Proof for Twin Prime Conjecture

Mesut Kavak[a]

The question about Twin Primes is pretty clear:

"Twin primes are prime numbers that differ by 2. Are there infinitely many twin primes?"

[a]kavakmesut@outlook.com.tr

I. Introduction

If any number group containing "n" consecutive and odd non-prime numbers is selected among the infinite number groups, this group must be between 2 prime numbers according to this condition. For n=4, choose a group of numbers as follows, consisting of 4 consecutive odd numbers, n_1, n_2, n_3 and n_4, which are between 2 prime numbers such as p_1 and p_2.

$$\mathbf{p_1} \quad n_1 \quad n_2 \quad n_3 \quad n_4 \quad \mathbf{p_2}$$

II. Solution

Theorem *At least one of these non-prime consecutive odd numbers of "n" must be an odd multiple of 3; because the distribution of odd multiples of 3 in the set of odd numbers depends on the function f(x)=6x+3, and therefore in the set of odd numbers there are always 2 consecutive odd numbers between every two consecutive odd multiples of 3.*

$$n_0 \quad n_x \quad \mathbf{p_1} \quad n_1 \quad n_2 \quad n_3 \quad n_4 \quad \mathbf{p_2} \quad n_y \quad n_5$$

- If the odd number n_2 is considered an odd multiple of 3, the prime number p_2 must be the next consecutive odd multiple of 3.

  Since p_2 is a prime number, this is only possible if it falls on a non-prime number in a group such as n=5.

  For n=4, different groups must be formed.

- If the odd number n_3 is considered an odd multiple of 3, then the prime number p_1 must be the previous consecutive odd multiple of 3.

  Since p_1 is a prime number, this is only possible if it falls on a non-prime number in a group such as n=5.

  For n=4, different groups must be formed.

- If the odd number n_1 is considered an odd multiple of 3, the odd number n_4 must be the next consecutive odd multiple of 3.

  Also, the odd number n_5 must be the second consecutive odd multiple of 3 immediately after the odd number n_4, and the odd number n_0 must be the previous odd multiple of 3 before the odd number n_1.

- If the odd number n_4 is considered an odd multiple of 3, the odd number n_5 must be the next consecutive odd multiple of 3.

  Also, for this acceptance, the odd numbers n_0 and n_1 must be previous consecutive odd multiples of 3; so *"The odd numbers* n_1 *and* n_4 *are the best choice to be odd multiples of 3."*

III. Result

The odd number n_5 can be followed by an infinite number of consecutive odd numbers "n"; therefore, for any value of "n" after the group n = 4, the number of elements of an odd set of numbers "n" is unimportant; but the odd number n_y is always prime or not, which is important. With this information, the $n_y = n_5 - 2 = (6x+3) - 2$ equation forms the (1) equation.

$$n_y = 6x + 1 \tag{1}$$

So it can be said for n_y;

I. n_y out of (1) with the condition $x \in \mathbb{Z}^+ \wedge x > 0$ can never be just a prime or a non-prime number.

II. It is not prime over (1) for an "x" value that does this; but it is prime for odd numbers formed between two numbers n_y and n_{y+1} which are the result of two consecutive numbers x and x + 1.

III. After all, when $n_y = p_3$ it is a group of twin primes between odd numbers n_4 and n_5; therefore twin primes are "infinite" even for the groups which have the same number of elements and different numbers that these groups can be written even for only a single value "n".

Result *"Twin primes are prime numbers that differ by 2, and there are an infinite number of twin primes."*

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IV. Appendix

Actually, I felt the need to make 2 additional proofs about the construction of this proof, although it was not necessary because it was easy to animate.

a. The first of these is about the existence of n groups. Topics related to these groups are as follows.

- Existence of an infinite set of numbers "containing n odd non-prime numbers between 2 prime numbers" for a positive and identical odd integer n.
- Whether there are infinite "groups of n" formed for varying n values.

b. The other is about the (1) equality. The issues related to this equality are as follows.

- Whether it returns a prime number only.
- Whether returns non-prime results only.

a. The Groups

I.

For example, for the group $n = 5$, let's take consecutive multiples of 3, 5, 7, 9, and 11, although any odd numbers can be used. Since a is an odd number, any multiple of odd numbers is $a(2x+1)$ for the required x; well

- $(6x_1+3)+2 = 10x_2+5$
  it becomes $x_1 = 5x$
- $(6x_1+3)+4 = 14x_3+7$
  it becomes $x_1 = 7x$
- $(6x_1+3)+6 = 18x_4+9$
  it becomes $x_1 = 9x$
- $(6x_1+3)+8 = 22x_5+11$
  it becomes $x_1 = 11x$

The results of $6x_1+3$, which are odd multiples of 3, become $30x+3$, $42x+3$, $54x+3$ and $66x+3$ for the specified values of x_1. If these are made equal to each other,

$$(11 \cdot 9 \cdot 7 \cdot 5 \cdot x) = u$$

and if the first number of the group is a multiple of 3, the consecutive multiples of the group numbers are $6u+3$, $6u+5$, $6u+7$, $6u+9$ and $6u+11$, respectively. For more distinct consecutive odd multiples, we can increase the number of numbers used in a group in the same way infinitely.

II.

A group as above does not always have to be between 2 primes; because the numbers that make up the group are also important. This also does not mean that groups between 2 primes are not infinite. Let's talk about the infinity of groups.

In fact, its proof requires no mathematical operations; because as long as the primes are not consecutive, there will be infinitely different groups for varying n values.

Since all odd non-prime numbers are (2x+1)(2y+1) for every integer value of x and y numbers

$$(2x+1)(2y+1) = 1$$

In an equation such as, x and y, which only take certain values, do not occur, like in a function with 1 unknown from the 1st degree. This is exactly a hyperbola; therefore, the distribution of prime and non-prime odd numbers does not depend on consecutive rules between any two consecutive primes.

b. The Function

If the function f(x)=6x+1 returns only non-prime numbers and *consecutive* all odd numbers

$$(2x+1)(2y+1) = 6z+1$$

equality must be satisfied for each x and y value, respectively; because odd non-prime numbers are odd multiples of prime and odd non-prime numbers.

Having a z-value for every x and y-value means that odd non-prime numbers have something in common with all of them. Otherwise, it can sometimes give a prime. For this, with the equation m=(2x+1)(2y+1)

$$\frac{m-1}{2} = 3n$$

equation is analyzed. Equality means that any number that is equal to 6n+1 must be divisible by 3 at the end of the operation; in that case

$$2(6a+3)+1 = 6b+3$$

equality must be achieved. If edited

$$b-2a = \frac{2}{3}$$

This means that; when the even number m-1 is divisible by 2, the odd number formed must be a multiple of 3, and the odd number before or after the even number m-1 can never be a number divisible by 3. m-1 is always between two consecutive odd multiples of 3; *then numbers that are multiples of 3 of all other odd numbers* cannot be between two consecutive odd multiples of 3, so there must be primes in between.

Result *"The function f(x)=6x+1 sequentially returns for each value of x not only a single non-prime number, consecutive or not."*

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