Testing Special Relativity With an Infinite Arm Interferometer

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Abstract

The Michelson-Morley experiment and its resolution by the special theory of relativity form a foundational truth in modern physics. In this paper I propose an equivalent relativistic experiment involving a single-source interferometer having infinite arms. Further, we debate the possible outcomes from such an experiment and in doing so uncover a conflict between special relativity and the symmetry of nature. I demonstrate this conflict by the method of *reductio ad absurdum*.

Keywords— Michelson-Morley, symmetry of nature, special relativity

1 Introduction

The paradigm shifting Michelson-Morley (MM) experiment [1] and its famous null result have changed the way modern science interprets the nature of space and time. Having conclusively demonstrated the absence of any *luminferous aether* [2], the MM null result also created a paradox of *unequal path lengths* (we will discuss this in detail) that was ultimately reconciled [3] by the application of Albert Einstein's special theory of relativity [4].

Let us now investigate and generalise the geometry and sequence of events within a MM interferometer as follows:

1. We begin with the geometry of two flat triangles that are relevant to the discussions at hand.
2. Then we consider a thought experiment involving travelling waves that reflect and interfere with each other within the confines of a circular boundary. Further, we establish that our thought experiment is equivalent to an MM interferometer having infinite arms and moving through space under inertial rules.
3. Finally we debate the physical implementation of our thought experiment in order to arrive at our conclusion.

2 Euclidean Geometry

On a flat surface, we draw any angle θ at origin Q bounded by two equal length line segments $QB = QB' = h$. We join points B and B' to points A and C such that the line segment AC is perpendicular to QB and centred at Q. We will restrict our arguments to the domain $x < h$. Fig. 1 illustrates.

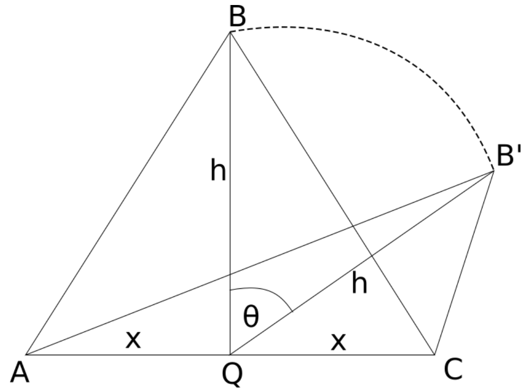

Figure 1: Triangles ABC and $AB'C$ rendered on a flat surface

From fig. 1, we establish the following geometric truths:

1. If $x > 0$, physical measurements will verify the theoretical statement $AB + BC \neq AB' + B'C$ is true for all $\theta \neq 0, \pi, 2\pi...$
2. Since h is constant, curve BB' will take the form of a circle as $0 \leq \theta \leq 2\pi$.

3 A Thought Experiment

Imagine an ideal homogeneous flat surface S1 enclosed by an ideal rigid boundary of geometrically circular shape (radius = h) and capable of transporting a travelling wave of the form,

$$\frac{1}{c^2}\frac{\delta^2 y}{\delta t^2} = \frac{\delta^2 y}{\delta x^2} \tag{1}$$

where the terms are as follows:

1. x represents the displacement of the measurement point from the origin of the wave measured along surface S1,
2. c represents the velocity of the wave measured along surface S1,
3. y represents the instant displacement of the wave measured perpendicular to surface S1.
4. t represents the time elapsed since the instant that the wave was created.

From directly above, we may project fig. 1 onto S1 without distortion such that the boundary of S1 is defined by curve BB', a circle of radius h about point Q. Now let us agree that surface S1 supports the geometry of fig. 1 over all $0 \leq \theta \leq 2\pi$ and $0 \leq x < h$.

We choose any point A on S1 and disturb the equilibrium causing an isotropic sinusoidal wave (wavelength = λ) to emanate from that point. As this primary wave expands, its wavefront will interact with S1's boundary generating innumerable secondary waves as it does so. Each reflection event along curve BB' generates its own isotropic wave and from physical measurements of fig. 1, we find that if $x \neq 0$ the statement $AB + BC \neq AB'_1 + B'_1C... \neq AB'_i + B'_iC$ is true (See fig. 2 which is a generalisation of fig. 1 over all $0 \leq \theta \leq 2\pi$). Let us invoke the following assumptions as we debate the nature of the interference pattern at point C:

1. The wave we generate originates from a single point and comprises exactly one complete cycle of a sinusoidal travelling wave
2. λ remains constant in accordance with the law of conservation of energy [5]

3. Reflections are instantaneous and lossless

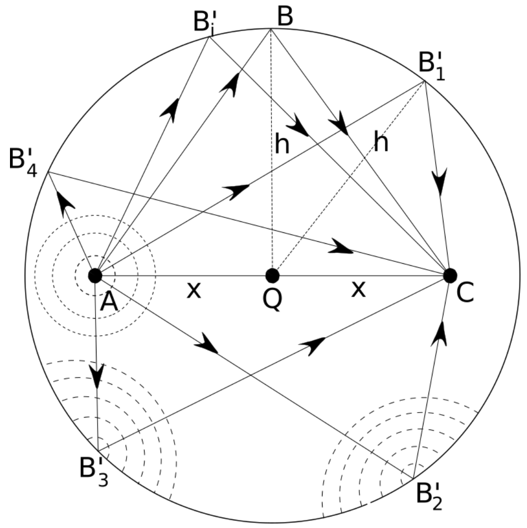

Figure 2: A single isotropic sinusoidal wave is emitted from point A and reflects from the circular boundary generating innumerable secondary wavefronts.

4 The Michelson-Morley Experiment

Now we turn to theoretical aspects of the MM experiment in order to establish it's equivalence with our thought experiment.

4.1 Frames of Reference

For the purpose of further discussion, we refer to fig. 1 and establish the following euclidean frames of reference:

1. A stationary reference frame I_0 centered at point Q.
2. A moving reference frame I_1 that translates from point A to point C with some constant velocity v relative to arbitrarily selected origin Q.

4.2 Geometry and Sequence of Events

First let us consider the structure of an MM interferometer [2](see fig. 3). By fixing $\angle B_1'QB_2' = \pi/2$, line segments QB_1' and QB_2' form the arms of the interferometer. The apparatus may be rotated about point Q and consequently each arm subtends its own angle θ measured from a perpendicular to line segment AC.

Now let us imagine this interferometer moving through space under inertial rules. Reference frame I_1 is fixed to the interferometric source and moves with constant velocity v relative to reference frame I_0 from point A to point C. The event cycle begins with the source at point A marking the simultaneous emission of a pair of photons (wavelength=λ). As the entire apparatus moves with some constant ($AQ = QC$) velocity v relative to origin Q along line segment AC, the photons are emitted at point A, reflect from mirrors B_1 and B_2 to finally arrive simultaneously (in phase with each other) at point C.

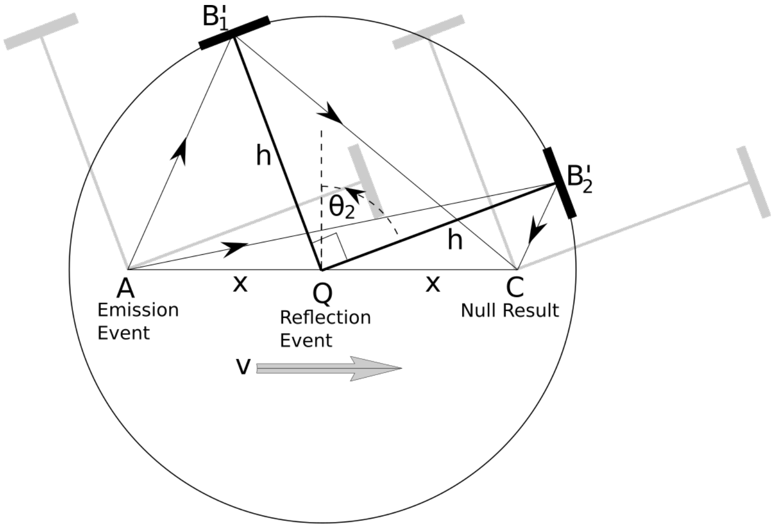

Figure 3: Geometry and event sequence within the Michelson-Morley experiment depicting the general case $x \neq 0$. Equivalent to our thought experiment and identical to fig. 1, we find $AB_1' + B_1'C \neq AB_2' + B_2'C$ but yet we agree that the MM outcome is a null result at point C.

As is true in our thought experiment, it is straightforward to recognise that in one *emission-reflection-result* cycle of an MM interferometer and for all $0 \leq v < c$, the locus of all points in space where a reflection event can occur is a physical circle of radius h about origin Q. In terms of scope, the event sequence in our thought experiment is equivalent to one cycle of an MM interferometer having infinite arms (See fig. 2). It is also a well established fact of modern science [6] that the MM experiment presents a null result for all $0 \leq v < c$, where c represents the velocity of light.

5 Generalisation

For purposes of generalisation, let us first accept the following statements as true:

1. Experimental Truth T1: Under inertial conditions, the MM interferometer presents a null result over all $0 \leq v < c$.
2. Theoretical Truth T2: The thought experiment presented is equivalent to one *emission-reflection-result* cycle of a MM interferometer having infinite arms and moving under inertial rules relative to origin Q.

Invoking symmetry in physics [7], we combine truths T1 and T2 to arrive at prediction P1:

Were it possible to realise a physical implementation of the thought experiment presented, then we expect equivalent physical outcomes to those expected from an MM interferometer generalised over all $0 \leq \theta < 2\pi$.

6 Practical Implications

The thought experiment presented may be brought a step closer to realisation by imagining a pair of isotropic radio antennae placed within a reflective boundary of circular shape.

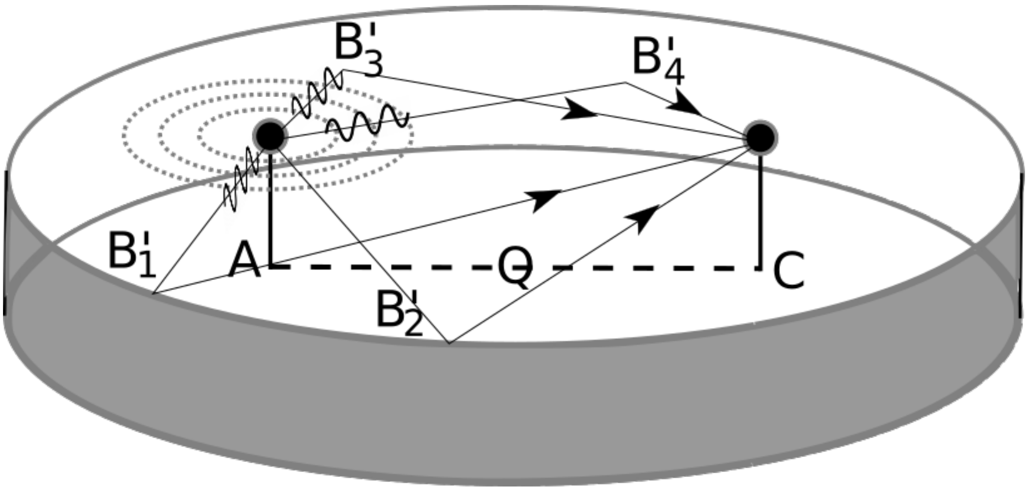

Figure 4: Two isotropic antennae placed diametrically opposite each other within a circular reflective boundary. When viewed from directly above, this physical setup is identical to fig. 2 and equivalent to a Michelson-Morley interferometer (having infinite arms) moving through space under inertial rules.

An isotropic source of electromagnetic waves is placed at some random point A within a circular shaped reflective boundary of arbitrary radius h. An isotropic receiver is placed diametrically opposite (point C). By energising the system, and according to the equivalency arguments of sec. 4 (from whence flowed truths T1 and T2) we have created an equivalent of the MM experiment with an interferometer having infinite arms. Let us refer to this physical setup as the Infinite Arm Interferometer (IAI). When viewed from directly above, we note that within the IAI, if $x > 0$ then $AB + BC \neq AB'_1 + B'_1C ... \neq AB'_i + B'_iC$ and we are presented with an equivalent inequality in path lengths as observed in the MM setup.

Since x and h in our thought experiment are equivalent to v and c in the MM experiment, the velocity of reference frame I_1 within the IAI with respect to origin Q can be expressed as a fraction of the velocity of light equal to x/h. Both x and h can be obtained by physical measurements of the apparatus using a measuring rod. Thus by setting $x \approx h$, we have arranged the condition $v \approx c$ and we are able use our apparatus to test the predictions of special relativity [4] namely lorentz contraction and time dilation up to the velocity of light.

7 On the Need for Experiment

Let us now discuss the possible outcomes from a physical implementation of the IAI experiment and debate whether at this stage, it is even necessary to conduct such an experiment. The following are the only possible outcomes from such an experiment:

1. Upon energising the system, we observe a null result only if $x = 0$. In this case, we conclude that the MM interferometer and IAI are experimentally distinct from each other. But this experimental truth contradicts theoretical truth T2 that asserts that the IAI and MM interferometers are equivalent. Should this outcome emerge from experiment, we are faced with a contradiction [8] between an experimental truth and a theoretical truth.

2. Next we consider the case that when energised, the IAI returns a null result over all $0 < x \leq h$ in agreement with prediction P1 *and* the space-time distortions (lorentz contraction and time dilation) predicted by special relativity are also manifested proportional to the lorentz factor [4],

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Equation 2 predicts that if $v \approx c$, then lorentz contraction and time dilation grow to infinite magnitudes. If this is true, then we (stationary reference frame I_0) would expect the IAI null result event to be approached asymptotically in space and in time. Recall now that this observational perspective of reference frame I_0 may be arranged simply by setting $x \approx h$. Given that the IAI is physically nothing more than a pair of radio antennae placed within a circular reflector, the expectation that setting $x \approx h$ (equivalently $v \approx c$) and energising the system will manifest infinitely large distortions in space and time strains the very bounds of reason. Thus we conclude that within the IAI, if the predictions of special relativity are true, they are absurd.

3. Finally we consider the case that upon energising the system, the IAI returns a null result over all $0 \leq v < c$ in agreement with prediction P1, but manifests no evidence of lorentz contraction and time dilation. In this case we are faced with the following questions:

   (a) Given equivalent sequences of events within equivalent geometries, has nature abandoned her impartiality [7] and *preferred* to implement lorentz contraction and time dilation in the operation of a two arm MM interferometer [3] but **not** in an equivalent interferometer having infinite arms?

   (b) If not by special relativity, then do we reconcile the paradox of unequal path lengths presented by the IAI null result?

8 Conclusion

Having discussed every possible outcome from a physical conduct of the IAI experiment, we are faced with irrevocable fundamental questions whatever the result. Therefore the physical conduct of the IAI experiment becomes unnecessary at this stage. However, it is now most necessary for a practitioner of special relativity to falsify truth T2 and prove by argument alone that the equivalency arguments of sec. 4 are flawed.

9 Statements and Declarations

The author has no competing interests to declare that are relevant to the content of this article. There are no data associated with this article.

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