Detection of the companions Algol-D and Algol-E and the speed of gravitational waves

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In the long-term data of the air pressure one finds the GW of the triple star system Algol at 8.073 μHz, which is phase-modulated with four different frequencies. Two frequencies can be assigned to Earth's orbit around the Sun and Algol-C. The other two could be generated by two previously undiscovered stars Algol-D and Algol-E. The high modulation index of all four PM can only be explained if the propagation speed of the gravitational waves is significantly lower than the speed of light.

1. Introduction

The star system *Algol* – also called *Beta Persei* – consists of (at least) three stars *Algol-A* ($m = 3.17m_\odot$), *Algol-B* ($m = 0.7m_\odot$) and *Algol-C* ($m = 1.76m_\odot$), which are only 93 light years away. The orbital period of the close pair A–B is 2.86732 days. The third star C orbits the binary system within 680 days [1,2]. The properties of this trio are well known because they have long been observed with electromagnetic waves.

Analyzing the gravitational waves (GW) from *Algol*, the picture changes: the system consists of two oscillators packed into a tiny volume (from an astronomical point of view), which generate and radiate GWs of different frequencies.

- The binary system A–B generates $f_{GWa} = 8.073104$ μHz.
- The binary system AB–C generates $f_{GWb} = 34.033$ nHz.

Possibly the two oscillators influence each other, because the wavelength of the GW exceeds the distance between the stars. Here, only f_{GWa} is examined. Since short interferometers such as LIGO or Virgo are "blind" to frequencies below 20 Hz, much longer antennas are required. Studies of the Earth's resonances revealed that long-term recordings of barometers contain signals of unknown origin [3]. In the records of weather stations that are far apart, one finds identical spectral lines with some properties of gravitational waves. The following investigation assumes that the atmosphere – just like any other object – reacts to GW and is therefore a suitable antenna for receiving GW. Perhaps, the earth's atmosphere is an inexpensive antenna for GW in the frequency range below 1000 Hz, which is hardly disturbed by earthquakes.

The search for GW with sufficient spectral resolution requires air pressure data with a minimum duration of twenty years (equation (1)). The German weather service DWD stores sufficiently long data chains [4], which may be used after some preparatory work. In order to reduce the influence of local peculiarities and data gaps of individual weather stations, the measured values of as many barometers as possible, which are distributed all over Germany and have been in operation almost without gaps for at least ten years, are summed up. Between the years 2000 and 2009, 64 data chains were found, between 2010 and 2019 only 51 data chains. Because the wavelength of the searched GW is at least a factor of 10^6 longer than the mutual distances between the barometers, all instruments react to the GW in phase. The coherent addition improves the S/N of the signals significantly and makes spectral lines visible that disappear in the noise when analyzing a single data chain.

Even nearby celestial bodies make the earth and atmosphere vibrate, the frequencies of which are tabulated in the tidal potential catalog HW95 [5]. According to this table, the moon should cause a strong excitation at 7.987 μHz, which is *not* detectable in the air pressure records. A 21 times fainter line at 8.07476 μHz differs significantly from f_{GWa} and is also untraceable. Possibly the GW of nearby binary systems affect the atmosphere stronger than nearby planets.

2. Identification of f_{GWa}

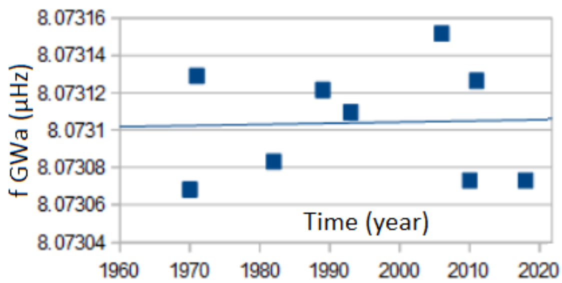

Figure 1): *Expected frequencies f_{GWa} of the Algol A–B system, calculated from orbital period measurements over the last decades. Assumption: $f_{GWa} = 2f_{orbit}$. The variable mass transfer between the stars could cause the striking differences.*

Between the years 1970 and 2018, the star pair A–B was frequently studied with EM waves, from which a mean orbital period of 2.86732 days was calculated. The drift was never determined because the values do not show any trend. If $f_{GWa} = 2f_{orbit}$ applies, a gravitational wave should be measurable at the frequency 8.0731 μHz (Figure 1). In order to achieve a frequency resolution Δf better than 1 nHz, data for Fourier analysis (FFT) must be collected over a long period of time. Filters with a narrow bandwidth take a long time to settle down. According to Küpfmüller [8], the minimum time is

$$T_{min} \cdot \Delta f \geq 0.5 \qquad (1)$$

As there is no peak in the air pressure spectrum at the expected frequency f_{GWa}, the noise level has to be reduced. The simplest option is to switch on the receiver with full sensitivity when the amplitude of the GW being sought is particularly large. Suppressing the noise during the rest of the time improves the S/N of the signal we are looking for [9]. Since the properties of the antenna are not yet known, a function V must be found empirically with which to multiply the data chain. Experiments have shown that the ansatz

$$V = 1 + a \cdot sin(2\pi t f_{day} + \varphi) \tag{2}$$

leads to success. Notes on the parameters of this equation: The rotation frequency of the earth f_{day} has to be calculated in the sidereal system so that the angle pulsar – center of the earth – barometer is constant during the measurement period of 20 years; the position of the sun is irrelevant. The factors a and φ are determined in such a way that the amplitude of the spectral line is maximum at f_{GWa} (Figure 2). A clear optimum results for $a = 1.05$ and $\varphi = 1.22$. If you modulate the received data with the calculated V, you get the spectrum shown in Figure 2. The FWHM is slightly larger than calculated with equation (1). This is reason to examine the properties of this line in detail.

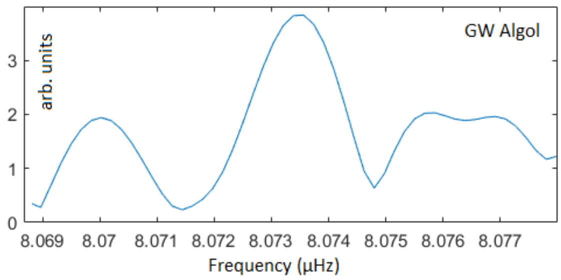

Figure 2): *Spectrum of the mean air pressure in Germany around the frequency* f_{GWa}. *The total duration of the database (20 years) limits the spectral resolution to 1.6 nHz.*

Technical note: Frequency and/or amplitude of the continuous signal at f_{GWa} may change in rhythm of several different frequencies (multiple modulation). Then next to f_{GWa} characteristic spectral lines appear (sidebands). If these are eliminated by filters, the corresponding modulations are removed. This does not affect the drift of the carrier frequency. Communications technology knows different methods (MSH method [9] or filters) to decode a specific modulation and to suppress others. The original goal of this paper – to measure only f_{GWa} and $\dot{f}_{GWa}$ – had to be expanded because measurements show extremely low-frequency modulations that cannot be eliminated by increasingly complex filters.

3. Signal processing and first results

In order to determine the frequency and drift of the GW, a narrow frequency range around f_{GWa} of the signal supplied by the antenna is shifted to the intermediate frequency $f_{ZF} = 7$ μHz using the heterodyne method. The frequency and drift of the auxiliary oscillator is iterated to reach a constant value of f_{ZF} throughout the analysis period (see Figure 3 with amplitude $a_D = 0$). The provisional result $f_{GWa} = 8.0735 \times 10^{-6}$ Hz and $\dot{f}_{GWa} = 3 \times 10^{-21}$ Hz/s shows:

- f_{GWa} is 0.4 nHz higher than the expected value determined with EM waves.
- The frequency drift $\dot{f}_{GWa}$ is unexpectedly small.

- A *constant* frequency cannot be achieved with any choice of parameters. If the bandwidth of the signal processing is constricted to 0.3 nHz, the results deviate more and more from the mean value. This bandwidth is much broader than one would expect for a linear drift. The analysis shows a non-linear, reproducible change of the frequency f_{ZF} within this bandwidth, which does not look random and is also not a simple sine function.

The method of selective integration (Appendix A) provides an answer: If you sum up f_{ZF} over the entire measurement period of 20 years with a phase-sensitive integrator, the envelope is not a straight line: Injecting a constant frequency, the slope would be strictly linear. A possible explanation is: f_{GWa} is phase modulated. This makes the envelope wavy and generates fluctuations in the same rhythm as the modulation frequency (compare Figure 4).

Result of the first measurement: f_{GWa} is not constant, the frequency drift is – within the scope of the preliminary measurement – not linear. The curvature appears to be periodic with a time constant exceeding 10 years. Such a slow modulation is difficult to detect because the baseline data period is only 20 years.

4. Measurement of frequency modulation and drift

There are assumptions that a fourth star *Algol D* orbits the known *Algol* trio with the period $T_D \approx 20$ years [7]. Then the pair of stars A-B would have to move in this rhythm and the frequency of the GW is phase-modulated (PM)[1].

To check, expand the iteration scheme (Figure 3): The auxiliary oscillator A of the heterodyne method (f_{Osz}) is phase modulated by a second auxiliary oscillator B with the initial value $f_D = 1/T_D \approx 1.5$ nHz. The parameters f_D, a_D and ϕ_D are iterated with the aim of making the very slow and non-linear frequency drift of f_{GWa} disappear. The solution provides clues as to the cause of this PM.

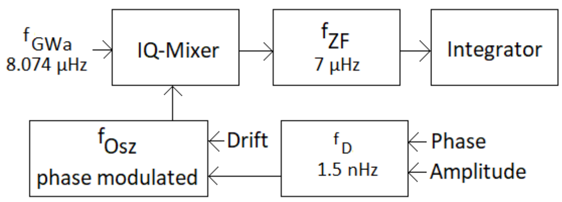

Figure 3): *Principle of the MSH method: The parameters phase and amplitude of the auxiliary oscillator f_{Osz} are iterated with the aim of a constant frequency f_{ZF}.*

The frequency of the local oscillator-A is calculated using the formula

$$f_{Osz} = f_{GWa} + f_{ZF} + t \cdot \dot{f}_{GWa} + a_D \cdot sin(2\pi t f_D + \phi_D) \tag{3}$$

The parameters have the following meaning:

[1] f_{GWa} may be modulated with different frequencies. If you are only looking for very low frequencies, you limit the bandwidth after the IQ mixer to 0.4 nHz and dampen higher-frequency modulations whose sidebands are outside the narrow filter range. Complete erasure is not possible.

f_{GWa} is the frequency of the suspected GW ($f_{GWa} \approx 8.0735 \times 10^{-6}$ Hz)
f_{ZF} is basically arbitrary. An unusually high value is chosen here because it allows the envelope of the selective integrator to be calculated more precisely. The shape of the envelope including the fringes is the key element of the following evaluation.
$\dot{f}_{GWa}$ is the frequency drift of f_{GWa} ($\dot{f}_{GWa} \approx 3 \times 10^{-21}$ Hz/s)
a_D is the modulation index of the PM assuming that the fourth star *Algol-D* describes a circular orbit around the well-known *Algol* trio.
f_D is the orbital frequency of the suspected companion *Algol-D*.
From the phase ϕ_D it follows when *Algol-D* is in front of or behind the *Algol* trio.

The aim is to determine the values of the five variables in equation (3) in such a way that the amplitude at the output of the selective integrator increases proportionally to the time. Then the intermediate frequency is unmodulated. The initial values of the parameters a_D, f_D and ϕ_D have to be found by trial and error since there are no clues and the variables influence each other.

By iterating equation (3) one finds a first solution that does not completely eliminate the ripple at the integrator output. This suggests that there is (at least) another companion *Algol-E*. In order to determine its orbital elements, equation (3) is supplemented with the addend $a_E \cdot \sin(2\pi t f_E + \phi_E)$. With the three additional parameters, the ripple may be further reduced, but not completely eliminated. The ripple at the integrator output disappears completely after the equation (3) is phase-modulated with a total of four frequencies.

The MSH method has a single parameter that can be chosen arbitrarily and influences the result: the bandwidth of the IF filter between the IQ mixer and the selective integrator (Figure 3). The iteration was calculated several times with different bandwidths (40 pm$<$BW$<$140 pm). The result is clear and reproducible:

1. Orbit of the Earth: $a_Y = 0.349686 \pm 0.00013$, $f_Y = (31.69789 \pm 0.000002) \times 10^{-9}$ Hz und $\phi_Y = 4.10554 \pm 0.00005$. (P $\approx$ 0.99967 years)

2. For *Algol-C* the following applies: $a_C = 3.06012 \pm 0.0001$, $f_C = (17.04600 \pm 0.00004) \times 10^{-9}$ Hz und $\phi_C = 0.162350 \pm 0.00003$ (P $\approx$ 1.85894 years)

3. For *Algol-D* we get: $a_D = 1.94034 \pm 0.0001$, $f_D = (2.2651 \pm 0.0014) \times 10^{-9}$ Hz und $\phi_D = 1.19229 \pm 0.00003$ (P $\approx$ 13.9895 years)

4. For *Algol-E* we get: $a_E = 0.97206 \pm 0.00004$, $f_E = (1.53493 \pm 0.00002) \times 10^{-9}$ Hz und $\phi_E = 0.790656 \pm 0.00001$ (P $\approx$ 20.6443 years)

5. The frequency drift of the GWa is $\dot{f}_{GWa} = +23.6(3) \times 10^{-20}$ Hz/s.

6. If these modulations are eliminated, f_{GW} becomes constant and the output amplitude of the phase-sensitive integrator increases almost exactly proportional to time. The very small remaining deviation from linearity ($\approx 10^{-14}$) suggests that *Algol-AB* has no other companions with P$<$ 200 years. Even slower changes in frequency cannot be clearly demonstrated due to the short period of data collection of only 20 years.

7. The perfectly linear increase in the amplitude at the integrator output proves that the amplitude of the GWa is constant throughout the measurement period.

8. A phase modulation with the frequency $f_{GWb} = 34.033$ nHz is not observed. It follows that the slow GW source (f_{GWb}) does not affect the higher frequency GW source (f_{GWa}) despite the small distance.

5. Interpretation from an astronomical point of view

Translating the above determined abstract results of the iteration of equation (3) into astronomical terms, the following relationships apply: All dates refer to the start of the analyzed data chains on 2000-01-01 (MJD = 51544) and apply under the condition that the corresponding celestial bodies describe circular orbits. The phase shift ϕ indicates the point in time at which the measurable instantaneous frequency is at its maximum. Then one has to add the frequency shift Δf produced by the Doppler effect to the average frequency f_{GWa}. The compilation of the results given above may be evaluated independently because all PM are linearly superimposed.

5.1. The Earth Orbit

The earth's orbit around the sun generates the phase modulation with the highest frequency $f_Y = 31.7$ nHz. The largest blue shift of f_{GWa} is measured at $365 \cdot \phi_Y / 2\pi = 239.$ day of the year. The highest red shift is measured on the 56th day; if the GW comes from the source *Algol*, we expect the maximum value on day 46 [6]. This corresponds to a measurement error of 2.7%.

The modulation index of a PM is defined as: $a_Y = \Delta f_Y / f_Y$. From this follows $\Delta f_Y = 11.084$ nHz – an unexpectedly large value that cannot be explained with previous assumptions. So far it has been assumed – without proof – that a GW propagates at the speed of light. Then the maximum value of the Doppler shift would be

$$\Delta f_Y = f_{GWa} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) \approx f_{GWa} \cdot \frac{v_{orbit}}{c} \approx f_{GWa} \cdot 10^{-4}. \tag{4}$$

In fact, one measures the much higher value $\Delta f_Y \approx f_{GWa} \cdot 13.73 \times 10^{-4}$! A measurement error of this magnitude can be ruled out after careful examination. What may cause the discrepancy? The formulas of the PM and the Doppler effect are well founded and confirmed a million times. What remains is the correction of the assumption, that GWs propagate at the speed of light – which has never been verified. The calculation of the receiving frequency f_{GWa} uses the longitudinal Doppler effect, in which the frequency is corrected relativistically. For maximum blueshift applies

$$f_{GWa} + \Delta f_Y = f_{GWa} \sqrt{1 - \left(\frac{v_{orbit}}{c}\right)^2} \cdot \frac{1}{1 - \frac{v_{orbit}}{v_{GW}}} \approx \frac{f_{GWa}}{1 - \frac{v_{orbit}}{v_{GW}}} \tag{5}$$

This results in

$$\frac{v_{orbit}}{v_{GWa}} = 1 - \frac{f_{GWa}}{f_{GWa} + \Delta f_Y} = 1.37 \times 10^{-3} \tag{6}$$

This intermediate result is used to calculate

$$v_{GWa} = \frac{v_{orbit}}{1.37 \times 10^{-3}} = \frac{27 \times 10^3\ m\ s^{-1}}{1.37 \times 10^{-3}} = 19.7 \times 10^6\ \frac{m}{s}. \tag{7}$$

The result is much smaller than the speed of light and is valid for $f_{GWa} \approx 8\ \mu$Hz. Possibly v_{GW} depends on the frequency (dispersion) and the gravitational field in the area. That's still speculation. After all, values smaller than c were measured earlier [11].

5.2. Algol-C

The decoding of the GW, PM confirms that *Algol-C* orbits the GW source *Algol-AB* with orbital period $P \approx 1.85894$ years = 679 days. Assuming a circular orbit, it follows from the phase angle $\phi_C = 0.162350$ that this companion on the $679 \cdot \phi_C/2\pi = 17.5$–th day after 2000-01-01 (MJD = 51544), i.e. on time $\text{MJD}_{\text{blue}} = 51561$ generated maximum blueshift of f_{GWa}. 340 days later ($\text{MJD}_{\text{rot}} = 51901$) it caused maximum redshift.

Previous observations suggest that the orbital plane of the inner pair *Algol-AB* has an inclination of $\approx 82^o$. If one assumes that the inclination of the orbital plane of *Algol-C* is 90^o, one would see *Algol-C* at time MJD = 51730 in front of *Algol-AB* and at time MJD = 52070 behind *Algol-AB*.

$\Delta f_C = 52.16$ nHz follows from the modulation index $a_C = \Delta f_C / f_C$ of the PM. The maximum value of the periodic frequency shift of f_{GWa} is the result of the Doppler effect, because the GW source rotates around the center of gravity of the *Algol* system. The calculation of the radial velocity is the task of classical astronomy. Considering the GW source *Algol-AB* as a star and *Algol-C* as a companion, Kepler's third law provides the orbital equation for the two-body system.

$$4\pi^2 (r_{AB} + r_C)^3 = G T^2 (m_A + m_B + m_C) \tag{8}$$

The radii refer to the center of gravity of the trio and because of the center of gravity theorem, we use $(m_A + m_B) \cdot r_{AB} = m_C \cdot r_C$ (the probable companions *Algol-D* and *Algol-E* are ignored here). With the measured values $m_A = 3.17 \cdot m_\odot$, $m_B = 0.70 \cdot m_\odot$, $m_C = 1.76 \cdot m_\odot$ and $m_C/(m_A + m_B) = 0.456$ [1], we get $r_{AB} = 1.261 \times 10^{11}$ m and $v_{AB} = 13.5 \times 10^3$ m/s. The GW source *Algol-AB* rotates with this orbital speed around the common center of gravity ABC (the orbital speed of *Algol-C* is about twice as large and cannot be measured because *Algol-C* emits no GW).

Analogous to the equations (6) and (7) we get

$$v_{GWa} = \frac{13.5 \times 10^3 \ m \ s^{-1}}{6.42 \times 10^{-3}} = 2.1 \times 10^6 \ \frac{m}{s}. \tag{9}$$

This result (about 0.7% of the speed of light) is the reference for the following calculations.

5.3. Algol-D

The GW of frequency f_{GWa} is also phase modulated with $f_D = 2.2651$ nHz. We assume a fourth star *Algol-D* orbiting the source of the GW with the orbital period $P \approx 14$ years. This star has not yet been observed with EM waves.

Assuming a circular orbit, it follows from the phase angle $\phi_D = 1.19229$ that this companion on $5110 \cdot \phi_D/2\pi = 970$–th day after 2000-01-01 (MJD = 51544), i.e. on Time $\text{MJD}_{\text{blue}} = 52514$ generated maximum blueshift of f_{GWa}. 2557 days later ($\text{MJD}_{\text{rot}} = 55071$) it caused maximum redshift.

If one assumes that the inclination of the orbital plane of *Algol-D* is 90^o, one would see *Algol-D* at time MJD = 53792 in front of *Algol-AB* and at time MJD = 56349 behind *Algol-AB*.

$\Delta f_D = 4.4$ nHz follows from the modulation index $a_D = \Delta f_D / f_D$ of the PM. This maximum value of the periodic frequency shift as a result of the Doppler effect is very small because *Algol-D* is far away from the center. This star has never been observed, its mass m_D is unknown. It circles the center of gravity, whose counterweight is formed by $m_A + m_B + m_C = 5.63 m_\odot$. m_D influences the orbit radius and may be determined in such a way that v_{GWa} either reaches the result of Section 5.1 or the result of Section 5.2. By linking the equations (5), (6) and (7) with Kepler's 3rd law, one obtains:

- For $m_D = 3.9 \cdot m_\odot$, we get $v_{GWa} = 19.7 \times 10^6 \ \frac{m}{s}$
- For $m_D = 0.303 \cdot m_\odot$, we get $v_{GWa} = 2.1 \times 10^6 \ \frac{m}{s}$

In the following section, we use $m_D = 0.303 \cdot m_\odot$.

5.4. Algol-E

The GW of frequency f_{GWa} is also phase modulated with $f_E = 1.53493$ nHz. The cause could be a fourth star *Algol-E* orbiting the source of the GW with the orbital period $P \approx 20.6$ years. This star has not yet been observed with EM waves.

Assuming a circular orbit, it follows from the phase angle $\phi_E = 0.790656$ that this companion on $7540 \cdot \phi_E/2\pi = 949$–th day after 2000-01-01 (MJD = 51544), i.e. on Time $\text{MJD}_{\text{blue}} = 52493$ generated maximum blueshift of f_{GWa}. 2557 days later ($\text{MJD}_{\text{rot}} = 56263$) it caused maximum redshift.

If one assumes that the inclination of the orbital plane of *Algol-E* is 90^o, one would see *Algol-E* at time MJD = 54378 in front of *Algol-AB* and at time MJD = 58148 behind *Algol-AB*.

$\Delta f_E = 1.5$ nHz follows from the modulation index $a_E = \Delta f_E / f_E$ of the PM. This maximum value of the periodic frequency shift as a result of the Doppler effect is very small because *Algol-E* is far away and its mass is probably very small. This star has never been observed, its mass m_E is unknown. It orbits the center of gravity, whose counterweight is formed by $m_A + m_B + m_C + m_D = 5.93 m_\odot$. m_E influences the orbit radius and may be determined in such a way that v_{GWa} either reaches the result of Section 5.1 or the result of Section 5.2. By linking the equations (5), (6) and (7) with Kepler's 3rd law, one obtains:

- For $m_E = 1.7 \cdot m_\odot$, we get $v_{GWa} = 19.7 \times 10^6 \ \frac{m}{s}$
- For $m_E = 0.12 \cdot m_\odot$, we get $v_{GWa} = 2.1 \times 10^6 \ \frac{m}{s}$

Presumably $m_E = 0.12 \cdot m_\odot$ applies.

6. Summary

From a communications point of view, decoding the phase modulations of f_{GWa} is a standard task, because the signal has a sufficient S/N. The opposite is true for the interpretation of the results from an astronomical point of view: The only way to explain the high values for the frequency deviation (Δf) is the assumption that gravitational waves at low frequencies around 8 μHz do not travel at the speed of light, but considerably more slowly. Even more amazing is that the value seems to depend on whether the transmitter or the receiver is moving in the radiation field. Both observations contradict the statements of the theory of relativity.

It is also unclear whether v_{GW} depends on the frequency, because the results at much higher frequencies around 60 Hz show no contradiction to the assumption $v_{GW} = c$ [9, 10]. Those measurements were performed using exactly the same methods as at 8 μHz. The diversity of antennas cannot cause these differences. Further measurements will provide more clarity. The frequently postulated identity of the propagation of light and gravitational waves requires a fundamental examination.

7. Data source

All barometric data is stored on the DWD [4] servers.

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A. Appendix Selective Integration

A phase sensitive integrator acts as an extremely narrow band filter with selectable frequency and a "memory" for the phase of already processed data.

The Data is a sequence of single values z_n and z_{n+1} measured at the separation of the sampling time T_s. If one analyzes the oscillations of a fixed frequency f, the phase angle increases with each step by $\alpha = 2\pi f T_s$. Although each value z_n results from the sum of many single frequencies, the selective integration hardly reacts to noise and neighboring frequencies. The realization of the integrator chosen here is based on the addition theorem of trigonometric functions. The basis are the two CORDIC equations

$$x_{n+1} = z_n + cos(\alpha)x_n + sin(\alpha)y_n \tag{10}$$

$$y_{n+1} = cos(\alpha)y_n - sin(\alpha)x_n \tag{11}$$

The choice of parameters determines the sequence of calculated values x_n and y_n:

- Without an injected signal ($z_n = 0$) and with initial values $x_1 = 0$ and $y_1 = 1$, the equations (10) and (11) calculate a table of values for $x = sin(2\pi t f)$ and $y = cos(2\pi t f)$. The amplitudes are constant.
- Setting $x_1 = y_1 = 0$ and feeding a monochromatic signal z_n of frequency f, the equations calculate an oscillation of a frequency whose amplitude increases proportionally in time.

- If the programmed and injected frequency differ or if the phase or amplitude of the injected signal z_n changes, the output signal of the integrator varies and the linear increase of the envelope is lost. If the signal is in phase opposition, the output signal decreases proportionally to time.
- If you feed a phase-modulated signal z_n, the envelope changes in rhythm with the modulation frequency.
- If only noise is fed in, the equations (10) and (11) calculate a low bandwidth frequency near f whose amplitude varies irregularly.

Figur 4 shows a typical result when the integrator processes a signal that is phase modulated with a very small modulation index $\eta \approx 10^{-4}$. This signal consists of the carrier frequency f_0 and the two adjacent sidebands $f_0 - f_m$ and $f_0 + f_m$. PM is difficult to measure using other methods because the amplitudes of the sidebands are much smaller than the amplitude of the carrier.

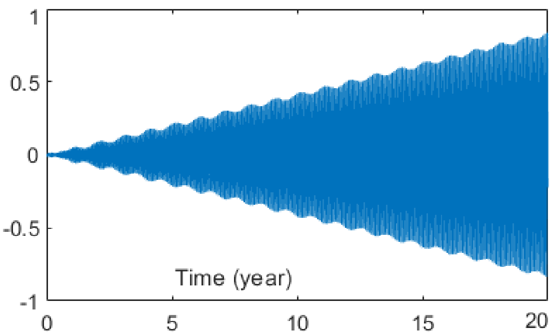

Figure 4): *Integrated amplitude of a PM signal from GWb of star system Algol. The ripple in the envelope is caused by alternating constructive and destructive interference of the carrier frequency and sidebands. Without modulation, the amplitude increase would be linear.*