

My ZFC Inconsistency Odyssey

By Jim Rock

Abstract: My progress in understanding ZFC's inconsistency is chronicled through the papers reviewed in this document.

Introduction: For a long while I have thought that ZFC was inconsistent, but getting other people to see this has been challenging. We will take a journey through the papers listed below to see how the inconsistency arguments developed and matured gradually. The basic elements common to all the papers are shown first. There generally was only one point of contention: Argument #1 step two. In the three papers we gradually come to see the truth of this step in ZFC, upon which the entire inconsistency argument depends.

For all rational numbers a in $[0, 1]$ let the collection of all Ra sets be $\{y \text{ is a rational number} \mid 0 \leq y \leq a\}$

- A. The entire collection of Ra sets form a hierarchy of sets.
- B. Each set contains all the elements in sets below it in the set hierarchy.
- C. Each set contains a single element that is not in any set below it in the set hierarchy.
- D. We take the largest element out of each set in the entire collection.
- E. The set containing zero becomes the null set.
- F. The previous largest element of every Ra set is now missing from each set. However, all Ra sets remain in the same position in the hierarchy of sets in descending order as $\{y \text{ is a rational number} \mid 0 \leq y < a\}$

Argument #1: Each Ra contains a largest element.

- 1) Each Ra contains the former largest elements of the subsets below it in the set hierarchy.
- 2) Each Ra contains element(s) not in any subset below it in the set hierarchy.
- 3) Let c and d be two elements of a single Ra set with $c > d$.
- 4) d is an element of Rc , which is a proper subset of Ra .
- 5) For any two elements in Ra the smaller element is contained in a proper subset of Ra .
- 6) By steps 2) and 5) each Ra set contains a single largest element not in any set below it in the hierarchy.

Argument #2: No Ra contains a largest element.

- 1) Suppose there is a largest element a' in some individual Ra .
- 2) $a' < (a' + a)/2 < a$.
- 3) Let $b = (a' + a)/2$.
- 4) Then b is in Ra and $a' < b$.
- 5) a' is in Rb a proper subset of Ra .

When a largest element is assumed in Argument #2, it leads to a contradiction; so there is no largest element. Every Ra element is in one of the proper subsets below Ra in the set hierarchy. It is a valid proof by contradiction.

Argument #1 Step 2) explored. We initially consider this step to be false[1] and only gradually recognize its truth within ZFC. First we can see [2] there has to be element(s) in each set that are not in the subsets below it in the hierarchy or the entire hierarchy would collapse. But this needs further elucidation to be accepted by the skeptics. Illumination comes in the next paper[3].

The Ra sets are in descending hierarchical order. In every position below each individual Ra for every rational $x < a$ is an Rx set $\{y \text{ is a rational number} \mid 0 \leq y < x\}$.

Since each Ra and its collection of Rx subsets are part of the entire descending set hierarchy, **there exists at least one Ra set element $s \geq$ (all values of) x . Otherwise, there is no descending set hierarchy.**

As shown in Argument #1 steps 3), 4) and 5) for any two elements in Ra , the smaller element is contained in a proper subset of Ra . **There is at most one Ra set element** missing from all the Rx subsets.

Thus, **there must be a single largest Ra set element** missing from all the Rx subsets.

[1]<https://vixra.org/abs/2302.0145> [2]<https://vixra.org/abs/2303.0105> [3]<https://vixra.org/abs/2304.0008>

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