On the Perception of Scale:

Connecting the Scales of Reality & Expanding Our Mathematical Tools

- Opening Remarks

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On the Perception of Scale in Science and Mathematics

Abstract

Over the last several centuries, science has discovered objects in the world along a continuum of scale. In one direction, we have found planets and stars, galaxies and galaxy clusters. In the other direction we have found cells and proteins, atoms and neutrinos. In order to locate and model this world, we use the 3 traditional directions of length, width and height. However inherent in all our measurements is the scale of what we are measuring – a continuum we do not directly see with our eyes. A key reason we do not understand this direction as part of our world is that we do not know how to measure along this continuum. We do not understand how to measure along this direction because we lack the mathematical tools to do so. Those tools require a numeric representational system with more power than our traditional decimal or positional based numerals. A system that can provide a single value for complex numbers and has built into it more operations than addition/subtraction, multiplication/division, and exponentiation/logarithms (from whence the new system gets its additional power).

The author presents some opening remarks on what is anticipated to be a much larger discussion, looking at a perspective of reality where objects at all levels of scale exist and interact together and considers some implications of utilizing more powerful mathematical tools than we have today.

Keywords: Science, Scale, Dimension, Numeric system, Complex values, Philosophy of Science

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Prologue

The history of science has been consistent in both a positive advancement of science and of significant jumps in knowledge as new ways of thinking about and observing our world occur. There are calls for such a jump today [39]. In particular is the question of connecting actions at the largest and smallest scales. There are calls for new theories and even for new mathematics [31]. There are calls for a paradigm shift [41]. What if a major paradigm shift is required? Such a shift would not just extend what we already know but would involve a true shift in how we understand reality. Could we be missing something crucial in our current understanding of reality and the universe that would lead to such a jump in understanding? We shall consider a shift involving a new perspective that is a unification or convergence of current physical theories with a new area of mathematics.

Part I: Scaling Up

1. Reality Crosses Scale

While there are many different disciplines of science, there is a basic assumption that all these different areas of study and experimentation apply to the same physical reality. Over the past few centuries, through the use of tools and technology, science has shown us objects we cannot see directly – in particular larger objects and smaller objects. It seems intuitive to understand that actions on the many different scales of science occur together. There are actions that occur on the sub-atomic level, the atomic level, the molecular level, the protein level, the cellular level, the tissue level, the organ level, ‘our’ level, the earth ecology and weather level, the solar system level, the stellar level, the galactic level, the galaxy cluster level. It would also seem intuitive that actions are occurring across all these levels ‘at the same time’. Sub-atomic particles whiz around, atoms shift their position in a molecule, a molecule moves within a cell, a cell interacts with other cells within a tissue, which will affect an organ, an organ moves with a body (eg. our heart beats), a body is impacted by the weather, our earth and sun move around our galaxy and our galaxy moves within a galaxy cluster. Actions are occurring at all levels at any moment.

Consider how we could measure the actions of our organs, blood vessels, cells, proteins and molecules ‘inside’ us. We typically think of our body as consisting of a three-dimensional volume, however, to measure the actions at all levels (organs, blood vessels, cells, proteins and molecules ‘inside’ us) requires we locate each of these objects at the appropriate scale, with each level taking up the same three-dimensional volume as our bodies at our scale. This means we have to stack each three-dimensional (3-D) scale ‘on-top’ of the other in order to model the actions of all these levels and associated objects together. This is precisely how to construct a four-dimensional mathematical space (see Figure 2 below).

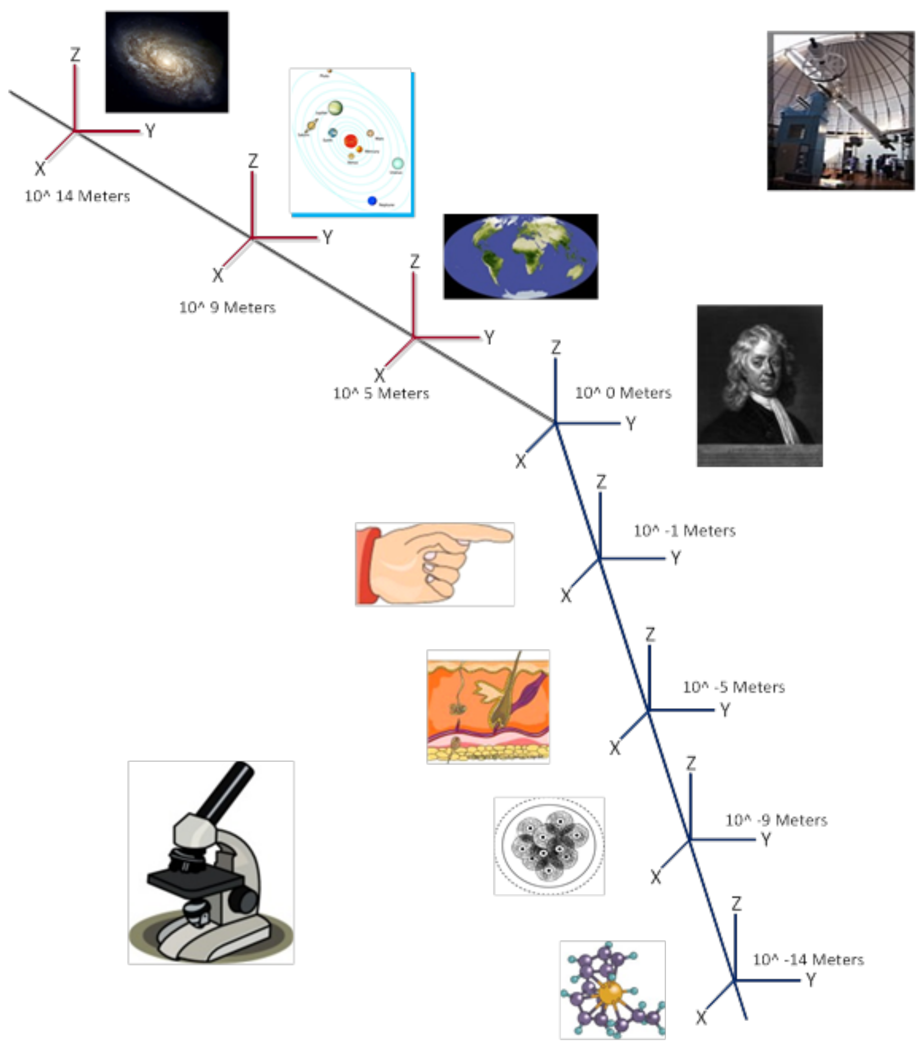

Figure 2: The 4-Dimensional Layout of Scale-Space

So, we could say we actually live in a four-dimensional (4-D) world, not a three dimensional one – without changing anything we sense or measure about the world. As science has discovered objects across scale, it has explicitly required we expand what we consider to be 'our world' through the discoveries of what we can 'see' of our world. To do so requires we expand our existing 3-D perspective, based strictly on what we see with our eyes and overt senses, to include the actions of our world at all levels consisting of the 'recently' discovered objects we cannot directly see at larger and smaller scales.

2. Three Dimensional Scientific Disciplines

If we consider the many different areas of scientific study at the different levels of scale, we find that each has historically developed to operate in a three-dimensional frame of reference. Particles move in three dimensions, atoms move in three dimensions, molecules and cells each move in three dimensions, as does our heart and do humans and rain clouds and our planet, our sun, and our galaxy. The study of particles and the study of molecules are observations in three dimensions at different levels. The study of weather and of our solar system are of actions in three dimensions at different levels.

Our studies of each level involve measurements and actions of objects at that level operating in three dimensions of space. While it is implicit that each new level is part of the same reality, we find we have investigated each level relatively separately. So, while 'reality' occurs at all these different levels, we seem able to only measure objects and actions at one level at a time. We conceptualize each level separately so that we can measure the actions at that level. As we historically identified different levels, first exploring our planet, then finding other planets, then tissues, cells, molecules and atoms, we worked to understand the objects and activities at each new level.

The physical sciences have used the reductionist method to drill down to smaller and smaller levels, attempting to explain larger levels through the actions at smaller levels. Now physicists work on the levels of atoms, and sub-atomic particles, expecting all actions at higher levels to derive from the actions at these smaller levels. There has recently also been an increase in the study of larger and larger objects, with theories that explain larger objects (eg, black holes) with the smaller (eg, sub-atomic particles). However, these connections have also led to problems when attempting to harmonize the theories of the largest and smallest objects and forces [31].

We are told that solid matter is "really" mostly empty space – at the atomic level, where everything is supposed to result from. However, reality is the occurrence of activity at all levels – the whole of reality as we know it. This may seem a direct and simple observation, yet it is at odds with the reductionist perspective that has driven the physical sciences to the plank level as the ultimate source of action in reality. To consider a multi-level conceptualization of reality, where actions occur at all levels simultaneously, requires knowledge of all these different levels. That we are increasing our understanding of how different activities of reality occur across all levels of scale belies any reductionist call and also any call for a theory of everything centered on the smallest scale only.

3. Stitching 3-D Scientific Disciplines Together

Can we believe that when we touch a finger to a pane of glass that all actions are on the atomic or sub-atomic level? Our eyes and senses show us touching the glass. How can we reject our own senses and consider this action only to be on some tiny level? Shouldn't an explanation of touching a pane of glass explain what we see directly, as well as the actions at multiple levels? In this simple action, events occur at our visual level, at the level of our skin, at the level of the cells in our skin and at the proteins and molecules and atoms of our body with the crystalline structure of molecules and atoms of the glass all occurring at the same time. Combining these levels, as noted previously, involves a 4-dimensional model and touching a pane of glass involves a surface that includes all these levels of scale with the objects at each level constituting the act of our touching the pane of glass. That surface, of touching at all these different levels, cannot be contained in any single level of scale, since it crosses many levels.

Over the last few centuries, we have begun to perceive objects across these many levels of scale. We have come to understand that we live on a spinning earth, which is circling a sun, which is moving within the Milky Way galaxy, which is moving in a cluster of galaxies. And we have come to understand that we are comprised of organs and blood vessels that continually pump blood, which moves cells made of proteins that supply oxygen and nutrients to other cells. And all these cells and proteins are made up of molecules and atoms and particles that are constantly moving on their levels. All these levels operate together, in some interconnected fashion, in contrast to current attempts to understand all these different levels as stemming from any one level. We have presumed we only need to apply our direct sensory directions to this expanding vista of objects beyond our direct senses. It is time to remove this presumption and begin to stitch all perceived objects together into a fuller picture of our world that incorporates objects and actions across all these levels of scale. In order to accomplish this, we need to be explicit in identifying the scale of objects at all levels involved in an event, in addition to the traditional position of the objects relative to other objects of the same scale. We are, therefore, left with the conclusion that reality crosses scale and therefore must involve ‘space’ in which the actions of objects at all these different levels can fit. Since we measure objects and their actions at each level in three dimensions, reality must include an adequate expanded space in which all objects at all scales can move. To model all these objects will require a 4-D space to put them all in. This additional dimension runs across scale, so maybe we call this the ‘Scale Dimension’ (see Figure 3 below).

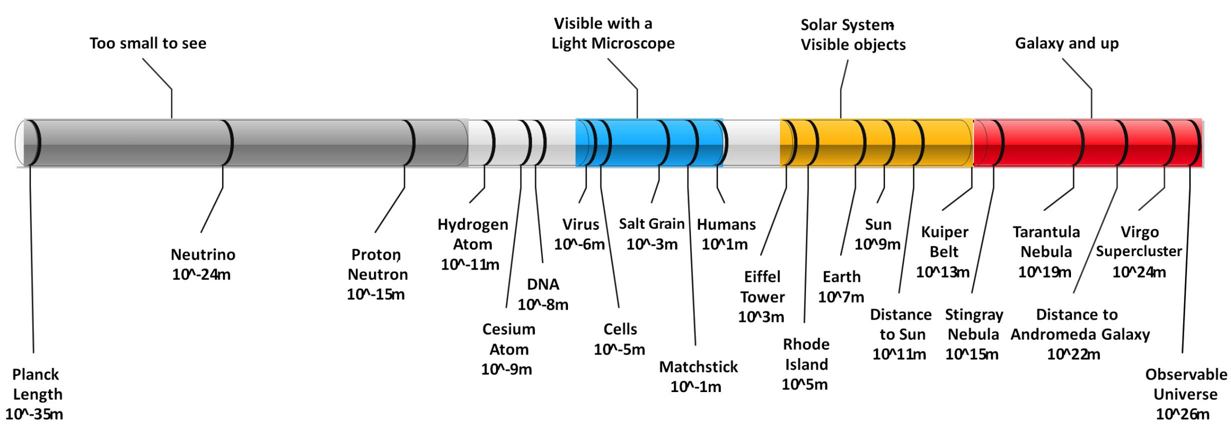

Figure 3: The Dimension of Scale, each level involving a 3-D Space

4. The Limits of Our Measurement Tools

From the above discussion we might wonder how we could measure across scales – or if our scientific tools are adequate for measuring across multiple levels of scale. Given that scientific disciplines tend to investigate specific levels, we appear to find it much more convenient to deal with each level separately. Even the recent discipline of Multi-Scale Modeling uses multiple models, each at a different scale [38]. A possibly compelling reason for missing scale as an additional spatial dimension comes not directly from science, but from the tools of science – the tools by which science investigates our world. These tools are the mathematical tools science uses to measure and model reality. If scale is a dimension of reality, how might we measure 'scale'? How might we measure across different levels of scale? More to the point, are our current mathematical tools used in science capable of measuring across scale? If they are not, then we would have an appropriate reason why the continuum of scale has not been adequately included in our paradigm of reality by so many intelligent minds over the last 450+ years.

A first observation is that we do measure across scale in science today, at least in a limited way. However, such measurements are mostly just to connect to the next level above or below to the one we are investigating. In developing theories of black holes (and even the Big Bang beginning of the universe), physicists have attempted to connect the very large with the very small and, via efforts to visualize what would happen if we were sucked into a black hole, all the levels in between. However, crossing these multiple intermediate levels of scale appears to be a difficult proposition. How could we measure the distance between an atom in the pen on the table in front of us and the sun? If we live in a strict three-dimensional reality, this should be a straight forward measurement. Take the three-dimensional position of the atom and the three-dimensional position of the sun and determine the distance between them using a three-dimensional distance measurement.

But the difference in scale of the two objects presents difficulties: On the scale of the atom, what is the position of the sun? On the scale of the sun, what is the position of the atom? To get a three-dimensional distance answer, we need to translate the position of one object into the scale of the other, say by taking the 'position' of the sun as the center of mass of the sun on the scale of the atom. The position of the sun, at the atomic scale, involves all the atoms of the sun, not a single atomic position. How do we define the center in such a way as to provide a three-dimensional position to the level of accuracy equivalent to the position of the atom?

Note that what we are actually doing is attempting to reduce the distance question to a measurement at only one level of scale, which would then be a 3-dimensional distance. The concept of distance, using our current paradigm, is within a defined level of scale, not something that crosses scale. This would seem to support the concept that distance, involving objects separated by multiple levels of scale, is not measurable via three dimensions of space. More to the point, the distance between objects of significantly different levels of scale does not seem to be measurable using our current tools of measurement.

5. Can We Measure Scale?

We can calculate a distance by equating the position of each object to a position at the same level of scale (as when attempting to find the distance between the sun and an atom above), however this removes any potential ‘distance’ in scale between the objects. Are we justified in stating this three-dimensional distance as the *entire* distance? We translate the position of objects at different scales to positions at one level of scale, thus removing any possible scale distance, so this might not be the entire distance. What if we considered an additional scale distance, when attempting to determine the distance between objects at vastly different scales? Then the difference in scale itself would need to be accommodated, beyond the standard metric of three-dimensional distance. What would this measure be called – beyond length, width, and depth? We could call it ‘scale distance’, at least for now. In what units would it be measured? We measure length, width, and depth via the same unit: length. Don’t we just have to identify a method of using ‘length’ that measures different levels of scale? But we find that different levels of scale require different scales of length, such as a meter, angstrom, or lightyear. The requirement that we use different scales of length for different scales of objects suggests we cannot measure scale using a standard ‘length’ unit. If we could, then we would likely not have missed this measure of reality.

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A mathematical dimension is the same as any other mathematical dimension, allowing full freedom of motion in any (or all) dimensional direction(s). Reality, however, does not have to work the same way. We easily perceive three dimensions with our eyes and physical movements. Yet, on a flat surface, movement forward and back or left to right is easy, while movement up and down is not. So, our experience of reality is not equivalent in the three dimensions we already hold to. It took instruments, like the microscope and telescope, to see across the different levels of scale. This means we do not perceive this scale dimension the same as our three comfortable dimensions. Given that our ability to travel in our three comfortable dimensions is not the same in each direction, we should consider that this is likely also true of the direction of scale.

At first glance, it would seem that it is very difficult to move in the direction of scale. Moving in this direction would seem equivalent to growing larger or smaller. We should note that we do, over the course of our lives, grow from the scale of a cell to that of a person. So we do move in scale, if not our entire body in a short period of time. And it takes energy to move in this direction – something needed for any change along our current physical dimensions. Consider that a chemical or atomic bomb could be considered a fast-moving event across scale. A chemical bomb starts at the molecular level and expands upward, not simply in three-dimensional directions, but also upward in scale. An atomic bomb involves an exponential step in energy over a chemical bomb and expands starting from a smaller scale. It traverses a longer scale distance than a chemical bomb (both starting from a smaller scale and expanding to a larger scale). So, we can find examples in the real world today of travel across scale. Note that a time interval is required to traverse this distance (something else necessary for travel across a physical distance). Even an atomic bomb takes time to move from the atomic level to that of our level or of islands and clouds.

We are also confronted with a lack of ability to identify a unit of 'length' with such cross-scale action. This, it would seem, is the crux of the problem with believing scale is a dimension: Are there units of measure to this scale? If not, then we can safely return to our previous belief in a simple 3-dimensional reality. However, we must consider that maybe we currently just do not have the tools to identify or measure this scale dimension. Having identified objects that exist at many levels of scale, we have developed the concept that scale is a continuum of reality. We have not, however, understood that this continuum of reality needs to be modeled using a fourth physical dimension. What would be a 'unit of scale'? How would it compare to our existing units of measure?

Again, it is only in the past few hundred years that we have begun to discover this continuum, with most of the discovery not much more than 200 years old. Identification of reality as three dimensional, based upon our direct senses, is at least a couple thousand years old and, as the Copernican revolution showed, shedding old ideas can be a lengthy and difficult process.

6. Finding the Dimension of Scale

There is an enjoyable book you might have heard about, called 'Flatland: A Romance of Many Dimensions' by Edwin A. Abbott (in 1884 [15]). It is a satirical novella about two dimensional beings living on a plane, with jibes at social castes, justice, and expanding beliefs. One such being encounters a three dimensional being and is introduced to the three-dimensional universe. It takes some convincing for him to believe in a three-dimensional universe, eventually even considering higher-dimensional worlds. When he returns to his own two-dimensional world, they do not believe him and lock him up. The novella is much more than this quick sketch and is worth a read.

The story has been used as an analogy for ourselves and visiting some unseen fourth dimension. But what if the situation presented is not correct and while we think we are two dimensional beings, we are actually three-dimensional ones. We can only see two dimensions, yet we find ourselves comprised of multiple levels of objects 'inside' ourselves. We do not understand these levels to comprise a third dimension into which we extend. We think all these internal objects exist within the line that separates our insides from the outside. How might we determine that, instead of multiple levels of objects all taking up the same area and somehow stuffed into the area of our bodies, we actually extend into a third dimension and that is how all these levels of different objects can exist in the same area we think of as our bodies?

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How is it that each level of objects that make up our body (eg. organs, cells, proteins, molecules) takes up the same 3-dimensional space? How can different objects take up the same 3-dimensional space? We are quite used to the concept that two objects cannot both be in the same location at the same time. Why do we violate this when considering objects at smaller and smaller levels? How can cells be both 'inside of us' and take up the same volume as ourselves? This can be explained if we only think we live in a 3-dimensional world, yet we actually are 4-dimensional beings.

There are two classic videos of traveling across scale: Cosmic Zoom [44] and Powers of Ten [45] both inspired by Kees Boeke's book "Cosmic View: The Universe in 40 Jumps" [42]. More recently is a book that looks at traversing the levels of scale by Gott, J. Richard and Vanderbei, Robert J. "Sizing Up the Universe: The Cosmos in Perspective" [43] and two worthy interactive web sites by the Huang brothers http://htwins.net/scale/ [46], http://www.scaleofuniverse.com [47]. These all give a feeling about what it would look like to travel through different levels of scale and see the objects at these different levels. Two things to note in all these videos (and books): 1) As we progress up or down, we see different objects (especially noticable when progressing down). This is characteristic of travel in our normal three dimensions – we see different objects as we travel. 2) From a standard unit of length perspective, travel up or down in scale involves traveling in 'Powers of Ten'. One unit upward would be an increase of 10 of our 'standard units' and two units upward would be an increase of 100 of our 'standard units'. This means a linear movement in the scale direction involves a power (or exponential) change in the lengths we measure at that scale. What this translates into is a linear movement in scale will appear to us as an exponential movement according to our standard units of length.

Part II: Our Mathematical Tools

7. The Need for New Mathematical Tools

If all we have are our current mathematical tools, should we expect to be able to make all possible measurements and perform all possible experiments? We act as if we have all the correct tools to perform any possible measurement or experiment. Should we really assume we have all the correct tools in order to understand the universe? Should we even expect that science has all the correct tools for understanding reality? Maybe we are using inappropriate tools while attempting to progress further in science. Maybe our faith in our tools has caused us to miss an aspect of reality that our tools are unable to adequately handle.

Scientists have marveled at the effectiveness of mathematics for science, as Eugene Wigner did in his 1960's article on "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [35]. He does, however, limit science to the study of "inanimate nature", which places other limits on his discussion, excluding the biologic organs and structure of living animals. Given this belief in such "unreasonable effectiveness", we should expect to already have all the mathematical tools required for science. There is no problem handling a fourth dimension – we simply add another positional value to the three we have been using for centuries – pound in that nail. We can measure any distance, can't we? We can measure anything, can't we? What tools could we possibly be missing? The strange part of our story is that the highly praised tools of science, those of mathematics, are where we could be falling short.

If scale is a continuum of reality, then we should be able to measure it and produce equations that demonstrate actions and influences that cross multiple levels of scale. We might think we just need to identify a 'length' to scale – to which we apply our traditional numeric values and mathematical tools. However, as with movement 'up' in our current 3-D world, all directions may not be equally accessible to us, including on a measurement and/or mathematical level. We saw that in attempting to measure the distance between the surface of the sun and a molecule near us, we needed to reduce the problem to one at the same level of scale. Why should we have to collapse measurements across levels of scale down to one level of scale, in order to measure something apparently as simple as distance? If this is strictly a 3-D spatial world, we should be able to measure the direct distance between these objects, regardless of their scale. However, if scale is a continuum which we must address differently than our traditional directions, then we should make sure we have the appropriate tools.

Consider that our current science would not be possible if the only mathematical measuring tools we had were Roman numerals. This system of representing numeric values cannot represent the

Real numbers (like pi, or sqrt(2)). The methods of calculation, using Roman numerals, are cumbersome and lengthy – and are inadequate for calculus or the mathematics taught at the elementary school level today. Our science of today must have (at least) the power of the decimal numeric system to exist. This level of mathematical tools is required for today's mathematical calculations and mathematically based science. Might an analogous situation exist for the next step in scientific knowledge? Might we need more sophisticated mathematical tools, in particular more powerful numeric tools, to take us to the next level?

Scale does not appear to be a direction we can travel in or measure directly. If scale does not appear to us the same as our traditional 3 dimensions, then simply adding a new position for a 4^{th} dimension may not be sufficient for this obscure scale dimension (we need something different from our current hammer and nail). While this dimension might appear easy to model mathematically in the same way as our traditional 3, if it does not work the same or does not measure the same as our traditional 3, then maybe the mathematical tools required are different. This should open up the question about whether we have the appropriate tools to accommodate the characteristics of this continuum. A related question arises: Why should we think we have reached the limits of all possible mathematical tools – or numeric systems? We have continued to expand our mathematics, although not our basic (measuring) mathematical tools. As noted earlier, physicists have been calling for new mathematics. Maybe we need to update our more basic mathematical tools, in particular our measurement tools via our numeric system, to address the science of the future.

8. Measurements and Complex Numeral Systems

Mathematics must use numerals to represent measurements and quantities. We cannot manipulate nor measure nor calculate quantities without numeral systems. And, as indicated earlier, the power of science has a significant dependency upon the power of the numeric systems used (eg. decimals vs Roman numerals). This provides a view into how deeply science depends upon our numeral systems – it is at a very foundational level. If we consider that limitations of our numeral systems could impact science and technology in such a potentially significant way, the question needs to be asked: Why should we believe the decimal numeral system, or positional numeral systems in general, are the end-all of numeral systems?

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We cannot use theoretic values for a measurement (2π radians is a theoretic measurement, not an actual measurement), and manipulation of measurements must conform to operations that are manageable by humans or human inventions (such as computers). Although theoretically possible, it is not practically possible to multiply by π if π is only accurately represented by an infinite decimal expansion. So, there is an important distinction between a Number system (such as Integers, Rationals, or Reals) and a Numeral system (such as Roman numerals, Fractions, or Decimals).

We should note that science uses 'complex numbers' for many calculations and equations. Our numeral system for representing complex numbers theoretically appears to produce unique values for all complex numbers. Yet there are always two parts to any complex value, one part of which we cannot resolve into a useable numeral value. We found a 'work-around' to representing complex values through defining them as 'x + iy'. However this is not a full representation of a complex value precisely because it includes an undefined term i = sqrt(-1). This is not a question of accuracy, as with π, since no numeric value at any level of accuracy exists for i.

From a practical standpoint, we use complex numbers for all sorts of calculations, but because we cannot resolve the imaginary part into an actual value, we need to ignore at least the undefined term, if not the entire imaginary part, when we use complex values for quantities or measurements. Ignoring the entire imaginary part allows many theoretic calculations involving measurements to produce different complex values, yet result in the same real quantity (5 + yi equates to the Real value 5, regardless of what 'y' or 'yi' are). This is a logical problem for physical theories, since calculations in a theory could produce different complex values, yet the theory would predict all these different values to be considered the same real quantity or measurement. Even if we account for 'y', we still cannot account for 'i' and thus cannot identify what a single complex value looks like, let alone what it could measure. We are left with two separate parts of a complex number that we can only evaluate as two parts. Five and π are understood as a single value and can be used in measurements and calculations as a single value. Not so with complex numbers – as we represent them today. If the currently accepted scientific philosophy is that all we can know of the physical world is through measurements, and we realize that our numeral systems are not capable of entirely specifying all practical quantities, then we have a direction to look into before any scientific theories can be considered complete.

On the theoretic mathematics side, we have become used to understanding complex numbers as 2-dimensional numbers. This situation appears to have ‘gelled’ into the idea that this is a property of the complex numbers. However, it is really the result of the numeric methods we use to represent complex numbers – with 2-parts, one of which involves an ‘always unknown’ value. Since we are unable to resolve the imaginary part into an actual numeral value, we leave it apart and unresolved, separating it from the part we can handle. Why should this be the case – forever?

It is possible the tool we are missing is a more ‘complex’ numeric system, which can handle Complex numbers. Note that this might also imply these numbers fit on a Complex continuum (something we currently see as ‘Real’). In this situation, we require a tool more than a Real hammer and this tool implies there are different aspects to what a model can include (a Complex continuum might have different capabilities than a Real continuum). Maybe we need a new tool, like a single valued complex numeral system, in order to adequately approach new phenomena. Since there would be (infinitely) more numbers on a Complex number line than a Real number line has, such a line would be more dense than a Real line or continuum. There would be characteristics of this continuum that a Real number line could not handle. Generating areas and volumes using this new continuum, would be more dense (more numbers or quantifying values) than our current areas and volumes. Maybe we need this denser continuum to be able to handle scale. Since we only have a Real continuum, maybe it is the hammer we see the world with. So, only using this tool, we can only see complex numbers as a pair of Real numbers (and an unknown value) – but we need another tool.

Resolving this situation might provide new mathematical tools with new capabilities – like measuring things we cannot today. Resolving this situation should produce a numeric system with considerably more power than our current ones provide. It should be capable of performing calculations rather simply that are currently either extremely time-consuming or impossible today. If it can measure across scale, then it should be able to handle cross-scale equations.

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To resolve the theoretic issue with complex numbers, we need a method of representing them as singular numeral values – as a single representation of a complex number value. This requires a representational solution to the question 'what is the square root of -2?'. With such a solution we would be able to measure and manipulate complex values - as single values, like we do Real, Rational, or Integer values. We need to break it into parts when needed, but there is no reason why we must always represent it by two components - except due to the limitations of our current mathematical tools. Without such a numeral solution, we will continue to work with complex numbers using incomplete numeral representations, will continue to mistake properties of our complex numeral system with the number system being represented, and will leave science without a solution for measuring fully complex quantities.

What if we could find a means of fully representing a value for that pesky 'i' (or of any value that, when squared, produces a negative real number)? This value certainly does not fall into the mathematical notations of today. So maybe mathematics needs to take a new step here. Maybe the 1500-year-old numerals we use today are not sufficient to represent 'modern' complex values. The symbols used to define 'imaginary' values could be consolidated with the 'real' part of a complex number and be reduced to a single value. What might we find, through the simplification of many equations made complicated due to 2-part complex values?

This representational discovery or invention could open up a new universe of possibilities for mathematics. It might also alter the interpretation of physical equations that 'toss out' the imaginary value for quantities and measurements given that we can only use real numeral values. Now we could have a value or measure, that included the imaginary part. Now a complex value could be handled in its complete form, possibly opening up measurements across scale not possible before. There would be complex measurements, not real measurements + imaginary placeholders – potentially identifying measurements we cannot make today.

Integer numerals can be enumerated using a basic unit (one) and the reversing operations of addition and subtraction. So, 1 + 1 = 2, 4 + 1 = 5, and 2 – 3 = (-1). Rational numerals can be specified using integers plus the reversing operations of multiplication and division, and fractions can enumerate all rational numbers (eg. ½, 2/3, 47/35, -5/7). Positional base numerals, like decimals, can represent Real numbers, by add the reversing operations of exponentials and logarithms (eg. $5x10^2 + 1x10^1 + 2x10^0 = 512$). Using these three pairs of reversing operations, we are able to represent all Real numbers, even if only theoretically (infinite decimals proving a practical limitation). We have a singular value for each Real number that can be used as a quantity or measurement – with an assessment of accuracy (or an 'error term').

Now, what about complex numbers – why should they be any different? Maybe we need a fourth pair of reversing operations in order to fully represent, as single values without any unknown placeholder, complex numbers. A possibility would be integration and differentiation added into the definition of a complex numeral value. In order to represent negative square roots, we might need to define an undefined area of mathematics – that of negative bases. As the ability to represent negative base numbers is currently undefined in mathematics, there is a bit of theoretical work to perform here – maybe even a little inventing.

As Donald Knuth worked on more than 60 years ago, maybe we need to develop – to invent – numerals using negative bases, which can represent negative square roots. Euler's great equation ($e^{(\pi i)} + 1 = 0$) might provide a clue to how to construct complex numerals, using 'e' as a base. When used for integration and differentiation, base 'e' allows for continuous integration or differentiation and does not 'bottom out' as typical bases do (eg. Derivative of $x^2 = 2x$ and derivative of $2x = c$, bottoming out – while derivative of $e^{(2x)} = 2e^{(2x)}$ and so the exponent does not decrease). A complex numeral might involve the positional placement of integrated and/or differentiated 'digits' in some similar way as exponential digits are used for decimal and positional base numerals. The capability of incorporating integration and/or differentiation into numerals suggests integration and differentiation operations should be simplified. Consider that modeling upward in scale generally involves integration, suggesting such a numeral system could 'take on' the difficulties of the scale continuum. It is very possible that such a numeral system may not be representable using traditional paper and pencil methods – requiring the use of computers.

It is not a huge step to consider systems beyond a complex numeral system - beyond where we do not quite see yet. So, we may still be in the early stages of understanding the extent of what mathematics can provide and science can utilize. Where mathematics needs to go could be well outside the 'standard model' of current mathematics (with only a Real line continuum) - and there might be tremendous dividends for science as well.

9. Mathematical Tools vs Theoretical Tools

If we do not have the correct tools, then we should expect that we will not be able to construct correct models. However, we might have the correct tools, yet the models we have found do not match reality. Realist physicists, tussling with how to understand quantum mechanics, might feel like the latter situation applies. We have very accurate measurements and models, yet they do not make sense somehow (think particle-wave duality or the uncertainty principle). There are calls for new mathematics, however these tend to be extensions to existing mathematics. What if there is an entirely new area of mathematics we have not yet discovered (or invented), that would change our tool set? It might provide new ways to take measurements and hence new experiments. It might provide new ways of calculation and new methods of analysis.

Back to the physicists and our models… Is a 3-D space the correct model for representing the world we see around us – especially if we consider much larger and much smaller objects? Or is this the only hammer we have? What if we think we are Flatland creatures, but actually have a third dimension to our world, a depth to us, that we are not correctly perceiving (see Figure 4)? We would have a 2-D ‘exterior’ surface to us and we would also have another 2-D surface ‘inside’ of us. Maybe we have discovered (what we think are) 2-D molecules or atoms ‘inside’ of us. Yet we are not accounting for, in our models, the third dimension of depth – something we seem unable to measure. Since we believe we only have 2-D measuring tools, we will only think we can measure 2-D objects in 2-D ways. Measurements in some ‘third’ dimension we cannot see in any direct way would seem ridiculous. However, our models would show anomalies, since we think we are measuring along a 2-D surface, yet the objects we are measuring are not on (or ‘in’) the same 2-D plane we believe is our world. This would be a case of not having the correct models (and believing we had the correct tools). Yet our models prelude us from thinking there is a depth to measure.

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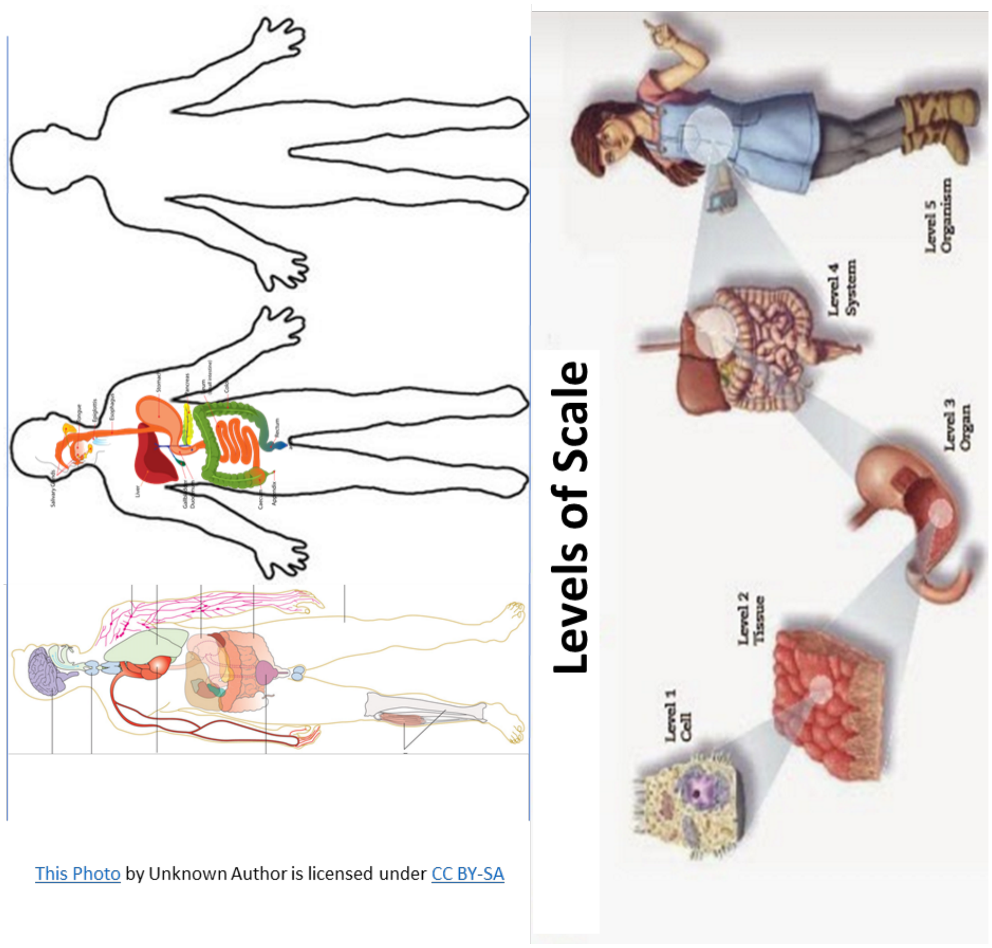

Figure 4: How Our 3-D Bodies Extend Across Scale

This is rather different from not having the correct tools for the job. Different from what most physicists seems to think, it could be that anomalies with our models are a consequence, not just of an incorrect model, but of incorrect tools to build a correct model. Consider if the hammer we have is our decimal (or positional) numeric system, which can handle Real numbers that fit on a continuum (the Real number line). We will see all continuums as Real ones that can be entirely handled by decimal (or positional) numeric values. If we come across something more than this, such as complex numbers, we couch such a different tool in terms of the hammer we know – Real numbers that have decimal values.

By not understanding the limits of our mathematical tools, our models would be missing anything the new tools could account for that our current tools cannot. This could be the situation with how we currently represent Complex numbers. We are unable to represent a complex number as a single value (as we can with Real numbers, like 3.14159), so we manage with what we know and represent a complex value using two Real numbers – x + iy. However, this means our models cannot account for Complex values that require a value for 'i'.

10. Closing Remarks and Call for Next Steps

What is being presented here is that a new comprehensive area of science needs to be built. This new discipline will combine and connect many current scientific disciplines across multiple levels of scale – a convergence across scale. It will not be a physicist, nor a chemist, nor a meteorologist, nor an astronomer who is the new scientist – it will be those people who can coordinate all these levels together that will produce the next step in science.
The modeling that will be required will include measurements and equations that incorporate processes across scales, upward and downward. New capabilities, technologies, and theories will be required that measure across scale.

Understanding four-dimensional scale-space will require new physical models using a yet-to-be-devised complex numeral system. This numeral system will be able to measure aspects of reality we currently are unable to. It will simplify a number of calculations, some of which are extremely difficult or practically impossible today. A very large practical impact will involve the updating of our computers and their operating systems to make use of this new numeral system.

It is possible there is much more to mathematics that we do not even have a clue about. Our current numeral systems, while very powerful and capable of producing our current tremendous technology, are not close to being the end-all of what can be represented (by a 'bit'). We have achieved a numeral system involving only 3 reversing pair operations and showing us only 3 dimensions of space (essentially what we can directly perceive). A fourth dimension may lie right under our nose, yet our mathematical tools limit what we measure and therefore what we perceive as our world. There is no reason to believe this expansion of numeral systems and the resulting scientific measurables they can introduce has an end. We are likely a long way from being able to represent all possible numbers or all possible measurable quantities.

On the Perception of Scale in Science and Mathematics

Bibliography

Mathematics

1. Benacerraf, Paul & Putnam, Hilary, Editors; 1964 (1983); *Philosophy of Mathematics, Selected Readings, 2nd Ed.*; Cambridge University Press, Cambridge, UK
2. Russell, Bertrand; 1971; *Introduction to Mathematical Philosophy*; Touchstone Book; NYC
3. Boyer, Carl B.; 1949 (1959); *The History of the Calculus and its Conceptual Development*; Dover Publications; NYC
4. Derbyshire, John; 2006; *Unknown Quantity, A Real and Imaginary History of Algebra*; Joseph Henry Press; Washington, DC
5. Eves, Howard; 1990; *Foundations and Fundamental Concepts of Mathematics*, 3rd; Dover Publications; Mineola, NY
6. Alexander, Amir; 2014; I*nfinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World*; Audible
7. Seife, Charles; 2000; *Zero: The Biography of a Dangerous Idea*; Penguin (Audible)
8. Greenberg, Marvin Jay; 1973; *Euclidean and Non-Euclidean Geometries, Development and History*; W. H. Freeman & Co; San Francisco, CA
9. Lindgren, C. Ernesto S. & Slaby, Steve M.; 1968; *Four-Dimensional Descriptive Geometry*; McGraw-Hill Boos; NYC
10. Flanigan, Francis J.; 1972; *Complex Variables, Harmonic and Analytic Functions*; Dover Publications, NYC
11. Rucker, Rudolf v. B.; 1977; *Geometry, Relativity, and the Fourth Dimension*; Dover Publications; NYC
12. Dantzig, Tobias; 1930 (1954); *Number, The Language of Science*, 4th Ed; Doubleday Anchor Books, Garden City, NY
13. Mlodinow, Leonard; 2009; *Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspeace*; Audible
14. Dyson, George; 2012; *Turing's Cathedral: The Origins of the Digital Universe*; Random House Audio
15. Abbott, Edwin A.; 1884; *Flatland: A Romance of Many Dimensions*; Dover, NYC

Science

16. Rohrlich, Fritz; 1987; *From Paradox to Reality: Out basic concepts of the physical world*; Cambridge University Press; Cambridge, UK
17. Wick, David; 1995; *The Infamous Boundary: Seven Decades of Controversy in Quantum Physics*; Birkhauser; Boston
18. Watson, Peter; 2016; *Convergence The Idea at the Heart of Science*; Simon & Schuster; NYC

19. Kuhn, Thomas S.; 1962 (1970); *The Structure of Scientific Revolutions*; University of Chicago Press; Chicago

20. Brockman, John; 2014; *The Universe: Leading Scientists Explore the Origin, Mysteries, and Future of the Cosmos*; The Edge Foundation, Tantor

21. Smolin, Lee; 2013; *Time Reborn: From the Crisis in Physics to the Future of the Universe*; Spin Networks, Tantor & NYC (NY): Mariner Books; Houghton Mifflin Harcourt

22. Clark, Stuart; 2016; *The Unknown Universe: A New Exploration of Time, Space and Cosmology*; Audible

23. Baggott, Jim; 2013; *Farewell to Reality: How Modern Physics Has Betrayed the Search for Scientific Truth*; Audible

24. Tegmark, Max; 2014; *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*; Max Tegmark, Random House Audio

25. Hawking, Stephen, Mlodinow, Leonard; 2010; *The Grand Design*; Random House Audio

26. Randall, Lisa; 2011; *Knocking on Heaven's Door: How Physics and Scientific Thinking Illuminate the Universe and the Modern World*; Tantor

27. Rovelli, Carlo; 2018; *The Order of Time*; Penguin Audio

28. Close, Frank; 2011; *The Infinity Puzzle: Quantum Field Theory and the Hunt for an Orderly Universe*; Tantor (2012)

29. Penrose, Roger; 2011; *Cycles of Time: An Extraordinary New View of the Universe*; Random House

30. Castelvecchi, Davide; 2015; *Is String Theory Science?: A debate between physicist and philosophers could redefine the scientific method and our understanding of the universe*; Nature;

31. Hossenfelder, Sabine and McGaugh, Stacy S.; 2018; *Is Dark Matter Real?*; Scientific American; Vol. 319, No. 2, pgs. 36-43

32. von Hippel, Matthew; 2019; *The Particle Code*; Scientific American; Vol. 320, No. 1, pgs. 31-35

33. Ananthaswamy, Anil; 2019; *Best-Yet Measurements Deepen Cosmological Crisis*; Scientific American; March 22, 2019

34. Ananthaswamy, Anil; 2018; *What Does Quantum Theory Actually Tell Us about Reality?*; Scientific American; September 3, 2018

35. Wigner E. *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*. Communications in Pure and Applied Mathematics. 1960 February;13(1):1-14

36. Hamming RW. *The Unreasonable Effectiveness of Mathematics*. The American Mathematical Monthly. 1980 February;87(2):81-90

37. Grattan-Guinness I. *Solving Wigner's Mystery: The Reasonable (Though Perhaps Limited) Effectiveness of Mathematics in the Natural Sciences*. The Mathematical Intelligencer. 2008 Summer;30(3):7-17

38. Marialore, Sulpizi, Roland, Faller, Sergio, Pantano; *Multiscale modeling on biological systems*; Biochemical and Biophysical Research Communications; Volume 498, Issue 2, 29 March 2018, Page 263

39. Falk, Dan; 2018; *Why some scientists say physics has gone off the rails*; NBC News MACH; June 2, 2018

40. Brody, Thomas; Edited by Luis de la Pena, Peter Hodgson; 1993; *The Philosophy Behind Physics*; Springer-Verlag; NYC

41. Laszlo, Ervin; *A New Scientific Revolution: Foundations of the New Paradigm*; Inner Traditions; 2014

Scale, incl. Videos/Websites

42. Boeke, Kees; 1957; *COSMIC VIEW: The Universe in 40 Jumps*; John Day Co.; NYC

43. Gott, J. Richard and Vanderbei, Robert J.; 2010; *Sizing Up the Universe: The Cosmos in Perspective*; National Geographic

44. National Film Board of Canada (Illustr. Szasz, Eva); 1968; *Cosmic Zoom*; Canada

45. Eames, Charles; 1977; *Powers of Ten*;

46. Huang, Cary & Michael; 2010; *The Scale of the Universe*

47. Huang, Cary & Michael; 2012; *The Scale of the Universe 2*