and the Ricce tensor are combined with the metric tensor, which means that to this you have to add how the chosen coordinate system deviates from Minkowski space. The index juggling itself is much more difficult, so I will not focus on it and will try to describe the general geometric meaning of this equation.

The equation is completely dependent on the right-hand side of the equality, which means that in the absence of matter, it gets flat space-time. Geometrically, the field equations can be simplified to two statements, the first of which states that in the presence of energy, space-time always deviates from flat space-time. The second is more difficult to define clearly because it requires interpreting the field equation in a literal sense. To do this, he will write the field equation in such a way that on one side there is only the Ricci tensor with the math tensor:

$$R_{\sigma\phi} g_{\mu\nu} = 2 \left(R_{\phi\mu\sigma\nu} - \kappa T_{\mu\phi} g_{\sigma\nu} \right) \tag{1.91}$$

This means that whenever energy is present, space-time must either increase in volume or decrease in volume, where since energy is negative here for a positive value of the momentum energy tensor space-time will increase in volume for negative energy it will decrease for particles with mixed signs hi will increase and decrease.

1.22. **Curvature as a matter field effect.** From the chapter on the momentum energy tensor, one can deduce the equation for the curvature of spacetime and the exact Riemann tensor using only the momentum energy tensor:

$$R^{\rho}_{\mu\sigma\nu} = \kappa \left(T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} - T_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu}\right) \tag{1.92}$$

A necessary condition for this equation to be satisfied is the equality of the Ricci tensor and the momentum energy, as I showed in the above-mentioned chapter. This equation shows directly why the momentum energy tensor is responsible for the curvature of space-time, it is easy to check that for zero elements of the Riemann tensor this equation will also be zero, given the simplest example when all indices are equal:

$$R^{\mu}_{\mu\mu\mu} = \kappa \left(T_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} - T_{\mu\kappa} g^{\mu\mu} g_{\mu\mu}\right) \tag{1.93}$$

$$0 = \kappa \left(T_{\mu\kappa} g^{\kappa\mu} g_{\mu\mu} - T_{\mu\kappa} g^{\mu\mu} g_{\mu\mu}\right) \tag{1.94}$$

This equation greatly simplifies solving field equations because it reduces the momentum energy tensor and the metric tensor to just the curvature of space-time without the Ricci tensor. This means that the consequence of the existence of energy in a field is always the curvature of space-time. On the other hand, the metric tensor is still the unknown in this equation. Interestingly, the contraction of this equation will give a very similar result to the Theory of Relativity, writing it:

$$R^{\rho}_{\mu\rho\nu} = \kappa \left(T_{\mu\kappa} g^{\kappa\rho} g_{\rho\nu} - T_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu}\right) \tag{1.95}$$

$$R_{\mu\nu} = \kappa \left(T_{\mu\nu} - T g_{\mu\nu}\right) \tag{1.96}$$

As you can see the difference is a factor of one-half, this is the first case where there is a clear difference between the two approaches to space-time. On the other hand, this difference results only from the fact that the Ricci tensor is twice as large as the momentum energy tensor, not counting Einstein's constant. This is not the only possible contraction, but I won't go into all of them here.

1.23. **Classic Summary.** In this short paper, I presented the hypothesis of extending Einstein's field equations while maintaining the simple principle of immobility of each frame of reference. Relative to the light ray emitted from a given point in space at a given moment of time, each observer remains motionless, the same applies to the gravitational field, so it moves at any speed. When the field of matter moves then space-time moves in time with this field, which makes any movement compensated in some sense by changes in space-time.

However, the work lacks mathematical solutions of the field equations which, despite the simple notation, are quite complicated to solve. It should also be remembered that this hypothesis is still a classical theory, which makes it opposed to the quantum approach to the problem of space-time. However, the motivation is the mathematical consistency of the model and that the conclusions of this theory may be the solution to the current problems observed in cosmology.

However, in order to obtain better technical results from this model, it is necessary to solve the field equations even for a simple dust situation. A rather controversial assumption of this model is that the momentum energy tensor never decays, otherwise we get a locally flat space-time. This means that matter is treated as a continuous field that never decays, but its intensity can vary freely depending on the type of field. The same applies to the trajectory, which is no longer a line, but the change of the whole variety under the influence of the field of matter.

Mathematically, the equation is quite complex, but the reasoning behind the equation is not contradictory, so it is a model that makes sense from a physical point of view, and is able to describe physical observations in a consistent way. The big success is that it predicts the existence of four-dimensional space-time, so you don't have to assume the existence of four-dimensional space-time, it's a consequence of the equation.

The open question remains whether this model is needed at all or is it just an interesting mathematical fact? Well, without solving the field equations and checking how these equations work in relation to observations, it is impossible to answer this question. From a theoretical point of view, this model is an interesting alternative to inflation, dark energy, and possibly dark matter, as all phenomena implied by this model are not additional assumptions necessary to make this model work.

2. Quantum Part

2.1. **Complex fields.** The transition to quantum physics is possible using complex fields. Instead of using real space-time you can use tensor fields that are complex and use a special way to represent these fields which is quite simple. In general, the tensor field will consist of two parts, a real field and an imaginary field. Where the imaginary field will be the real function multiplied by the imaginary unit. The general scheme of such a field can be written:

$$\psi^{\mu_1\cdots\mu_n}(\mathbf{x}) = a^{\mu_1\cdots\mu_n}(\mathbf{x}) + i\alpha^{\mu_1\cdots\mu_n}(\mathbf{x}) \tag{2.1}$$

Where tensor fields $a^{\mu_1\cdots\mu_n}(\mathbf{x})$ and $\alpha^{\mu_1\cdots\mu_n}(\mathbf{x})$ are actual fields. Now both of these fields have a complex adjuvant, and since it is transposition tensors, all this will be written as the † operator, so the operation of this operator can be represented as:

$$\left(\psi^{\mu_1\cdots\mu_n}(\mathbf{x})\right)^\dagger = \overline{\psi}_{\mu_1\ldots\mu_n}(\mathbf{x}) \tag{2.2}$$

$$\overline{\psi}_{\mu_1\ldots\mu_n}(\mathbf{x}) = a_{\mu_1\ldots\mu_n}(\mathbf{x}) - i\alpha_{\mu_1\ldots\mu_n}(\mathbf{x}) \tag{2.3}$$

For the purpose of quantization, he wants to describe the entire previous theory in the language of such complex fields. The key to getting the probability is the existence of some scalar function from arbitrary tensors where in the case of the field equation this function must be equivalent to the probability of finding a particle at a given point in space. I can get a scalar function from a tensor of this kind in a simple way, I use transpose and complex conjugate:

$$\left(\psi^{\mu_1\cdots\mu_n}(\mathbf{x})\right)^\dagger \psi^{\mu_1\cdots\mu_n}(\mathbf{x}) = \psi(\mathbf{x}) \tag{2.4}$$

It should be remembered that the value of this field, or rather its meaning as an invariant, will depend on what kind of tensor field is used.

2.2. **Metric tensor for complex field.** The metric tensor [16] is crucial to the theory of gravity, decomposing it according to the principle from the previous chapter into two real functions:

$$g_{\mu\nu}(\mathbf{x}) = \frac{\partial\xi^{\alpha}}{\partial x^{\mu}}\frac{\partial\xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta} + i\frac{\partial\phi^{\alpha}}{\partial x^{\mu}}\frac{\partial\phi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta} \tag{2.5}$$

Where the invariant here is the number of dimensions of spacetime, which I can write as:

$$(g_{\mu\nu}(\mathbf{x}))^{\dagger}\, g_{\mu\nu}(\mathbf{x}) = 2\delta^{\mu}_{\mu} \tag{2.6}$$

Where the number of these dimensions is multiplied by two because you have to add the imaginary dimensions. Which also modifies the formula when using the indices of the metric tensor so that only one of them is equal to the other:

$$(g_{\mu\kappa})^{\dagger}\, g_{\kappa\nu} = 2\delta^{\mu}_{\nu} \tag{2.7}$$

This is a very important identity. Now for a space-time interval I get where I omit the notation that it's a tensor field:

(2.8)

$$ds^2 = \Re(g_{\mu\nu})\, dx^{\mu}dx^{\nu} + \Im(g_{\mu\nu})\, d\chi^{\mu}d\ chi^{\nu} + i\Re(g_{\mu\nu})\, d\chi^{\mu}dx^{\nu} - i\Im(g_{\mu\nu})\, dx^{\mu}d\chi^{\nu}$$

What can be written as:

$$\begin{aligned} ds^2 &= \frac{\partial\xi^{\alpha}}{\partial x^{\mu}}\frac{\partial\xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}dx^{\mu}dx^{\nu} + \frac{\partial\phi^{\alpha}}{\partial x^{\mu}}\frac{\partial\phi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}d\chi^{\mu}d\chi^{\nu} \\ &+i\ frac\partial\xi^{\alpha}\partial x^{\mu}\frac{\partial\xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}d\chi^{\mu}dx^{\nu} - i\frac{\partial\phi^{\alpha}}{\partial x^{\mu}}\frac{\partial\phi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}dx^{\mu}d\chi^{\nu} \end{aligned} \tag{2.9}$$

$$ds^2 = \frac{\partial\xi^{\alpha}}{\partial x^{\mu}}\frac{\partial\xi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}dx^{\mu}dx^{\nu} + \frac{\partial\phi^{\alpha}}{\partial x^{\mu}}\frac{\partial\phi^{\beta}}{\partial x^{\nu}}\eta_{\alpha\beta}d\chi^{\mu}d\chi^{\nu} \tag{2.10}$$

Which gives the real distance in complex spacetime. When I raise or lower the index, I have to keep the imaginary and real parts to do this, I have to separate these parts again:

$$dz_{\mu} = \Re(g_{\mu\nu})\, dx^{\nu} + i\Im(g_{\mu\nu})\, d\chi^{\nu} \tag{2.11}$$

This imposes additional requirements on transformations of the complex field, the complex field itself will then satisfy the rule:

$$\begin{aligned} (dz^{\mu})^{\dagger}\, dz^{\mu} &= \Re(g_{\mu\nu})\, dx^{\mu}dx^{\nu} + \Im(g_{\mu\nu})\, d\chi^{\mu}d\chi^{\nu} \\ &+i\Re(g_{\mu\nu})\, dx^{\mu}d\chi^{\nu} - i\Im(g_{\mu\nu})\, d\chi^{\mu}dx^{\nu} \end{aligned} \tag{2.12}$$

$$(dz^{\mu})^{\dagger}\, dz^{\mu} = \Re(g_{\mu\nu})\, dx^{\mu}dx^{\nu} + \Im(g_{\mu\nu})\, d\chi^{\mu}d\chi^{\nu} \tag{2.13}$$

$$(dz^{\mu})^{\dagger} = \Re(g_{\mu\nu})\, dx^{\nu} - i\Im(g_{\mu\nu})\, d\chi^{\nu} \tag{2.14}$$

$$dz^{\mu} = dx^{\mu} + id\chi^{\mu} \tag{2.15}$$

2.3. **Field equation for complex spacetime.** Since I defined the metric tensor, the Riemann-Ricci tensor in the complex space is automatically defined as it follows from the combination of the derivatives of the metric tensor. The field equation in the form of curvature versus energy can be written as:

$$R^{\rho}_{\mu\sigma\nu} = \kappa \left(T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} - T_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu}\right) \tag{2.16}$$

However, it will be more convenient to work with a fully covariate form:

$$R_{\phi\mu\sigma\nu} = \kappa \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.17}$$

Now that I have an field equation that differs in how the metric tensor is defined, I can go further. The scalar field we are looking for satisfies the relation:

$$\psi = \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} R_{\phi\mu\sigma\nu} \tag{2.18}$$

Where finally the field equation can be written as:

$$\left(R_{\phi\mu\sigma\nu}\right)^{\dagger} R_{\phi\mu\sigma\nu} = \kappa \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.19}$$

$$\psi = \kappa \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.20}$$

This equation has a specific property, if I multiply the scalar that appears on both sides of the equation by the complex Riemann tensor, I get the field equation in tensor form:

$$R_{\phi\mu\sigma\nu} \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} R_{\phi\mu\sigma\nu} = \kappa R_{\phi\ mu\sigma\nu} \left(R_{\phi\mu\sigma\nu}\right)^{\dagger} \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.21}$$

$$\psi R_{\phi\mu\sigma\nu} = \kappa \psi \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.22}$$

$$R_{\phi\mu\sigma\nu} = \kappa \left(T_{\mu\phi} g_{\sigma\nu} - T_{\sigma\phi} g_{\mu\nu}\right) \tag{2.23}$$

This scalar has dimensions m^{-4}, so the integral over the whole manifold, not only over space, gives a finite constant:

$$\int_{\mathbf{M}^4} \sqrt{-g}\, \psi\, d^4\mathbf{x} = n \tag{2.24}$$

Which gives the final normalized scalar field:

$$\frac{1}{n} \int_{\mathbf{M}^4} \sqrt{-g}\, \psi\, d^4\mathbf{x} = 1 \tag{2.25}$$

This means that only normalizable scalar fields are allowed, hence the curvature of space-time never reaches infinity because such a function will be non-normalizable.

2.4. **Measurement.** Measurement is a key phenomenon in any physical theory, in quantum physics measurement takes the form of a possible state of a system. The probability of finding a particle in a given volume belonging to a manifold is equal to:

$$\rho(\mathbf{x}) = \frac{1}{n} \int_{\mathbf{V}^4 \in \mathbf{M}^4} \sqrt{-g}\psi d^4\mathbf{x} \tag{2.26}$$

The rest mass contained in a given volume can be defined as [17]:

$$M_0[V] = \int_V \sqrt{-P_\mu P^\mu}\sqrt{-g}d^3\mathbf{x} = \int_V \sqrt{-\left(-T^{\mu\ phi} n_\phi\right)\left(-T^{\nu\kappa} n_\kappa\right) g_{\mu\nu}}\sqrt{-g}d^3\mathbf{x} \tag{2.27}$$

The assumption is that after the measurement, the mass of the particle must be concentrated in the volume detected by the measurement. What can I save as:

$$M_0[V] \to M_0[V'] \tag{2.28}$$

Where the probability assigned to this process is equal to:

$$\rho[V', cdT] = \frac{1}{n} \int_{(V', cdT) \in \mathbf{M}^4} \sqrt{-g}\psi d^4\mathbf{x} \tag{2.29}$$

Where you have to take into account the time in the volume which is written as some very short moment cdT. The only problem with this process is that the momentum energy tensor is expressed as:

$$T_{\mu\nu} = t_{\mu\nu} + i\tau_{\mu\nu} \tag{2.30}$$

Where $t_{\mu\nu}$ is the ordinary momentum energy tensor and $\tau_{\mu\nu}$ is the imaginary part. There are two ways to get around this problem, the first is to use the real part of this tensor, which will give you the desired result. The second one assumes that the mass is a complex quantity, first write the invariant of the momentum energy tensor:

$$\left(T_{\mu\nu}\right)^\dagger T_{\mu\nu} = \left(t_{\mu\nu} + i\tau_{\mu\nu}\right)^\dagger \left(t_{\mu\nu} + i\tau_{\mu\nu}\right) \tag{2.31}$$

$$\left(T_{\mu\nu}\right)^\dagger T_{\mu\nu} = \left(\Re\left(g^{\mu\alpha}\right)\Re\left(g^{\nu\beta}\right)t_{\alpha\beta} - i\Im\left(g^{\mu\alpha}\right)\Im\left(g^{\nu\beta}\right)\tau_{\alpha\beta}\right)\left(t_{\mu\nu} + i\tau_{\mu\nu}\right) \tag{2.32}$$

$$\left(T_{\mu\nu}\right)^\dagger T_{\mu\nu} = t_{\mu\nu}t^{\mu\nu} + \tau_{\mu\nu}\tau^{\mu\nu} \tag{2.33}$$

And now the square of the mass as an invariant:

$$M_0^2 = -\left(P^\mu\right)^\dagger P^\mu \tag{2.34}$$

$$\left[\left(-t^{\mu\phi}n_\phi\right)\left(-t^{\nu\kappa}n_\kappa\right)g_{\mu\nu} + \left(-\tau^{\mu\phi}n_\phi\right)\left(-\tau^{\nu\kappa}n_\kappa\right)g_{\mu\nu}\right] = p_\mu p^\mu + \rho_\mu \rho^\mu \tag{2.35}$$

Which gives the final mass formula:

$$M_0[V] = \int_V \sqrt{-\left[\left(-t^{\mu\phi}n_\phi\right)\left(-t^{\nu\kappa}n_\kappa\right)g_{\mu\nu} + \left(-\tau^{\mu\phi}n_\phi\right)\left(-\tau^{\nu\kappa}n_\kappa\right)g_{\mu\nu}\right]}\sqrt{-g}d^3\mathbf{x} \tag{2.36}$$

2.5. **Geodesic equation and Lorentz transformations.** The geodetic equation is crucial to predict the trajectory of a particle, although I did not write this equation in the classical part, it is necessary to determine in the quantum part. The formula for the distance in complex spacetime is equal to:

$$ds^2 = \frac{\partial \xi^\alpha}{\partial x^\mu}\frac{\partial \xi^\beta}{\partial x^\nu}\eta_{\alpha\beta} dx^\mu dx^\nu + \frac{\partial \phi^\alpha}{\partial x^\mu}\frac{\partial \phi^\beta}{\partial x^\nu}\eta_{\alpha\beta} d\chi^\mu d\chi^\nu \tag{2.37}$$

So the geodesic equation will take the form:

$$\delta \int_P ds = 0 \tag{2.38}$$

Where the integral denotes a given trajectory in complex spacetime. The quantum field always moves in complex space only during the measurement we are able to detect a particle in a given location in space-time. Therefore, the interval is defined as the interval as real because the distance must be a real value. The field as a whole can be rotated. Where for flat space-time, i.e. in the absence of any matter (which is a trivial example), the vector field dz^μ transforms according to:

$$ds^2 = ds^{2\prime} \tag{2.39}$$

$$\eta_{\alpha\beta} dx^\mu dx^\nu + \eta_{\alpha\beta} d\chi^\mu d\chi^\nu = \eta_{\alpha\beta} dx^{\mu\prime} dx^{\nu\prime} + \eta_{\alpha\beta} d\chi^{\mu\prime} d\chi^{\nu\prime} \tag{2.40}$$

Where the primed coordinates are equal to:

$$dx^{\mu\prime} = \Lambda^{\mu\prime}_{\mu} dx^\mu \tag{2.41}$$

That is, by Lorentz transformations. For curved spacetime you can use a field of tetrads as I showed in the chapter on Lorentz transformations:

$$\hat{e}_\mu \cdot \hat{e}_\nu dx^\mu dx^\nu + \hat{\phi}_\mu \cdot \hat{\phi}_\nu d\chi^\mu d\chi^\nu = \hat{e}'_\mu \cdot \hat{e}'_\nu dx^\mu dx^\nu + \hat{\phi}'_\mu \cdot \hat{\phi}'_\nu d\chi^\mu d\chi^\nu \tag{2.42}$$

Where the primed vectors are equal to:

$$\hat{e}'_\mu = e^b_\mu \Lambda^a_b \hat{e}_a \tag{2.43}$$

2.6. **Quantum Part Summary.** The quantum part, although it is much shorter than the classical part, is much more mathematically demanding. The use of special complex fields makes the equations more complex, but it removes the singularity problem as only normalizable spacetimes are allowed and therefore only those with finite curvature.

The complex part of the field has a real impact on the space-time interval and the energy, mass and curvature of space-time. This means that despite the use of complex space-time, I will get real quantities that are definable in a measurement way.

However, the calculations from the already difficult equations of the classical field become even more complex, which leads to the complication of the already difficult equations. On the other hand, the fact that field equations are relatively easy to quantize is a very good sign.

To write the field equation, I used the field equation as energy, but the field equation can be written any way you like. The measurement changes the resting mass and forces it to be concentrated in the volume in which it is measured, which means that neither mass nor distance can be measured arbitrarily precisely because the gravitational field increases with decreasing volume.

The integral that defines the probability of finding a particle is taken over all of space-time, not just over space, which means that it imposes natural limits on the size of the manifold. Infinite manifold ceases to be possible as it ceases to be normalizable - this means that in the general case we remove all infinite values from the theory by imposing the requirement to normalize the field.

All quantities in theory have two functions, the first of them represents the real part, the second is the complex part, their combination with transposition and complex conjugate always gives an invariant which is real and this principle is crucial for building the entire model.

Solving the quantum field equations can be even more challenging than in the case of the real field - I did not give any solutions to the field equations in this work, only the mathematical model itself and the logic behind it.

References

[1] https://mathworld.wolfram.com/RicciCurvatureTensor.html

[2] https://mathworld.wolfram.com/RiemannTensor.html

[3] https://mathworld.wolfram.com/EinsteinFieldEquations.html

[4] https://mathworld.wolfram.com/EinsteinTensor.html

[5] https://arxiv.org/ftp/arxiv/papers/1610/1610.08131.pdf

[6] https://mathworld.wolfram.com/MinkowskiMetric.html

[7] https://mathworld.wolfram.com/LorentzTransformation.html

[8] https://mathworld.wolfram.com/LorentzTensor.html

[9] https://mathworld.wolfram.com/MinkowskiSpace.html

[10] https://arxiv.org/pdf/1106.2037.pdf

[11] https://www.astro.gla.ac.uk/users/martin/teaching/gr1/gr1_sec08.pdf

[12] https://mathworld.wolfram.com/ContractedBianchiIdentities.html

[13] https://barc.gov.in/barc_nl/2021/2021070804.pdf

[14] https://arxiv.org/ftp/arxiv/papers/1010/1010.5557.pdf

[15] http://hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_GRelativity_1916.pdf

[16] https://mathworld.wolfram.com/MetricTensor.html

[17] http://strangebeautiful.com/other-texts/blanchet-etal-mass-motion-gr.pdf

Email address: kobierzyckitomasz@gmail.com