New Prime Number Theory

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Abstract

This paper introduces a novel approach to estimating the distribution of prime numbers by leveraging insights from partition theory, prime number gaps, and the angles of triangles. Application of this methodology to infinite sums and nth terms, and propose several ways of defining the nth term of a prime number. By using the Ramanujan infinite series of natural numbers, they are able to derive an infinite sequence of prime numbers. Overall, this work represents a significant contribution to the field of prime number theory and sheds new light on the relationship between prime numbers and other mathematical concepts.

1 Introduction

Prime numbers and number theory were long considered to be abstract and pure mathematical topics without any practical applications beyond their inherent beauty and complexity. However, in the 1970s,it was discovered that prime numbers could be used as the foundation for creating public key cryptography algorithms, as well as for hash tables and pseudo random number generators. Additionally, rotor machines with prime or co-prime numbers of pins on each rotor were developed to create a complete cycle of possible rotor positions, allowing for more secure communication. In addition to cryptography and computer science, prime numbers have also been utilized in music composition. Some musicians have leveraged the unique properties of prime numbers to produce pieces that go beyond traditional rules and structures, creating a sense of innovation and freedom. Furthermore, the search for the largest prime number has become a popular pursuit among mathematicians and computer scientists, with organizations such as the Electronic Frontier Foundation offering financial rewards for its discovery. Although finding the largest prime number may not have any direct practical applications in the real world, it inspires a motivated audience to delve into the study of prime numbers, which could lead to new breakthroughs and advancements in mathematics and computer science. .While the study of prime numbers may not have direct applications in practical contexts, it can still

attract a wide range of enthusiasts who appreciate the beauty and complexity of mathematics. By delving into the properties and relationships of prime numbers, scholars can uncover essential structures and principles that underlie the vast and fascinating world of mathematics .The discovery of a new solution to the prime number theorem is an exciting achievement in the field of mathematics. The sequence of prime numbers has long been a source of fascination for mathematicians, and finding new and exact solutions to the sum, and infinite sum, is a significant contribution to our understanding of this sequence.[6][7][8] The use of the theory of partition and gap prime number in this discovery highlights the importance of these mathematical concepts in the study of the distribution of prime numbers. The results of this work have the potential to advance our understanding of the prime number theorem and to inform future research in number theory and related fields.It is inspiring to see the dedication and persistence of mathematicians in their quest to understand the mysteries of prime numbers. This new result is a testament to the power of human curiosity and the pursuit of knowledge.[2][1][5]
Srinivasa Ramanujan's discovery of the Ramanujan Summation in the early 1900s has had a profound impact on the field of physics, specifically in the study of the Casimir Effect. The effect, as predicted by Hendrik Casimir, suggests that two uncharged conductive plates in a vacuum will experience an attractive force due to the presence of virtual particles created by quantum fluctuations. To model the amount of energy between the plates, Casimir used the Ramanujan Summation, highlighting the significance of this mathematical technique.

The Ramanujan Summation has continued to prove its worth as a valuable tool in understanding the behavior of physical systems, even almost a century after its discovery. Its application in the Casimir Effect is just one example of how this technique has contributed to advancements in various branches of physics.

,I will using this result and find the sum of infinite of prime number series . My plan to use the Ramanujan Summation to find the sum of the infinite prime number series is also a promising avenue for research. The prime number theorem is an important topic in number theory, and finding new and exact solutions to its problems can further our understanding of this mathematical concept. [11][10][9][4]

2 Drive a Formula For Prime-Counting Function One

It is very interesting the prime number starting from 2 and it has two partitions.
2 = 2
2 = 1+1
P (2) =2 let we take a set $S = \{1, 2\}$
Choice the values for making sum of table 1 , 2 (right sides) from the set S always and L.H.S side of the table we take the prime number .

Now we create a table one

prime no	partition
2	1+1
3	1+1+1
5	1+1+1+1+1
7	1+1+1+1+1+1+1
11	1+1+1+1+1+1+1+1+1+1+1
. . .	....
sp	2n+1(n-1)+2(n-2)+2(n-3)+4(n-4)...

Table 1: prime sum table one

Making formula

$$\pi(n) = 2n + 1(n-1) + 2(n-2) + 2(n-3) + 4(n-4)...$$

$$\pi(n) = 2n + \sum_{i=1}^{n-1} \Delta g_i (n-i) \qquad \Delta g_i = p_{i+1} - p_i$$

$$\pi(n) = 2n + \sum_{i=1}^{n-1} \Delta g_i f(c_i) \qquad f(c_i) = (n-i)$$

2.1 Other Form Of Formula

Step Fuction . A function f:[a,b]→R is called a step function if it is piecewise constant, i.e. if there are numbers

$$a = x_o < x_1 < x_2 ... < x_n = b$$

such that f is constant on each half open interval with $\{x_{i-1}.x_i)$. For a step function we define the integral to be
$\int_a^b f(x)dx = \sum_{i=1}^n f(x_{i-1})(x_i - x_{i-1})$
The collection of numbers $\{x_o, x_1, x_2 ... x_n\}$ are called a partition for the step function f . [12]
Definite Integrals of piecewise functions
To integrate a piecewise function with constant values over each interval, it is necessary to split the integration at the exact boundary points between the intervals. A constant piecewise function is a function that maintains a consistent value across each "piece" or interval of its domain. This means that the function retains the same output value for any input value within a particular interval..[3]
let we draw a sequence.
$x_i = x_{i-1} + \Delta g_i$ i=1,2,3,4...(n-1)
Let $x_o = 0$ $where\ \Delta g_i = p_{i+1} - p_i$

$x_1 = x_o + \Delta g_1 = 0 + 1 = 1$

$x_2 = x_1 + \Delta g_2 = 1 + 2 = 3$

$x_3 = x_2 + \Delta g_3 = 3 + 2 = 5$ and so on

$x_p = \{0, 1, 3, 5...x_{i-1}, x_i\}$ $\qquad f(c_i) = (n - i)$

make sub-interval

$\{0, 1\}, \{1, 3\}, \{3, 5\}..., \{x_{i-1}, x_i\}$

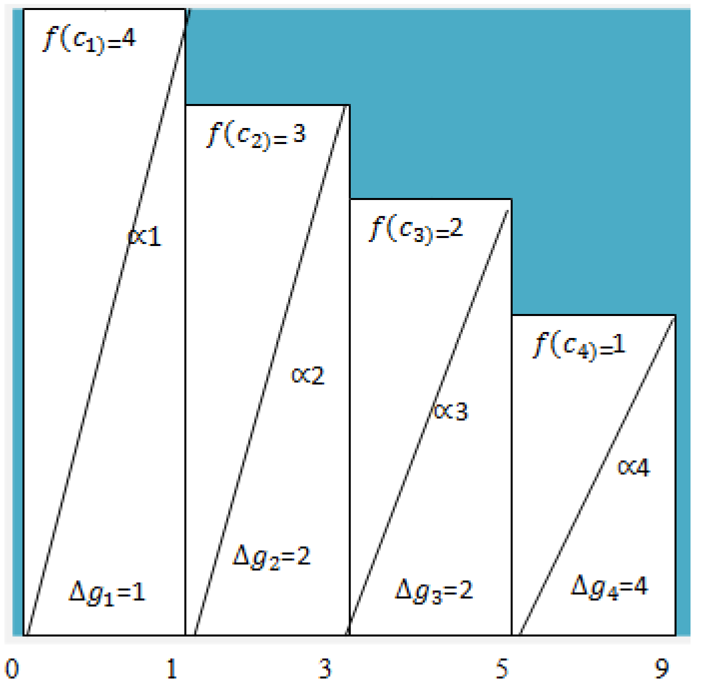
$$f(c) = \begin{cases} f(c_1) & 0 \leq x \leq 1 \\ f(c_2) & 1 \leq x \leq 3 \\ f(c_3) & 3 \leq x \leq 5 \\ f(c_4) & 5 \leq x \leq 9 \\ ..... & \\ f(c_i) & x_{i-1} \leq x \leq x_i \end{cases}$$

Integral form formula

$$\pi(n) = 2n + \int_{x_o}^{x_1} f(c_1)dx + \int_{x_1}^{x_2} f(c_2)dx \cdots + \int_{x_{i-1}}^{x_i} f(c_i)dx$$

$\pi(n) = 2n + \int_a^b f(c)dx$

2.2 Formula in Trigonometry Form/Name Dua Sum



To better understand the behavior and distribution of prime numbers, we can create a graph that visualizes the sum of prime munber. This graph would have the prime gaps plotted on the x-axis and the function values plotted on the y-axis.

By plotting the function values on the graph, we can observe the formation of multiple triangles, each corresponding to a consecutive sum of primes that equals a certain value. These triangles can help us better understand how prime numbers are distributed and how they relate to each other.

Overall, this graph can provide valuable insights into the nature of prime numbers and help us identify patterns and trends that would otherwise be difficult to discern.

$$\pi(n) = 2n + \sum_{i=1}^{n-1} (f(c_i))^2 tan\alpha_i$$

$tan\alpha_i = \frac{\Delta g_i}{f(c_i)}$ $\qquad f(c_i) = (n-i)$ $\qquad i = 1, 2, 3...(n-1)$

with fixed prime gap working

$\Delta g = \frac{final\ angle - initial\ angle\ \alpha_1}{n-1}$ $\quad \alpha_{i+1} = \alpha_i + \Delta g \ \ \Delta g \ \ fixed\ i = 1, 2, 3...n$

3 Drive a Formula for Prime Counting Function Two

Now we create a table two

prime no	partition	
2	2 fix	
3	2 starting	+1
5	2+2	+1
7	2+2+2	+1
11	2+2+2+2+2	+1
...	....	
Sp	2n+2(n-1)+2(n-2)+4(n-3)+2(n-4)... +2+n	

Table 2: prime sum two

Making formula

$\pi(n) = 2n + 2(n-1) + 2(n-2) + 4(n-3) + 2(n-4)... + 2 + n$
$\pi(n) = 3n + 2(n-1) + 2(n-2) + 4(n-3) + 2(n-4)... + 2$
"By replacing n with n-1 in the equation, we can adjust for the absence of the first row, giving us the following expression"
$\pi(n) = 3(n-1) + 2(n-2) + 2(n-3) + 4(n-4) + 2(n-5)... + 2$
$\pi(n) = 3n - 3 + 2(n-2) + 2(n-3) + 4(n-4) + 2(n-5)... + 2$
$\pi(n) = 3n - 1 + 2(n-2) + 2(n-3) + 4(n-4) + 2(n-5)...$

$\pi(n) = (3n-1) + \sum_{i=2}^{n-1} \Delta g_i (n-i) \qquad \Delta g_i = p_{i+1} - p_i \qquad i = 2, 3, 4...(n-1)$

$$\pi(n) = (3n-1) + \sum_{i=2}^{n-1} \Delta g_i f(c_i) \qquad f(c_i) = (n-i) \qquad i = 2, 3, 4...(n-1)$$

Note sum always positive take

3.1 Integral Formula Form

To integrate a piecewise function with constant values over each interval, it is necessary to split the integration at the exact boundary points between the intervals. A constant piecewise function is a function that maintains a consistent value across each "piece" or interval of its domain. This means that the function retains the same output value for any input value within a particular interval. let we draw a sequence.
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$x_1 = x_o + \Delta g_2 = 0 + 2 = 2$
$x_2 = x_1 + \Delta g_3 = 2 + 2 = 4$

$x_3 = x_2 + \Delta g_4 = 4 + 4 = 8$ and so on
$x_p = \{0, 2, 4, 8 ... x_{i-2}, x_{i-1}\}$ $\qquad f(c_i) = (n - i)$

Make subinterval

$\{0, 2\}, \{2, 4\}, \{4, 8\} ..., \{x_{i-2}, x_{i-1}\}$

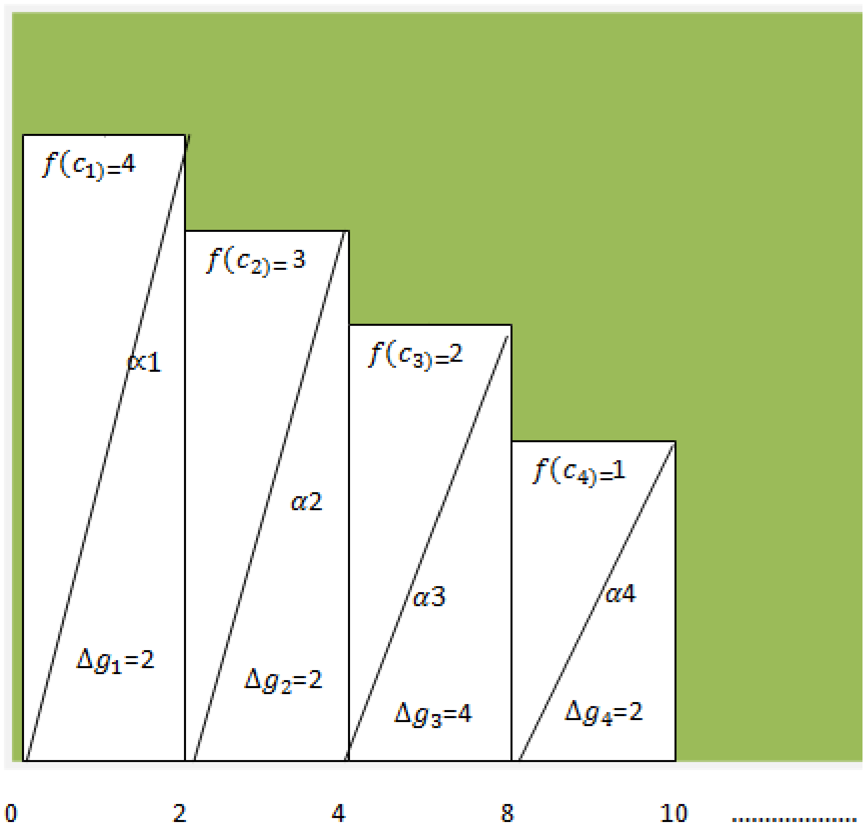
$$f(c) = \begin{cases} f(c_2) & 0 \leq x \leq 2 \\ f(c_3) & 2 \leq x \leq 4 \\ f(c_4) & 4 \leq x \leq 8 \\ f(c_5) & 8 \leq x \leq 10 \\ ..... & \\ f(c_i) & x_{i-2} \leq x \leq x_{i-1} \end{cases}$$

Integral form formula

$$\pi(n) = (3n - 1) + \int_{x_o}^{x_1} f(c_2) dx + \int_{x_1}^{x_2} f(c_3) dx ... + \int_{x_{i-2}}^{x_{i-1}} f(c_i) dx$$

$\pi(n) = (3n - 1) + \int_a^b f(c) dx$

3.2 Formula in Trigonometry Form/Name Noor sum



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with fixed prime gap working

$\Delta g = \frac{final\ angle - initial\ angle \alpha_2}{n-1}$ $\alpha_{i+1} = \alpha_i + \Delta g$ Δg $fixed\ i = 2, 3 ... (n-1)$

4 For Ramanujan Summation

For those of you who are unfamiliar with this series, which has come to be known as the Ramanujan Summation after a famous Indian mathematician named Srinivasa Ramanujan, it states that if you add all the natural numbers, that is 1, 2, 3, 4, and so on, all the way to infinity, you will find that it is equal to -1/12.we drived some results from this

$$S(N)_\infty = \frac{-1}{12}$$

$$S(E)_\infty = 2(\frac{-1}{12}) = \frac{-1}{6}$$

$$S(O)_\infty = S(N)_\infty - S(E)_\infty = \frac{1}{12}$$

$$S(E)_\infty - S(O)_\infty = \frac{-1}{6} - \frac{1}{12} = \frac{-1}{4}$$

$$2 - 1 + 4 - 3 + 6 - 5 ... = \frac{-1}{6} - \frac{1}{12} = \frac{-1}{4}$$

$$1 + 1 + 1 + 1 + 1 \cdots = \frac{-1}{4}$$

Very important result

$$1 + 1 + 1 + 1 \cdots = \sum_{n=1}^{\infty} 1 = \frac{-1}{4}$$

4.1 Sum of Table Two

The value of the largest element in set S, also known as the "supermum", is 2. Therefore, when partitioning odd numbers, we always include 2 as the first term so on and 1 as the last term. The result is obtained by adding together the L.H.S and R.H.S of the table two.

$\pi(\infty) = 2 + 3 + 5 + 7 ... = (2 + 2 + 2 + 2 ...) + (1 + 1 + 1 + 1 ...)$

$$\pi(\infty) = 2 \sum_{n=1}^{\infty} 1 + \sum_{n=1}^{\infty} 1 = \frac{-3}{4}$$

$$\pi(\infty) = \frac{-3}{4} \sim \frac{-\pi}{4}$$

5 Results And Application

Using the Dua sum formula, we can obtain the sum of the 5th and 6th prime numbers. The results of this calculation are as follows:

Working with fix gap e.g

$\Delta g = \frac{90^0 - \alpha_1}{n-1}$ $\qquad \alpha_{i+1} = \alpha_i + \Delta g \qquad \Delta g \quad fixed$

$$\alpha_1 = \arctan \frac{1}{5-1} = 14^\circ$$

$$\Delta g = \frac{90^0 - 14^0}{5-1} = 19^0$$

Simmerly find next valuess

$$\alpha_2 = \alpha_1 + \Delta g = 14^o + 19^o = 33^o$$

For 5th values prime sum is equal to

$$\pi(5) = 10 + (4)^2 tan14^o + (3)^2 tan33^o + (2)^2 tan52^o + (1)^2 tan71^o = 27.954$$

Simmerly next

$$\pi(6) = 41.12$$

- Application is dependent on the initial angle and final angle values that are chosen. In other words, the selection of initial angle and final angle values is crucial for the proper functioning and output of an application. These values determine the range or scope of the application and can greatly impact the results obtained. Therefore, careful consideration and selection of the initial and final angles values is essential for the success of any application.
- Noor sum infinite formula , is better formula i think. $\sim \frac{-\pi}{4}$
- For fast results we use noor sum formula.
- Use dua or noor (in trigonometry form for using frist gap) sum formula the *results are nearly → prime sum*.
- In order to achieve improved results, it is recommended to use the average of the initial angle values. By doing so, the outcome is likely to be more favorable.
- we also use radian or degree angles

e.g , Integral Formula Form Working

$$\pi(4) = 8 + \int_o^1 (4-1)dx + \int_1^3 (4-2)dx + \int_3^5 (4-3)dx$$

$$\pi(4) = 8 + 3 + 4 + 2 = 17$$

6 Conclusion

Interdisciplinary research is becoming increasingly important in today's world, as it allows experts from different fields to collaborate and leverage their expertise to solve complex problems.this paper introduces a distinctive methodology for estimating prime number distribution using partition theory, prime number gaps, and triangle angles, and applies it to analyze infinite sums and nth terms in various shapes. The insights gained from this approach shed new light on the nature of prime numbers and their connections with other mathematical concepts. .

7 Acknowlegment

I dedicate my work to my sweet daughters Dua and Noor.To my beloved wife, who has been my constant source of inspiration, encouragement, and unwavering support throughout my academic pursuit. Your love and support have sustained me through the challenges and hardships of this journey, and I dedicate this thesis to you as a token of my gratitude and appreciation. Thank you for being my rock and for always believing in me.

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