

Causation and correlation

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Abstract

In this paper, I will introduce the causation's magnitude allowing to compute the importance of causes in the cause-and-effect relationship from correlation matrix.

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1 Introduction

This paper is based on the results obtained in papers [5] and [6]. The purpose of this paper is to relate the notions of causality and correlation. The paper begins by introducing the causation's magnitude computing the contribution of the causes in the cause-and-effect relationship which will be computed from a correlation matrix only. The paper ends with the application of the causation's magnitude in order to obtain a causal graph from a correlation matrix.

2 The causation's magnitude obtained from correlations

In this section, we will consider the result obtained in paper [6] page 4 and interpret the percentage of conditioning as the causation's magnitude.

Definition:

Consider a set of events $E = \{X_j, \Omega_1, \Omega_2\}$ and the correlation matrix $\tilde{K}_{(X_j, \Omega_1, \Omega_2)^2}$. We define **the causation's magnitude** $\lambda_{\Omega_1 \rightarrow X_j}$ of a events's set Ω_1 which acts on the event X_j as follows:

$$\lambda_{\Omega_1 \rightarrow X_j} = \frac{\ln(1 - \tilde{K}_{X_j; \Omega_1} \tilde{K}_{\Omega_1}^{-1} \tilde{K}_{\Omega_1; X_j})}{\ln(1 - \tilde{K}_{X_j; \Omega_1 \Omega_2} \tilde{K}_{(\Omega_1, \Omega_2)}^{-1} \tilde{K}_{\Omega_1 \Omega_2; X_j})}$$

Properties:

1. $0 \leq \lambda_{\Omega_1 \rightarrow X_j} \leq 1$ with $\lambda_{\emptyset \rightarrow X_j} = 0$ and $\lambda_{\Omega_1 \cup \Omega_2 \rightarrow X_j} = 1$
2. We can consider a **strong effect** of Ω_1 on X_1 when $\lambda_{\Omega_1 \rightarrow X_j} \geq 0.75$
3. if $0 \leq \ln(1 - \tilde{K}_{X_j; \Omega_1 \Omega_2} \tilde{K}_{(\Omega_1, \Omega_2)}^{-1} \tilde{K}_{\Omega_1 \Omega_2; X_j}) \leq 0.1$, we can consider the event X_j to be independent of the set $\Omega_1 \cup \Omega_2$. Above a value of 0.1 we can compute the contribution of the set causes Ω_1 which acts on the event X_j .

3 Causal graph obtained from correlations matrix

To obtain the causal graph from a correlation matrix, we will use the method of successive highest causation's magnitudes (see paper [6] page 6).

We will present the contribution of the causes on the form of a bar graph and then will set a causation's magnitude value of 0.75 to be exceeded for each node assigned to the events in order to obtain the causal graph.

Once the causal graph is obtained, we will compute the density of the graph as in paper (see paper [6] page 7)

4 Correlation matrix Attitude and causal graph

We will consider in the following the attitude 's correlation matrix found in the R software.

$$\tilde{K}_{(X_1, X_2, X_3, X_4, X_5, X_6, X_7)^2} = \begin{pmatrix} 1.000000 & 0.8254176 & 0.4261169 & 0.6236782 & 0.5901390 & 0.1564392 & 0.1550863 \\ 0.8254176 & 1.0000000 & 0.5582882 & 0.5967358 & 0.6691975 & 0.1877143 & 0.2245796 \\ 0.4261169 & 0.5582882 & 1.0000000 & 0.4933310 & 0.4454779 & 0.1472331 & 0.3432934 \\ 0.6236782 & 0.5967358 & 0.4933310 & 1.0000000 & 0.6403144 & 0.1159652 & 0.5316198 \\ 0.5901390 & 0.6691975 & 0.4454779 & 0.6403144 & 1.0000000 & 0.3768830 & 0.5741862 \\ 0.1564392 & 0.1877143 & 0.1472331 & 0.1159652 & 0.3768830 & 1.0000000 & 0.2833432 \\ 0.1550863 & 0.2245796 & 0.3432934 & 0.5316198 & 0.5741862 & 0.2833432 & 1.0000000 \end{pmatrix}$$

where the events are:

1. $X_1 = \textit{rating}$
2. $X_2 = \textit{complaints}$
3. $X_3 = \textit{privileges}$
4. $X_4 = \textit{learning}$
5. $X_5 = \textit{raises}$
6. $X_6 = \textit{critical}$
7. $X_7 = \textit{advance}$

4.1 Causation's magnitude for the node X_1

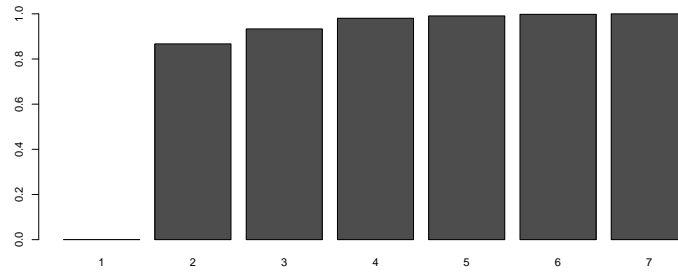


Figure 1: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_1} = 0$
2. $\lambda_{X_2 \rightarrow X_1} = 0.866971$
3. $\lambda_{\{X_2, X_4\} \rightarrow X_1} = 0.9333113$
4. $\lambda_{\{X_2, X_4, X_7\} \rightarrow X_1} = 0.9803893$
5. $\lambda_{\{X_2, X_4, X_7, X_3\} \rightarrow X_1} = 0.9908109$
6. $\lambda_{\{X_2, X_4, X_7, X_3, X_5\} \rightarrow X_1} = 0.997756$
7. $\lambda_{\{X_2, X_4, X_7, X_3, X_5, X_6\} \rightarrow X_1} = 1$

4.2 Causation's magnitude for the node X_2

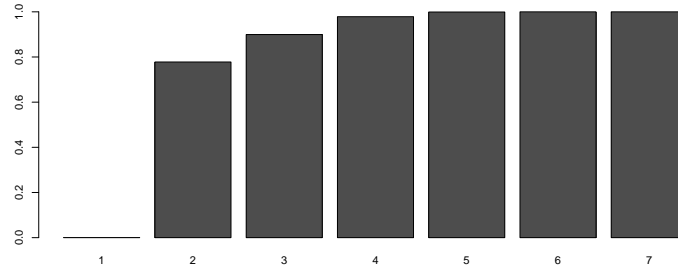


Figure 2: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_2} = 0$
2. $\lambda_{X_1 \rightarrow X_2} = 0.7778966$
3. $\lambda_{\{X_1, X_3\} \rightarrow X_2} = 0.8994167$
4. $\lambda_{\{X_1, X_3, X_5\} \rightarrow X_2} = 0.9784996$
5. $\lambda_{\{X_1, X_3, X_5, X_7\} \rightarrow X_2} = 0.9991813$
6. $\lambda_{\{X_1, X_3, X_5, X_7, X_6\} \rightarrow X_2} = 0.9999381$
7. $\lambda_{\{X_1, X_3, X_5, X_7, X_6, X_4\} \rightarrow X_2} = 1$

4.3 Causation's magnitude for the node X_3

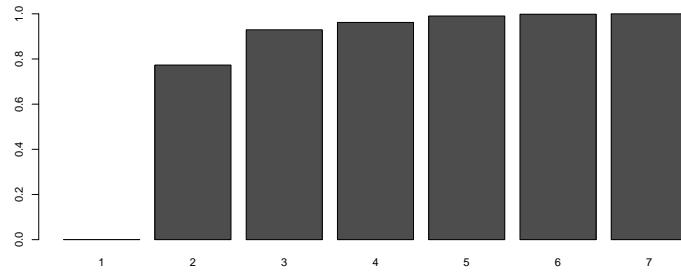


Figure 3: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_3} = 0$
2. $\lambda_{X_2 \rightarrow X_3} = 0.773189$
3. $\lambda_{\{X_2, X_7\} \rightarrow X_3} = 0.9293297$
4. $\lambda_{\{X_2, X_7, X_4\} \rightarrow X_3} = 0.961864$
5. $\lambda_{\{X_2, X_7, X_4, X_1\} \rightarrow X_3} = 0.9903198$
6. $\lambda_{\{X_2, X_7, X_4, X_1, X_5\} \rightarrow X_3} = 0.9982807$
7. $\lambda_{\{X_2, X_7, X_4, X_1, X_5, X_6\} \rightarrow X_3} = 1$

4.4 Causation's magnitude for the node X_4

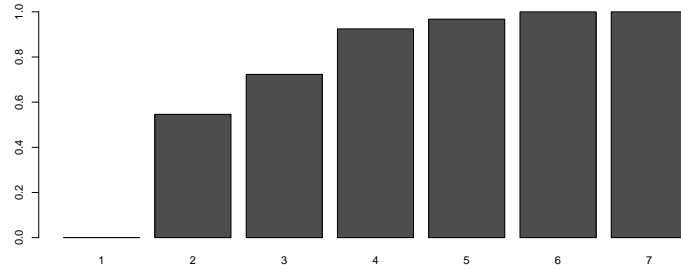


Figure 4: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_4} = 0$
2. $\lambda_{X_5 \rightarrow X_4} = 0.5461211$
3. $\lambda_{\{X_5, X_1\} \rightarrow X_4} = 0.7230502$
4. $\lambda_{\{X_5, X_1, X_7\} \rightarrow X_4} = 0.924563$
5. $\lambda_{\{X_5, X_1, X_7, X_6\} \rightarrow X_4} = 0.9674856$
6. $\lambda_{\{X_5, X_1, X_7, X_6, X_3\} \rightarrow X_4} = 0.9999058$
7. $\lambda_{\{X_5, X_1, X_7, X_6, X_3, X_2\} \rightarrow X_4} = 1$

4.5 Causation's magnitude for the node X_5

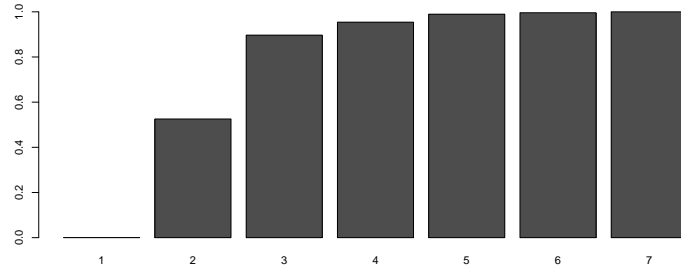


Figure 5: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_5} = 0$
2. $\lambda_{X_2 \rightarrow X_5} = 0.5254475$
3. $\lambda_{\{X_2, X_7\} \rightarrow X_5} = 0.896712$
4. $\lambda_{\{X_2, X_7, X_6\} \rightarrow X_5} = 0.9541725$
5. $\lambda_{\{X_2, X_7, X_6, X_4\} \rightarrow X_5} = 0.9895531$
6. $\lambda_{\{X_2, X_7, X_6, X_4, X_1\} \rightarrow X_5} = 0.9959638$
7. $\lambda_{\{X_2, X_7, X_6, X_4, X_1, X_3\} \rightarrow X_5} = 1$

4.6 Causation's magnitude for the node X_6

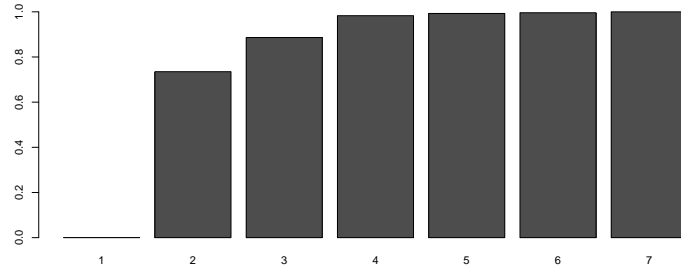


Figure 6: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_6} = 0$
2. $\lambda_{X_5 \rightarrow X_6} = 0.7349938$
3. $\lambda_{\{X_5, X_4\} \rightarrow X_6} = 0.886297$
4. $\lambda_{\{X_5, X_4, X_7\} \rightarrow X_6} = 0.9829557$
5. $\lambda_{\{X_5, X_4, X_7, X_1\} \rightarrow X_6} = 0.9927832$
6. $\lambda_{\{X_5, X_4, X_7, X_1, X_2\} \rightarrow X_6} = 0.9960153$
7. $\lambda_{\{X_5, X_4, X_7, X_1, X_2, X_3\} \rightarrow X_6} = 1$

4.7 Causation's magnitude for the node X_7

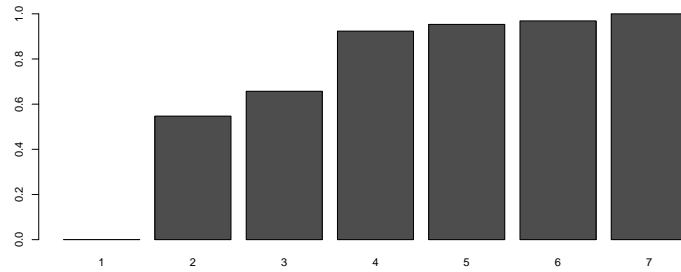
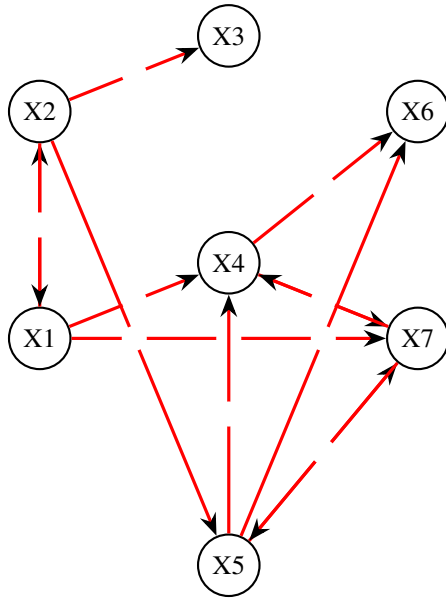


Figure 7: Causation's magnitude

1. $\lambda_{\emptyset \rightarrow X_7} = 0$
2. $\lambda_{X_5 \rightarrow X_7} = 0.5471036$
3. $\lambda_{\{X_5, X_1\} \rightarrow X_7} = 0.657135$
4. $\lambda_{\{X_5, X_1, X_4\} \rightarrow X_7} = 0.9234174$
5. $\lambda_{\{X_5, X_1, X_4, X_6\} \rightarrow X_7} = 0.9532082$
6. $\lambda_{\{X_5, X_1, X_4, X_6, X_2\} \rightarrow X_7} = 0.9687386$
7. $\lambda_{\{X_5, X_1, X_4, X_6, X_2, X_3\} \rightarrow X_7} = 1$

4.8 Causal graph obtained from the attitude's correlation matrix

By using the method of successive highest causation's magnitude (see paper [6] page 6) and by setting a causation's magnitude value of 0.75 to exceed, we get the following causal graph:



We have as causation's magnitude value:

1. $\lambda_{X_2 \rightarrow X_1} = 0.866971$
2. $\lambda_{X_1 \rightarrow X_2} = 0.7778966$
3. $\lambda_{X_2 \rightarrow X_3} = 0.773189$
4. $\lambda_{\{X_5, X_1, X_7\} \rightarrow X_4} = 0.924563$
5. $\lambda_{\{X_2, X_7\} \rightarrow X_5} = 0.896712$
6. $\lambda_{\{X_5, X_4\} \rightarrow X_6} = 0.886297$
7. $\lambda_{\{X_5, X_1, X_4\} \rightarrow X_7} = 0.9234174$

and we can compute the graph's density:

$$Density = \frac{13}{7.6} = \frac{13}{42} \approx 0.31$$

5 Conclusion

In this paper we have exposed the causation's magnitude computed from correlations. From an example, we have shown the steps to follow to obtain a causal graph from a correlation matrix.

[1] *Elements of information theory*. Author: Thomas M.Cover and Joy A.Thomas. Copyright 1991 John Wiley and sons.

[2] *Optimal stastical decisions*. Author: Morris H.DeGroot. Copyright 1970-2004 John Wiley and sons.

[3] *Matrix Analysis*. Author: Roger A.Horn and Charles R.Johnson. Copyright 2012, Cambridge university press.

[4] *Causality: Models, reasoning and inference*. Author:Judea Pearl .Copyright 2000, Cambridge university press.

[5] *Directed dependency graph obtained from a continious data matrix by the highest successive conditionings method*. Author: Ait-taleb Nabil.Years:2021. Published: Vixra

[6] *Directed dependency graph obtained from a correlation matrix by the highest successive conditionings method*. Author: Ait-taleb Nabil. Years:2021.Published: Vixra