The Dimensionless Equations of the Universe

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Abstract

In this paper in an elegant way will be present the Dimensionless Equations of the Universe. All these equations are simple,elegant and symmetrical in a great physical meaning. We will propose the Dimensionless unification of the fundamental interactions and the Dimensionless unification of atomic physics with cosmology. We find the formulas for the cosmological constant and we will prove that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy.

Keywords

Theory of everything , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant , Unification of the microcosm and the macrocosm , Poincaré dodecahedral space

1. Introduction

Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi}+1=0$$

The expression who connects the six basic mathematical constants,the number 0,the number 1,the golden ratio φ,the Archimedes constant π,the Euler's number e and the imaginary unit i is:

$$e^{\frac{i\pi}{1+\varphi}}+e^{\frac{-i\pi}{1+\varphi}}+e^{\frac{i\pi}{\varphi}}+e^{\frac{-i\pi}{\varphi}}=0$$

In [1] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. The fine-structure constant α is defined as:

$$\alpha=\frac{q_e^2}{4\pi\varepsilon_0\hbar c}$$

Also the fine-structure constant is universal scaling factor:

$$\alpha=\frac{2\pi r_e}{\lambda_e}=\frac{\lambda_e}{2\pi\alpha_0}=\frac{r_e}{l_{pl}}\frac{m_e}{m_{pl}}=\sqrt{\frac{r_e}{\alpha_0}}$$

Dr. Rajalakshmi Heyrovska in [2] has found that the golden ratio φ provides a quantitative link between various known quantities in atomic physics. Fine-structure constant can also be formulated in [3] , [4] and [5] exclusively in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \quad (1)$$

with numerical values:

$$\alpha^{-1}=137.0359991647........$$

The numerical value is the average of all the measurements. The formula is the exact formula for the fine-structure constant α. Another beautiful forms of the equations are:

$$\frac{1}{\alpha}=\frac{360}{\varphi^2}-\frac{2}{\varphi^3}+\frac{1}{3^5\varphi^5} \quad (2)$$

$$\frac{1}{\alpha}=\frac{360}{\varphi^2}-\frac{2}{\varphi^3}+\frac{3^{-5}}{\varphi^5} \quad (3)$$

We proposed in [5] , [6] and [7] the simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π:

$$\alpha^{-1}=\frac{2706}{43}\pi\ln 2 \quad (4)$$

with absolutely accurate numerical values:

$$\alpha^{-1}=137.035999078...$$

The equivalent expression for the fine-structure constant is:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2 \quad (5)$$

So the equivalent expressions for the fine-structure constant with the madelung constant $b_2(2)$ are:

$$\alpha^{-1}=-\frac{2706}{43}b_2(2) \quad (6)$$

In Physics,the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron,from the system of units. The value of μ is a solution of the equation:

$$3\cdot\mu^4-5508\cdot\mu^3-841\cdot\mu^2+10\cdot\mu-2111=0$$

We propose in [8] the exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu=11^{47/32}\cdot5^{5/2}\cdot9349^{5/76}\cdot\varphi^{-21/16} \quad (7)$$

So the exact mathematical expression for the proton to electron mass ratio is:

$$\mu^{32}=(\varphi^5-\varphi^{-5})^{47}\cdot(2\cdot\varphi-1)^{160}\cdot(\varphi^{19}-\varphi^{-19})^{40/19}\cdot\varphi^{-42} \quad (8)$$

$$\mu^{32}=\varphi^{-42}\cdot F5^{160}\cdot L5^{47}\cdot L19^{40/19} \quad (9)$$

with numerical value:

$$\mu=1836.15267343...$$

Also we propose the exact mathematical expression for the proton to electron mass ratio:

$$\mu = 165\sqrt[3]{\frac{\ln^{11}10}{7}} \tag{10}$$

with numerical value:

$$\mu=1836.15267392...$$

Other equivalent expressions for the proton to electron mass ratio are:

$$\mu^3=7^{-1}\cdot 165^3\cdot \ln^{11}10$$

$$7\cdot\mu^3=(3\cdot 5\cdot 11)^3\cdot \ln^{11}(2\cdot 5) \tag{11}$$

Other exact mathematical expression for the proton to electron mass ratio is:

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15} \tag{12}$$

with numerical value:

$$\mu=1836.15267343...$$

Also in [8] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42) \tag{13}$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12 \tag{14}$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231 \tag{15}$$

$$\mu-807\cdot\alpha=1205\cdot\pi-518\cdot\varphi-411\cdot e \tag{16}$$

In [9] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot 10^2\cdot\cos(\mu\cdot\alpha^{-1})+13^2=0 \tag{17}$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \tag{18}$$

This exponential form can also be written with the beautiful form:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=13^2\cdot e^{i\pi} \tag{19}$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \tag{20}$$

So other beautiful formula that connects the fine-structure constant,the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2\cdot(5\cdot\varphi^{-2}+\varphi^{-5})^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+(5\cdot\varphi^2-\varphi^{-5})^2=0 \tag{21}$$

The formula that connects the fine-structure constant,the proton to electron mass ratio and the mathematical constants π,φ,e and i is:

$$10^2 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5\cdot\varphi^2-\varphi^{-5})^2\cdot e^{i\pi} \tag{22}$$

In physics,the gravitational coupling constant α_G is a constant that characterizes the gravitational pull between a given pair of elementary particles. The gravitational coupling constant α_G is defined as:

$$\alpha_G = \frac{Gm_e^2}{\hbar c}$$

There is so far no known way to measure α_G directly. The value of the constant gravitational coupling α_G is only known in four significant digits. The approximate value of the constant gravitational coupling α_G is $\alpha_G=1.7518099\times10^{-45}$. Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha\frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha\alpha_0}\right)^2$$

The gravitational coupling constant $\alpha_{G(p)}$ for the proton is produced similar to the electron,but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton $\alpha_{G(p)}$ is defined as:

$$\alpha_{G(p)} = \frac{Gm_p^2}{\hbar c}$$

The approximate value of the constant gravitational coupling of the proton is $\alpha_{G(p)}=5.9061512\text{x}10^{-39}$. Also other expression for the gravitational coupling constant is:

$$\alpha_{G(p)} = \frac{m_p^2}{m_{pl}^2} = \mu^2\alpha_G = \frac{\alpha\mu}{N_1} = \frac{\alpha^2}{N_1^2\alpha_G}$$

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1904. But Weyl and Eddington suggested that the number was about 10^{40} and was related to cosmological quantities. The electric force F_c between electron and proton is defined as:

$$F_c = \frac{q_e^2}{4\pi\varepsilon_0 r^2}$$

The gravitational force F_g between electron and proton is defined as:

$$F_g = \frac{Gm_e m_p}{r^2}$$

So from these expressions we have:

$$N_1 = \frac{F_c}{F_g}$$

$$N_1 = \frac{q_e^2}{4\pi\varepsilon_0 G m_e m_p}$$

$$N_1 = \frac{k_e q_e^2}{G m_p m_e}$$

$$N_1 = \frac{\alpha\hbar c}{G m_e m_p}$$

So the ratio N_1 of electric force to gravitational force between electron and proton is defined as:

$$N_1 = \frac{\alpha}{\mu\alpha_G} = \frac{\alpha\mu}{\alpha_{G(p)}} = \frac{\alpha}{\sqrt{\alpha_G \alpha_{G(p)}}} = \frac{k_e q_e^2}{G m_e m_p} = \frac{\alpha\hbar c}{G m_e m_p}$$

The approximate value of the ratio of electric force to gravitational force between electron and proton is $N_1=2.26866072\times10^{39}$. The ratio N_1 of electric force to gravitational force between electron and proton can also be written in expression:

$$N_1 = \frac{5}{3}2^{130} = 2,26854911 \times 10^{39}$$

According to current theories N_1 should be constant. The ratio N_2 of electric force to gravitational force between two electrons is defined as:

$$N_2 = \mu N_1 = \frac{\alpha}{\alpha_G} = \frac{N_1^2 \alpha_{G(p)}}{\alpha} = \frac{k_e q_e^2}{G m_e^2} = \frac{\alpha\hbar c}{G m_e^2}$$

The approximate value of N_2 is $N_2=4.16560745\times10^{42}$. According to current theories N_2 should grow with the expansion of the universe. Avogadro's number N_A is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. The exact value of the Avogadro's number is $N_A=6.02214076\times10^{23}$. The value of the Avogadro's number N_A can also be written in expressions:

$$N_A=84446885^3=6.02214076\times10^{23}$$

$$N_A=2^{79}=6.04462909\times10^{23} \quad (23)$$

In [10] was presented the exact mathematical formula that connects 6 dimensionless physical constants The length Planck l_{pl} can be defined by three fundamental natural constants,the speed of light at vacuum c,the reduced Planck constant and the gravity constant G as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl} c} = \frac{h}{2\pi m_{pl} c} = \frac{m_p r_p}{4 m_{pl}}$$

The Bohr radius α_0 is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius α_0 is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

For the reduced Planck constant ħ apply:

$$\hbar=\alpha\cdot m_e\cdot \alpha_0\cdot c$$

So from these expressions we have:

$$\hbar^2=\alpha^2\cdot m_e^2\cdot \alpha_0^2\cdot c^2$$

$$(\hbar\cdot G/c^3)=\alpha^2\cdot m_e^2\cdot \alpha_0^2\cdot(G/\hbar\cdot c)$$

$$(\hbar\cdot G/c^3)=\alpha^2\cdot \alpha_0^2\cdot(G\cdot m_e^2/\hbar\cdot c)$$

$$l_{pl}^2=\alpha^2\cdot \alpha_G\cdot \alpha_0^2$$

So the new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \tag{24}$$

Jeff Yee proposed in [11] that the mole and charge are related by deriving Avogadro's number from three constants,the Bohr radius,the Planck length and Euler's number. The Avogadro's number N_A can be calculated from the Planck length l_{pl},the Bohr radius α_0 and Euler's number e:

$$N_A = \frac{\alpha_0}{2el_{pl}}$$

We will use this expression and the new formula for the Planck length l_{pl} to resulting the unity formula that connects the fine-structure constant α and the gravitational coupling constant α_G:

$$\alpha_0 = 2 \cdot e \cdot N_A \cdot l_{pl}$$

$$\alpha_0 = 2eN_A\alpha\sqrt{\alpha_G}\alpha_0$$

$$2eN_A\alpha\sqrt{\alpha_G} = 1$$

Therefore the unity formula that connect the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \tag{25}$$

The unity formula is equally valid:

$$\alpha^2 \cdot \alpha_G = (2 \cdot e \cdot N_A)^{-2} \tag{26}$$

This formula is the simple unification of the electromagnetic and the gravitational interactions. So from this expression the new formula for the Avogadro number N_A is:

$$N_A = \left(2e\alpha\sqrt{\alpha_G}\right)^{-1} \tag{27}$$

The exact mathematical formula that connect the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$ are:

$$\alpha_{G(p)} = \mu^2 \cdot \alpha_G$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \tag{28}$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \tag{29}$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_{G(p)} \tag{30}$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_{G(p)} \tag{31}$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2 \tag{32}$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \tag{33}$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \tag{34}$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \tag{35}$$

$$\mu^2=4\cdot e^2\cdot \alpha_G\cdot \alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \qquad (36)$$

$$\mu=4\cdot e^2\cdot \alpha\cdot \alpha_G\cdot \alpha_{G(p)}\cdot N_A^2\cdot N_1 \qquad (37)$$

3. Dimensionless unification of the fundamental interactions

The value of the strong coupling constant,like other coupling constants,depends on the energy scale. As the energy increases, this constant decreases as shown in Figure 1.

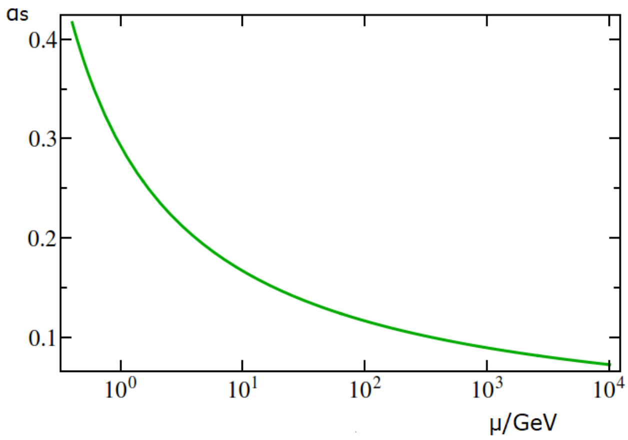

Figure 1. *Strong coupling constant as a function of the energy.*

The last measurement [12] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order,which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously,the value of the strong coupling constant at the Z boson mass is extracted as $\alpha_s(m_Z)=0.1170\pm0,0019$. For quarks in quantum chromodynamics,a strong interaction constant is introduced:

$$\alpha_s = \frac{q_{qg}^2}{4\pi\hbar c} = \frac{q_{qg}^2\varepsilon_0\alpha}{q_e^2} = \frac{\varepsilon_0 q_{qg}^2}{q_{pl}^2}$$

where g_{qg} is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks,which is achieved in high-energy particle collisions,a logarithmic reduction of α_s and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. In [13] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{Euler'\ number}{Gerford's\ constant} = \frac{e}{e^{\pi}} = e^{1-\pi} = 0,11748.. \qquad (38)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s=e\cdot e^{-\pi}=e\cdot i^{2i}=i^{-2i/\pi}\cdot i^{2i}=i^{2i-(2i/\pi)}=i^{2i(\pi-1)/\pi}$$

In the papers [14] , [15] , [16] and [17] was presented the unification of the fundamental interactions. The decays of the delta baryons is:

$$\Delta^+ \rightarrow p + \pi^0$$

The lifetime of the delta baryons is:

$$\tau_\Delta \approx 6\times10^{-24}\ s$$

The decays of the sigma baryons is:

$$\Sigma^{+} \rightarrow p + \pi^{0}$$

The lifetime of the delta baryons is:

$$\tau\Sigma \approx 8\times10^{-11}\ s$$

The coupling constant ratio can then be estimated for this situation [18]:

$$\frac{\alpha_w}{\alpha_s} = \sqrt{\frac{\tau_\triangle}{\tau_\Sigma}} = 10^{-7}e$$

$$\frac{\alpha_w}{\alpha_s} = 10^{-7}e \tag{39}$$

From this expression we can result the world average value of the weak coupling constant αw:

$$\alpha w=e\cdot\alpha s\cdot10^{-7}$$

$$\alpha w=e^{2-\pi}\cdot10^{-7}$$

$$\alpha w=e\cdot e\cdot i^{2i}\cdot10^{-7}$$

$$\alpha w=e^{2}\cdot i^{2i}\cdot10^{-7}$$

So the recommended theoretical current world average value for the weak coupling constant is:

$$\alpha w=(e\cdot i^{i})^{2}\cdot10^{-7}=3.19310\cdot10^{-8} \tag{40}$$

From expression can result other equivalent expressions:

$$\alpha w\cdot\alpha s^{-1}=e\cdot10^{-7}$$

$$\alpha s\cdot\alpha w^{-1}=e^{-1}\cdot10^{7}$$

$$e\cdot\alpha s=10^{7}\cdot\alpha w \tag{41}$$

From this expression apply:

$$e^{\pi}\cdot\alpha s\cdot\alpha s=10^{7}\cdot\alpha w$$

$$e^{\pi}\cdot\alpha s^{2}=10^{7}\cdot\alpha w$$

$$\alpha s^{2}=10^{7}\cdot e^{-\pi}\cdot\alpha w$$

$$\alpha s^{2}=i^{2i}\cdot10^{7}\cdot\alpha w \tag{42}$$

From this expression and Euler's identity resulting the beautiful formulas:

$$e^{i\pi}+1=0$$

$$(10^{7}\cdot\alpha s^{-2}\cdot\alpha w)^{i}+1=0$$

$$(10^{-7}\cdot\alpha s^{2}\cdot\alpha w^{-1})^{i}+1=0$$

$$10^{-7i}\cdot\alpha s^{2i}\cdot\alpha w^{-i}+1=0$$

$$10^{-7i}\cdot a_s^{2i}\cdot a_w^{-i}=i^2$$

$$a_s^{2i}=i^2\cdot 10^{7i}\cdot a_w^i$$

$$\frac{a_s^{2i}}{a_w^i}=i^2 10^{7i} \tag{43}$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear forces:

$$e\cdot a_s=10^7\cdot a_w$$

$$a_s^2=i^{2i}\cdot 10^7\cdot a_w$$

(Dimensionless unification of the strong nuclear and the weak nuclear force interactions)

Jesús Sánchez in [19] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos a^{-1}=e^{-1} \tag{44}$$

Another elegant expression is the following exponential form equations:

$$e^{i/a}-e^{-1}=-e^{-i/a}+e^{-1}$$

$$e^{i/a}+e^{-i/a}=2\cdot e^{-1} \tag{45}$$

Also the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos(10^3\cdot a^{-1})=\varphi^2\cdot e^{-1}$$

$$e\cdot\cos(10^3\cdot a^{-1})=\varphi^2 \tag{46}$$

Another elegant expression is the following exponential form equation:

$$e^{1000i/a}+e^{-1000i/a}=2\cdot\varphi^2\cdot e^{-1} \tag{47}$$

From these expressions resulting the following equations:

$$\cos^{-1}a^{-1}\cdot\cos(10^3\cdot a^{-1})=\varphi^2$$

$$\cos(10^3\cdot a^{-1})=\varphi^2\cdot\cos a^{-1} \tag{48}$$

We will use the expressions to resulting the unity formulas that connects the strong coupling constant a_s and the fine-structure constant a:

$$\cos a^{-1}=e^{-1}$$

$$a_s=e^{1-\pi}$$

$$\cos a^{-1}=(e^{\pi}\cdot a_s)^{-1}$$

$$\cos a^{-1}=e^{-\pi}\cdot a_s^{-1}$$

$$e^{\pi}\cdot a_s\cdot\cos a^{-1}=1 \tag{49}$$

Other forms of the equations are:

$$\cos a^{-1}=(i^{-2i}\cdot a_s)^{-1}$$

$$i^{-2i} \cdot \alpha_s \cdot \cos\alpha^{-1} = 1$$

$$\cos\alpha^{-1} = i^{2i} \cdot \alpha_s^{-1}$$

$$\alpha_s \cdot \cos\alpha^{-1} = i^{2i} \qquad (50)$$

So the beautiful formulas for the strong coupling constant α_s are:

$$\alpha_s = e^{-\pi} \cdot \cos^{-1}\alpha^{-1}$$

$$\alpha_s = i^{2i} \cdot \cos^{-1}\alpha^{-1}$$

Now we need to study the following equivalent equations:

$$\cos\alpha^{-1} = \frac{e^{-\pi}}{\alpha_s}$$

$$\cos\alpha^{-1} = \frac{i^{2i}}{\alpha_s}$$

$$\cos\alpha^{-1} = \frac{\alpha_s^{-1}}{e^{\pi}}$$

$$\cos\alpha^{-1} = \frac{\alpha_s^{-1}}{i^{-2i}}$$

The figure 2 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^{π}.

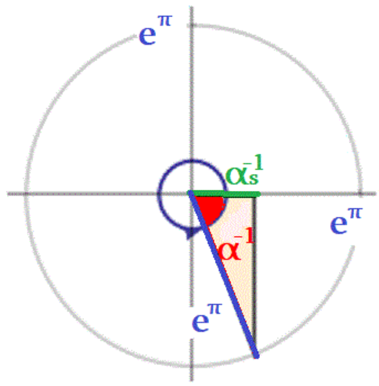

Figure 2. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius e^{π}.*

The figure 3 and 4 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

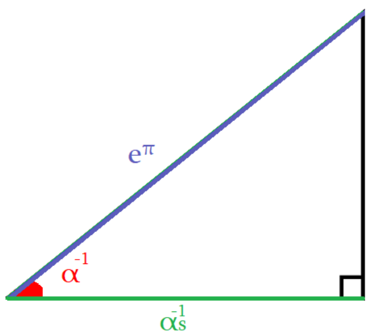

Figure 3. *The strong coupling constant α_s and the fine-structure constant α are in a right triangle with the variable acute angle α^{-1} radians. The adjacent side is the variable side α_s^{-1} while the hypotenuse is constant e^{π}.*

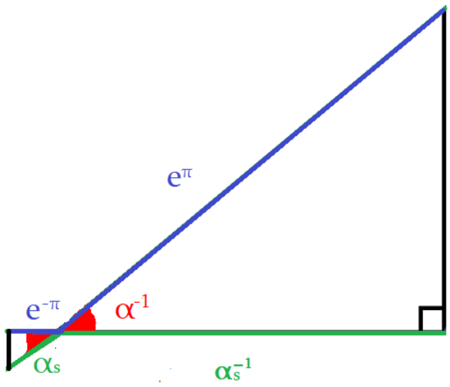

Figure 4. *Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.*

From expressions resulting the formulas that connects the strong coupling constant α_s and the fine-structure constant α:

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot e^{-1}$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi}\cdot\alpha_s)^{-1}$$

$$e^{i/\alpha}-(e^{\pi}\cdot\alpha_s)^{-1}=-e^{-i/\alpha}+(e^{\pi}\cdot\alpha_s)^{-1}$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot e^{-\pi}\cdot\alpha_s^{-1}$$

$$e^{\pi}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2 \tag{51}$$

Other forms of the equations are:

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(i^{-2i}\cdot\alpha_s)^{-1}$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot i^{2i}\cdot\alpha_s^{-1}$$

$$e^{i/\alpha}-i^{2i}\cdot\alpha_s^{-1}=-e^{-i/\alpha}+i^{2i}\cdot\alpha_s^{-1}$$

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i} \tag{52}$$

These equations are applicable for all energy scales. So the beautiful formulas for the strong coupling constant α_s are:

$$\alpha_s=2\cdot e^{-\pi}\cdot(e^{i/\alpha}+e^{-i/\alpha})^{-1}$$

$$\alpha_s=2\cdot i^{2i}\cdot(e^{i/\alpha}+e^{-i/\alpha})^{-1}$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic forces:

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i}$$

(Dimensionless unification of the strong nuclear and the electromagnetic interactions)

The figure 5 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^7\cdot e^{\pi-1}$.

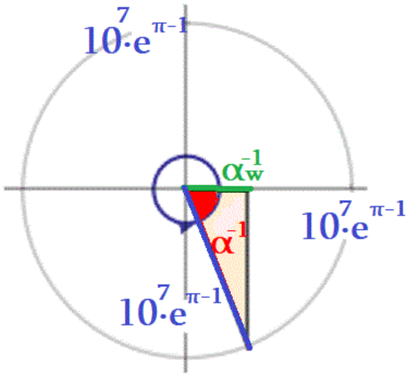

Figure 5. *The angle in* α^{-1} *radians. The rotation vector moves in a circle of radius* $10^7 \cdot e^{\pi-1}$.

The figure 6 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.

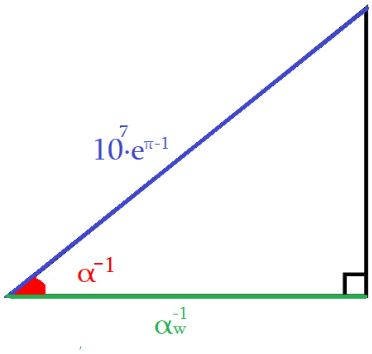

Figure 6. *Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions*

We will use the expressions to resulting the unity formula that connect the weak coupling constant α_w and the fine-structure constant α:

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$e^{\pi} \cdot \alpha_s \cdot \cos\alpha^{-1} = 1$$

$$e^{\pi} \cdot 10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = e$$

$$e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = 1$$

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = e^{1-\pi} \tag{53}$$

Other forms of the equations are:

$$\alpha_w \cdot \cos\alpha^{-1} = e \cdot i^{2i} \cdot 10^{-7}$$

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = e \cdot i^{2i} \tag{54}$$

So the formulas for the weak coupling constant α_w are:

$$\alpha_w = (e^{\pi-1} \cdot 10^7 \cdot \cos\alpha^{-1})^{-1}$$

$$\alpha_w = e^{1-\pi} \cdot 10^{-7} \cdot \cos^{-1}\alpha^{-1}$$

$$\alpha_w = e \cdot i^{2i} \cdot (10^7 \cdot \cos\alpha^{-1})^{-1}$$

$$\alpha_w=e\cdot i^{2i}\cdot 10^{-7}\cdot \cos^{-1}\alpha^{-1}$$

Resulting the unity formulas that connects weak coupling constant α_w and the fine-structure constant α:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$e^{\pi}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2$$

$$e^{\pi}\cdot 10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}$$

$$e^{i/\alpha}-(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}=-e^{-i/\alpha}+(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}$$

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e^{1-\pi} \qquad (55)$$

Other form of the equations is:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i} \qquad (56)$$

So the formulas for the weak coupling constant α_w are:

$$\alpha_w=2\cdot[e^{\pi-1}\cdot 10^7\cdot(e^{i/\alpha}+e^{-i/\alpha})]^{-1}$$

$$\alpha_w=2\cdot e^{1-\pi}\cdot 10^{-7}\cdot(e^{i/\alpha}+e^{-i/\alpha})^{-1}$$

$$\alpha_w=2\cdot e\cdot i^{2i}\cdot[10^7\cdot(e^{i/\alpha}+e^{-i/\alpha})]^{-1}$$

$$\alpha_w=2\cdot e\cdot i^{2i}\cdot 10^{-7}\cdot(e^{i/\alpha}+e^{-i/\alpha})^{-1}$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i}$$

(Dimensionless unification of the weak nuclear and the electromagnetic interactions)

The figure 7 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7.

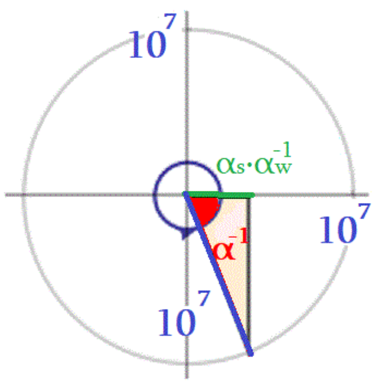

Figure 7. *The angle in α^{-1} radians.The rotation vector moves in a circle of radius 10^7.*

We will use the expressions to find the expression that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$\cos\alpha^{-1}=e^{-1}$$

$$\cos\alpha^{-1}=\alpha_s\cdot\alpha_w^{-1}\cdot 10^{-7}$$

So the unity formula that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α is:

$$10^7\cdot\alpha_w\cdot\cos\alpha^{-1}=\alpha_s \tag{57}$$

Now we need to study the following equivalent equations:

$$\cos\alpha^{-1}=\frac{10^{-7}\alpha_w^{-1}}{\alpha_s^{-1}}$$

$$\cos\alpha^{-1}=\frac{\alpha_s}{10^7\alpha_w}$$

$$10^7\cos\alpha^{-1}=\frac{\alpha_s}{\alpha_w}$$

$$\cos\alpha^{-1}=\frac{\alpha_s\alpha_w^{-1}}{10^7}$$

The figure 8 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.

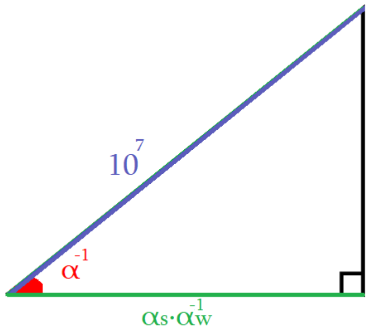

Figure 8. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.*

Resulting the beautiful formulas that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot e^{-1}$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot 10^{-7}\cdot\alpha_s\cdot\alpha_w^{-1}$$

$$\alpha_w\cdot\alpha_s^{-1}\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot 10^{-7}$$

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot\alpha_s \tag{58}$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot\alpha_s$$

(Dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions)

It was presented in [10] the mathematical formulas that connects the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G \tag{59}$$

$$\alpha=\mu\cdot N_1\cdot\alpha_G \tag{60}$$

$$\alpha\cdot\mu=N_1\cdot\alpha_{G(p)} \tag{61}$$

$$\alpha^2=N_1^2\cdot\alpha_G\cdot\alpha_{G(p)} \tag{62}$$

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1 \tag{63}$$

$$\mu^2=4\cdot e^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{64}$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A^2 \tag{65}$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=1 \tag{66}$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{67}$$

$$\mu^2=4\cdot e^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \tag{68}$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{69}$$

Also resulting the expressions:

$$\cos(\alpha^{-1})=e^{-1}$$

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1$$

$$4\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=e^{-2}$$

$$\cos^2\alpha^{-1}=4\cdot\alpha^2\cdot\alpha_G\cdot N_A^2$$

$$\alpha^{-2}\cdot\cos^2\alpha^{-1}=4\cdot\alpha_G\cdot N_A^2 \tag{70}$$

This unity formula is equally valid:

$$\alpha^{-1}\cos\alpha^{-1}=2N_A\sqrt{\alpha_G} \tag{71}$$

Also resulting another elegant exponential form equations:

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot e^{-1}$$

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1$$

$$4\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=e^{-2}$$

$$16\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=(e^{i/\alpha}+e^{-i/\alpha})^2 \tag{72}$$

This unity formula is equally valid:

$$\alpha^{-1}\left(e^{\frac{i}{\alpha}}+e^{\frac{-i}{\alpha}}\right)=4N_A\sqrt{\alpha_G} \tag{73}$$

The figure 9 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

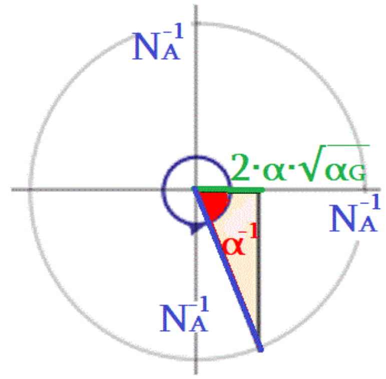

Figure 9. *The angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.*

Also resulting the expression with power of two:

$$2^{160}\cdot e^2\cdot \alpha^2\cdot\alpha_G=1 \tag{74}$$

$$\alpha^{-2}\cdot\cos^2\alpha^{-1}=2^{160}\cdot\alpha_G \tag{75}$$

$$2^{162}\cdot\alpha^2\cdot\alpha_G=(e^{i/\alpha}+e^{-i/\alpha})^2 \tag{76}$$

Other form of the equations is:

$$\alpha^{-1}\cos\alpha^{-1}=2^{34}\sqrt{\alpha_G} \tag{77}$$

The figures 10 and 11 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

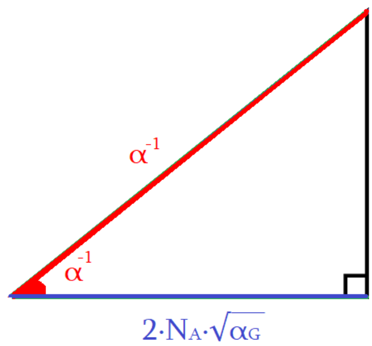

Figure 10. *First geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

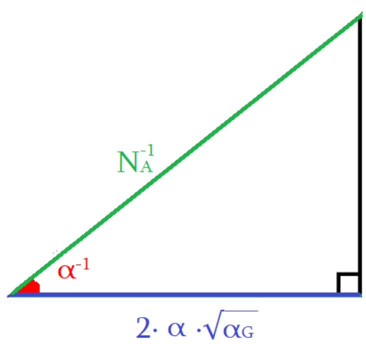

Figure 11. *Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2$$

(Dimensionless unification of the gravitational and the electromagnetic interactions)

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.

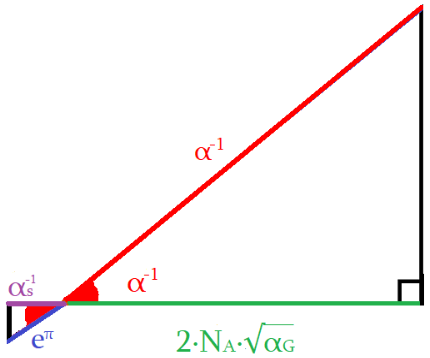

Figure 12. *Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions*

We will use the previous expressions to resulting the unity formulas that connect the strong coupling constant a_s,the fine-structure constant a and the gravitational coupling constant a_G:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot e^2 \cdot (e^\pi \cdot a_s)^2 \cdot a_G \cdot N_A^2 = 1$$

$$4 \cdot e^{2\pi} \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \qquad (78)$$

Other form of the equation is:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \qquad (79)$$

Also resulting the mathematical formulas that connects the strong coupling constant a_s,the proton to electron mass ratio μ,the fine-structure constant a,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the gravitational coupling constant a_G of the electron and the gravitational coupling constant of the proton $a_{G(p)}$:

$$4 \cdot e^{2\pi} \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \qquad (80)$$

$$\mu^2 = 4 \cdot e^{2\pi} \cdot a_s^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \qquad (81)$$

$$\mu \cdot N_1 = 4 \cdot e^{2\pi} \cdot a_s^2 \cdot a^3 \cdot N_A^2 \qquad (82)$$

$$4 \cdot e^{2\pi} \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \qquad (83)$$

$$\mu^3 = 4 \cdot e^{2\pi} \cdot a_s^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \qquad (84)$$

$$\mu^2 = 4 \cdot e^{2\pi} \cdot a_s^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \qquad (85)$$

$$\mu = 4 \cdot e^{2\pi} \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \qquad (86)$$

Other equivalent forms of the equations are:

$$4 \cdot a_S^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \quad (87$$

$$i^{4i} \cdot \mu = a_S^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \quad (88)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot a_S^2 \cdot a^3 \cdot N_A^2 \quad (89)$$

$$4 \cdot a_S^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = i^{4i} \quad (90)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot a_S^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (91)$$

$$i^{4i} \cdot \mu^2 = 4 \cdot e^{2\pi} \cdot a_S^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (92)$$

$$i^{4i} \cdot \mu = 4 \cdot a_S^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (93)$$

$$a_S \cdot \cos a^{-1} = i^{2i}$$

$$2 \cdot N_A \cdot a_G^{1/2} = a^{-1} \cdot \cos a^{-1}$$

$$2 \cdot a_S \cdot a \cdot a_G^{1/2} \cdot N_A = i^{2i}$$

$$2 \cdot a_S \cdot a \cdot N_A \cdot a_G^{1/2} \cdot a_S \cdot \cos a^{-1} = i^{2i} \cdot i^{2i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a_S^2 \cdot a_G^{1/2} \cdot N_A = i^{4i}$$

$$4 \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_S^4 \cdot a_G \cdot N_A^2 = i^{8i} \quad (94)$$

$$a_S \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

$$2 \cdot N_A \cdot a_G^{1/2} = a^{-1} \cdot (e^{i/a} + e^{-i/a})$$

$$2 \cdot a_S \cdot a \cdot N_A \cdot a_G^{1/2} = i^{2i}$$

$$a_S \cdot (e^{i/a} + e^{-i/a}) \cdot 2 \cdot a_S \cdot a \cdot N_A \cdot a_G^{1/2} = 2 \cdot i^{2i} \cdot i^{2i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a_S^2 \cdot a_G^{1/2} \cdot N_A = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_S^4 \cdot a_G \cdot N_A^2 = i^{8i} \quad (95)$$

Also resulting the expressions with power of two:

$$2^{80} \cdot a_S \cdot a \cdot a_G^{1/2} = i^{2i}$$

$$2^{160} \cdot a_S^2 \cdot a^2 \cdot a_G = i^{4i} \quad (96)$$

$$2^{80} \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_S^2 \cdot a_G^{1/2} = i^{4i}$$

$$2^{160} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_S^4 \cdot a_G = i^{8i} \quad (97)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot a_S^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_S^4 \cdot a_G \cdot N_A^2 = i^{8i}$$

(Dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions)

The figure 13 below shows the geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions.

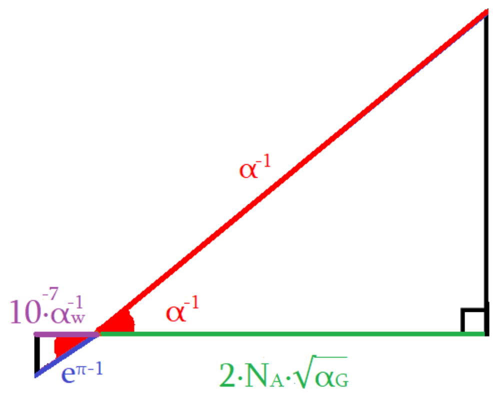

Figure 13. *Geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions*

We will use the previous expressions to resulting the unity formulas that connects the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant α_G:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$2\cdot e^{\pi}\cdot\alpha_s\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A=1$$

$$2\cdot\alpha_s\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A=i^{2i}$$

$$2\cdot e^{\pi}\cdot 10^7\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A=e$$

$$4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=e^2 \quad (98)$$

$$2\cdot 10^7\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A=i^{2i}\cdot e$$

$$4\cdot 10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=i^{4i}\cdot e^2 \quad (99)$$

Also resulting the mathematical formulas that connects the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=e^2 \quad (100)$$

$$e^2\cdot\mu^2=4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \quad (101)$$

$$e^2\cdot\mu\cdot N_1=4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha^3\cdot N_A^2 \quad (102)$$

$$4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=e^2 \quad (103)$$

$$e^2\cdot\mu^3=4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \quad (104)$$

$$e^2\cdot\mu^2=4\cdot 10^{14}\cdot e^{2\pi}\cdot\alpha_w^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \quad (105)$$

$$e^2\cdot\mu=4\cdot e^{\pi}\cdot 10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \quad (106)$$

Other equivalent forms of the equations are:

$$4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=i^{4i}\cdot e^2 \quad (107)$$

$$i^{4i}\cdot e^2\cdot \mu^2=4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha^2\cdot \alpha_{G(p)}\cdot N_A^2 \quad (108)$$

$$i^{4i}\cdot e^2\cdot \mu\cdot N_1=4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha^3\cdot N_A^2 \quad (109)$$

$$4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1=i^{4i}\cdot e^2 \quad (110)$$

$$i^{4i}\cdot e^2\cdot \mu^3=4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha\cdot \alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \quad (111)$$

$$i^{4i}\cdot e^2\cdot \mu^2=4\cdot 10^{14}\cdot e^{2\pi}\cdot \alpha_W^2\cdot \alpha_G\cdot \alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \quad (112)$$

$$i^{4i}\cdot e^2\cdot \mu=4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha\cdot \alpha_G\cdot \alpha_{G(p)}\cdot N_A^2\cdot N_1 \quad (113)$$

$$\alpha_W^{-1}\cdot \alpha_S^2=i^{2i}\cdot 10^7$$

$$\alpha_S^2=i^{2i}\cdot 10^7\cdot \alpha_W$$

$$2\cdot \alpha\cdot \cos\alpha^{-1}\cdot \alpha_S^2\cdot \alpha_G^{1/2}\cdot N_A=i^{4i}$$

$$2\cdot \alpha\cdot \cos\alpha^{-1}\cdot i^{2i}\cdot 10^7\cdot \alpha_W\cdot \alpha_G^{1/2}\cdot N_A=i^{4i}$$

$$2\cdot 10^7\cdot \alpha\cdot \cos\alpha^{-1}\cdot \alpha_W\cdot \alpha_G^{1/2}\cdot N_A=i^{2i}$$

$$4\cdot 10^{14}\cdot \alpha^2\cdot \cos^2\alpha^{-1}\cdot \alpha_W^2\cdot \alpha_G\cdot N_A^2=i^{4i} \quad (114)$$

$$\alpha\cdot (e^{i/\alpha}+e^{-i/\alpha})\cdot \alpha_S^2\cdot \alpha_G^{1/2}\cdot N_A=i^{4i}$$

$$\alpha\cdot (e^{i/\alpha}+e^{-i/\alpha})\cdot i^{2i}\cdot 10^7\cdot \alpha_W\cdot \alpha_G^{1/2}\cdot N_A=i^{4i}$$

$$10^7\cdot \alpha\cdot (e^{i/\alpha}+e^{-i/\alpha})\cdot \alpha_W\cdot \alpha_G^{1/2}\cdot N_A=i^{2i}$$

$$10^{14}\cdot \alpha^2\cdot (e^{i/\alpha}+e^{-i/\alpha})^2\cdot \alpha_W^2\cdot \alpha_G\cdot N_A^2=i^{4i} \quad (115)$$

Also resulting the expression with power of two:

$$2^{80}\cdot 10^7\cdot \alpha_W\cdot \alpha\cdot \alpha_G^{1/2}=i^{2i}\cdot e$$

$$2^{160}\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha^2\cdot \alpha_G=i^{4i}\cdot e^2 \quad (116)$$

$$2^{80}\cdot 10^7\cdot \alpha\cdot (e^{i/\alpha}+e^{-i/\alpha})\cdot \alpha_W\cdot \alpha_G^{1/2}=i^{2i}$$

$$2^{160}\cdot 10^{14}\cdot \alpha^2\cdot (e^{i/\alpha}+e^{-i/\alpha})^2\cdot \alpha_W^2\cdot \alpha_G=i^{4i} \quad (117)$$

We reached the conclusion of the dimensionless unification of the weak nuclear,the gravitational and electromagnetic forces:

$$4\cdot 10^{14}\cdot \alpha_W^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=i^{4i}\cdot e^2$$

$$10^{14}\cdot \alpha^2\cdot (e^{i/\alpha}+e^{-i/\alpha})^2\cdot \alpha_W^2\cdot \alpha_G\cdot N_A^2=i^{8i}$$

(Dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions)

The figure 14 below shows the variation of the coupling constants of the four fundamental interactions of physics as a function of energy.

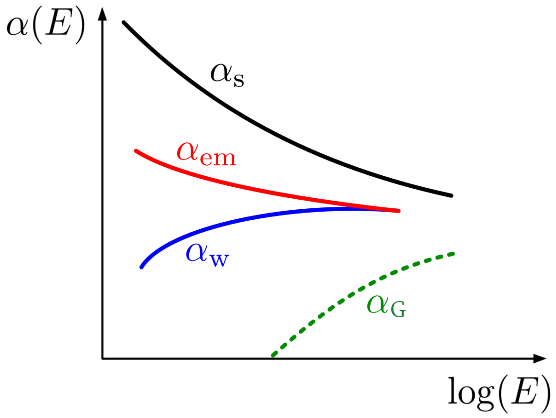

Figure 14. *Variation of the coupling constants of the four fundamental interactions of physics as a function of energy.*

The figure 15 below shows the geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions

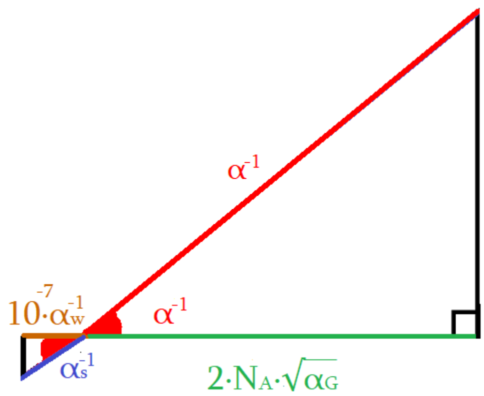

Figure 15. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions*

We will use the previous expressions to resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant α_G:

$$\alpha_w^{-1}\cdot\alpha_s^2=i^{2i}\cdot 10^7$$

$$2\cdot\alpha_s\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}=i^{2i}$$

$$\alpha_w^{-1}\cdot\alpha_s^2=2\cdot 10^7\cdot\alpha_s\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}$$

$$\alpha_w^{-1}\cdot\alpha_s=2\cdot 10^7\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}$$

$$2\cdot 10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot\alpha_s^{-1}=1$$

$$\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot\alpha_s^{-1}=(2\cdot 10^7\cdot N_A)^{-1} \tag{118}$$

$$2\cdot 10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}=\alpha_s$$

$$\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot\alpha_s^{-2}=(2\cdot 10^7\cdot N_A)^{-2} \tag{119}$$

So the beautiful unity formula that connects the strong coupling constant α_s,the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant α_G is:

$$(2\cdot 10^7\cdot N_A\cdot\alpha_w\cdot\alpha)^2\cdot\alpha_G=\alpha_s^2$$

$$4\cdot 10^{14}\cdot N_A^2\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G=\alpha_s^2 \tag{120}$$

Sometimes the gravitational coupling constant for the proton $\alpha_{G(p)}$ is used instead of the gravitational coupling constant α_G for the electron:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G$$

$$\alpha_G^{1/2}=\alpha_{G(p)}^{1/2}\cdot\mu^{-1}$$

$$\alpha_s\cdot\mu\cdot(\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2})^{-1}=2\cdot10^7\cdot N_A$$

$$\alpha_s\cdot\mu=2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}$$

$$2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}\cdot\alpha_s^{-1}\cdot\mu^{-1}=1$$

$$2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}\cdot\alpha_s^{-1}=\mu\cdot\alpha_s$$

$$\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}\cdot\alpha_s^{-1}=(2\cdot10^7\cdot N_A)^{-1}\cdot\mu \qquad (121)$$

So the beautiful unity formula that connect the strong coupling constant α_s,weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant $\alpha_{G(p)}$ for the proton is:

$$(2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha)^2\cdot\alpha_{G(p)}=\mu^2\cdot\alpha_s^2$$

$$4\cdot10^{14}\cdot N_A^2\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}=\mu^2\cdot\alpha_s^2 \qquad (122)$$

$$\cos\alpha^{-1}=2\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A$$

$$2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}=\alpha_s$$

$$2\cdot\alpha\cdot\alpha_G^{1/2}\cdot N_A\cdot2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}=\alpha_s\cos\alpha^{-1}$$

$$4\cdot10^7\cdot\alpha^2\cdot\alpha_G\cdot\alpha_w\cdot N_A^2=\alpha_s\cos\alpha^{-1}$$

$$\alpha^{-1}\cdot(e^{i/\alpha}+e^{-i/\alpha})=4\cdot N_A\cdot\alpha_G^{1/2}$$

$$2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}=\alpha_s$$

$$2\cdot10^7\cdot N_A\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot4\cdot N_A\cdot\alpha_G^{1/2}=\alpha_s\cdot\alpha^{-1}\cdot(e^{i/\alpha}+e^{-i/\alpha})$$

$$8\cdot10^7\cdot N_A^2\cdot\alpha_w\cdot\alpha^2\cdot\alpha_G=\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})$$

Resulting the mathematical formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2 \qquad (123)$$

$$\mu^2\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \qquad (124)$$

$$\mu\cdot N_1\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^3\cdot N_A^2 \qquad (125)$$

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1 \qquad (126)$$

$$\mu^3\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \qquad (127)$$

$$\mu\cdot\alpha_s=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2 \qquad (128)$$

$$\mu\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \qquad (129)$$

Also resulting the expressions with power of two:

$$2^{80}\cdot10^7\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot\alpha_s^{-1}=1$$

$$2^{160}\cdot 10^{14}\cdot a_w^2\cdot a^2\cdot a_G\cdot a_s^{-2}=1 \quad (130)$$

$$a_s=2^{80}\cdot 10^{7}\cdot a_w\cdot a\cdot a_G^{1/2}$$

$$a_s^2=2^{160}\cdot 10^{14}\cdot a_w^2\cdot a^2\cdot a_G \quad (131)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$a_s^2=4\cdot 10^{14}\cdot a_w^2\cdot a^2\cdot a_G\cdot N_A^2$$

$$8\cdot 10^{7}\cdot N_A^2\cdot a_w\cdot a^2\cdot a_G=a_s\cdot(e^{i/a}+e^{-i/a})$$

(Dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions)

In [20] and [21] it presented the theoretical value of the Gravitational constant $G=6.67448\times 10^{-11}\ m^3/kg\cdot s^2$. This value is very close to the CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. For Milne, the space was not a structured object but merely a frame of reference in which relations such as this could accommodate Einstein's conclusions:

$$G=\frac{c^3}{\mathrm{M}_U}T_U$$

According to this relationship, G increases with time. Dirac hypothesized that the constant of universal attraction G varies with time. Dirac's hypothesis went so far as to claim that such coincidences could be explained if the very physical constants changed with T_U, especially the gravitational constant G, which must decrease with time:

$$G\approx\frac{1}{t}$$

The gravitational constant is defined as:

$$G=\alpha_G\frac{\hbar c}{m_e^2}$$

The expressions for the gravitational constant G in terms of Planck units are:

$$G=\frac{c^3 l_{pl}^2}{\hbar}=\frac{\hbar c}{m_{pl}^2}=\frac{l_{pl}c^2}{m_{pl}}=\frac{c^5 t_{pl}^2}{\hbar}$$

A surprisingly close relationship between gravity and the electrostatic interaction. The gravitational constant G and the Coulomb constant ke are expressed in terms of Planck units as:

$$G=\frac{K_e q_e^2}{a m_{pl}^2}$$

Also another beautiful expression that proves the close relationship between gravity and electrostatic interaction is:

$$G=\frac{\alpha c^4 l_{pl}^2}{K_e q_e^2}$$

The 2018 CODATA recommended value of gravitational constant is $G=6.67430\times10^{-11}$ $m^3/kg{\cdot}s^2$ with standard uncertainty 0.00015×10^{-11} $m^3/kg{\cdot}s^2$ and relative standard uncertainty $2.2\text{x}10^{-5}$. In August 2018,a Chinese research group announced new measurements based on torsion balances, $6.674184(78)\times10^{-11}$ $m^3/kg{\cdot}s^2$ and $6.674484(78)\times10^{-11}$ $m^3/kg{\cdot}s^2$ based on two different methods [22]. Now we will find the formulas for the gravitational constant G using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant a_G can be written in the form:

$$4\cdot e^2\cdot N_A{}^2\cdot a^2\cdot a_G=1$$

$$a_G=(2\cdot e\cdot a\cdot N_A)^{-2} \tag{132}$$

Therefore from this expression the formula for the gravitational constant is:

$$G=(2e\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \tag{133}$$

The gravitational coupling constant a_G can be written in the forms:

$$4\cdot e^{2\pi}\cdot a_s{}^2\cdot a^2\cdot a_G\cdot N_A{}^2=1$$

$$a_G=(2\cdot e^{\pi}\cdot a_s\cdot a\cdot N_A)^{-2} \tag{134}$$

$$4\cdot a_s{}^2\cdot a^2\cdot a_G\cdot N_A{}^2=i^{4i}$$

$$a_G=i^{4i}\cdot(2\cdot a_s\cdot a\cdot N_A)^{-2} \tag{135}$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G=(2e^{\pi}\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \tag{136}$$

$$G=i^{4i}(2\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2} \tag{137}$$

The gravitational coupling constant a_G can be written in the form:

$$4\cdot10^{14}\cdot e^{2\pi}\cdot a_w{}^2\cdot a^2\cdot a_G\cdot N_A{}^2=e^2$$

$$a_G=(2\cdot e^{\pi-1}\cdot10^7\cdot a_w\cdot a\cdot N_A)^{-2} \tag{138}$$

$$4\cdot10^{14}\cdot a_w{}^2\cdot a^2\cdot a_G\cdot N_A{}^2=i^{4i}\cdot e^2$$

$$a_G=i^{4i}\cdot e^2\cdot(2\cdot10^7\cdot a_w\cdot a\cdot N_A)^{-2} \tag{139}$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G=\left(2e^{\pi-1}10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{140}$$

$$G=i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{141}$$

The gravitational coupling constant a_G can be written in the form:

$$4\cdot10^{14}\cdot N_A{}^2\cdot a_w{}^2\cdot a^2\cdot a_G=a_s{}^2$$

$$\alpha_G = \alpha_s^2 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A)^{-2} \tag{142}$$

Therefore from this expression the formula for the gravitational constant is:

$$G = \alpha_s^2 \left(2\,10^7 \alpha_w \alpha \mathrm{N_A}\right)^{-2} \frac{\hbar c}{m_e^2} \tag{143}$$

Now we will find the theoretical value of the Gravitational constant G using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant α_G can be written in the form:

$$\alpha^{-2} \cdot \cos^2 \alpha^{-1} = 4 \cdot \alpha_G \cdot N_A^2$$

$$\alpha_G = (2 \cdot \alpha \cdot N_A)^{-2} \cdot \cos^2 \alpha^{-1} \tag{144}$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2\alpha \mathrm{N_A})^{-2} \cos^2 \alpha^{-1} \frac{\hbar c}{m_e^2} \tag{145}$$

Using the 2018 CODATA recommended value of the the fundamental constants resulting the theoretical value of the Gravitational constant G:

$$G = 6.67448 \times 10^{-11}\ m^3/kg \cdot s^2 \tag{146}$$

4. Dimensionless unification of atomic physics and cosmology

In [23] and [24] resulting in the dimensionless unification of atomic physics and cosmology. The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha\sqrt{\alpha_G} r_e}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha} r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \qquad (147)$$

with numerical value:

$$\alpha_g = 1.886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot \alpha^{-6}$$

$$\alpha_g^2 = \left(\frac{\alpha_G}{\alpha^2}\right)^3$$

Two approaches for Archimedes constant π are:

$$6 \cdot 7^{103} \cdot \pi^5 \approx 2^{300} \qquad (148)$$

$$6 \cdot \pi^5 \approx 150^{3/2} - 1 \qquad (149)$$

A approach for the Gelfond's constant e^π is:

$$e^\pi \simeq \frac{55}{\pi}\sqrt{\frac{2}{\ln \pi}} \qquad (150)$$

A approximation expression that connects the golden ratio φ,the Archimedes constant π and the Euler's number e is:

$$2^2 11^2 e \simeq 3^4 \varphi^5 \sqrt[3]{\pi} \qquad (151)$$

Two approximations expressions that connects the golden ratio φ,the Archimedes constant π,the Euler's number e and the Euler's constant γ are:

$$4e^2 \gamma \ln^2(2\pi) \simeq \sqrt{3^3}\varphi^5 \qquad (152)$$

$$\sqrt{3^5} e \gamma \ln(2\pi) \sqrt[3]{\pi} \simeq 11^2 \qquad (153)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π,the Euler's number e and the Euler's constant γ is:

$$\alpha_g = [e \cdot \gamma \cdot \ln^2(2 \cdot \pi)]^{-1} \times 10^{-60} = 1.886837 \times 10^{-61} \qquad (154)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \qquad (155)$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant π is:

$$\alpha_g = \frac{\sqrt{3^5}\sqrt[3]{\pi}}{11^2} \times 10^{-60} = 1,886837 \times 10^{-61} \tag{156}$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's constant γ is:

$$\alpha_g = \frac{7\varphi\gamma^2}{2} \times 10^{-60} = 1,886826 \times 10^{-61} \tag{157}$$

The expression that connects the gravitational fine-structure constant α_g with the Archimedes constant and the golden ratio φ is:

$$\alpha_g = \frac{2\pi}{3\varphi^5} \times 10^{-60} = 1,888514 \times 10^{-61} \tag{158}$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3} \tag{159}$$

Also apply the expressions:

$$(2\cdot e\cdot \alpha^2\cdot N_A)^3\cdot \alpha_g=1$$

$$8\cdot e^3\cdot \alpha^6\cdot \alpha_g\cdot N_A^3=1$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=i^{6i}\cdot(2\cdot \alpha_s\cdot \alpha^2\cdot N_A)^{-3} \tag{160}$$

Also apply the expression:

$$(2\cdot \alpha_s\cdot \alpha^2\cdot N_A)^3\cdot \alpha_g=i^{6i}$$

$$8\cdot \alpha_s^3\cdot \alpha^6\cdot \alpha_g\cdot N_A^3=i^{6i}$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=i^{6i}\cdot e^3\cdot(2\cdot 10^7\cdot \alpha_w\cdot \alpha^3\cdot N_A)^{-3} \tag{161}$$

Also apply the expression:

$$(2\cdot 10^7\cdot \alpha_w\cdot \alpha^3\cdot N_A)^3\cdot \alpha_g=i^{6i}\cdot e^3$$

$$8\cdot 10^{21}\cdot \alpha_w^3\cdot \alpha^9\cdot \alpha_g\cdot N_A^3=i^{6i}\cdot e^3$$

Resulting the unity formulas for the gravitational fine-structure constant α_g:

$$\alpha_g=(10^7\cdot \alpha_w\cdot \alpha_G^{1/2}\cdot e^{-1}\cdot \alpha_s^{-1}\cdot \alpha^{-1})^3 \tag{162}$$

Also apply the expressions:

$$\alpha_g=10^{21}\cdot \alpha_w^3\cdot \alpha_G^{3/2}\cdot \alpha_s^{-3}\cdot \alpha^{-3}\cdot e^{-3}$$

$$\alpha_g\cdot \alpha_s^3\cdot \alpha^3\cdot e^3=10^{21}\cdot \alpha_w^3\cdot \alpha_G^{3/2}$$

So the unity formula for the gravitational fine-structure constant α_g is:

$$\alpha_g^2=(10^{14}\cdot \alpha_w^2\cdot \alpha_G\cdot e^{-2}\cdot \alpha_s^{-2}\cdot \alpha^{-2})^3 \tag{163}$$

Also apply the expressions:

$$\alpha_g^2=10^{42}\cdot\alpha_w^6\cdot\alpha_G^3\cdot e^{-6}\cdot\alpha_s^{-6}\cdot\alpha^{-6}$$

$$e^6\cdot\alpha_s^6\cdot\alpha^6\cdot\alpha_g^2=10^{42}\cdot\alpha_w^6\cdot\alpha_G^3$$

$$\alpha_g^2\cdot(e\cdot\alpha_s\cdot\alpha)^6=(10^{14}\cdot\alpha_w^2\cdot\alpha_G)^3$$

Resulting the unity formula for the gravitational fine-structure constant α_g:

$$\alpha_g=i^{6i}\cdot(10^7\cdot\alpha_w\cdot\alpha_G^{1/2}\cdot\alpha_s^{-2}\cdot\alpha^{-1})^3$$

$$\alpha_g=10^{21}\cdot i^{6i}\cdot(\alpha_w\cdot\alpha_G^{1/2}\cdot\alpha_s^{-2}\cdot\alpha^{-1})^3$$

$$\alpha_g=10^{21}\cdot i^{6i}\cdot\alpha_w^3\cdot\alpha_G^{3/2}\cdot\alpha_s^{-6}\cdot\alpha^{-3} \tag{164}$$

Also apply the expressions:

$$\alpha_g^{1/3}\cdot\alpha_s^2\cdot\alpha\cdot\alpha_w^{-1}\cdot\alpha_G^{-1/2}=i^{2i}\cdot10^7$$

$$\alpha_g\cdot\alpha_s^6\cdot\alpha^3=10^{21}\cdot i^{6i}\cdot\alpha_w^3\cdot\alpha_G^{3/2}$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g^2=i^{6i}\cdot(10^{14}\cdot\alpha_w^2\cdot\alpha_G\cdot\alpha_s^{-4}\cdot\alpha^{-2})^3$$

$$\alpha_g^2=10^{42}\cdot i^{12i}\cdot(\alpha_w^2\cdot\alpha_G\cdot\alpha_s^{-4}\cdot\alpha^{-2})^3$$

$$\alpha_g^2=10^{42}\cdot i^{12i}\cdot\alpha_w^6\cdot\alpha_G^3\cdot\alpha_s^{-12}\cdot\alpha^{-6} \tag{165}$$

Also apply the expressions:

$$\alpha_g^2\cdot\alpha_s^{12}\cdot\alpha^6\cdot\alpha_w^{-6}\cdot\alpha_G^{-3}=i^{12i}\cdot10^{42}$$

$$(\alpha_s^6\cdot\alpha^3\cdot\alpha_g)^2=(10^{14}\cdot i^{4i}\cdot\alpha_w^2\cdot\alpha_G)^3$$

$$\alpha_s^{12}\cdot\alpha^6\cdot\alpha_g^2=10^{42}\cdot i^{12i}\cdot\alpha_w^6\cdot\alpha_G^3$$

So the unity formulas for the gravitational fine-structure constant α_g are:

$$\alpha_g=\left(\frac{10^7\alpha_w\sqrt{\alpha_G}}{e\alpha_s a}\right)^3 \tag{166}$$

$$\alpha_g^2=10^{42}\left(\frac{\alpha_G a_w^2}{e^2\alpha_s^2\alpha^2}\right)^3 \tag{167}$$

$$\alpha_g=10^{21}i^{6i}\left(\frac{\alpha_w\sqrt{\alpha_G}}{\alpha_s^2\alpha}\right)^3 \tag{168}$$

$$\alpha_g^2=10^{42}i^{12i}\left(\frac{\alpha_G\alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{169}$$

Laurent Nottale in [25] suggests a large-number relation:

$$\alpha\frac{m_{pl}}{m_e}=\left(\frac{L}{l_{pl}}\right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L=\sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d=\sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d=\sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g:

$$\alpha\frac{m_{pl}}{m_e}=\left(l_{pl}\sqrt{\Lambda}\right)^{-\frac{1}{3}}$$

$$\alpha_g=l_{pl}\sqrt{\Lambda}$$

$$\alpha_g=\sqrt{\frac{G\hbar\Lambda}{c^3}}$$

So the cosmological constant Λ equals:

$$\Lambda=\alpha_g^2 l_{pl}^{-2}$$

$$\Lambda=\frac{l_{pl}^4}{r_e^6}$$

$$\Lambda=\alpha_g^2\frac{c^3}{G\hbar}$$

$$\Lambda=\frac{G}{\hbar^4}\left(\frac{m_e}{a}\right)^6$$

Resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g=(2\cdot e\cdot\alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=(2\cdot e\cdot\alpha^2\cdot N_A)^{-6} \quad (170)$$

$$(2\cdot e\cdot\alpha^2\cdot N_A)^6\cdot l_{pl}^2\cdot\Lambda=1 \quad (171)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 \mathrm{N_A}\right)^{-6} \frac{c^3}{G\hbar} \tag{172}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-6} \tag{173}$$

$$(2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \tag{174}$$

For the cosmological constant equals:

$$\Lambda = i^{12i} \left(2\alpha_s a^2 N_A\right)^{-6} \frac{c^3}{G\hbar} \tag{175}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-6} \tag{176}$$

$$(2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \tag{177}$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 \left(2\,10^7 \alpha_w a^3 N_A\right)^{-6} \frac{c^3}{G\hbar} \tag{178}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} \left(\frac{\alpha_G a_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \tag{179}$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{180}$$

For the cosmological constant equals:

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \frac{c^3}{G\hbar} \tag{181}$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \tag{182}$$

$$\alpha_s^{12}\cdot\alpha^6\cdot l_{pl}^2\cdot\Lambda=10^{42}\cdot i^{12i}\cdot\alpha_G^3\cdot\alpha_w^6 \tag{183}$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \frac{c^3}{G\hbar} \tag{184}$$

In [26] we presented the Equation of the Universe:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \tag{185}$$

5. Dimensionless Unification of the Microcosm and the Macrocosm

In [27] , [28] and [29] we presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities.This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length l,time t,speed υ and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{\max}} = \frac{t_{min}}{t_{\max}} = \frac{\upsilon_{min}}{\upsilon_{\max}} = \frac{\mathrm{T}_{min}}{\mathrm{T}_{max}} \tag{186}$$

Energy E, mass M, action A,momentum P and entropy S have another min/max ratio,which is the square of α_g:

$$\alpha_g^2 = \frac{\mathrm{E}_{min}}{\mathrm{E}_{\max}} = \frac{\mathrm{M}_{min}}{\mathrm{M}_{\max}} = \frac{\mathrm{A}_{min}}{\mathrm{A}_{\max}} = \frac{\mathrm{P}_{min}}{\mathrm{P}_{\max}} = \frac{S_{min}}{S_{max}} \tag{187}$$

Force F has min/max ratio which is α_g^4:

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \tag{188}$$

Mass density has min/max ratio which is α_g^5:

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \tag{189}$$

In [30] we presented the Unification of the Microcosm and the Macrocosm. The length Planck l_{pl} defined as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The classical electron radius is given as:

$$r_e = \alpha^2 \alpha_0 = \frac{\hbar\alpha}{m_e c} = \frac{\lambda_c \alpha}{m_e c^2} = \frac{\mu_0}{4\pi}\frac{q_e^2}{m_e} = \frac{k_e q_e^2}{m_e c^2} = \frac{\alpha^3}{4\pi R_\infty}$$

The Bohr radius α_0 is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

Thus respectively the Compton wavelength λ_c of the electron with mass m_e is given by the formula:

$$\lambda_c = \frac{2\pi r_e}{\alpha} = \frac{h}{m_e c}$$

The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi \alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha \frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha \alpha_0}\right)^2$$

We proposed to be a lattice structure, in which its unit cells have sides of length $2 \cdot e \cdot l_{pl}$. Perhaps for the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot l_{pl} \tag{190}$$

From expressions apply:

$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} \cdot l_{min} = 2 \cdot l_{pl}$$

$$\cos \alpha^{-1} = \frac{2 l_{pl}}{l_{min}} \tag{191}$$

The figures 16 below show the geometric representation of the fundamental unit of length.

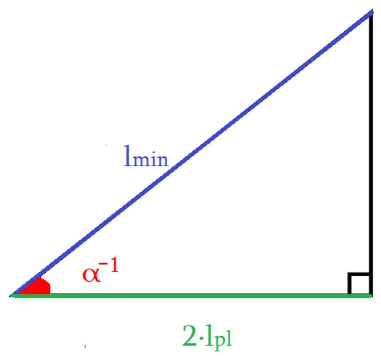

Figure 16. *Geometric representation of the fundamental unit of length.*

For the Bohr radius α_0 apply:

$$\alpha_0 = N_A \cdot l_{min}$$

$$\alpha_0 = 2 \cdot e \cdot N_A \cdot l_{pl} \tag{192}$$

The figures 17 below show the geometric representation of the relationship between the Bohr radius and the Planck length.

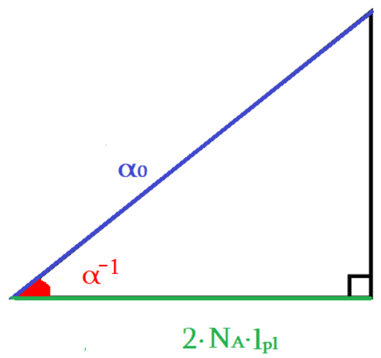

Figure 17. *Geometric representation of the relationship between the Bohr radius and the Planck length.*

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

For the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_\mathrm{H}^2}{R_d^2}$$

$$\Omega_\Lambda = \left(\frac{L_\mathrm{H}}{R_d}\right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\%$$

So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (193)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_\mathrm{H}^2}{2R_d^2} \quad (194)$$

The figure 18 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

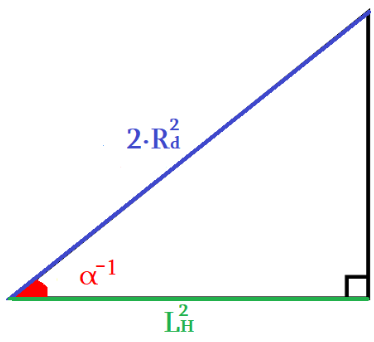

Figure 18. *Geometric representation of the relationship between the de Sitter radius and the Hubble length.*

The maximum distance l_{max} corresponds to the distance of the universe $l_U=c\cdot H_0^{-1}$. Therefore:

$$l_{max}=l_U=c\cdot H_0^{-1}$$

Length l has the max/min ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (195)$$

The maximum distance l_{max} corresponds to the distance of the universe:

$$l_{max}=L_H=c\cdot H_0^{-1}=\alpha_g^{-1}\cdot l_{min} \quad (196)$$

The figure 19 shows the geometric representation of the relationship between the maximum distance and the Planck length.

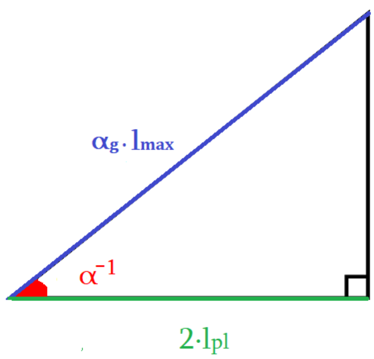

Figure 19. *Geometric representation of the relationship between the maximum distance and the Planck length.*

The value of the maximum distance l_{max} is:

$$l_{max}=4.656933\times10^{26}\text{ m}$$

The mass Planck m_{pl} can be defined by three fundamental natural constants,the speed of light in vacuum c,the reduced Planck constant ħ and the gravity constant G as:

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_{pl}c} = \frac{\mu_0 q_{pl}^2}{4\pi l_{pl}}$$

In [31] J.Forsythe and T. Valev found an extended mass relation for seven fundamental masses. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum "Gravity Atom" whose gravitational potential is equal to electrostatic potential , Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. We found a similar mass relation for seven fundamental masses:

$$M_n=\alpha^{-1}\cdot\alpha_g^{(2-n)/3}\cdot m_e \tag{197}$$

$$n=0,1,2,3,4,5,6$$

For $n=0$ M_0 is the minimum mass M_{min}:

$$M_0=\alpha^{-1}\cdot\alpha_g^{(2-0)/3}\cdot m_e$$

$$M_0=\alpha^{-1}\cdot\alpha_g^{2/3}\cdot m_e \tag{198}$$

For $n=1$ M_1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass,most likely a yet unobserved light particle:

$$M_1=\alpha^{-1}\cdot\alpha_g^{(2-1)/3}\cdot m_e$$

$$M_1=\alpha^{-1}\cdot\alpha_g^{1/3}\cdot m_e \tag{199}$$

For $n=2$ M_2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass:

$$M_2=\alpha^{-1}\cdot\alpha_g^{(2-2)/3}\cdot m_e$$

$$M_2=\alpha^{-1}\cdot m_e \tag{200}$$

For $n=3$ M_3 is the Planck mass:

$$M_3=\alpha^{-1}\cdot\alpha_g^{(2-3)/3}\cdot m_e$$

$$M_3=\alpha^{-1}\cdot\alpha_g^{-1/3}\cdot m_e \tag{201}$$

For $n=4$ is the central mass of a hypothetical quantum “Gravity Atom”.

$$M_4=\alpha^{-1}\cdot\alpha_g^{(2-4)/3}\cdot m_e$$

$$M_4=\alpha^{-1}\cdot\alpha_g^{-2/3}\cdot m_e \tag{202}$$

For $n=5$ is of the order of the Eddington mass limit of the most massive stars:

$$M_5=\alpha^{-1}\cdot\alpha_g^{(2-5)/3}\cdot m_e$$

$$M_5=\alpha^{-1}\cdot\alpha_g^{-1}\cdot m_e \tag{203}$$

For $n=6$ is the mass of the Hubble sphere and the mass of the observable universe.

$$M_6=\alpha^{-1}\cdot\alpha_g^{(2-5)/3}\cdot m_e$$

$$M_6=\alpha^{-1}\cdot\alpha_g^{-4/3}\cdot m_e \tag{204}$$

The similar mass relation for seven fundamental masses is:

$$M_n=\alpha_g^{-n/3}\cdot M_{min} \tag{205}$$

$$n=0,1,2,3,4,5,6$$

For $n=0$ M_0 is the minimum mass:

$$M_0=M_{min} \tag{206}$$

For $n=1$ M_1 is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass,most likely a yet unobserved light particle:

$$M_1=\alpha_g^{-1/3}\cdot M_{min} \tag{207}$$

For n=2 M2 is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass:

$$M_2=\alpha_g^{-2/3}\cdot M_{min} \tag{208}$$

For n=3 M3 is the Planck mass m_{pl} :

$$M_3=\alpha_g^{-1}\cdot M_{min} \tag{209}$$

For n=4 is the central mass of a hypothetical quantum "Gravity Atom".

$$M_4=\alpha_g^{-4/3}\cdot M_{min} \tag{210}$$

For n=5 is of the order of the Eddington mass limit of the most massive stars:

$$M_5=\alpha_g^{-5/3}\cdot M_{min} \tag{211}$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe.

$$M_6=\alpha_g^{-2}\cdot M_{min} \tag{212}$$

The following applies to the minimum mass M_{min}:

$$\mathrm{M}_{\min}c^2 = \frac{\hbar}{t_{max}}$$

$$\mathrm{M}_{\min}c^2 = \hbar \mathrm{H}_0$$

$$\mathrm{M}_{\min} = \frac{\hbar \mathrm{H}_0}{c^2} \tag{213}$$

$$\mathrm{M}_{\min} = \frac{\hbar}{c l_{max}} \tag{214}$$

So apply the expressions:

$$\mathrm{M}_{\min} = \frac{\hbar}{c}\sqrt{\Lambda} \tag{215}$$

$$\mathrm{M}_{\min} = \frac{m_{pl}^2}{\mathrm{M}_{max}} \tag{216}$$

$$\mathrm{M}_{\min} = \frac{m_{pl}^2}{\mathrm{M}_{max}} \tag{217}$$

Therefore for the minimum mass M_{min} apply:

$$\mathrm{M}_{\min} = \alpha_g m_{pl} \tag{218}$$

$$\mathrm{M}_{\min} = \frac{\alpha_G}{\alpha^3} m_e \tag{219}$$

$$\mathrm{M}_{\min} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \tag{220}$$

$$M_{min}=(2\cdot e\cdot N_A)^{-2}\cdot \alpha^{-1}\cdot m_e \tag{221}$$

For the value of the minimum mass M_{min} apply:

$$M_{min}=4.06578\times 10^{-69}\ \text{kg}$$

Mass M have max/min ratio,which is the square of α_g:

$$\alpha_g^2=\frac{M_{min}}{M_{max}} \tag{222}$$

For the maximum mass M_{max} applies:

$$M_{max}=\frac{F_{max}l_{max}}{c^2} \tag{223}$$

$$M_{max}=\frac{m_{pl}^2}{M_{min}} \tag{224}$$

$$M_{max}=\alpha^{-1}\cdot \alpha_g^{-4/3}\cdot m_e \tag{225}$$

$$M_{max}=\alpha^3\cdot \alpha_G^{-2}\cdot m_e \tag{226}$$

For the value of the maximum mass M_{max} apply:

$$M_{max}=1.153482\times 10^{53}\ \text{kg}$$

Also apply the expressions:

$$m_{p\ell}\cdot L_{max}=m_{max}\cdot l_{p\ell} \tag{227}$$

$$l^2_{max}\cdot M_{min}=l^2_{min}\cdot M_{max} \tag{228}$$

R. Adler in [32] calculated the energy ratio in cosmology,the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies,the rest energy of the electron E_e,and the binding energy of the hydrogen atom E_H. The rest energy of the electron E_e is defined as:

$$E_e=m_ec^2$$

The binding energy of the hydrogen atom E_H is defined as:

$$E_H=\frac{m_ee^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{E_H}{E_e}=\frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales,the Planck energy density ρ_{pl},and the density of the dark energy ρ_Λ. The Planck energy density is defined as:

$$\rho_{pl}=\frac{E_{pl}}{l_{pl}}=\frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density ρ_Λ is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \qquad (229)$$

The Planck time t_{pl} is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl}c^2}$$

For the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot l_{pl}$$

So for the minimum time t_{min} apply:

$$t_{\min} = \frac{l_{min}}{c}$$

$$t_{\min} = \frac{2el_{pl}}{c}$$

$$t_{min} = 2 \cdot e \cdot t_{pl}$$

From expressions apply:

$$\cos\alpha^{-1} = e^{-1}$$

$$\cos\alpha^{-1} \cdot t_{min} = 2 \cdot t_{pl}$$

$$\cos\alpha^{-1} = \frac{2t_{pl}}{t_{min}} \qquad (230)$$

The figures 20 below show the geometric representation of the fundamental unit of time.

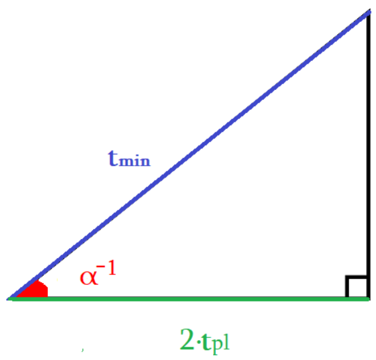

Figure 20. *Geometric representation of the fundamental unit of time.*

The maximum time period t_{max} is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe $t_U=H_0^{-1}$. Therefore:

$$t_{max}=t_U=H_0^{-1}$$

Time t has the min/max ratio which is.

$$\alpha_g = \frac{t_{min}}{t_{max}}$$

$$\alpha_g=t_{min} \cdot H_0 \quad (231)$$

$$\alpha_g=2 \cdot e \cdot t_{pl} \cdot H_0 \quad (232)$$

From [33] the gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. The time quantum in the brain t_B,the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (233)$$

6. Poincaré dodecahedral space

In [34] and [35] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy. In 2003 J.-P. Luminet in [36] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called “well-proportioned spaces” because they have a similar extent in all three dimensions. The sum of the contributions to the total density parameter Ω_0 at the current time is $\Omega_0=1.02\pm0.02$. Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda=0.73$,$\Omega_D=0.23$ and $\Omega_B=0.04$ [37]. The figure 21 shows the Geometric representation of the density parameter for the baryonic matter .

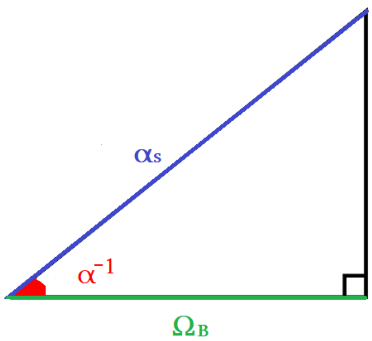

Figure 21. *Geometric representation of the the density parameter for the baryonic matter*

The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B=0.044\pm0.004$. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\% \quad (234)$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^{i}+1=0 \quad (235)$$

$$\Omega_B^{i}=i^2 \quad (236)$$

$$\Omega_B^{2i}=1 \quad (237)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B = e^{-1} \cdot \alpha_s \tag{238}$$

$$\Omega_B = \alpha_w^{-1} \cdot \alpha_s^2 \cdot 10^{-7} \tag{239}$$

$$\Omega_B = 2^{-1} \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) \tag{249}$$

$$\Omega_B = 2 \cdot N_A \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \tag{241}$$

$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) \tag{242}$$

$$\Omega_B = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \tag{243}$$

$$\Omega_B = 10^{-7} \cdot \alpha_g^{1/3} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_w^{-1} \cdot \alpha_G^{-1/2} \tag{244}$$

In [38] we presented the solution for the Density Parameter of Dark Energy. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\% \tag{245}$$

So apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \tag{246}$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot \cos\alpha^{-1} \tag{247}$$

So apply the expression:

$$\cos\alpha^{-1} = \frac{\Omega_\Lambda}{2} \tag{248}$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/\alpha} + e^{-i/\alpha} \tag{249}$$

The figure 22 shows the geometric representation of the density parameter for dark energy.

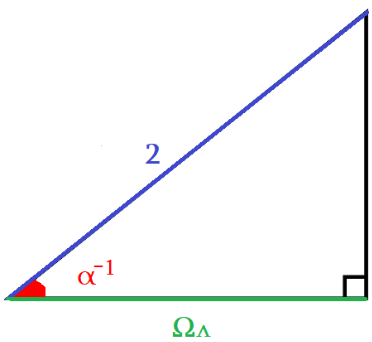

Figure 22. *Geometric representation of the the density parameter for the dark energy*

So apply the expression:

$$\cos\alpha^{-1} = \frac{L_H^2}{2R_d^2} \tag{250}$$

The figure 23 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

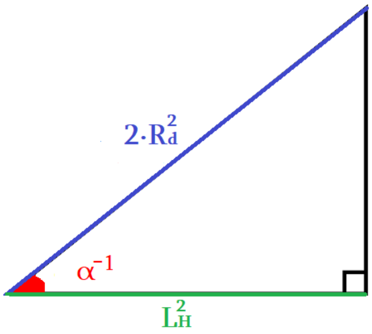

Figure 23. *Geometric representation of the relationship between the de Sitter radius and the Hubble length*

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2\cdot 10^{-7} a_s \cdot a_w^{-1} \quad (251)$$

$$\Omega_\Lambda = 2\cdot i^{2i} \cdot a_s^{-1} \quad (252)$$

$$\Omega_\Lambda = 2\cdot e\cdot 10^{-7}\cdot i^{2i} \cdot a_w^{-1} \quad (253)$$

$$\Omega_\Lambda = 2\cdot 10^{-7}\cdot a_s \cdot a_w^{-1} \quad (254)$$

$$\Omega_\Lambda = 4\cdot a \cdot a_G^{1/2}\cdot N_A \quad (255)$$

$$\Omega_\Lambda = i^{8i}\cdot a^{-2}\cdot a_s^{-4}\cdot a_G^{-1}\cdot N_A^{-2} \quad (256)$$

$$\Omega_\Lambda = 10^{7}\cdot i^{4i}\cdot a^{-1}\cdot a_w^{-1}\cdot a_G^{-1/2}\cdot N_A^{-1} \quad (257)$$

$$\Omega_\Lambda = 8\cdot 10^{7}\cdot N_A^{2}\cdot a_w\cdot a^{2}\cdot a_G \cdot a_s^{-1} \quad (258)$$

The figure 24 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

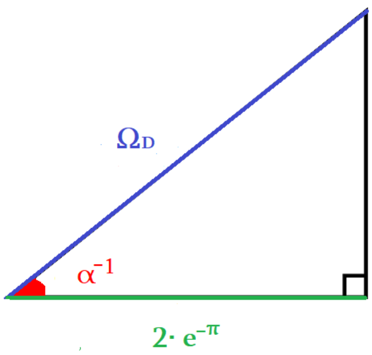

Figure 24. *Geometric representation of the density parameter of dark matter.*

The figure 25 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.

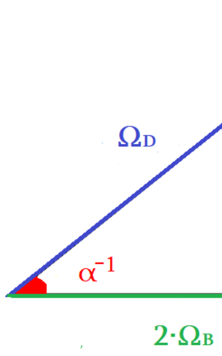

Figure 25. *Geometric representation of the relationship between the density parameter of dark and baryonic matter.*

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D=0.23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2\cdot e^{1-\pi}=2\cdot e\cdot i^{2i}=0.2349=23.49\% \quad (259)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2\cdot a_s \quad (260)$$

$$\Omega_D=2\cdot 10^7\cdot e^{-1}\cdot a_w \quad (261)$$

$$\Omega_D=2\cdot (i^{2i}\cdot 10^7\cdot a_w)^{1/2} \quad (262)$$

$$\Omega_D=4\cdot i^{2i}\cdot (e^{i/a}+e^{-i/a})^{-1} \quad (263)$$

$$\Omega_D=10^7\cdot a_w\cdot (e^{i/a}+e^{-i/a}) \quad (264)$$

$$\Omega_D=4\cdot 10^7\cdot a_w\cdot a\cdot a_G^{1/2}\cdot N_A \quad (265)$$

$$\Omega_D=16\cdot 10^7\cdot N_A^2\cdot a_w\cdot a^2\cdot a_G\cdot (e^{i/a}+e^{-i/a})^{-1} \quad (266)$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D=2\cdot e\cdot \Omega_B \quad (267)$$

The relationship between the density parameter of dark energy,dark matter and baryonic matter is:

$$\Omega_D\cdot \Omega_\Lambda=4\cdot \Omega_B \quad (268)$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_\Lambda$$

$$\Omega_0=e^{-\pi}+2\cdot e^{1-\pi}+2\cdot e^{-1} \quad (269)$$

$$\Omega_0=1.0139 \quad (270)$$

The figure 26 shows that the shape of the universe is Poincaré dodecahedral space.

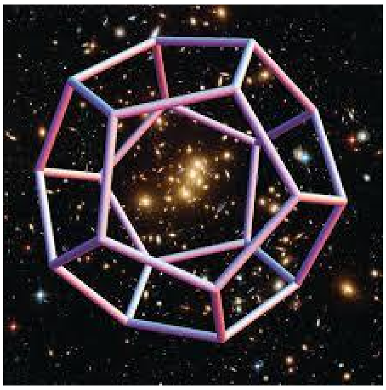

Figure 26. *The shape of the universe is Poincaré dodecahedral space.*

In [39] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP's observations with no fine-tuning required. Circle searching (Cornish,Spergel and Starkman,1998) may confirm the model's topological predictions,while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0 \approx 1.013 > 1$$

In [40] we proposed a possible solution for the cosmological parameters. The density parameter for normal baryonic matter is:

$$\Omega_B = e^{-\pi} = i^{2i} = 0.04321 = 4.32\% \tag{271}$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-\pi} = 6 \cdot i^{2i} = 0.25928 = 25.92\% \tag{272}$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-\pi} = 17 \cdot i^{2i} = 0.73463 = 73.46\% \tag{273}$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-\pi} = 24 \cdot i^{2i} = 1.03713 \tag{274}$$

In [41] we proposed a possible solution for the Equation of state in cosmology. In cosmology,the equation of state of a perfect fluid is characterized by a dimensionless number w,equal to the ratio of its pressure p to its energy density ρ:

$$w = \frac{p}{\rho}$$

From the dimensionless unification of the fundamental interactions the state equation w has value:

$$w = -24 \cdot e^{-\pi} = -24 \cdot i^{2i} = -1.037134 \tag{275}$$

From the dimensionless unification of the fundamental interactions for the measurable ordinary energy E(O) apply:

$$E(O) = i^{2i} \cdot m \cdot c^2$$

Also from the dimensionless unification of the fundamental interactions for the sum of the dark energy with the dark matter density of the universe E(D) apply:

$$E(D) = 23 \cdot i^{2i} \cdot m \cdot c^2$$

So for the total energy E apply:

$$E=K\cdot m\cdot c^2$$

$$E=E(O)+E(D)$$

$$E=i^{2i}\cdot m\cdot c^2+23\cdot i^{2i}\cdot m\cdot c^2$$

$$E=(i^{2i}+23\cdot i^{2i})\cdot m\cdot c^2$$

$$E=24\cdot i^{2i}\cdot m\cdot c^2 \qquad (276)$$

Other forms of the equation are:

$$E=12\cdot i^{2i}\cdot m\cdot c^2+12\cdot i^{2i}\cdot m\cdot c^2$$

$$E=12\cdot i^{2i}\cdot m\cdot c^2-i^2\cdot 12\cdot i^{2i}\cdot m\cdot c^2$$

$$E=12\cdot i^{2i}\cdot m\cdot c^2-12\cdot i^{2i}\cdot m\cdot (i\cdot c)^2$$

$$12\cdot i^{2i}\cdot m\cdot (i\cdot c)^2+E=12\cdot i^{2i}\cdot m\cdot c^2 \qquad (277)$$

7. Conclusions

We presented new exact formula for the fine-structure constant α in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

We propose in a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot\pi\cdot\ln 2$$

We propose the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32}=\varphi^{-42}\cdot F5^{160}\cdot L5^{47}\cdot L19^{40/19}$$

We propose the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3=7^{-1}\cdot 165^3\cdot\ln^{11}10$$

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}$$

We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42)$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231$$

$$\mu-807\cdot\alpha=1205\cdot\pi-518\cdot\varphi-411\cdot e$$

The new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0$$

The new formula for the Avogadro's number N_A is:

$$N_A = \left(2e\alpha\sqrt{\alpha_G}\right)^{-1}$$

The mathematical formulas that connect dimensionless physical constants are:

$$a_{G(p)}=\mu^2\cdot a_G$$

$$a=\mu\cdot N_1\cdot a_G$$

$$a\cdot\mu=N_1\cdot a_{G(p)}$$

$$a^2=N_1{}^2\cdot a_G\cdot a_{G(p)}$$

$$4\cdot e^2\cdot a^2\cdot a_G\cdot N_A{}^2=1$$

$$\mu^2=4\cdot e^2\cdot a^2\cdot a_{G(p)}\cdot N_A{}^2$$

$$\mu\cdot N_1=4\cdot e^2\cdot a^3\cdot N_A{}^2$$

$$4\cdot e^2\cdot a\cdot\mu\cdot a_G{}^2\cdot N_A{}^2\cdot N_1=1$$

$$\mu^3=4\cdot e^2\cdot a\cdot a_{G(p)}{}^2\cdot N_A{}^2\cdot N_1$$

$$\mu^2=4\cdot e^2\cdot a_G\cdot a_{G(p)}{}^2\cdot N_A{}^2\cdot N_1{}^2$$

$$\mu=4\cdot e^2\cdot a\cdot a_G\cdot a_{G(p)}\cdot N_A{}^2\cdot N_1$$

We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10\cdot(e^{i\mu/a}+e^{-i\mu/a})^{1/2}=13\cdot i$$

We presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{Euler'\ number}{Gerford's\ constant} = \frac{e}{e^{\pi}} = e^{1-\pi} = 0,11748..$$

It presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot a_s=10^7\cdot a_w$$

$$a_s{}^2=i^{2i}\cdot 10^7\cdot a_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$a_s\cdot(e^{i/a}+e^{-i/a})=2\cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7\cdot a_w\cdot(e^{i/a}+e^{-i/a})=2\cdot e\cdot i^{2i}$$

The dimensionless unification of the strong nuclear,the weak nuclear and electromagnetic interactions:

$$10^7\cdot a_w\cdot(e^{i/a}+e^{-i/a})=2\cdot a_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/\alpha} + e^{-i/\alpha})^2$$

The dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

$$\alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha}) \cdot \alpha_s^4 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of of the weak nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot \alpha^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^2 \cdot \alpha_w^2 \cdot \alpha_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha})$$

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron,the gravitational coupling constant of the proton $\alpha_{G(p)}$,the strong coupling constant α_s and the weak coupling constant α_w:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$\mu^2 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1$$

$$\mu \cdot \alpha_s = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2$$

$$\mu \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1$$

We found the formula for the Gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} e^2 \left(2\, 10^7 \alpha_w \alpha N_A\right)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = \alpha_s^2 \left(2\, 10^7 \alpha_w \alpha N_A\right)^{-2} \frac{\hbar c}{m_e^2}$$

It presented the theoretical value of the Gravitational constant $G=6.67448\times10^{-11}$ $m^3/kg \cdot s^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring 6.674184×10^{-11} $m^3/kg\cdot s^2$ and 6.674484×10^{-11} $m^3/kg\cdot s^2$-of-swinging and angular acceleration methods, respectively. We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{2i} \left(\frac{\alpha_G a_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6$$

$$a_s^{12} \cdot a^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} \left(2\alpha_s a^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} e^6 \left(2\, 10^7 \alpha_w a^3 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G\hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

We presented the law of the gravitational fine-structure constant a_g followed by ratios of maximum and minimum theoretical values for natural quantities. Length l,time t,speed υ and temperature T have the same max/min ratio which is.

$$\alpha_g = \frac{l_{min}}{l_{\max}} = \frac{t_{min}}{t_{\max}} = \frac{\upsilon_{min}}{\upsilon_{\max}} = \frac{T_{min}}{T_{max}}$$

Energy E, mass M, action A,momentum P and entropy S have another max/min ratio,which is the square of α_g:

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}}$$

Force F has max/min ratio which is α_g^4:

$$\alpha_g^4 = \frac{F_{min}}{F_{max}}$$

Mass density has max/min ratio which is α_g^5:

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}}$$

Perhaps for the minimum distance l_{min} apply:

$$l_{min}=2\cdot e\cdot l_{pl}$$

The maximum distance l_{max} is:

$$l_{max}=L_H=c\cdot H_0^{-1}=\alpha_g^{-1}\cdot l_{min}$$

For the minimum mass M_{min} apply:

$$M_{min} = \alpha_g m_{pl}$$

$$M_{min} = \frac{\alpha_G}{\alpha^3} m_e$$

For the the maximum mass M_{max} apply:

$$M_{max}=\alpha^3\cdot\alpha_G^{-2}\cdot m_e$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120}$$

Perhaps for the minimum time t_{min} apply:

$$t_{min}=2\cdot e\cdot t_{pl}$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter,dark matter and dark energy:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\%$$

$$\Omega_\Lambda=2\cdot e^{-1}=0.7357=73.57\%$$

$$\Omega_D=2\cdot e^{1-\pi}=2\cdot e\cdot i^{2i}=0.2349=23.49\%$$

The sum of the contributions to the total density parameter at the current time is $\Omega_0=1.0139$. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry,and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold,such as the Poincaré dodecahedral space,all of which are quotients of the 3-sphere. These results prove that the weather space is finite. The state equation w has value:

$$w=-24 \cdot e^{-\pi}=-24 \cdot i^{2i}=-1.037134$$

For as much as $w<-1$,the density actually increases with time [42].

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