Inhomogeneous distribution of the universe's matter density as a physical basis for MOND's acceleration a_0

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Abstract

One of the most effective theories for dark matter is Milgrom's Modified Newtonian Dynamics, where a modified law of gravity based on a fixed acceleration scale a_0 is postulated that provides a correct description of the gravitational fields in galaxies. However, the significance of a_0 is unknown, and the whole theory is generally viewed as a phenomenological description of the observations. Based on Newton's gravitational law as applied to a uniform continuous mass we posit a non-homogeneous distribution of mass at cosmological scales that would gives rise to a constant acceleration and agrees with MOND's a_0. The implications for MOND as a viable theory of dark matter and for the problem of dark energy are briefly discussed.

Modified Newtonian Dymanics (MOND) is a Newtonian-derived hypothetical model of gravity proposed 40 years ago by Mordehai Milgrom to explain the multiple gravitational anomalies observed in galaxies and galaxy clusters [1-3]. They are summarized and conventionally explained through the existence Dark Matter, an elusive new form of matter that interacts only gravitationally and is not included in the Standard Model of Particle Physics. While no such particles have yet been found, the search goes on and MOND usually plays a secondary role in the list of candidate explanations for dark matter. One of the reasons is that a_0, the distinctive feature of MOND, does not correspond to any physical entity, and –it is argued- was postulated solely as a means to obtain a gravitational law that fits the observations. It is sometimes dubbed a phenomenological explanation.

While a_0 agrees to within one order of magnitude with the acceleration calculated at the border regions of the observable universe from the simple Newtonian formula, and is also found to

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agree with the currently accepted values of Hubble's constant and with the square root of the cosmological constant Λ, in both cases multiplied by the speed of light *c*, no physical representation has yet been devised and most physicists would agree that it behaves as another constant of nature, whose role would be to relate fundamental gravitational phenomena in the low-acceleration regime.

The Newtonian ball model of gravity

A generally accepted assumption of all current astrophysical models is the Cosmological Principle, the idea that the universe at large scales is both homogeneous and isotropic. While it may still be isotropic and strong constraints have been set on the range of variation in matter density, the homogeneity condition has little theoretical supporting evidence. Based on a recent toy model of Newtonian gravity [4, 5] that postulates that spacetime is a dynamical network of nodes joined by constantly changing and reformatting virtual force-vectors that can elongate and reorient in the presence of mass, and following original ideas due to Isaac Newton, we shall argue that the universe can be modelled as a nearly homogeneous continuous distribution of mass that obeys simple dynamics embodied in the Universal Law of Gravitation. As Newton amazingly found in the late 1600s [6], when a continuous distribution of mass with constant density is allowed to evolve according to such law, an acceleration appears that is null at the center and increases outwards in linear proportion to radial distance until it reaches, for a distance equal to the radius of the ball, the exact same value as predicted by conventional Newtonian gravity.

$$F_B = G\,M\,m\,r / R^3$$

as opposed to a point-mass gravitational field:

$$F_N = G\,M\,m / R^2$$

where F_B (force in the Newtonian ball model) and F_N (Newton's conventional point-mass gravitational force) are the force on a test particle with mass m placed at a distance r from the center of the R-ball, or at a distance R from the central point-mass M, respectively. The acceleration for the ball with mass M is then

$$Acc_B = G\,M\,r / R^3$$

and solving for G

$$G = Acc_B\,R^3 / M\,r$$

We now define G' as $4\pi G$ and substitute it for G above, following the ideas of the previously mentioned toy model [5]. The resulting expression is mathematically equivalent, though it may facilitate the visualization of upcoming considerations.

$G' = (Acc_B\, 4\pi\, R^3) / (M\, r)$ $\qquad [G' := 4\pi G]$

And multiplying both parts of the right-hand quotient by a factor of three,

$$G' = 3\ Acc_B\ 4/3\ \pi\ R^3 / M\ r$$

and since $4/3\ \pi\ R^3 / M$ is the inverse of the matter density for the spherical volume,

$$G' = 3\ (Acc_B / r) \cdot (1/\rho)$$

$$G' = 3\ Acc_B / r \cdot \rho \qquad (1)$$

where ρ is now the average, not necessarily constant matter density of the universe.

Looking at equation (1) we see that in such a ball model of the universe, if ρ is constant, then the quotient (Acc_B / r) must be constant, which agrees with the Newtonian view but does not help us understand the existence of a constant acceleration pervading the whole universe that at the same time agrees with the Newtonian acceleration at its border regions, as MOND postulates and available evidence strongly suggests.

We therefore let ρ vary with radial distance, however small the constant of proportionality may be, and assume that it is the product in the denominator of Equation (1) ($r \cdot \rho$) that is constant. In other words, we let density to decay as the inverse of radial distance. We immediately see then that since both G' and the product ($r\cdot\rho$) are constant, so must be Acc_B, and this acceleration agrees with MOND's universal acceleration a_0 and with the calculated Newtonian acceleration at the border regions of the ball to whithin one order of magnitude, as can be easily checked. Indeed, feeding in the accepted values for the mass of the observable universe (10^{53} Kg), radial distance (10^{26} m) and G, it turns out that the acceleration perceived at the border regions of the observable universe is about $3.4 \cdot 10^{-10}\ m\cdot s^{-2}$, quite close to the reported value for a_0 ($1.2\cdot10^{-10}$). And according to the Newtonian ball model, assuming $r\cdot\rho$ constant, this same aceleration would be present as a background curvature in the whole universe.

Since both G and G' are approximately of the order of Acc_B –both lay around 10^{-10} in MKS units-- density ρ decays as 1/r with a constant of proportionaly of the order of 10^{-26}, the currently accepted value for the average density of the universe, in Kg/m^3. (We can neglect here the difference between G and G').

Observational evidence for the distribution of mass density in the universe is scant. The large-scale average density of the universe, known as the cosmic density parameter, Ω, depends on its composition and, according to the ΛCDM model, is very close to the critical mass density Ω_c, the one required to make the universe flat. The density of normal, baryonic matter would amount to about 28% of the global density ($\Omega_m = 0.28$).

As for its distribution as a function of radial distance, it is generally assumed that the average density of matter would follow the general trend of decreasing as the radius increases, reflecting the overall dilution of matter on larger scales, but what is observed is a complex hierarchichal structure, the so-called cosmic web, that hinders precise measurements.
Several authors [7 - 11] notably Peebles, Karanchentsev, Nuza and others have probed into the mass distribution in the vicinity of our Milky Way and found that, on average, its density is significantly lower than the average for the whole universe. We would thus be in a local region of low density, the Local Void, which makes the observations not representative of the universe. The interpretation of the results is also compounded by the influence of dark matter and structure formation, two processes of which we know little.
In two important studies [7, 8] the authors examined the distribution of the mean density of matter in spheres of various radii around our Galaxy –the so-called Local Universe- and found that matter density up to about 50 Mpc seems to decay with distance. The authors conclude that density is on average lower that the global density for the universe ($\Omega_{m,local} = 0.08$ vs $\Omega_m = 0.28$) and tends to an asymptotic minimum value. However, looking at the data in the figures, we speculate that they might also be consistent with a 1/r decay law in that range. For larger distances up to 90 Mpc, uncertainties are too large to draw any conclusions.

Another interesting observation is the striking ressemblance of equation (1) with the Friedman equation. For flat space (k = 0), the Friedman equation can be expressed as

$G' = 4\pi G = 3/2 \cdot H^2 / \rho$ which certainly reminds us of Eq 1:

$G' = 3\, Acc_B / r \cdot \rho$ and since dimensions of Accel / r equals $1/T^2$ we have

$G' = 3 \cdot (1 / t)^2 \cdot 1 / \rho$,

If we then interpret 1/t as the constant rate of expansion H,

$G' = 3H^2 / \rho$

which differs from the Friedman equation only by a factor of 2. The reason for the discrepancy we ignore, but it has happened in other realms of physics when a classical, non-relativistic approach has been later superseded by the appropriate relativistic version, e.g., the old estimation of the bending of light by gravity before Einstein differed from the relativistic solution by a factor of two.

Thus the hypothesis of a decreasing matter density seems a reasonable one and, from old standard Newtonian mechanics it would lead to a constant background cosmic acceleration that agrees with MOND's a_0 and might explain rotation curves in galaxies without modifying the laws of gravity nor General Relativity. The observed accelerations below a certain threshold turn out to agree with MOND and are the geometric average of the Newtonian and the background a_0. This would be a real physical phenomenon caused by the interaction of two competing accelerations, not only a mathematical construct.
Some other shortcomings of MOND might also be solved this way. For instance, ideas on the

correct application of MOND to galaxy clusters through an averaging of the gravitational fields due to the nearby galaxies with those generated by the cluster itself have been proposed [* Our paper Fluid Spacetime]. No explanation so far can account the observations in colliding clusters like the Bullet.

We therefore conclude that

1. In a modified Newtonian ball model of the universe, a continuously decreasing matter density that scales as 1/r, as opposed to the uniform distribution from the Cosmological Principle, would give rise to a constant universal physical acceleration that agrees with MOND's a_0.

2. This could provide a physical basis for MOND and support it as a viable interpretation of the dark matter problem.

3. The resulting matter density distribution may be hard to verify experimentally, for the densities involved, as well as the variations incurred might be extremely low.

Cosmological acceleration as a basis for the universe's expansion

We now turn our attention to the mysterious empirical relation observed between a_0 and the parameters that reflect the universe's expansion, H_0 and Λ.

Indeed, the numerical value of MOND's a_0 has been found to be approximately

$$a_0 \sim (c / 2\pi) \cdot H_0 \sim (c^2 / 2\pi) \cdot \mathrm{SQRT}(\Lambda/3)$$

Why is that? What is the intimate relation of a_0 to the accelerated expansion of the universe?

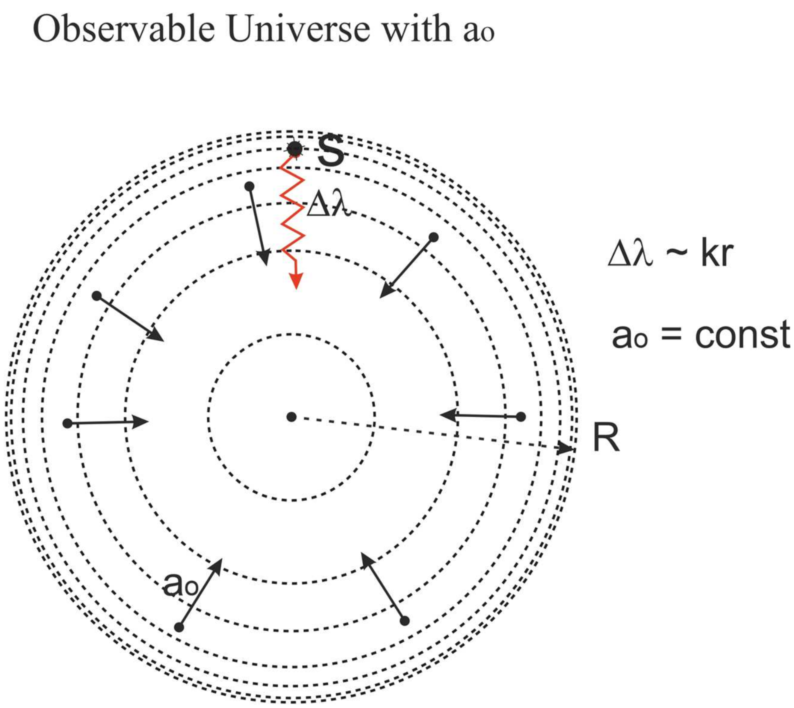

Fig 1. A non-homogeneous universe with matter density that decays as 1/r generates a constant acceleration a_0 that gives rise to a redshift proportional to radial distance. No expansion is needed to explain redshift. Varying intervals of space layers representing acceleration are greatly exaggerated.

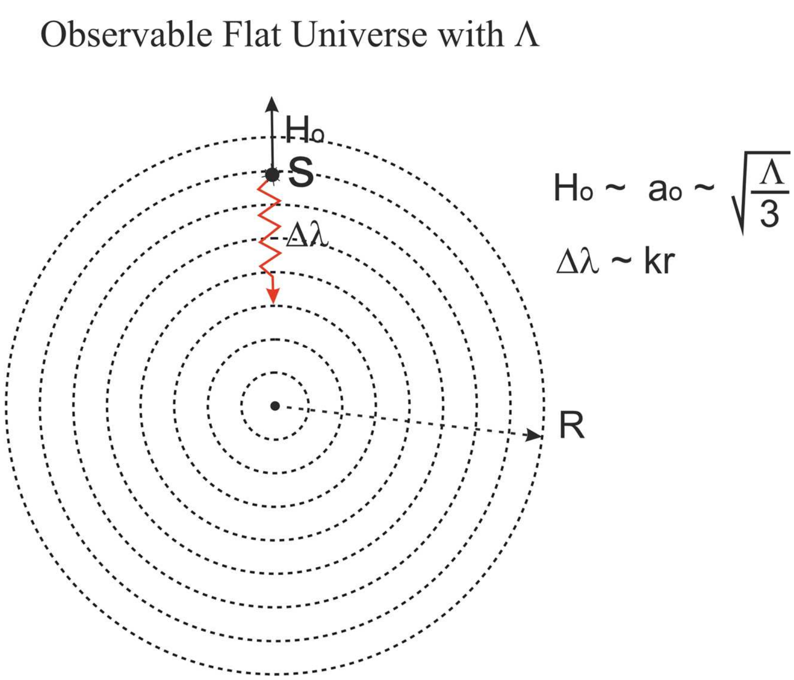

Fig 2. Current models of the universe postulate a constant acceleration for all unbound stellar bodies that agrees with a_0, H, and Λ. Recessional velocities and redshift that increase with distance indicate that an expansion is taking place.

Let's take a look at the modified Newtonian ball model of gravity as applied to the whole universe. The postulated real universe (Fig 1) is made of a spacetime network (*Ref Toy model) with a constant acceleration (a_0), so that the separation between neighbouring nodes and layers of the network decays linearly with radial distance, i.e., there is a constant gradient of the deformation in the radial direction that corresponds to the constant acceleration a_0. In our curent cosmological models in contrast, flat space is assumed (Fig 2) and the intervals between neighboring nodes and layers of the network are constant, there is no definite acceleration in space, or it is very small and negligible, but we then observe that faraway galaxies seem to recede with a constant acceleration that turns out to be the same or very close to the previously mentioned a_0 (Fig 2). From Hubble's Law and previous observations on the redshift of moving stellar bodies, recessional velocity was found to increase linearly with distance. Furthermore, the speed seems to be accelerating since at least several billion years, and since all observed galaxies are doing approximately the same and receding from us, the natural conclusion is that the universe is expanding, and it is doing so actually at an accelerated rate.

But from general Relativity's Equivalence Principle (EP), the situation with flat universe and cosmic expansion (Fig 2) is completely indistinguishable from a static universe with a constant gravitational field pervading it (Fig 1). All gravity-related phenomena –including redshift- must be present equally in both situations. A decission as to which is happening must come from external observations or reasoning. For any observer bound to a galaxy and placed a large distance away from us, it would be impossible to tell which one of these is happening:

1. The galaxy is accelerating away from the center of the universe by a_0 in flat space. (Fig 2). And since all neighboring galaxies are reporting the same, with velocities that increase with distance at a constant rate, the universe must be expanding all around them. Or else,

2. The galaxy is rotationally bound but static, immersed in a gravitational field with intensity a_0 (Fig 1). And since all other galaxies seem to experience exactly the same, the conclusion would be that a constant acceleration due to gravity is pervading the whole universe.

It is worth noting that redshift would not allow them to tell which is true, even if they were informed of the redshift detected from far away buddy observers. Or course, if the observer was smart enough, she would reason that the first alternative would be weird and very costly, so she might conclude that all the gravitational phenomena she is experiencing, including redshift, are due to a curvature of spacetime from a constant gravitational field a_0.

This would explain why a_0 scales so precisely with the Hubble parameter H_0 and the cosmological constant Λ. They are but two ways to interpret the same phenomenon. Which one do we choose depends on external reasoning and observation. If there are other clues suggesting that we are in a curved space or we lack any ideas on what is the origin of the energy that drives the expansion, we might conclude that the accelerated outward motion is apparent and the real thing is a constant gravitational field that generates the redshift proportional to radial distance.

We notice that this does not invalidate redshift as an accurate indicator of velocity for stellar bodies in general. A redshift that scales with linearly with velocity is recorded that generally will not be affected by the universal background acceleration. Only when very large distances and cosmological scales are involved will the discrepancy be noticeable. We recall that for short distances, spacetime is approximately flat in the universe, the cosmological acceleration is very weak and thus the discrepancies are negligible when redshift is used to measure velocities at galactic, sub-cosmological scales.

Discussion and Q&A

The first comment that comes to mind is how plausible a non-uniform distribution of matter is, given the fundamental character of homogeneity, as well as isotropy, in modern cosmology. The answer is that we don't know, and it is not easy to either verify or rule it out. Even assuming that inhomogeneities in the mass distribution are constrained by some observations, including the CMB and the wide field observations of distant galaxies, small inhomogeneities that are still larger that the ones needed here cannot be presently ruled out by observations. As for the theoretical arguments, given that we can only record with certainty a minute fraction of the matter assumed as present in the universe, this highlights our limitations to determine theoretical estimates and boundaries for the mass density distribution.

On the other hand, as mentioned before, a central concentration of mass density would intuitively make sense when one considers masses governed only by gravity. Some authors have reported decreasing densities with distance in the local universe, although these are not representive of the whole. Lastly, an agreement with observations without the need for dark matter or dark energy might count as supporting evidence. But a definitive answer must come from direct observations, and we have seen how such small accelerations and densitiies represent a challenge for our current technology.

-Assuming that mass density varies with radial distance in a model that was meant and tuned to describe uniformly distributed masses, does it not disprove the argument ?

The Newtonian model for gravity in solid spheres is valid not only for spheres with uniform density, but for any sphere in which density depends only on radial distance, i.e, for any spherically symmetrical distribution of matter.

-How well does the model support MOND as an effective theory for dark matter ?

MOND has been considered by most authors either as a modification of the laws of gravity awaiting proper justification, or as a mere mathematical description based on the introduction of a new free variable, a_0 to fit the observations. We claim that it is neither.
In so far as the main drawback of MOND has been its speculative nature and the arbitrary splitting of the gravitational law in two domains, corresponding to accelerations higher and lower than a_0, the view that a_0 is a real acceleration based on a plausible distribution of matter and the original Newtonian laws of gravity makes it much more plausible. It would explain not only the value of a_0, but also the fact that MOND kicks in at a definite threshold acceleration. MOND asserts that the real observed acceleration in MOND regime is the geometric average of the calculated one and a_0, and this can now be understood as the influence of a constant, backgrond acceleration playing its role only when the gravitational field from the mass is comparable or lower than a_0.

-How does this model affect other parts of the current ΛCDM cosmological framework such as dark energy and the Big Bang?

It is currently difficult to forsee the impact that a confirmation of the physical nature of a_0 and a non-homogeneous distribution of matter would bring about. As shown above, dark energy would certainly be one of the prime targets –or beneficiaries. Though still in the minority, many physicists have stated the suspicion that dark matter and dark energy might be related, but no consistent hypothesis has so far been provided. The idea that the observed expansion of the universe is an apparent phenomenon due to a constant cosmological acceleration seems a disturbing hypothesis. And yet, most of the observations would remain valid. It would mean just a change in perspective, a more effective reference frame that ultimaltely simplifies our models. Perhaps it would also make Einstein happy, for he could finally do away with the cosmological constant, that dreaded parameter that he called the worst blunder of his life.

-Why don't astrophysical observations support the view that there is a center of the universe? Why is redshift approximately the same in all directions?

Unless we are very near the actual center of the universe –which seems unlikely though not impossible- there should be ways to tell where the center of teh universe is, or at least in what direction it lies. Distances to faraway galaxies are hard to assess. Our measurements rely on our current models for an expanding universe and are given in 'h^{-1}·Mpc' units, which makes it hard to agree on what do they mean in a static universe. But worse is that uncertainties grow with larger distances, and current technology has limited power to resolve and analyze distances at cosmic scales. On the other hand, redshift should provide some clues as to where is the center or in what direction it lies. Unfortunately, light reaching us from the center regions of the univers is expected to be also redshifted, just as when it comes from the periphery. The same reasoning applies as above. When looking towards the center of the univers, a constant gravitational field is present, with us in the high potential side of it. The situation is equivalent to us moving away from the center, which in turn is equivalent as the observed stars in the center moving away from us. Therefore, the light received from stationary galaxies in the central region would be also redshifted.
However, when looking in the centripetal direction, the number of galaxies and teh mass density as a function of redshift (assuming we look at distances large enough to avoid local variations) the pattern of redshit should be one of increasing number of galaxies and mass density as we approach to the center, followed by a steep decrease when loking through into the other side from the center. Conversely, when looking at the galaxies in the periphery away from the center,

the number of galaxies and matter density should always decrease, as does the average density at larger radii. The precision needed is nevertheles a serious challenge, for the variations in matter density at large scales are expected to be extremely small, very likely of the order of 10^{-26} as mentioned before. But the possibility should still be there.

Lastly, it can be argued that this discussion is limited to the non-relativistic case, where time is absolute and space is likewise treated quite naively. The observation would be pertinent, but we are not all that sure that absolute spacetime is not a real feature of the universe. Special and General Relativity are the only correct description of the universe we live in, but this might be only as long as we do not take actual deformations of spacetime into account. Einstein himself warned us in his famous Leiden address of 1920 [*] that spacetime might in the end turn out to be a real entity, albeit possibly an unmeasurable one. And perhaps by real, in this context, he meant absolute. At any rate, the present discussion seeks to adress very real problems in the astrophysical realm that for the most part occur at sub-relativistic speeds. The approach is classical, intuitive and Newtonian because this is the only way our imagination can be put to work, but we expect that a precise formulation of the present ideas could be adapted to the postulates of Relativity, as Einstein seemed willing to accept.

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