Gravity extensions equation and complex spacetime

Tomasz Kobierzycki

July 15, 2023

Abstract

In this hypothesis, a possible extension of Einstein's field equations will be presented which is reduced to his field equation by contractions. Basic concepts such as the description of the inertial system or the definition of a physical observer are discussed. The field equation predicts the existence of exactly four-dimensional space-time, as only in four-dimensional space-time does this equation have an equal number of unknowns for each term of the equation. The equation itself can be written in two mixed and fully covariant forms:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{0.1}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2} R_{\sigma\phi} g_{\mu\nu} = \kappa T_{\mu\phi} g_{\sigma\nu} \tag{0.2}$$

This model relates the field of matter to the curvature of space-time in a direct way, if matter is not present at a given point in space, it is simply flat space-time, which makes it a requirement that the momentum energy tensor does not equal zero in the presence of space-time curvature. In this work, i not give the exact solutions of the equations, only their derivation and their form in a particular case. This paper is in two parts, first is classical part, second one is quantum part that uses a complex spacetime to make sense of probability in field equations. It turns field equation into complex field equation and then from it it creates a scalar that represents probability of finding particle in spacetime.

Contents

1 Classic part

1.1 Description of the inertial system

According to Newton's laws of motion, motion with constant velocity or lack of it gives an inertial frame of reference. This definition can be simplified even more that it is a system in which there are no forces associated with the motion of this system. Such a system does not feel the forces associated with its movement. The key question is whether an observer under the influence of a gravitational field can be treated as an inertial observer?

According to the equivalence principle, the gravitational field cannot be locally distinguished from the acceleration, on the other hand, omitting the tidal forces, one can look at the inertial system as the system of any falling observer in the gravitational field. This observer locally has no weight, no force acts on him. There are two possibilities, either the observer is in uniform motion or the observer is at rest. The first possibility can be ruled out for obvious reasons only the second possibility remains, the observer in the gravitational field is motionless. This means that every observer in the gravitational field that is not subjected to any kind of apparent force connected with e.g. standing on the surface of the gravitational field source or any other force is treated as an inertial system. And according to this observer's perspective, this gravitational field needs to be described. This means that any non-inertial observer cannot see the true cause of motion because there are forces in his system, so they exclude him from being an inertial system. The description of the laws of physics and thus motion must always be seen from the perspective of an inertial observer as only he perceives the true cause of motion, which also applies to the gravitational field.

An inertial frame of reference is defined as one which, under the influence of a physical field, remains completely still from its own perspective, where this stillness is defined only locally. This locality makes us ignore the tidal forces that will naturally accompany the gravitational field, and thus the physical field that is the source of motion. However, the definition of the physical field itself, i.e. the gravitational field, the field that causes motion, is more delicate. It results from the definition of the inertial system itself, physical fields are a field from the perspective of which no inertial observer can be described as an inertial observer, so he must be in motion relative to this field. This means that the field itself cannot give us the whole picture of how motion physically occurs. Only the perspective of a field that is the source of motion and an inertial observer that is able to detect true non-relative motion as a combination gives us a description of physical reality. Any motion that is relative depends on the system in which it is measured, the physical field or simply the gravitational field cannot be dependent on the system in which it is measured, it must be a source of motion for every inertial observer.

1.2 Light signals and their interpretation

The basis of the Theory of Relativity (Special) is the constancy of the speed of light for each inertial system. This creates a transformation of the frame of reference so that the speed of light is conserved for each observer. For two observers observing an event of length ds, you can write this transformation as a requirement $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx'^\mu dx'^\nu$ where prime coordinates are the second observer. This means that the light signal plays a key role in building the concept of distance in space-time. Consider a light signal propagating from the point $\mathbf{x}$ then the space-time distance (interval) can be written as:

$$ds^2(\mathbf{x}) = \eta_{\mu\nu} dx^\mu(\mathbf{x}) dx^\nu(\mathbf{x}) \tag{1.1}$$

This means that distance is fundamentally linked to the ability to send a light signal from a given point in spacetime. How a given observer measures axis of time and space is not absolute, but the magnitude is. So what is its physical meaning? A light signal sent from a given point in space-time determines that event happening at that point in space-time. This means events are understood as light signals propagating from every existing point in space at every possible point in time. Of course, these signals do not have to be physically sent, it's just a geometric fact, so for a light signal, the interval is always zero $ds^2 = 0$. An event for itself has no distance from its beginning. Despite this, different observers will perceive differently how these events occur, the light signal does not "perceive" the distance between itself and every other event. The light signal is immediately located in that place and time in which the event is present from the perspective of the given observer. This gives a fairly obvious interpretation of how an inertial observer defines his laws of motion and how he perceives time and space.

Since the observer is always stationary relative to the light signal emitted from any point in time and space, this gives an additional important rule in determining motion, the observer is always stationary from the perspective of any light interval, i.e. an interval with zero distance in time and space. The inertia of this observer is always defined with respect to the event itself, or more precisely with respect to space-time. The observer itself is always motionless relative to any event that happens in space-time. So any truly inertial frame of reference is defined by the impossibility of motion with respect to the event. Thus, space-time consists of inertial observers and this inertia results from the invariance of the speed of light for each observer, i.e. events in space-time.

1.3 Observer definition

The observer is understood as a frame of reference capable of measuring time (clock) and distance (ruler). The units of measurement must always be chosen to express distance in time or space. This means that if I measure distance in meters and seconds I have to express both units in meters or seconds, which is achieved by multiplying time by the speed of light (meters) or dividing distance by the speed of light (seconds). This is a fairly basic assumption in the Theory of Relativity.

What is crucial for extending the field equations is the exact physical definition of the phenomenon for a given observer. A physical phenomenon is simply such a phenomenon that meets the previous assumption, the observer is completely at rest relative to the event, which means that it is defined as an inertial frame of reference. The previous definition, of course, only makes sense in the case of flat space-time, so it is not a general case. To go to the general case, it is necessary to define an observer in a gravitational field as still an observer motionless relative to an event in which a gravitational field is present. Before discussing free fall from the perspective of an inertial observer, one key point needs to be addressed.

Space-time in the mathematical description must adhere to the principle according to which the observer remains inertial to the event, this means that locally, as in the Theory of Relativity, the observer locally measures flat space-time, which is not true globally. The whole point of this paper is to show that there are other field equations that reproduce this principle but with an additional condition. This condition is that the gravitational field is fully dependent on the existence of a field of matter and/or energy at every point in space-time. In the absence of matter at any point in space-time, these equations become equations for flat space-time, which means that literally the source of deviations from flat space-time must be the presence of matter at the point where this space-time deviates from it, otherwise we get flat Minkowski space. There is an additional principle that is central to this whole model and its assumptions, the equivalence of gravity and the field of matter. Which I will discuss in more detail later in the section on the non-zero momentum energy tensor requirement.

1.4 Free fall

Fall of freedom is a basic gravitational phenomenon, the assumption is that during free fall the falling observer remains motionless in relation to the gravitational field. Additionally, the gravitational field is equivalent to the field of matter or energy. From these two assumptions, only one interpretation of what happens in free fall can be gained that is consistent with experience.

Let us consider a thought experiment, I have an inertial frame U and a gravitational field source U', the mass of the frame U' is much greater than the mass of the inertial frame U so that the gravitational influence of the first frame is negligible. Since the system U is at rest, it means that the system U' is in motion. The first frame is approximately point-like and the U' frame is a spherical mass expressed by the energy density functions $\rho = \frac{\int_0^R m(r)drc^2}{\frac{4}{3}\pi r^3}$ such that the integral of $m(r)$ is equal to the initial mass or rest mass of the system $\int_0^R m(r)dr = m_0$. Where R is the surface radius of this mass. An inertial observer perceives that the motion of the U' system is directed spherically in all directions as the relative size of this object "increases", the U' system expands in all directions, but when this system expands enough for the U system to hit it with surfaces the size of both will not change himself. This means that both objects must have experienced exactly the same expansion in space. This can be described by the Ricci tensor and the equivalence of this tensor to the field of matter. However, the units of the energy tensor and the Ricci tensor are not the same, it is necessary to use Einstein's constant, the whole thing can be written as:

$$R_{00} = n\kappa T_{00} \tag{1.2}$$

Where the numeric constant n is some number. The key here is that only the time-time component of the Ricci tensor and the momentum energy tensor are taken into account. Due to the fact that the geodesic lines move away from each other or the volume form increases over time, this constant must have a negative sign, so $n = -a$ where a is a certain number. This means that the observer U is stationary but time is expanding with the gravitational field source U'. Objectively, both observers remain at rest, while time expands along with the gravitational field. Writing the whole thing as an equation:

$$R_{00} = -a\kappa \frac{\int_0^R m(r)drc^2}{\frac{4}{3}\pi r^3} \tag{1.3}$$

This equation is crucial for the whole of this work, it shows the equivalence between the material field and the expansion of space-time and thus the inertia of both systems U and U' as systems that are physically stationary.

1.5 Spacetime

According to the principle written in the previous chapter, gravitational systems remain inertial. It should be remembered, however, that these systems are truly inertial, they must meet not only the stationary in the gravitational field in the classical sense, but also in the sense of the necessity of stationarity in relation to the light ray sent from a given point in space-time. According to the Theory of Relativity, this requirement can be written as a transition from flat to curved spacetime:

$$ds^2\left(\mathbf{x}\right) = g_{\mu\nu}\left(\mathbf{x}\right) dx^{\mu}\left(\mathbf{x}\right) dx^{\nu}\left(\mathbf{x}\right) \tag{1.4}$$

$$g_{\mu\nu}\left(\mathbf{x}\right) = \frac{\partial \xi^{\alpha}}{\partial x^{\mu}} \frac{\partial \xi^{\beta}}{\partial x^{\nu}} \eta_{\alpha\beta} \tag{1.5}$$

On the other hand, the space-time metric itself must be not so much arbitrary as fulfilling the principle described in the previous chapter. To obtain this, we need a gravitational field equation whose solution is a space-time metric satisfying this equation. Before proceeding to the derivation of this equation, it is important to introduce all the rules that must be met so that such a space-time satisfies the principle of stationarity of any system defined in this space-time.

This means that each system must be defined as inertial, while the structure of space-time itself depends on the field of matter or energy at a given point. From the previous chapter it can be deduced that the momentum energy tensor does not disappear at any point in spacetime, it is necessary that the Ricci tensor does not disappear with it. This is quite a simple rule, the farther from the center of mass, the lower the density of matter, the closer it is, the greater, the surface of this matter is only conventionally understood as the limit beyond which the mass does not increase because the amount of matter, and more precisely the rest mass, does not increase. On the other hand, the density of matter will continue to decrease indefinitely no matter how far we move away from the central mass, which means that the gravitational field does not disappear and the field of matter does not disappear with distance.

Since it follows from the assumptions that we are studying the deviation of space-time from Minkowski space, the principle that the inertial observer should be motionless relative to the hypothetical light signal sent from a given point still applies. Combining all these principles into one principle, we get a space-time consistent with this general idea of inertial systems. The last step to success in this reasoning is to find field equations that satisfy this principle, fortunately there is only one way to derive such an equation and doing so mathematically complicates Einstein's field equations. However, the assumption is that it gives a better description of the gravitational field and reduces to the field equations in a suitable way.

1.6 Finding the field equation

The field equation must reduce to the Einstein field equations but this is only one requirement, the second requirement is the stationarity of all observers relative to the light signal sent from a given point in space-time, the last requirement is that the momentum energy tensor must not vanish at each point otherwise we get Minkowski space. An equation that satisfies all these conditions is very difficult to find without any mathematical clue, it turns out that all assumptions are satisfied if we take only the first law. The field equations must be reducible to Einstein's field equations.

The Einstein field equations have ten unknowns, they consist of the Ricci tensor, the Ricci scalar, the metric tensor, and the momentum energy tensor. I'll start by writing these equations without the momentum energy tensor, i.e. I'll write the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{1.6}$$

The key question is can there be a tensor that reduces to the Einstein tensor? Well, yes, the Einstein tensor can also be written as two contractions of the Riemann tensor and contractions of the Ricci tensor with the metric tensor:

$$G^{\rho}_{\mu\rho\nu} = R^{\rho}_{\mu\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.7}$$

Such writing of the Einstein tensor automatically yields a tensor whose contraction leads to the Einstein tensor. Now to get the tensor that I am looking for I only need to change the index ρ to another index, in this case I will use the index σ which will give me a new tensor:

$$G^{\rho}_{\mu\sigma\nu} = R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{1.8}$$

For obvious reasons, this tensor reduces to the Einstein tensor, the problem is that this tensor has 256 components, of which only 20 are independent in four-dimensional space-time. Interestingly, this tensor has only the same number of unknowns in four-dimensional spacetime, which can be written as an equality, where on one side there are the number of independent components of the Riemann tensor on the other side twice the number of components for a symmetric second-order tensor:

$$\frac{n^2\left(n^2-1\right)}{12} = n\left(n+1\right) \tag{1.9}$$

The solution to this equation is four-dimensional space-time, or zero-dimensional space-time, I omit solutions with a negative number for obvious reasons. The last step to complete the field equation is the other side of the equation, namely the momentum energy tensor.

1.7 Field equation

The last element of the full field equation is the momentum energy tensor, in this case I need to write a tensor whose contraction leads to the momentum energy tensor itself. I need to use the ρ and σ indices the same way as in the Einstein tensor except that I want to get the momentum energy tensor not its trace multiplied by the metric tensor. So a contraction is needed that leads to the Knocker delta, I can write this part as:

$$T^{\rho}_{\mu\sigma\nu} = T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{1.10}$$

It can be easily verified that in fact the contraction of the index ρ and σ leads to the Knocker delta $g^{\kappa\rho} g_{\rho\nu} = \delta^{\kappa}_{\nu}$ acting on the momentum energy tensor will change its indices to be consistent with the rest of the equation, so I finally get the field equation:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} = \kappa T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \tag{1.11}$$

It can be checked again that this equation reduces to Einstein's equations with the difference that there is not only one possible contraction, it means that the field equation can not only be reduced to one equation but to several different equations. Their common feature is that for a vacuum they all reduce to one equation, the Ricci tensor is equal to zero. This results in their common feature, returning to the extended equation, as will be shown later in this work, this equation meets all the requirements that are necessary for the observer to always be motionless in relation to the event, i.e. simply space-time, which is equivalent to a light signal sent from a given point of it, and additionally satisfy this principle for the gravitational field.

This equation, however, is quite a complicated equation and is at the cost of being mathematically more complicated as a whole than Einstein's field equations on the other hand, it is possible that this equation solves problems that are observed in cosmology (dark matter and energy) and does not require addition to the gravitational field no additional ingredients to make certain predictions match them. The cost of this is the mathematical complication of the gravitational field equations, where already difficult field equations become even more difficult to solve. The principle on which these equations are based can also be highly controversial as gravity acts as an attraction here this attraction is a kind of illusion in fact we are talking about the expansion of time which looks like an attraction from a certain perspective. However, it gives a theoretically reasonable description of every observer as inertial, which also applies to the gravitational field.

1.8 Requirement for a non-zero momentum energy tensor

The extended Einstein tensor for a vacuum will always give a Minkowski space. This can easily be proved if I zero the components of the Riemann tensor by setting all indices equal to each other, I get:

$$-\frac{1}{2}R_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma} = 0 \tag{1.12}$$

This equation will end up with a Ricci tensor equal to zero, I can rearrange it in two ways, the first way is to write the equation as a Ricci scalar times the metric tensor is equal to zero but remember that there are components of the Ricci tensor in the equation, so I can use the metric tensor identity which will give E equality of diagonal elements Ricci tensor of zero:

$$R_{\sigma\sigma} = 0 \tag{1.13}$$

So I used the fact that the metric tensor gives the Knocker delta δ^{κ}_{σ}, that's part of the equation, but since the Ricci tensor is equal to zero (its diagonal elements) then whenever there are diagonal elements of the Ricci tensor, the Riemann tensor will also be equal to zero. You can prove from this equation that if all the diagonal elements of the Ricci tensor are equal to zero, the Riemann tensor will also be equal to zero. I'll write the field equation again only this time for the diagonal components of the Ricci tensor, where I take into account that if the Ricci tensor has non-diagonal components it gives zero so only the diagonal components of the metric tensor are allowed:

$$R^{\sigma}_{\mu\sigma\nu} - \frac{1}{2}R_{\sigma\sigma}g^{\sigma\sigma}g_{\mu\nu} = 0 \tag{1.14}$$

$$R^{\sigma}_{\mu\sigma\nu} = 0 \tag{1.15}$$

$$R_{\mu\nu} = 0 \tag{1.16}$$

Which ultimately gives the Ricci tensor, which is always zero, and therefore the Riemann tensor, which is always zero:

$$R^{\rho}_{\mu\sigma\nu} = 0 \tag{1.17}$$

Which proves that the momentum energy tensor must be nonzero at every point in space for the equation not to result in flat spacetime. The extended Einstein tensor is zero, so for vacuum it just gives Minkowski space. Here the two equations are completely different from each other in relativity there are solutions for vacuum here there are no solutions for vacuum other than flat space-time so another requirement that was assumed is satisfied by the field equation.

1.9 Conservation laws and the covariant derivative

With respect to the covariant derivative, the Einstein tensor and the momentum energy tensor are zero, which is a key conservation law in the Theory of Relativity. It can be easily shown that from the Einstein field equation it is possible to derive an extended field equation using only metric tensors whose covariant derivative is equal to zero, so they can be treated as constants, instead of deriving the formula from the Banach equation, it will be easier to do it from the Einstein equations. The trick is to choose the right summation indices of the Riemann tensor that yields the Ricci tensor. I can write this whole process except that it will use the covariant derivative altered by the metric tensor to have a superscript $\nabla^\nu = g^{\nu\alpha}\nabla_\alpha$ by writing the whole equation starting with the Einstein tensor :

$$\nabla^\nu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = 0 \tag{1.18}$$

$$\nabla^\nu \left(R^\rho_{\mu\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \right) = 0 \tag{1.19}$$

$$\nabla^\nu g_{\sigma\alpha} g^{\alpha\rho} \left(R^\rho_{\mu\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \right) = 0 \tag{1.20}$$

$$\nabla^\nu \delta^\rho_\sigma \left(R^\rho_{\mu\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\mu\nu} \right) = 0 \tag{1.21}$$

$$\nabla^\nu \left(R^\rho_{\mu\sigma\nu} - \frac{1}{2} R_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \right) = 0 \tag{1.22}$$

The same reasoning can be applied to the momentum energy tensor, except that I have to rewrite this tensor as a product of the Knocker delta, then this delta as a product of two metric tensors, which doesn't change the result but allows us to get the correct tensor. Writing the entire mathematical transformation:

$$\nabla^\nu T_{\mu\nu} = 0 \tag{1.23}$$

$$\nabla^\nu T_{\mu\kappa} \delta^\kappa_\nu = 0 \tag{1.24}$$

$$\nabla^\nu T_{\mu\kappa} g^{\kappa\rho} g_{\rho\nu} = 0 \tag{1.25}$$

$$\nabla^\nu g_{\sigma\alpha} g^{\alpha\rho} T_{\mu\kappa} g^{\kappa\rho} g_{\rho\nu} = 0 \tag{1.26}$$

$$\nabla^\nu \delta^\rho_\sigma T_{\mu\kappa} g^{\kappa\rho} g_{\rho\nu} = 0 \tag{1.27}$$

$$\nabla^\nu T_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} = 0 \tag{1.28}$$

So this field equation, just like Einstein's equation, satisfies the conservation principle, which is a very important feature. Otherwise, the model would be mathematically inconsistent.

1.10 Field equation for free fall

In the chapter on free fall, I wrote down an equation that I can now fully derive from the field equation. A special case of the field equations is one where the components of the Riemann tensor are zero and I get only the diagonal elements of the metric tensor, the momentum energy and the Ricci tensor. By writing this equation:

$$R^{\mu}_{\mu\mu\mu} - \frac{1}{2}R_{\mu\kappa}g^{\kappa\mu}g_{\mu\mu} = \kappa T_{\mu\kappa}g^{\kappa\mu}g_{\mu\mu} \tag{1.29}$$

$$-\frac{1}{2}R_{\mu\kappa}g^{\kappa\mu}g_{\mu\mu} = \kappa T_{\mu\kappa}g^{\kappa\mu}g_{\mu\mu} \tag{1.30}$$

$$-\frac{1}{2}R_{\mu\kappa}\delta^{\kappa}_{\mu} = \kappa T_{\mu\kappa}\delta^{\kappa}_{\mu} \tag{1.31}$$

$$-\frac{1}{2}R_{\mu\mu} = \kappa T_{\mu\mu} \tag{1.32}$$

$$R_{\mu\mu} = -2\kappa T_{\mu\mu} \tag{1.33}$$

Now I can go back to free fall, and from the perspective of an observer moving with the frame of reference, I have four equations to solve for dust:

$$R_{00} = -2\kappa\rho_0 c^2 \tag{1.34}$$

$$R_{11} = R_{22} = R_{33} = 0 \tag{1.35}$$

So the constant we're looking for is two, which was discussed in the chapter on free fall, and according to that chapter, the field equation gives exactly the expected result. Going back to the equation itself, this is the only way to write the field equation where the elements of all tensors are only diagonal without getting a flat space-time. The unknown in this equation is the metric tensor and the equation itself is quite complicated despite the relatively simple notation, the time-time component of the Ricci tensor differs from the Theory of Relativity, the rest remains the same as in solutions for vacuum. The interpretation of this equation is the same as for free fall. The minus sign means that the volumetric form, as the radius coordinate decreases, increases in proportion to the density, which becomes larger and larger. Of course, real mass is described by mass functions from the radius, not from the point concentrated mass - the point mass will be a singularity at zero radius equal to zero, but if the mass is a function that vanishes at zero then you can get rid of this singularity. This model is not free of singularities, which inherits in a sense from the Theory of Relativity, on the other hand, these singularities are completely dependent on the field of matter.

1.11 The universe and its expansion as a matter field effect

From the previous chapter, you can assume that the universe behaves like dust and calculate the value of the Ricci tensor for the present conditions. These calculations result in a result very close to the cosmological constant, with the difference that here it is not a constant but a value dependent on the density of the matter field, where dark matter must be taken into account in mass calculations. For dust as in the previous chapter, there is only one non-zero component of the Ricci tensor in the frame of reference that is not moving. This is a time-time component and its direct effect is to increase the distance between geodesics in time, which results from the negative sign of this component. Two observers in the universe, depending on how far they look, will always perceive the universe as expanding globally, not necessarily locally, this is not due to the existence of an additional energy field in this model, but to gravity itself, which I discussed in the chapter on free fall. I will now write the value of the Ricci tensor where I multiply the mass of the universe by $1 + \frac{85}{15}$ where the fraction represents the dark matter of which there is about 85% in the universe:

$$R_{00} = -2\kappa\rho_0 c^2 \tag{1.36}$$

$$\rho_0 = \rho_U \left(1 + \frac{85}{15}\right) \tag{1.37}$$

$$\rho_0 \approx 3 \cdot 10^{-27} [kg/m^3] \tag{1.38}$$

$$R_{00} \approx -1 \cdot 10^{-52} [m^{-2}] \tag{1.39}$$

Which is a result practically equal to the observed cosmological constant, with the difference that it is not a constant but a value depending on the field of matter. This means that, according to this model, the cosmological constant is not constant but variable depending on the density of matter in the universe. The lower the density, the lower the value, the higher the higher the value. On the local scale, this magnitude is so small that other gravitational fields dominate, while on the global scale of the universe, this magnitude will be visible as it is the effect of the global gravity of the universe. This means that, according to this model, the universe expands in the same way as any other gravitational field. Just like in free fall, the earth gets bigger and bigger as it expands, so time expands as the earth moves, which makes it appear stationary to an observer on its surface, but to any other observer in free fall it is. he is motionless and the earth is expanding with time all around, the same applies to the universe, we are in free fall relative to the universe, therefore we perceive its expansion in time as the expansion of space, where according to this model it is not space that expands but time what it gives the impression of matter moving away from each other inside the universe.

1.12 Non-vanishing matter fields and dark matter

The requirement that the momentum energy tensor is non-vanishing at every point in space so that the curvature is non-zero automatically generates seemingly extra matter in the universe. According to this model, it is not extra matter but matter itself is treated as a continuous field. This means that every physical object extends to infinity in space. Depending on the adopted model of matter, this will give different geometries of space-time. This means that the geodesic equation no longer describes the trajectory of a single object, but must be treated as the motion of a continuous field. This means that I have to consider not the trajectories but the motion of the whole region of space or the whole field in general. This means that matter can only be locally gravitationally defined as a constant amount of mass or energy.

Exactly the same requirement works for the momentum energy tensor, it must be defined over the entire manifold, in addition, if the manifold is not a Minkowski space, it must be nonzero over the entire manifold. The requirement for rest mass as total mass in the field is no longer met, rest mass is only locally conserved, global mass not equal to rest mass, rest mass is only the area of greatest concentration of the field if the field has one. This means that the field is treated as a continuous field of matter defined over the whole manifold, not just one region or point. Therefore, the total field mass or energy has no interpretation, only the local field energy or mass can have an interpretation. It is locally that causes gravitational effects, so it is responsible for the modification of space-time from Minkowski space. This is a possible explanation for dark matter as missing mass, the amount of total mass not equaling the rest mass.

This means that the momentum energy tensor must be defined in such a way that its value always agrees with the local and global curvature of space-time. This only says that the transition from the definition of matter to the definition of the field of matter is necessary for this model to make sense. The definition of rest mass must therefore be globally replaced by a field which only locally has a rest mass and only locally this mass remains constant. Since the field is equivalent to the gravitational field, the law of conservation of the field requires that this tensor gives zero covariant derivative. This requirement is crucial to the whole field equation and is described in the chapter on conservation laws.

1.13 Model and Principle Problems

The biggest problem with this model is that it is not a quantum model, it is still a classical theory of gravity which is in opposition to the quantum theory. The second problem of the model is the occurrence of singularities. It is impossible to remove singularities from this model if any mass is concentrated in a point, it leads to infinite curvature of space-time, in this case negatively infinite. The ideal model would be devoid of singularities, however, this model, through the mathematical description used, generates singularities depending on the field of matter, the key question is whether these singularities can be removed. Alternatively, whether these singularities can be surpassed. The key question is also whether any non-quantum theory of gravity will lead to singularities, so it is necessary to look for quantum theory or singularities in the real world that exist and have physical significance.

If singularities do not occur in the real world (I mean the points where the curvature of space-time becomes infinite) eventually quantum gravity is needed to get around this problem, but if singularities are real and occur in the physical world there must be accurate ways to understand the geometry of such objects. Singularities must be surpassable for the field of matter, geodesics cannot end at singularities, in relativity geodesics end at singularities the key question is whether geodesics end in this model the same as in relativity, unfortunately the exact answer to this question requires, firstly, exact solutions of the field equations which are not presented here, secondly, their analysis. A good clue to this is that singularities only appear due to the existence of a point mass, assuming that the mass is a continuous field that only concentrates in a given region of space and this concentration is equal to the rest mass, and at zero distance from the mass, the singularity disappears for the zero radius disappears with the mass, it means that in purely theoretical imprecise considerations, singularities can be avoided.

The mere fact that for a positive momentum energy tensor, the Ricci tensor is negative in the simple case of writing the field equation gives hope that the geodesic lines, instead of ending at the singularity, start there. Because if you reverse the motion away from the singularity, you get an expansion of space-time that slows down until you get to infinite distance from the source of mass, then you'll just be left with flat space-time. Of course, a formal mathematical proof of this reasoning is necessary to be sure of this, but the reasoning makes sense. Since the momentum energy tensor cannot be fully defined negatively, we also avoid the problem of the infinitely positive Ricci tensor which gives exactly the singularity at which every geodesic ends. This model, however, conflicts with quantum physics, and only time and experimentation can show which approach is true.

1.14 Fully covariant field equation

The field equation can be converted to a fully covariant form. To do this, just use the metric tensor to lower the index ρ, such an equation can be written as:

$$R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} = \kappa T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} \tag{1.40}$$

$$g_{\phi\rho}R^{\rho}_{\mu\sigma\nu} - \frac{1}{2}g_{\phi\rho}R_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} = \kappa g_{\phi\rho}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} \tag{1.41}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2}R_{\sigma\kappa}\delta^{\kappa}_{\phi}g_{\mu\nu} = \kappa T_{\mu\kappa}\delta^{\kappa}_{\phi}g_{\sigma\nu} \tag{1.42}$$

$$R_{\phi\mu\sigma\nu} - \frac{1}{2}R_{\sigma\phi}g_{\mu\nu} = \kappa T_{\mu\phi}g_{\sigma\nu} \tag{1.43}$$

This form of the field equation may in many cases be more useful than the previous one, it may simplify calculations. Of course, since this is a tensor equation, the form of this equation doesn't matter. For example, I can derive the field equations for all diagonal elements simply by zeroing the Riemann tensor, making all diagonal elements zero:

$$-\frac{1}{2}R_{\phi\phi}g_{\phi\phi} = \kappa T_{\phi\phi}g_{\phi\phi} \tag{1.44}$$

I omit the metric tensor because its values are identical on both sides of the equation, which gives exactly the same result as before:

$$R_{\phi\phi} = -2\kappa T_{\phi\phi} \tag{1.45}$$

From fully covariant form i can write whole field equation for simplest case:

$$R_{\sigma\mu\sigma\mu} - \frac{1}{2}R_{\sigma\sigma}g_{\mu\mu} = \kappa T_{\sigma\mu}g_{\sigma\mu} \tag{1.46}$$

$$R_{\sigma\sigma\sigma\sigma} - \frac{1}{2}R_{\sigma\sigma}g_{\sigma\sigma} = \kappa T_{\sigma\sigma}g_{\sigma\sigma} \tag{1.47}$$

$$-\frac{1}{2}R_{\sigma\sigma}g_{\sigma\sigma} = \kappa T_{\sigma\sigma}g_{\sigma\sigma} \tag{1.48}$$

$$R_{\sigma\sigma} = -2\kappa T_{\sigma\sigma} \tag{1.49}$$

$$R_{\sigma\mu\sigma\mu} = -\kappa T_{\sigma\sigma}g_{\mu\mu} \tag{1.50}$$

$$R_{\sigma\mu\sigma\mu} = \frac{1}{2}R_{\sigma\sigma}g_{\mu\mu} \tag{1.51}$$

So field equation in full form for simplest case is more complex than just free fall equation. It consists of sixteen equations, and first i need to solve it for Ricci tensor then for those twelve equations with Ricci tensor or energy tensor as they differ only by constant. Where it should be remembered that the last two equations are the twelve equations and the last three are the remaining four.

1.15 Summary of classical part

In this short paper, I presented the hypothesis of extending Einstein's field equations while maintaining the simple principle of stationarity of each frame of reference. Relative to the light ray emitted from a given point in space at a given moment of time, each observer remains motionless, the same applies to the gravitational field, so it moves at any speed. When the field of matter moves then space-time moves in time with this field, which makes any movement compensated in some sense by changes in space-time.

However, the work lacks mathematical solutions of the field equations which, despite the simple notation, are quite complicated to solve. It should also be remembered that this hypothesis is still a classical theory, which makes it opposed to the quantum approach to the problem of space-time. However, the motivation is the mathematical consistency of the model and that the conclusions of this theory may be a solution to the current problems observed in cosmology.

However, in order to obtain better technical results from this model, it is necessary to solve the field equations even for a simple dust situation. A rather controversial assumption of this model is that the momentum energy tensor never vanish, otherwise we get a locally flat space-time. This means that matter is treated as a continuous field that never vanish, but its intensity can vary freely depending on the type of field. The same applies to the trajectory, which is no longer a line, but the change of the whole manifold under the influence of the field of matter.

Mathematically, the equation is quite complex, but the reasoning behind the equation is not contradictory, so it is a model that makes sense from a physical point of view, and is able to describe physical observations in a consistent way. The big success is that it predicts the existence of four-dimensional space-time, so you don't have to assume the existence of four-dimensional space-time, it's a consequence of the equation.

The open question remains whether this model is needed at all or is it just an interesting mathematical fact? Well, without solving the field equations and checking how these equations work in relation to observations, it is impossible to answer this question. From a theoretical point of view, this model is an interesting alternative to inflation, dark energy, and possibly dark matter, as all phenomena implied by this model are not additional assumptions necessary to make this model work.

2 Quantum part

2.1 Extending idea to quantum physics

Key question is there a way to extend this idea to quantum physics? Short answer it yes, but it has key problem. That problem is that there is need to use complex spacetime rather than real one. Still there is a way to make out a probability function out of this complex spacetime. It means that there still can be defined a real physical implication of that complex spacetime, or saying otherwise from complex spacetime there is a way to extract information about real observer particle in gravity field. This extraction will use all information about physical systems from section before so it will be a lot simpler than trying to figure it all out form start. Goal is to apply same principles but this time to complex spacetime not a real one, so nothing really changes other than use of complex spacetime.

Field equation is turned into complex field equation and it has a complex conjugate, but I will use both complex conjugate and contravariant forms of all tensors in complex spacetime. Basics are that to mark change from real tensors to a complex one I will denote a complex tensor as $\mathcal{R}_{\mu\nu}$ where this is representation of Ricci tensor, all tensors will be written this way and additionally i will write complex conjugate as a bar over a tensor so for example complex conjugate of covariant Ricci tensor is written $\overline{\mathcal{R}}_{\mu\nu}$. Both covariant and contravariant tensors will have complex conjugate, I will start by definition of complex metric tensor and its action on complex vectors and from it will build rest of curvature tensors.

By building this model from start its easy to follow general rules and how really simple is going from real spacetime to a complex one. Goal is to find a complex scalar field and it's complex conjugate to get probability of finding particle in given region of spacetime. In general there are two ways to do it from fact that field equation has on one side energy part that never vanishes and on other side it has curvature part that never vanishes too if spacetime is curved or there is presence of matter that both are same thing, stated in two possible ways. Simpler way is to use energy part but still there is need to solve field equation first to find a working solutions. It's key that energy momentum tensor has to be complex like rest of tensors.

2.2 Complex spacetime

Flat complex spacetime is just spacetime defined as light signals that propagate in complex spacetime so spacetime interval can be written in two possible ways, first way is using normal complex numbers z and second one using it's complex conjugate $\overline{z}$. I will write those two spacetime intervals in flat spacetime:

$$ds^2(\mathbf{z}) = \eta_{\mu\nu} dz^{\mu}(\mathbf{z})\, dz^{\nu}(\mathbf{z}) \tag{2.1}$$

$$d\overline{s}^2(\overline{\mathbf{z}}) = \eta_{\mu\nu} d\overline{z}^{\mu}(\overline{\mathbf{z}})\, d\overline{z}^{\nu}(\overline{\mathbf{z}}) \tag{2.2}$$

Now its definition in flat spacetime, for complex spacetime I need to define metric tensor first, I will define both complex and it's complex conjugate. Definition is straight forward:

$$\mathcal{G}_{\mu\nu}(\mathbf{z}) = \frac{\partial \xi^{\alpha}}{\partial z^{\mu}} \frac{\partial \xi^{\beta}}{\partial z^{\nu}} \eta_{\alpha\beta} \tag{2.3}$$

$$\overline{\mathcal{G}}_{\mu\nu}(\overline{\mathbf{z}}) = \frac{\partial \overline{\xi}^{\alpha}}{\partial \overline{z}^{\mu}} \frac{\partial \overline{\xi}^{\beta}}{\partial \overline{z}^{\nu}} \eta_{\alpha\beta} \tag{2.4}$$

When in first case they depend on complex field, in second case on complex conjugate field. It means that now form metric tensors I can define another key component that is Christoffel symbols of second kind, where as rule follows I will use both complex and it's complex conjugate parts. Those te definitions are where I will use letter $\mathcal{C}$ not letter Γ:

$$\mathcal{C}^{\mu}_{\alpha\beta} = \frac{1}{2} \mathcal{G}^{\mu\kappa} \left(\frac{\partial \mathcal{G}_{\kappa\alpha}}{\partial z^{\beta}} + \frac{\partial \mathcal{G}_{\kappa\beta}}{\partial z^{\alpha}} - \frac{\partial \mathcal{G}_{\alpha\beta}}{\partial z^{\kappa}} \right) \tag{2.5}$$

$$\overline{\mathcal{C}}^{\mu}_{\alpha\beta} = \frac{1}{2} \overline{\mathcal{G}}^{\mu\kappa} \left(\frac{\partial \overline{\mathcal{G}}_{\kappa\alpha}}{\partial \overline{z}^{\beta}} + \frac{\partial \overline{\mathcal{G}}_{\kappa\beta}}{\partial \overline{z}^{\alpha}} - \frac{\partial \overline{\mathcal{G}}_{\alpha\beta}}{\partial \overline{z}^{\kappa}} \right) \tag{2.6}$$

Where for saving notation space i did skip writing that it's a field that depends on complex number or on complex conjugate of numbers. Now last part i want to write before moving forward is geodesic equations for complex spacetime and it's complex conjugate that is same equation but just with complex numbers that means complex Christoffel symbols of second kind:

$$\frac{d^2 z^{\mu}}{ds^2} + \mathcal{C}^{\mu}_{\alpha\beta} \frac{dz^{\alpha}}{ds} \frac{dz^{\beta}}{ds} = 0 \tag{2.7}$$

$$\frac{d^2 \overline{z}^{\mu}}{d\overline{s}^2} + \overline{\mathcal{C}}^{\mu}_{\alpha\beta} \frac{d\overline{z}^{\alpha}}{d\overline{s}} \frac{d\overline{z}^{\beta}}{d\overline{s}} = 0 \tag{2.8}$$

2.3 Riemann, Ricci tensors in complex spacetime

Before I just write definition of those tensor in title, want to start by simple rules for index raising and contraction. If i have two vectors one covariant and contravariant so z_μ and z^μ I still need their complex conjugate , then I need to match metric tensor with complex form or it's complex conjugate. I can do it by writing four equations for each possible part:

$$z_\alpha \mathcal{G}^{\alpha\mu} = z^\mu \tag{2.9}$$

$$\mathcal{G}_{\mu\alpha} z^\alpha = z_\mu \tag{2.10}$$

$$\overline{z}_\alpha \overline{\mathcal{G}}^{\alpha\mu} = \overline{z}^\mu \tag{2.11}$$

$$\overline{\mathcal{G}}_{\mu\alpha} \overline{z}^\alpha = \overline{z}_\mu \tag{2.12}$$

Where i switch indexes from fact that metric tensor is symmetric, its not only symmetric it's contraction with itself gives Knocker delta:

$$\mathcal{G}^{\mu\alpha} \mathcal{G}_{\alpha\nu} = \delta^\mu_\nu \tag{2.13}$$

$$\overline{\mathcal{G}}^{\mu\alpha} \overline{\mathcal{G}}_{\alpha\nu} = \delta^\mu_\nu \tag{2.14}$$

Now when I have basic identities of complex metric tensor that are same as like in real spacetime I can move to defining curvature tensors. I will start by Riemann curvature tensor , form fact that I only change Christoffel symbols from real to complex one like rest of equations it stays same:

$$\mathcal{R}^\rho_{\mu\sigma\nu} = \partial_\sigma \mathcal{C}^\rho_{\nu\mu} - \partial_\nu \mathcal{C}^\rho_{\sigma\mu} + \mathcal{C}^\rho_{\sigma\kappa} \mathcal{C}^\kappa_{\nu\mu} - \mathcal{C}^\rho_{\nu\kappa} \mathcal{C}^\kappa_{\sigma\mu} \tag{2.15}$$

$$\overline{\mathcal{R}}^\rho_{\mu\sigma\nu} = \overline{\partial}_\sigma \overline{\mathcal{C}}^\rho_{\nu\mu} - \overline{\partial}_\nu \overline{\mathcal{C}}^\rho_{\sigma\mu} + \overline{\mathcal{C}}^\rho_{\sigma\kappa} \overline{\mathcal{C}}^\kappa_{\nu\mu} - \overline{\mathcal{C}}^\rho_{\nu\kappa} \overline{\mathcal{C}}^\kappa_{\sigma\mu} \tag{2.16}$$

Where I did use notation for complex conjugate derivative $\overline{\partial}_\sigma = \frac{\partial}{\partial \overline{z}^\sigma}$ and for normal complex derivative it's same notation $\partial_\sigma = \frac{\partial}{\partial z^\sigma}$. Now i can move to Ricci tensor taht is just contraction of Riemann tensor:

$$\mathcal{R}_{\mu\nu} = \partial_\rho \mathcal{C}^\rho_{\nu\mu} - \partial_\nu \mathcal{C}^\rho_{\rho\mu} + \mathcal{C}^\rho_{\rho\kappa} \mathcal{C}^\kappa_{\nu\mu} - \mathcal{C}^\rho_{\nu\kappa} \mathcal{C}^\kappa_{\rho\mu} \tag{2.17}$$

$$\overline{\mathcal{R}}_{\mu\nu} = \overline{\partial}_\rho \overline{\mathcal{C}}^\rho_{\nu\mu} - \overline{\partial}_\nu \overline{\mathcal{C}}^\rho_{\rho\mu} + \overline{\mathcal{C}}^\rho_{\rho\kappa} \overline{\mathcal{C}}^\kappa_{\nu\mu} - \overline{\mathcal{C}}^\rho_{\nu\kappa} \overline{\mathcal{C}}^\kappa_{\rho\mu} \tag{2.18}$$

Important part is Riemann tensor in fully covariant form that is just mixed form with metric tensor:

$$\mathcal{R}_{\phi\mu\sigma\nu} = \mathcal{G}_{\phi\rho} \partial_\sigma \mathcal{C}^\rho_{\nu\mu} - \mathcal{G}_{\phi\rho} \partial_\nu \mathcal{C}^\rho_{\sigma\mu} + \mathcal{G}_{\phi\rho} \mathcal{C}^\rho_{\sigma\kappa} \mathcal{C}^\kappa_{\nu\mu} - \mathcal{G}_{\phi\rho} \mathcal{C}^\rho_{\nu\kappa} \mathcal{C}^\kappa_{\sigma\mu} \tag{2.19}$$

$$\overline{\mathcal{R}}_{\phi\mu\sigma\nu} = \overline{\mathcal{G}}_{\phi\rho} \overline{\partial}_\sigma \overline{\mathcal{C}}^\rho_{\nu\mu} - \overline{\mathcal{G}}_{\phi\rho} \overline{\partial}_\nu \overline{\mathcal{C}}^\rho_{\sigma\mu} + \overline{\mathcal{G}}_{\phi\rho} \overline{\mathcal{C}}^\rho_{\sigma\kappa} \overline{\mathcal{C}}^\kappa_{\nu\mu} - \overline{\mathcal{G}}_{\phi\rho} \overline{\mathcal{C}}^\rho_{\nu\kappa} \overline{\mathcal{C}}^\kappa_{\sigma\mu} \tag{2.20}$$

Now I have all formal things can move to complex field equation.

2.4 Complex field equation

I will start by writing complex field equation in two possible ways:

$$\mathcal{R}_{\phi\mu\sigma\nu} - \frac{1}{2}\mathcal{R}_{\phi\sigma}\mathcal{G}_{\mu\nu} = \kappa\mathcal{T}_{\phi\mu}\mathcal{G}_{\sigma\nu} \tag{2.21}$$

$$\overline{\mathcal{R}}_{\phi\mu\sigma\nu} - \frac{1}{2}\overline{\mathcal{R}}_{\phi\sigma}\overline{\mathcal{G}}_{\mu\nu} = \kappa\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{G}}_{\sigma\nu} \tag{2.22}$$

Now from this two field equations I can create two scalars, from fact that both sides are equal I can use both complex extended Einstein tensor or complex energy momentum part with complex metric tensor:

$$\mathcal{E}_{\phi\mu\sigma\nu} = \mathcal{R}_{\phi\mu\sigma\nu} - \frac{1}{2}\mathcal{R}_{\phi\sigma}\mathcal{G}_{\mu\nu} \tag{2.23}$$

$$\Psi(\mathbf{z}) = \mathcal{E}_{\phi\mu\sigma\nu}\mathcal{E}^{\phi\mu\sigma\nu} \tag{2.24}$$

$$\overline{\mathcal{E}}_{\phi\mu\sigma\nu} = \overline{\mathcal{R}}_{\phi\mu\sigma\nu} - \frac{1}{2}\overline{\mathcal{R}}_{\phi\sigma}\overline{\mathcal{G}}_{\mu\nu} \tag{2.25}$$

$$\overline{\Psi}(\overline{\mathbf{z}}) = \overline{\mathcal{E}}_{\phi\mu\sigma\nu}\overline{\mathcal{E}}^{\phi\mu\sigma\nu} \tag{2.26}$$

Now for energy momentum tensor part:

$$\Psi(\mathbf{z}) = \kappa^2\mathcal{T}_{\phi\mu}\mathcal{G}_{\sigma\nu}\mathcal{T}^{\phi\mu}\mathcal{G}^{\sigma\nu} = 4\kappa^2\mathcal{T}_{\phi\mu}\mathcal{T}^{\phi\mu} \tag{2.27}$$

$$\overline{\Psi}(\overline{\mathbf{z}}) = \kappa^2\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{G}}_{\sigma\nu}\overline{\mathcal{T}}^{\phi\mu}\overline{\mathcal{G}}^{\sigma\nu} = 4\kappa^2\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{T}}^{\phi\mu} \tag{2.28}$$

That gives final field equation when I replace scalar field with energy momentum part and extended complex Einstein tensor:

$$\mathcal{E}_{\phi\mu\sigma\nu}\mathcal{E}^{\phi\mu\sigma\nu}\overline{\mathcal{E}}_{\phi\mu\sigma\nu}\overline{\mathcal{E}}^{\phi\mu\sigma\nu} = 16\kappa^4\mathcal{T}_{\phi\mu}\mathcal{T}^{\phi\mu}\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{T}}^{\phi\mu} \tag{2.29}$$

Where from fact of equality between both sides I can write that both are equal to complex scalar field with it's complex conjugate:

$$\Psi(\mathbf{z})\,\overline{\Psi}(\overline{\mathbf{z}}) = \mathcal{E}_{\phi\mu\sigma\nu}\mathcal{E}^{\phi\mu\sigma\nu}\overline{\mathcal{E}}_{\phi\mu\sigma\nu}\overline{\mathcal{E}}^{\phi\mu\sigma\nu} \tag{2.30}$$

$$\Psi(\mathbf{z})\,\overline{\Psi}(\overline{\mathbf{z}}) = 16\kappa^4\mathcal{T}_{\phi\mu}\mathcal{T}^{\phi\mu}\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{T}}^{\phi\mu} \tag{2.31}$$

That follows form all equations before.

2.5 Probability as a scalar of field

Probability of finding particle in some given location of spacetime is given by an integral of spacetime volume, where whole integral over manifold is equal to some constant that is needed to make that scalar field normalized. So I can define that scalar field has this property that integral over whole manifold is equal to some constant:

$$\int_{\mathbf{C}^4}\int_{\overline{\mathbf{C}}^4}\sqrt{|\det\mathcal{G}_{\mu\nu}|}\sqrt{|\det\overline{\mathcal{G}}_{\mu\nu}|}\Psi(\mathbf{z})\,\overline{\Psi}(\overline{\mathbf{z}})\,d^4\mathbf{z}d^4\overline{\mathbf{z}}=\eta^2 \tag{2.32}$$

Now I can rewrite this equation as normalized equation so it will be equal to one:

$$\frac{1}{\eta^2}\int_{\mathbf{C}^4}\int_{\overline{\mathbf{C}}^4}\sqrt{|\det\mathcal{G}_{\mu\nu}|}\sqrt{|\det\overline{\mathcal{G}}_{\mu\nu}|}\Psi(\mathbf{z})\,\overline{\Psi}(\overline{\mathbf{z}})\,d^4\mathbf{z}d^4\overline{\mathbf{z}}=1 \tag{2.33}$$

Where I did denote complex manifold $\mathbf{C}^4$, that means I take whole manifold as a spacetime volume. That leads to principle that any volume of spacetime that is subset of whole manifold will be always equal to less than one. So probability of finding particle in given spacetime volume $\mathbf{V}^4$ is equal to:

$$\frac{1}{\eta^2}\int_{\mathbf{V}^4}\int_{\overline{\mathbf{V}}^4}\sqrt{|\det\mathcal{G}_{\mu\nu}|}\sqrt{|\det\overline{\mathcal{G}}_{\mu\nu}|}\Psi(\mathbf{z})\,\overline{\Psi}(\overline{\mathbf{z}})\,d^4\mathbf{z}d^4\overline{\mathbf{z}}=\rho\left(\mathbf{V},\overline{\mathbf{V}}\right) \tag{2.34}$$

This equation can be written in two other ways where I do replace probability functions with scalar field equations parts, units of scalar field are meters to power minus fourth. Volume of spacetime will be written then in units of meters to power fourth so probability is a dimensionless number as it should be, meters cancel out. It's easy to see that its true by writing scalar field with energy momentum tensor part:

$$\frac{16\kappa^4}{\eta^2}\int_{\mathbf{C}^4}\int_{\overline{\mathbf{C}}^4}\sqrt{|\det\mathcal{G}_{\mu\nu}|}\sqrt{|\det\overline{\mathcal{G}}_{\mu\nu}|}\mathcal{T}_{\phi\mu}\mathcal{T}^{\phi\mu}\overline{\mathcal{T}}_{\phi\mu}\overline{\mathcal{T}}^{\phi\mu}d^4\mathbf{z}d^4\overline{\mathbf{z}}=\rho\left(\mathbf{V},\overline{\mathbf{V}}\right) \tag{2.35}$$

Metric tensor is dimensionless and energy momentum tensor times Einstein constant is in units of meters to power minus two, so if I have two metric tensors and two energy tensors units are meters to power minus forth. It is same for extended Einstein tensor part that normally Einstein tensor is written in units of meter to power minus two same with it's extension so combining two of them will give meter to power minus forth:

$$\frac{1}{\eta^2}\int_{\mathbf{C}^4}\int_{\overline{\mathbf{C}}^4}\sqrt{|\det\mathcal{G}_{\mu\nu}|}\sqrt{|\det\overline{\mathcal{G}}_{\mu\nu}|}\mathcal{E}_{\phi\mu\sigma\nu}\mathcal{E}^{\phi\mu\sigma\nu}\overline{\mathcal{E}}_{\phi\mu\sigma\nu}\overline{\mathcal{E}}^{\phi\mu\sigma\nu}d^4\mathbf{z}d^4\overline{\mathbf{z}}=\rho\left(\mathbf{V},\overline{\mathbf{V}}\right) \tag{2.36}$$

2.6 Solving the singularity problem?

In previous subsection need for probability to be normalized gives an important constrain on possible solutions to field equation. If any part of field equation will lead to infinities it can't be normalized so both extended Einstein tensor and energy momentum tensor part will have to give finite values. It means that there are no solutions with infinities when using correct solutions that are physical in sense of well defined probability.

Next important question is how measurement changes field, there is a way to calculate probability of finding particle at some region of spacetime, but what happens to that particle when measured? If particle will be localized only in that one point of spacetime it gravity field would be gone and need that energy momentum tensor never vanishes would be broken. So to conserve gravity field there is need to do something else. I need to shift whole manifold to position where I did find particle, let's say that have a particle that energy momentum tensor is starting at some point of spacetime $\mathbf{z}_0$ where it's a complex coordinate. Where that point is where energy is mostly present. If I do find particle at location in point $\mathbf{z}_1$ I need to move whole field to that point. It gives a big problem if energy momentum tensor does not have a focus point but when dealing with one particle need is that it always has a focus point , there is need to have it's rest mass in some volume of spacetime. Then that rest mass extends creating gravity field. So whole process can be written as writing a scalar field before measurement and after:

$$\Psi(\mathbf{z})\overline{\Psi}(\overline{\mathbf{z}}) \rightarrow \Psi(\mathbf{z}+\mathbf{z}_{10})\overline{\Psi}(\overline{\mathbf{z}}+\overline{\mathbf{z}}_{10}) \tag{2.37}$$

$$\rho\left(\mathbf{V},\overline{\mathbf{V}}\right) \rightarrow \rho\left(\mathbf{V}+\mathbf{z}_{10},\overline{\mathbf{V}}+\overline{\mathbf{z}}_{10}\right) \tag{2.38}$$

Where those two displacement complex vectors are just final position vector change from focus point of a field:

$$\mathbf{z}_{10} = \mathbf{z}_1 - \mathbf{z}_0 \tag{2.39}$$

$$\overline{\mathbf{z}}_{10} = \overline{\mathbf{z}}_1 - \overline{\mathbf{z}}_0 \tag{2.40}$$

So focus point is point where there is center of rest mass of particle or it's energy that are equivalent. So when measurement is done whole manifold with one particle shifts to another location in spacetime. This process should happen by instant when particle is measured. It means that scalar field itself has to stay invariant and do not change only it's position does change under measurement.

2.7 Rotation symmetry of a field

It's a complex spacetime so it has not normal rotation symmetry. First I will start by flat spacetime symmetry, and I will start by rotation symmetry of only space part of metric that will give just $SU(3)$ symmetry. This can be written as taking a space only vector and rotating it by matrix, If I take only space parts of metric tensor will get just Knocker delta so whole equation is that after rotation vector length is left unchanged that is definition of rotation:

$$\delta_{ab} dz^a dz^b = \delta_{cd} U_a^c dz^a U_b^d dz^b \tag{2.41}$$

This matrix is just $SU(3)$ matrix. When now I go to complex spacetime, still a flat one. Rule is same but for Minkowski metric in complex spacetime so it yields complex Lorentz transformation in four dimensional spacetime:

$$\eta_{\mu\nu} dz^\mu dz^\nu = \eta_{\alpha\beta} U_\mu^\alpha dz^\mu U_\nu^\beta dz^\nu \tag{2.42}$$

It is a rotation symmetry $SU(1,3)$ so base symmetry is that symmetry of a field. I can write last two equations for complex conjugate, those matrix has to obey this equation for both of them:

$$\delta_{ab} d\overline{z}^a d\overline{z}^b = \delta_{cd} \overline{U}_a^c d\overline{z}^a \overline{U}_b^d d\overline{z}^b \tag{2.43}$$

$$\eta_{\mu\nu} d\overline{z}^\mu d\overline{z}^\nu = \eta_{\alpha\beta} \overline{U}_\mu^\alpha d\overline{z}^\mu \overline{U}_\nu^\beta d\overline{z}^\nu \tag{2.44}$$

2.8 Summary of quantum part

Extending field equations to complex spacetime gave a probability scalar field that is key component to understood this equation in quantum way. Another good result is that it gets rid of singularity problems by need of scalar field normalization. But opposition is that it's still a complex spacetime, if this idea is correct it would mean that we live in complex spacetime not a real one. Still physical interpretation is that complex spacetime has invariant property that is scalar field and it's a real scalar field and it connect world of quantum with real world as there is measurement done.

If field equations are complicated to solve in classical way, there are even more complicated to solve in quantum way, still all mathematical rules for this model to work was defined. Field itself has a base rotation symmetry that is $SU(1,3)$.

New part about physical implications of field is measurement process. Instead of collapsing scalar field and making particle localized in just one point of spacetime it makes field shift places to match particle rest energy or mass to a point of measurement. It means that field itself does not change as it is expected from invariant property it just changes it's localization.

Still model misses any solutions to field equations, so in that sense it's hard to say is this idea of quantazation of field correct. It's still possible that only classical field of equation is possible to solve there is no way to get rid of singularity by using complex spacetime. Then it would mean that model is only a classical model and can't be quantize. Otherwise this model if possible to solve could be solution to quantum gravity problem but it depends on classical part of field , on other hand if this method of quantization is correct it apply just to normal general relativity. So there are two question this model ask:

1. Is extension of Einstein equations a correct way of how gravity works in classical sense?

2. Is using complex spacetime and making it's a scalar field out of invariant property of field equation is a correct way to quantize gravity?

And those two questions are not answer by this work, but all reasoning and framework in mathematical sense is given.