A New Model Suggesting a Mass Difference Between Electron and Positron at 10 ppb

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Abstract

Background/Objectives: The primary objective is to investigate a new theoretical model approach about fundamental particles. Especially the electron and positron is considered. The model utilizing the concept of energy density limits and find an acceptable interpretation of a speed of light reference frame. Due to it's consistent nature this enable us to implement these limits without breaking Lorentz's invariance. This new model employs mass-less current loops at the speed of light, to construct a candidate for a stable, self-contained system, which can be perceived as either an electron or positron, depending on its configuration.

Methods: This is a pure theoretical work where all figures was generated by LaTeX constructs to illustrate the concepts. However there are referenced measurement results that are important for the discussion. The mathematics is on a basic level, although the paper is dense with deductions and formulas. Only calculus and general mathematical maturity is needed as well as knowledge about special relativity, electromagnetism and some basic atom and particle physics.

Results: We evaluate the resultant angular momentum and derive a formula that aligns with Bohr's renowned assumption about angular momentum in his atomic model. This method not only provides insights into the enigmatic number 137 in physics but also suggests a potential discrepancy between the masses of the electron and positron, with a relative error of 10 ppm in the measurement. This difference is too subtle for existing measurement techniques.

Conclusions: The main result in this paper are a model that basis its approach using the electromagnetic theory and deduces stable constellations, that resambles particles, within this model. This theory does introduce the controversial prediction that the particle and antiparticle mass differ using a deduction of a formula for the mass. It is also quite possible as we quantize the difference, that this prediction can be clarified by forthcoming measurement projects. Also we deduced a couple of soundness feature of the model, such as deriving the Bohr's condition for angular momentum in his atomic model and explain how this can be used to deduce the actual measured angular momentum. Also the invariance of angular momentum and charge is proven as a result of the model.

1 Introduction

We will mainly use the theory of electromagnetism in this paper, [6], and special relativity [1]. With this tools and a novel idea of introducing limits on what values an energy density can have, we find out that the mathematical consequences match many of the current theoretical properties of the electron and positron. To obstacle to overcome in this model is manage object we normally do not see in this theoretical realm as it is very challenging to implement limits of this kind and still manage to keep important properties like Lorentz's invariance. The main idea is to try and define a reference system at the speed of light and explore the consequences of putting the limits there. This insight seam to open up the gate for being able to explain how we have particles in the first place as the models introduced are stable, finite and quantasized. Note the peculiar fact the we so not use quantum mechanics [4] in this paper. This may put severe questions about the content, but as argued, Bohr himself did not either use quantum mechanics in his model and there nothing that stops that theory to take up the ideas here and incorporate them.

2 Methods

2.1 The main model assumptions

We will ground our analysis in a fundamental object: a uniform stream of charge, devoid of mass, that travels at the speed of light. This object's distinguishing characteristic is its electrostatic interaction exclusively within a reference frame moving at light speed; a mathematical construct we will define below. Moreover, when tangent lines are drawn between such objects, they align at the closest distance, with the connecting line being perpendicular to both points (a property that remains invariant under Lorentz transformations). Specifically for this scenario, we'll invoke Coulomb's law for these distinctive segments. We'll also work under the implicit assumption that each particle is composed of these foundational objects oriented in every conceivable direction. While this isn't a widely accepted natural law, it's worth noting that intersections between two such lines in our model are non-interactive. This allows us to overlay an ensemble of these systems, forming a macroscopic structure with the intention of replicating the standard Coulomb interaction. We propose that each loop maintains a consistent charge amount, implying that as the loop enlarges, its charge density diminishes. In essence, we're describing a scale-able, closed system. Lastly, we'll assume the existence of an energy density limit in this crafted reference frame, with values that are nearly identical for each charge sign.

2.2 The reference frame at the speed of light

Given that the geometric condition prompting an interaction remains invariant under Lorentz transformations, we'll narrow our focus to two interacting points. For simplicity and without loss of generality, we'll assume both streams move linearly, sharing identical velocity and direction. We'll then arrange a sequence of charges in both systems to ensure perfect alignment. We can also presume that only a length contraction in the direction normal to the streams is relevant in the Lorentz transformation; any contraction in the stream direction becomes negligible as both systems approach light speed and become effectively stationary. Since every reference frame in the sequence adheres to Lorentz invariance, we can transform each frame to negate orthogonal length contraction caused by the observer's moving reference frame. This implies that even the limiting interaction respects Lorentz invariance. Practically, this means that for any constellation of these currents moving relative to the observer, one can adjust the geometry according to length contraction, disregard the velocity, and apply the resulting distances directly into Coulomb's law. The two charge densities would then be determined using the universal total charge condition of the current loop and its length.

2.3 The charge stream model details

Let's conceptualize the streams as the limit of a sequence of objects represented by:

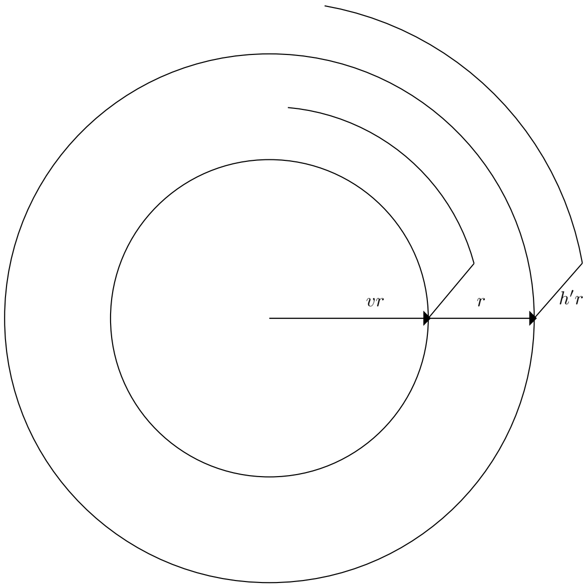
$$\lim_{n\to\infty, v\to c} \sum_{i=0}^{n} e_a \frac{\delta_{r_i^a(t)=\left(\frac{i}{n}+vt\right)\hat{x}}}{\sqrt{n}}$$

Here, we're distributing n mass-less electrons evenly on the interval [0,1] at $t = 0$ and propelling them at speed v along the x-axis. Let's also consider the corresponding parallel stream:

$$\lim_{n\to\infty, v\to c} \sum_{i=0}^{n} e_a \frac{\delta_{r_i^b(t)=\left(\frac{i}{n}+vt\right)\hat{x}+h\hat{y}}}{\sqrt{n}}$$

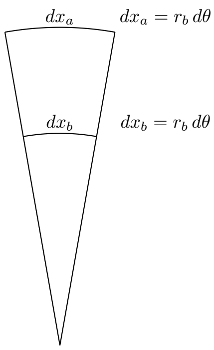
It's vital to note that every current loop possesses the same total charge, an assumption we regard as a universal invariant. As velocity rises, length contraction occurs. To achieve a consistent charge density in the observer's reference frame, the charges must be increasingly dispersed in it's sequence of frames. Therefore, any two parallel segments in this frame must perfectly correspond. In the same frame, adjacent charges are effectively an infinite distance apart, nullifying self-interaction. Charges not at their closest possible distance appear infinitely distant. Also, given two interacting streams, the likelihood they directly collide is essentially zero; we could argue that they synchronize to prevent interactions. Should this not hold true, we'd witness a discontinuous law. Thus, we'll consider only r_i^a and r_i^b as conventional electric charges, disregarding all others. We mandate that streams remain parallel, uniformly directed, and the interaction points positioned as closely as possible. For any geometric arrangement, one must seek parallel tangents that aren't offset—meaning the tangents drawn must be parallel, and the paired infinitesimal segments should be at their closest possible distance. Since interactions are considered in the rest reference system, one can focus solely on the electrostatic interaction for a particular geometric setup. This approach mirrors quantum electrodynamics, which also excludes internal magnetic terms in the standard Dirac equation [5] for entities like electrons in a hydrogen atom. Having defined energy in the rest frame, we can also set a limit for energy density—distinct for each type of charge.

2.4 First step, the loop.



Consider two concentric loops positioned one above the other, thereby forming a dual-cylindrical structure. In the inner loop, the charge is positive, while in the outer loop, the charge is negative (we will also study a reversed configuration later). We assume a constant charge density for these streams: e_a for the negative charge and e_b for the positive charge. We will not be strict in mathematical rigor; instead, we will consider all interaction terms as limits in the L_2 norm of their combinations. It's important to note that the configuration of two concentric circles maximizes the number of interaction points. This characteristic, when considering two loops of opposite signs constrained in a plane, is sufficient for our study.

Let $r_b = vr$ represent the inner radius and $r_a = r$ denote the outer radius. We will introduce a scaling such that the effective charge density in the outer cylinder satisfies a radian contribution constraint. Given that e_a represents the charge density at the outer radius and e_b denotes the charge density at the inner radius, the constraint is as follows:



Starting with the relationship between charge densities and radii, we have:

$$\begin{aligned} er_a d\theta &= (e_a r_a - e_b r_b) d\theta \\ &= (e_a - e_b v) r_a d\theta \\ &= (e_a - v e_b) r d\theta. \end{aligned} \tag{1}$$

From which it directly follows that:

$$e = e_a - v e_b. \tag{2}$$

Given our interest in scaling properties, it is natural to express e in terms of e_a as $e = ue_a$. Alternatively, for the dual setup, we have:

$$-e = be_a - e_b. \tag{3}$$

In this context, we'll use $e = ue_b$ as the scaling.

When combining two of these streams, the interaction can be thought of as the "square root" of a delta measure, which is clarified through the limiting argument discussed earlier. The relevant terms are:

$$e_a e_b r_a r_b d\theta, \quad e_a^2 r_a^2 d\theta, \quad e_b^2 r_b^2 d\theta. \tag{4}$$

For the subsequent energy relations, when referring to the "energy density," we are specifically considering its effects on paired segments r_i^a and r_i^b. Aggregating these effects over unit length yields:

$$e_a e_a, \quad e_a^2, \quad e_b^2. \tag{5}$$

To understand this more concretely, we will assume a normalized condition on the energy density for the singly scaled pairings.

In the next stage, we consider stacking multiple loops to form a torus with a radius R. It's worth noting that when two such tori are examined, they exhibit significant interactions only if situated in two parallel planes. At these intersections, the interaction is limited to a circle of radius R. When examining the torus as a surface along this axis, we scale the charge, while for the other configurations, we consider it as stacked circles where the effective charge on all those concentric loops is e. While this may seem non-intuitive, the intent of this model is not to offer an airtight theory, but to investigate its explanatory power.

Considering the aforementioned, the scale invariance of the charge condition ensures the resultant charge remains consistent. The attractive energy per radian of the loops, applying the modified Coulomb's law and referencing equation (4), is given by

$$V_l d\theta = k \frac{e_a e_b r_a r_b}{r_a - r_b} d\theta = k \frac{e_a e_b v r}{1 - v} d\theta. \tag{6}$$

It's important to note that the term $|1 - v|$ has a scaling relative to $2\pi R$, though this is glossed over in the analysis that follows.

When we stack loops directly atop one another with a pitch $\bar{h} = h'r$, the forces acting on one segment in a given direction are

$$F = k \frac{e_*^2 r_*}{(hr)^2} (1 + 2^{-2} + 3^{-2} + \ldots) = \zeta(2) \frac{k e_*^2}{(h'r)^2}. \tag{7}$$

This is a simplification; the behavior changes if we form a torus or helix. We'll therefore assume the aforementioned force transforms as ζ_h / h', leaving the precise form of ζ_h for later consideration. When calculating the force on both sides, the energy is found by integrating over $h'r$ as

$$V_{h,*} d\theta = 2\zeta_h \frac{k e_*^2 r_*^2}{h'r} d\theta. \tag{8}$$

Consequently, the total energy for one loop is

$$E = (V_{h,1} + V_{h,2} + 2V_l)2\pi = 2\pi kr\Big(\frac{2\zeta_h}{h'}(e_a^2 + e_b^2 v^2) - 2\frac{e_a e_b v}{|1-v|}\Big). \tag{9}$$

Utilizing the relation $e = e_a - ve_b$ in the expression for energy, we aim to find the stationary point by varying $x = ve_b$ and holding other parameters constant. Introducing constants A and B into the equation, we derive:

$$A(2e + 4x) - B(e + 2x) = 0. \tag{10}$$

This can be simplified to:

$$(2A - B)(2x + e) = 0 \implies x = ve_b = -\frac{e}{2}. \tag{11}$$

This implies that ve_v tends towards zero unless:

$$2A - B = 0 \Leftrightarrow \frac{2\zeta_h}{h'} = \frac{1}{|1-v|}. \tag{12}$$

Under this condition, ve_v can vary freely in terms of energy. To streamline our representation, let's use $e_b = ue_a$ and define $w = uv$. Taking into account $e = e_a(1-w)$, our energy expression becomes:

$$E = 2\pi kre_a^2\Big(\frac{2\zeta_h}{h'}(1 + w^2) - 2\frac{w}{|1-v|}\Big). \tag{13}$$

By integrating the condition (12), we get:

$$E = 2\pi kre_a^2\frac{2\zeta_h}{h'}(1 + w^2 - 2w). \tag{14}$$

Completing the square, we obtain:

$$E = 2\pi kr\frac{2\zeta(2)}{h'}(e_a(1-w))^2 = 4\pi kr\frac{\zeta(2)}{h'}e^2. \tag{15}$$

Note that this condition is invariant of how we combine the charges. To evaluate the energy density and apply limits on them, the system aims to scale down to minimize energy. Assuming condition (5) for evaluating this limit, the charge densities at loop a are:

$$\rho_a = ke_a^2\Big(\frac{2\zeta_h}{h'} - \frac{u}{1-v}\Big)\frac{1}{r}. \tag{16}$$

Utilizing (12), we have:

$$\rho_a = ke_a^2\frac{2\zeta_h}{h'}(1-u)\frac{1}{r}. \tag{17}$$

For loop b, the density is:

$$\rho_b = ke_a^2\Big(\frac{2\zeta_h}{h'}u^2 - \frac{u}{1-v}\Big)\frac{1}{r}. \tag{18}$$

Again, using (12), we derive:

$$\rho_b = ke_a^2 \frac{2\zeta_h}{h'}(u^2 - u)\frac{1}{r}. \tag{19}$$

To ensure that these two densities are at a positive and negative limit, using (17) and (19), we must have:

$$\rho_a = c_a, \tag{20}$$

$$\rho_b = -c_b. \tag{21}$$

To simplify this analysis, apply (12) and introduce:

$$C_* = c_* * C = c_* \frac{h'}{2\zeta_h ke_a^2} = c_* \frac{|1-v|}{ke_a^2}. \tag{22}$$

This leads to:

$$\frac{|1-u|}{r} = C_a = c_a C, \tag{23}$$

$$u\frac{|1-u|}{r} = C_b = c_b C. \tag{24}$$

It's noteworthy that this result remains unchanged regardless of how the charges combine to form e. Dividing (24) by (23), we get:

$$\frac{e_b}{e_a} = u = \frac{C_b}{C_a} = \frac{c_b}{c_a}. \tag{25}$$

Applying constraint (23), we obtain:

$$\frac{|1-u|}{r} = c_a C = c_a \frac{|1-v|}{ke_a^2}. \tag{26}$$

Reformulating, we derive:

$$|1-v| = \frac{ke_a^2|1-u|}{rc_a}. \tag{27}$$

In the dual we consider here, we simply take the anti-particle of the system. We denote this with $\mathcal{D}(\cdot)$. We find $\mathcal{D}(u) = \frac{c_a}{c_b}$, $\mathcal{D}(e_a) = -e_b$, and in this context:

$$\begin{aligned} |1-\mathcal{D}(v)| &= \frac{ke_b^2|1-\mathcal{D}(u)|}{\mathcal{D}(r)c_b} \\ &= \frac{ke_b e_a(1-u)}{\mathcal{D}(r)c_b} \\ &= \frac{ke_a^2(1-u)}{\mathcal{D}(r)c_a} \\ &= |1-v|\frac{r}{\mathcal{D}(r)}. \end{aligned} \tag{28}$$

Utilizing (27), we can express the condition for e as:

$$e = e_a(1 - uv) = e_a(1 - u) + e_a u(1 - v) = \Delta \left(1 + \frac{ke_b e_a}{rc_a}\right), \tag{29}$$

where we define $\Delta = e_a - e_b$.
The dual expression is:

$$\mathcal{D}(e) = -\Delta \left(1 + \frac{ke_b e_a}{\mathcal{D}(r)c_b}\right). \tag{30}$$

For $e = -e'$, it is necessary that:

$$\mathcal{D}(r) = \frac{r}{u}. \tag{31}$$

Therefore,

$$(1 - \mathcal{D}(v)) = -(1 - v)u, \tag{32}$$

and $\mathcal{D}(h') = h'u$. If we solve for r in (29), we get:

$$r = \frac{ke_b e_a}{c_a \left(\frac{e}{\Delta} - 1\right)}. \tag{33}$$

Starting with,

$$e = e_a|1 - u| + e_a u|1 - v|, \tag{34}$$

and using (23), we find:

$$e = D\frac{rc_a}{e_a} + e_a u|1 - v|, \tag{35}$$

where $D = \frac{h'}{2\zeta_h k}$.
Assuming h', v, and u to be constant, by minimizing the energy with respect to e, we find:

$$e_a = \sqrt{\frac{Drc_a}{u|1 - v|}} = \sqrt{\frac{\frac{h'}{2\zeta_h} rc_a}{ku|1 - v|}} = \sqrt{\frac{rc_a}{ku}}. \tag{36}$$

From (23), we have:

$$\frac{rc_a}{ku}|1 - u| = rc_a\frac{|1 - v|}{k}. \tag{37}$$

Rearranging, we get:

$$|1 - u| = u|1 - v|. \tag{38}$$

Thus,

$$e = 2e_a|1 - u| = 2\sqrt{\frac{rc_a}{ku}}|1 - u|. \tag{39}$$

Moreover,

$$e = 2\frac{e_a}{c_a}|c_a - c_b|. \tag{40}$$

The constraint (12) implies:

$$h' = |1 - v|2\zeta_h = \frac{|1 - u|2\zeta_h}{u}. \tag{41}$$

Furthermore, $\mathcal{D}(h') = h'u$. Given that the pitch $h'r$ remains invariant, the justification for having equal charge would be that the most energetically favorable configuration arises when a negative and positive charge align, leading to an equal pitch. Consequently, the negative and positive charge must be identical, and as we shall observe, this further implies that $\hbar$ remains consistent. Using equation (41) and squaring equation (39), we get:

$$e^2 = 4\frac{rc_a}{ku}|1 - u|^2. \tag{42}$$

The principle of special relativity allows us to deduce the masses per loop from (15) as:

$$\begin{aligned} E &= mc^2 \\ &= 4\pi kr\frac{\zeta_h}{h'}e^2 \\ &= 2\pi\frac{kr}{|1 - v|}e^2. \end{aligned}$$

Integrating (38), we derive:

$$mc^2 = \frac{2\pi krue^2}{|1 - u|}.$$

Thus, the equation for mass is:

$$m = \eta\frac{2\pi krue^2}{|1 - u|c^2}, \tag{43}$$

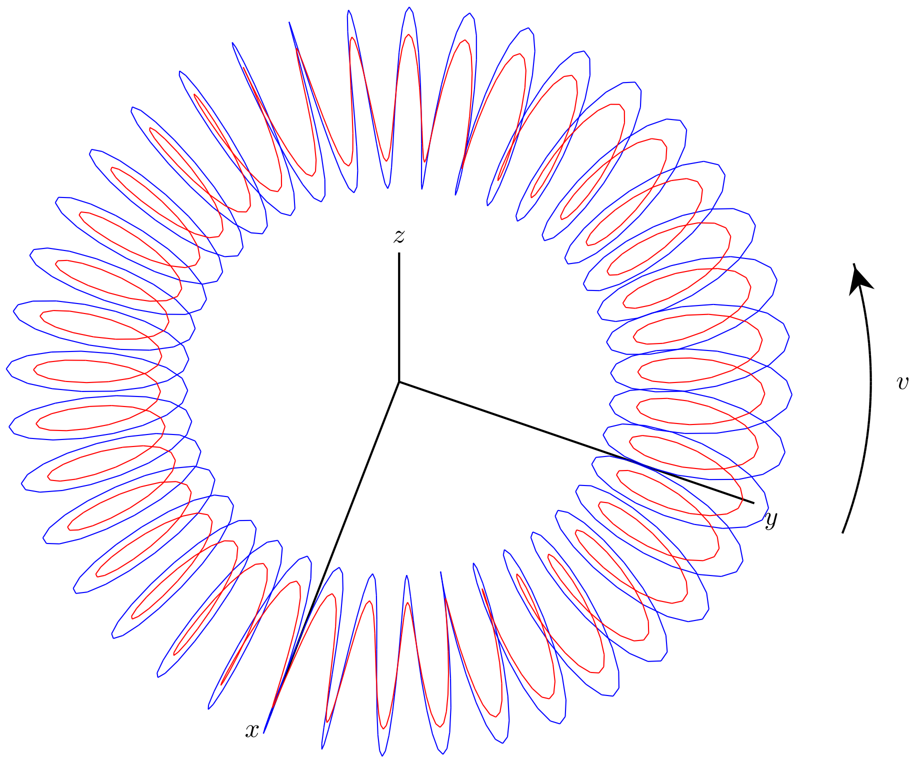
and its dual relationship is:

$$\mathcal{D}(m) = \frac{m}{u^2}. \tag{44}$$

It's worth noting that the unit here is kg/m, with η being an unknown unit. However, considering the loop as a delta measure, one can interpret it as the outcome of taking a limiting value of a scaled mass within a thin cylindrical shell. Therefore, we can set:

$$\eta = 1 \quad [m].$$

Such a clarification ensures that a discerning reader won't encounter any confusion related to units in their calculations. Hence, m will have the unit $[kg]$.



2.5 Stacking into a torus

Initially, our system comprised of loops stacked to create a cylindrical structure. Now, we shall modify this arrangement to connect all loops, forming a torus. For this configuration, the pitch is defined by:

$$h'r = h2\pi Rr,$$

which leads to:

$$h' = 2\pi Rh.$$

From our previous discussion, the dual condition is given by $R' = R$. While it's possible to stack the loops mathematically, creating a stable structure this way is challenging. A more feasible approach would be to transform the loops into helical paths along the helix, each moving with a velocity v. This configuration ensures stability. To see this we need to prove that the attractive optimum is having the two helices in it is enough to realize that any small deformation reduces the number of interaction points and hence the attractive part of the energy is reduced. Assuming the different charge loop are very close this essentially fix the possible paths into this geometrical constellation.

When considering the system's reference frame (i.e., moving with the particles along the main circular path), the structure still forms loops. Each helix repels another similar helix situated one pitch away. Given that the number of pitches is consistent (denoted by $h'r$), there are two distinct radii of the torus: R in the laboratory's system and R_0 in the rest frame. Their relationship is given by:

$$R = \frac{R_0}{\gamma}. \tag{45}$$

We'll analyze the interactions in the rest frame. Stacking n of these helical paths, we obtain:

$$nh'r = 2\pi R. \tag{46}$$

From which, the relation is:

$$h'r = \frac{1}{n}. \tag{47}$$

Given the variable distances between paths, the above cannot be strictly true due to a contraction in the closer $R - r$ distance. Thus, the correct relationship is:

$$nh'r = 2\pi(R - r) = \frac{2\pi R}{f}, \tag{48}$$

where the factor f is defined as:

$$f = \frac{1}{1 - \frac{r}{R}}. \tag{49}$$

This leads us to:

$$hrf = \frac{1}{n}. \tag{50}$$

The energy due to attraction remains unchanged, as it's independent of the loops' orientation. However, repulsion acts only at two points where the loops interact. This energy can be represented as an average, equivalent to using the center distance R. However, we must assess the energy density at the $R - r$ distance, where it is most intense. This is achieved by the transformation:

$$c_* \to \frac{c_*}{f}. \tag{51}$$

In this context, the unit of h is given by $[1/m]$.

2.6 Scaling

Consider the scaling properties of the system. Given that the number of loops per torus, denoted by n', remains fixed, only the loops will undergo scaling. Using relations from Equations 27, 33, and 15, we can derive the following scaling relationships:

$$e \to xe, \tag{52}$$
$$r \to \frac{r}{x}, \tag{53}$$
$$v \to v, \tag{54}$$
$$u \to u, \tag{55}$$
$$h \to hx, \tag{56}$$
$$rh \to rh, \tag{57}$$
$$E \to xE, \tag{58}$$
$$m \to xm. \tag{59}$$

To maintain the same scaling, we must also have:

$$R \to \frac{R}{x}, \tag{60}$$
$$R_0 \to \frac{R_0}{x}, \tag{61}$$
$$\frac{r}{R} \to \frac{R}{r}, \tag{62}$$
$$f \to f, \tag{63}$$
$$rhf \to rhf. \tag{64}$$

Given that the helix stretches in proportion to R, the total values for energy, mass, and charge remain unchanged:

$$E_{\text{tot}} \to E_{\text{tot}}, \tag{65}$$
$$m_{\text{tot}} \to m_{\text{tot}}, \tag{66}$$
$$e_{\text{tot}} \to e_{\text{tot}}. \tag{67}$$

2.7 Angular momentum

In a steady-state scenario, the torus will contain n pitches evenly distributed over its radius R_0. The charged paths are transported along the helix with a velocity v_h, resulting in a length contraction for the radius:

$$R = \frac{R_0}{\gamma(v_h)}. \tag{68}$$

The angular momentum for each loop is:

$$l = m\gamma(v_h)v_h R = mv_h R_0. \tag{69}$$

Considering the scaling of v_h, we deduce the following relationships:

$$R_0 \to R_0 x, \tag{70}$$
$$m \to \frac{m}{x}, \tag{71}$$
$$v_h \to v_h, \tag{72}$$
$$l \to l, \tag{73}$$
$$l_{\text{tot}} \to l_{\text{tot}}. \tag{74}$$

If L denotes the length of the helix, then in its rest frame, v_h is given by:

$$\frac{v_h}{c} = \frac{2\pi R_0}{L_0}. \tag{75}$$

Taking into account the necessity of removing angular momentum from the outer loop equal to that of the inner loop, we get:

$$l = mv_h R_0 |1 - v|. \tag{76}$$

From Equations 75 and 43, this becomes:

$$l = \eta \frac{A_0 R_0}{L_0} \frac{kue^2}{|1-u|c} |1 - v|, \tag{77}$$

where A_0 represents the area of the torus:

$$A_0 = 2\pi r 2\pi R_0. \tag{78}$$

Using 38, we deduce:

$$l = \eta \frac{A_0 R_0}{L_0} \frac{ke^2}{c}. \tag{79}$$

For the charge to be properly managed, we require:

$$\frac{A_0 2\pi R_0}{L_0} = 1. \tag{80}$$

Given that $n = \frac{1}{hr}$ (disregarding f), and using 38, we derive the expression:

$$\hbar = \eta \frac{ke^2}{hrc}. \tag{81}$$

Although we've considered one helix turn per pitch, we could potentially have any integer m factors to the number of pitches but after one loop the helix is shifted by a fraction of $1/m$ of the smaller circle. This aligns with the Bohr condition of angular momentum:

$$l_{\text{tot}} = m\hbar. \tag{82}$$

From this, we can solve for hr:

$$hr = \eta \frac{ke^2}{\hbar c} = \alpha \approx \frac{1}{137} = \frac{1}{n}. \tag{83}$$

As we account for the maximal energy density to be closer to the inner part of the torus, we can generalize the computation for the actual fine structure constant, α. This insight sheds light on Wolfgang Pauli's and many other physicists exploration regarding why $\frac{1}{\alpha}$ closely approximates a natural number, specifically 137, and the reason it doesn't exactly match. In the discussion below regarding this, we present a hypothetical formula that derives α from $\frac{1}{137}$. This approach also provides a deduction for Planck's constant, assuming we recognize that there are 137 pitches.

Finally there is an argument for why 137 is a prime. The reason is that if there was an integer factor. We could as well get a similar setup using one of the integer fractions and an excited Bohr condition for the angular momentum. Implying that the most energetically stable system is something less that the original number. This is a bit of hand waving. But still an important observation.

2.8 Defining the zeta factor

Consider N charges evenly distributed on a unit circle. To analyze the forces acting on a single charge, note that the charges are located at positions $e^{2\pi k/N}$ for $k = 0, \ldots, N-1$. The force at $k = 0$ is self-evident. In order to avoid canceling any of their contributions, the cumulative force is given by

$$V(N) = \sum_{k=1}^{N} \frac{h'r}{R} \frac{1}{|e^{2\pi ik/N} - 1|}. \tag{84}$$

Expanding the term in the denominator, we get

$$|e^{2\pi ik/N} - 1|^2 = (e^{2\pi ik/N} - 1)(e^{-2\pi ik/N} - 1) \tag{85}$$

$$= 2 - 2\cos(2\pi k/N). \tag{86}$$

This simplifies our expression for $V(N)$ to

$$V(N) = \frac{1}{\sqrt{2}} \sum_{k=1}^{N} \frac{h'r}{R} \frac{1}{\sqrt{1-\cos(2\pi k/N)}}. \tag{87}$$

Using the trigonometric identity for the double cosine, we have

$$1 - \cos(2\pi k/N) = 2\sin^2(\pi k/N). \tag{88}$$

Which gives

$$V(N) = \frac{1}{2} \sum_{k=1}^{N} \frac{h'r}{R} \frac{1}{\sin(\pi k/N)}. \tag{89}$$

From this, we derive

$$\zeta_h(N) = N \sum_{k=1}^{N} \frac{\pi\alpha}{\sin(\pi k/N)}. \tag{90}$$

Given that

$$\sin(\pi k/N) < \pi k/N, \tag{91}$$

including the N charges, we obtain

$$\zeta_h(N) > N^2 \alpha \ln(N). \tag{92}$$

A direct calculation with $N = 137$ provides

$$\zeta_h(137) \approx 137 \times 1382.5\alpha. \tag{93}$$

2.9 Numerology

The following expression is an adequate approximation for the fine structure constant,

$$\frac{\alpha}{1 + \frac{\alpha}{1-(2\pi-1)^2}} = \frac{1}{137}. \tag{94}$$

Further exploration yields another expression,

$$\frac{\alpha}{1 + \frac{\alpha}{1-\left(1-\frac{2\pi}{1-\frac{4\pi\alpha}{1+\frac{2\pi}{1-\frac{4\pi\alpha}{1-2\pi/(1+2\alpha)}}}}\right)^2}} = \frac{1}{137}. \tag{95}$$

From this, we can postulate,

$$\frac{\alpha}{1 - \frac{\alpha}{x^2-1}} = \frac{1}{137}, \tag{96}$$

where x satisfies

$$x = 1 - \frac{2\pi}{1 - \frac{4\pi\alpha}{1+\frac{2\pi}{1-\frac{4\pi\alpha}{x}}}}. \tag{97}$$

While seemingly numerological and fine-tuned through trial and error, we might motivate this by noting that as density is higher closer to the center, the limiting energy density is located there. This gives rise to the relation,

$$hrf = \alpha f = \frac{1}{137}, \tag{98}$$

where

$$f = \frac{R}{R-r} = \frac{1}{1-\frac{r}{R}}. \tag{99}$$

Thus,

$$\frac{\alpha}{1-\frac{r}{R}} = \frac{1}{137}. \tag{100}$$

Matching with the earlier expression, we find

$$\frac{r}{R} \approx \frac{\alpha}{(2\pi-1)^2-1}, \tag{101}$$

which can also be expressed as

$$\frac{r}{R} = \frac{rh}{hR} \tag{102}$$

$$= \frac{\alpha}{hR}. \tag{103}$$

This allows us to identify

$$x^2 = hR + 1. \tag{104}$$

Given that $x^2 = 28.7778$, it implies

$$hR = 27.7778, \tag{105}$$

which can be rewritten as

$$\alpha R = 27.7778r \implies R = 3807r. \tag{106}$$

Let $L_h = rh2\pi R_0$ represent the pitch distance and $L_r = 2\pi r$ denote the distance of one helix turn in the loop direction. Then

$$\frac{v_h}{c} = \frac{L_h}{\sqrt{L_h^2+L_r^2}} = \frac{rh2\pi R_0}{\sqrt{(2\pi r)^2+(rh2\pi R_0)^2}}. \tag{107}$$

Rearranging, we deduce

$$\frac{v_r}{c} = \frac{1}{\sqrt{1+(1/(hR_0))^2}}. \tag{108}$$

Solving for hR_0, we get

$$hR_0 = \sqrt{\gamma^2-1}. \tag{109}$$

Considering $hr = \alpha$,

$$\alpha\frac{r}{R_0} = \sqrt{\gamma^2-1}. \tag{110}$$

As $R = R_0\gamma$, this implies

$$hR = \gamma\sqrt{\gamma^2-1}. \tag{111}$$

With $hR \approx 27.7778$,

$$\gamma \approx 5.3. \tag{112}$$

Since α is invariant under the duality, and $\mathcal{D}(r) = \frac{r}{u}$, it follows that

$$\mathcal{D}(R) = \frac{R}{u}. \tag{113}$$

Given that $\mathcal{D}(R_0) = R_0$, we have

$$\mathcal{D}(\gamma) = \gamma u, \tag{114}$$

leading to

$$\mathcal{D}(\gamma m) = \frac{\gamma m}{u}. \tag{115}$$

2.10 On r

We need to have a measure of the size of the electron in order to judge how large r is and hence also from this find n. This is not known, so we punt on this issue. However, we note that being able to set up similar equations for quarks may mean that we can get a grip on c_a, c_b and from this deduce the radius of, e.g., the proton. At the moment, we can only resolve $c_a r$ from Eq. 39, and we will not, in this paper, shed light on this fundamental constant.

2.11 On mass

If we consider the total mass scale invariant, we obtain (using n copies and Eq. 43),

$$m_e = \gamma n m = \eta n \gamma \frac{2\pi k r u e^2}{|1-u|c^2} = \eta n \gamma \frac{\alpha 2\pi k u e^2}{h|1-u|c^2}. \tag{116}$$

Using the electron mass equation 116 and the condition for h, Eq. 41,

$$m_e = \eta n \gamma \alpha \frac{2\pi k u e^2}{\frac{|1-u|2\zeta_h}{u}|1-u|c^2} = \eta n \alpha \gamma \frac{\pi k e^2}{h \zeta_h c^2}\left(\frac{u}{|1-u|}\right)^2 = \eta n \gamma \alpha^2 \frac{\pi \hbar}{\zeta_h c}\left(\frac{u}{|1-u|}\right)^2, \tag{117}$$

plugging in Eq. 93, we find,

$$m_e \approx \eta \gamma \frac{\alpha}{1382.5} \frac{\pi \hbar}{c} \left(\frac{u}{|1-u|}\right)^2.$$

From Eq. 112, $\gamma = 5.3$ we deduce,

$$|1-u| \approx \epsilon = 5.83 \times 10^{-9}.$$

Also note that $u = 1 \pm \epsilon$. From Eq. 115,

$$m_{\text{positron}} = \mathcal{D}(m) = \frac{m}{u} = \frac{m}{1 \pm \epsilon} \approx m(1 \pm \epsilon).$$

Choosing the lower value, the positron mass is,

$$m_{\text{positron}} = 0.510\,998\,953\,0 \quad [\text{Mev}/c^2].$$

Measurements approximate the value to,

$$m^*_{\text{positron}} = 0.510\,998\,951\,9 \quad [\text{Mev}/c^2].$$

Considering the references [2] and [3], we observe that the current measurement errors are around 130 ppb and are not able to resolve the proposed mass difference at about 10 ppb. This would undoubtedly require solid measurement results, as it represents such a revolutionary and controversial finding. However, given the proximity, forthcoming measurements may address this question, either falsifying the model proposed in this paper or challenging current assumptions based on symmetry arguments.

2.12 An addition theorem of charge streams and a fundamental scaling property

On the other hand, if we overlay many of these geometric structures to span a spherically symmetric object, interaction will occur solely between parallel torus structures when they are sufficiently separated. There will be one such pair for every direction, implying that the standard Coulomb law naturally applies due to symmetry. Since the entire construction is defined at the limit between proper electromagnetic (EM) theoretical objects, it's expected that the magnetic field will properly manifest when we shift our reference frame. Therefore, we have successfully reconstructed our macroscopic understanding using these elementary building blocks.

Now, consider the scenario when we overlay two loops at a specific point. To maintain the overall limit balance, we require $c_a \to xc_a$ and $c_b \to xc_b$. For the charge to remain invariant, rc_a and rc_b need to be constant, as indicated by Eqs. 23 and 24. This implies $r \to r/x$, leading to $h \to hx$. Consequently, v remains invariant as both R and the pitch are unchanged. Furthermore, we have

$$E \to Ex \quad \text{and} \quad m \to mx.$$

As a result, for the individual systems, $l \to lx$. This suggests that we can naturally average the loops within the sphere. If we only add loops pointing towards the upper half uniformly, vector addition reveals that the overall angular momentum becomes the well-known

$$l_z = \frac{\hbar}{2}.$$

3 Results

The following results is developed as a consequence of the model:

- A satisfying argument for why we have particles and the nature of them.
- A mass formula for the electron and positron leading to the prediction that the masses differ.
- It is shown by quantifying the difference between them that currently this different is under the radar of the measurement error, but still so large that it is not impossible, with new experiments to find a difference hence falsifying or supporting this model approach.
- A formula for the angular momentum is derived and shown to match Bohr's angular momentum assumption in his atomic model. We also indicate how this is not a contradiction to known measurements of this quantity.
- The nature of why the inverse of the fine structure constant is close to 137 is explored and with this model the prime number 137 is the number of pitches in a helix. We argue why this is a prime and also what effect that makes the true measurement differ slightly.

4 Discussion

This approach has obviously some strong theoretical alignment as seen from the results section. But still it is a very young idea and goes against the common idea that we must base these kind of models deeply within the quantum mechanical framework. As Bohr used a more naive idea of a model, this model may as well be naive, but still useful as an inspiration for further developments. One need to notice how strongly the model at the same time diverges from the standard model. This, due to deep symmetry reasons, the masses of the electron and its anti particle should be the same. Also this model has a very attractive property to be mathematically relative simple to analyze, as shown in this paper. This fact may lead to a lot of progress, and in the end results in great new inventions and practical applications.

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Data Availability

The only data used are located in papers referenced in the text.

Ethical Statement

There are no ethical considerations needed as this is a purely theoretical work.

Conflict of Interest

The authors declare that they have no conflict of interest.

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