Exact Mass of the Charmed Lambda Baryon

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In 2006 Particle Data Group changed their FIT of the mass of the Charmed Lambda+ baryon from 2284.9 +-0.6 MeV, to 2286.46 +-0.14 MeV (an increase of 1.56 Mev), because they said the new BABAR measurement was so much better, meaning more accurate. True, the measurement is more accurate, but n-sphere factoring shows the new mass found by BABAR is probably not the mass of a Lambda(c) baryon (**udc** quark content), but rather it is likely the mass of a Xi(c) baryon (**dsc** quark content). This paper explains the arguments for the two quark content assignments, and gives an expression for the exact mass of each baryon. (Since Planck's constant was declared exact recently, n-sphere factoring of hadron experimental masses results in expressions yielding exact theoretical masses.)

1.0 Introduction

The masses of all hadrons can be represented as multiples of n-sphere surface volumes times Planck's constant's coefficient (h = 6.62607015). This is suspected of being true because when an n-sphere surface volume times h, is divided into a hadron's mass, many times the result is an integer or a small denominator fraction, the numerator of which is either a power of two, or is the sum of a few powers of two. For instance, the second experimental mass listed by PDG for the **cd** meson **D^+**, is 1870.0 +-0.5/+-1.0 MeV. Its n-sphere factoring is **7680/900 S7h = 1870.049 MeV,** where S7h is an abbreviation for the surface volume of a unit radius 7-sphere times h. (S7h was used as the unit of factorization in this case, because it is compatible with **cd** quark content. More on this later.) The numerator of the factoring fraction, 7680, is the sum of four consecutive powers of two: 7680 = 4096 + 2048 + 1024 + 512. All hadrons seem to factor with either powers of two, sums of powers of two, or sums of powers of two minus a smaller power of two in the numerator of the factoring fraction. As another case in point (the subject of this paper) the lambda(c)+ baryon with experimental mass 2284.7 MeV, factors with three consecutive powers of two in the numerator of its factoring fraction: 16384 + 8192 + 4096.

Why does n-sphere surface volume factoring work? The theory is that all the matter of a hadron is composed of higher dimensional matter, and occupies the volume in n-sphere surfaces of various dimensions depending on the 'quark content' of the hadron. Quarks are not considered particles, but volumes of n-sphere surfaces filled with matter. The up quark correspnds to matter filling the surface volume of a 2-sphere, the down quark corresponds to matter filling the surface volume of a 3-sphere, etc. Here's a list of the quarks and their corresponding n-sphere surface volume formulae.

Quark	n-Sphere Surface Volume Formula	
u	S2 =	$2 \pi r^1$
d	S3 =	$4 \pi r^2$
s	S4 =	$2 \pi^2 r^3$
c	S5 =	$(8/3) \pi^2 r^4$
b	S6 =	$\pi^3 r^5$
t	S7 =	$(16/15) \pi^3 r^6$

Note that this quark model can be extended indefinitely. How can n-dimensional matter exist 'in' our 3D space? Because the matter of which hadrons are composed only *intersects* 3D space (the Higgs field). Higher dimensional matter is not 'in' 3D space. No matter exists 'in' 3D space. The 3D interior of hadrons is devoid of matter. The surface of the proton and all other hadrons, does not define the surface of a 3D object, as is commonly thought, but rather, it defines the intersection between the hadron's higher dimensional matter and our 3D space (the Higgs field). 100% of the matter of which hadrons are composed resides in the higher dimensional space exterior to, and immediately adjacent to, the hadron's apparent 3D location. (This architecture is possible because 3D space has zero thickness in the fourth and higher dimensional directions, therefore 4D and higher dimensional space is next to *every* point in 3D space.)

Source of mass data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

2.0 n-Sphere Surface Volume Factoring

How does one factor a hadron with n-sphere surface volumes? If the quark content of the hadron is known, multiply all the n-sphere surface volume formula associated with each quark in the hadron together along with Planck's constant's coefficient (h=6.62607015). The unit of factorization derived is then divided into the hadron's mass. In the case of the **cd** meson **D**$^+$ mentioned above, its unit of factorization would be **cdh**, or S5S3h, or $(8/3\ \pi^2 r^4)(4\ \pi\ r^2)$h. Multiplying, one gets $(32/3\ \pi^3 r^6)$h as the unit of factorization. Substituting r=1, and solving, one gets 2191.4641 MeV for the value of that unit of factorization. It can be used to factor any **cd** meson. Dividing it into the experimental mass of the D+ meson (1870.0) one gets 0.85333. Multiplying that by 900 you get 768. (The 900 divisor works for this factoring because of the 3's in the decimal expansion of 0.85333.) The complete factoring is: 768/900 **cdh** = 1870.0494 MeV. The numerator, 768 is the sum of two powers of two: 768 = 512 + 256.

Notice that the factoring unit just derived for factoring **cd** mesons, **cdh** = $(32/3\ \pi^3 r^6)$h, has the same powers of π and r in it as **S7h** = $(16/15\ \pi^3 r^6)$h. Both have (π,r) powers of (3,6). The **S7h** unit is just ten times smaller than the **cdh** unit. Except for that they are identical, so **S7h** can be used interchangeably with **cdh**. This is true of all units of factorization that are derived by multiplying the associated n-sphere surface volume formulae of quarks together. For any combination of quarks multiplied together (that results in π and r powers found in an n-sphere surface volume formula) a single n-sphere surface volume formula having the same π and r powers can be used instead. To find out which n-sphere surface volume formula can be used to factor which quark combinations see *Appendix A, Quark Content Possibilities by Factoring Unit Used.*

3.0 Factorization of the 2284.7 MeV Mass with S8h

Since the charmed lambda baryons have quark content **udc**, their masses can be factored not only with **udch**, but also with **S8h** (S8 represents the surface volume formula of an 8-sphere. See Appendix D for its formula.)**,** because **udch** and **S8h** each have (π, r) powers of (4, 7) in their respective formulae. They differ only in their constants of multiplication - **udch** is 64 times larger than **S8h**. Below is the factoring of the 2284.7 experimental mass done with **S8h**.

FACTORING *THRMASS (MeV)*

$$\frac{28672}{2700}\ \mathbf{S8h} = \frac{(16384 + 8192 + 4096)}{2700}\ \mathbf{S8h} = 2284.6963$$

$$\frac{28672}{2700}\ \mathbf{S8h} = \frac{(2^{14} + 2^{13} + 2^{12})}{2700}\ \mathbf{S8h} = 2284.6963$$

COMPARISON of EXPMASS vs THRMASS

EXPMASS (MeV)		*THRMASS (MeV)*		*DIFF (MeV)*
2284.7	-	2284.6963	=	0.0037

This factoring is convincing, meaning probably correct, for two reasons. First, the theoretical mass given by the factoring expression (2284.6963 MeV) closely matches a highly accurate experimental mass measurement (2284.7 MeV). They differ by only 0.0037 MeV, and the statistical and systematic errors of the 2284.7 mass measurement are only 0.6 and 0.7 MeV respectively, which indicates it's a very accurate measurement. Secondly, concerning the factoring expression, the numerator of the fraction that is the multiplier of the unit of factorization (S8h) is the sum of *three consecutive and relatively large powers of two*: 16384 + 8192 + 4096. These two factors, along with the fact that the S8h unit is compatible with **udc** quark content makes it highly likely that the expression **28672/2700 S8h** represents the exact mass of the Λc+ baryon. The next section will show that the mass adopted by PDG in 2006 for the Λc+ baryon's mass (2286.46 MeV) is most likely the mass of a charmed Xi baryon (**Ξc)**, because it factors much better with S10h than with S8h, indicating it has quark content **dsc**, not **udc**. (Note: **dsc** = $(4\pi r^2)(2\pi^2 r^3)(8/3\ \pi^2 r^4) = 64/3\ \pi^5 r^9$, and S10h = $1/12\ \pi^5 r^9$, so both have (π,r) powers of (5,9) so **S10h** can be used to factor **dsc** baryons.)

Source of mass data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

4.0 Factorization of the 2286.46 MeV Mass with S10h

In 2006 Particle Data Group changed their FIT for the mass of the Λc+ from 2284.9 +-0.6 MeV to the more accurate 2286.46 +-0.14 MeV BABAR determined mass value. It was deemed a 'better' measurement because it was more accurate, but n-sphere surface volume factoring puts its classification as a charmed-lambda baryon in doubt, because a charmed lambda baryon has quark content **udc** and factors with **S8h**, whereas the 2286.46 BABAR mass factors much better with **S10h**, so probably has **dsc** quark content (if its a baryon), which means it is a charmed-xi baryon.

	FACTORING	*THRMASS (MeV)*
$\frac{9472}{700}$ **S10h** =	$\frac{(8192 + 1024 + 256)}{700}$ **S10h**	= **2286.4820**
$\frac{9472}{700}$ **S10h** =	$\frac{(2^{13} + 2^{10} + 2^{8})}{700}$ **S10h**	= **2286.4820**

COMPARISON of EXPMASS vs THRMASS

THRMASS (MeV)		*EXPMASS (MeV)*		*DIFF (MeV)*
2286.4820	-	**2286.46**	=	**0.0220**

As shown above, the S10h factoring of the 2286.46 mass is very convincing (meaning likely correct). The theoretical and experimental masses match closely. The error of the 2286.46 measurement is +-0.14 MeV, and the difference between the experimental and theoretical masses is only 0.022 MeV, which is just 16% of the 0.14 MeV error, and the numerator of the factoring fraction is the sum of three relatively large powers of two: 8192 + 1024 + 256. All these points add up to the conclusion that this is the correct factoring for the 2286.46 MeV mass. Factoring it with S8h results in a numerator that is not the sum of relatively large powers of two as the S10h factoring is. The S8h factoring is: (16384 + 8192 + 4096 + 22)/2700 S8h. The 22 in the numerator is not even close to a *small* power of two, and rounding the 22 in the numerator down to 16, or rounding it up to 32 places the theoretical mass out of the error range of the experimental mass's measurement, so S8h/2700 factoring for this mass is not correct. Other S8h factorings using divisors other than 2700, even other power of three based divisors such as 8100, have been tried to no avail, but the search was not exhaustive, so there is still a possibility an S8h factoring better than the S10h factoring could be found, but because of the closeness of the S10h factoring result to the 2286.46 MeV measurement and the large powers of two in the numerator of the S10h factoring's fraction, the chances of a *better* S8h factoring being found is small.

Source of mass data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

5.0 Summary and Conclusion

Because the **2284.7** MeV mass factors convincingly with **S8h** (which is **udc** compatible, or Lambda(c) baryon compatible), and the **2286.46** MeV mass doesn't factor convincingly with **S8h**, but does factor convincingly with **S10** (which is **dsc** compatible, or Xi (c) baryon compatible) the true mass of the charmed-lambda baryon is most probably (to eight digits of accuracy): **2284.6963** MeV, not **2286.46** MeV. The exact mass is given by the expression: **28672 S8h/2700 MeV**. (It is correct to say that expression represents the *exact* mass of the Λc^+ because 'h' was recently (2019) declared exact.)

The factoring, 28672 S8h/2700, of the Λc+ baryon begs the question, "Is the factoring of a hadron congruent in some way to its structure?" It's possible, especially when considering the factoring of similar hadrons. For instance, another charmed lambda baryon, the $\mathbf{\Lambda_c}(2860)^+$ also factors with a divisor of 2700, and with a sum of powers of two in its factoring fraction's numerator. (See Appendix A for details of its factoring.) It factors as (32768 + 2048 + 1024) S8h/2700, which is exactly 7168 S8h/2700 MeV greater than the mass of the Λc+ baryon, and 7168 = 4096 + 2048 + 1024 + 512. So, n-sphere factoring may give clues to hadron structure.

6.0 References

[1] P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

7.0 Appendices

Appendix A Factorization of $\mathbf{\Lambda_c}^+$ and $\mathbf{\Lambda_c}(2860)^+$
Appendix B Quark Content Possibilities by Factoring Unit Used
Appendix C Examples of n-Sphere Surface Volume Factoring of Some Hadron Masses
Appendix D n-Sphere Surface Volume Formulae
Appendix E Values of n-Sphere Surface Volume Units of Factorization

APPENDIX A

Factorization of Λ_c^+ and $\Lambda_c(2860)^+$

Λ_c^+ MASS

EXPMASS (MeV)
2284.7 0.6/0.7

FACTORING *THRMASS (MeV)*

$$\frac{28672}{2700}\ \mathbf{S8h} = \frac{(16384 + 8192 + 4096)}{2700}\ \mathbf{S8h} = 2284.6963$$

$$\frac{28672}{2700}\ \mathbf{S8h} = \frac{(2^{14} + 2^{13} + 2^{12})}{2700}\ \mathbf{S8h} = 2284.6963$$

$\Lambda_c(2860)^+$ MASS

EXPMASS (MeV)
2856.1 2.0/1.7 **PDG FIT**

FACTORING *THRMASS (MeV)*

$$\frac{35840}{2700}\ \mathbf{S8h} = \frac{(32768 + 2048 + 1024)}{2700}\ \mathbf{S8h} = 2855.8704$$

$$\frac{35840}{2700}\ \mathbf{S8h} = \frac{(2^{15} + 2^{11} + 2^{10})}{2700}\ \mathbf{S8h} = 2855.8704$$

$\Lambda_c(2860)^+$ - Λ_c^+ MASS DIFFERENCE

$$\frac{35840\ \mathrm{S8h}}{2700} - \frac{28672\ \mathrm{S8h}}{2700} = \frac{7168\ \mathrm{S8h}}{2700} = 571.1740\ \mathrm{MeV/c^2}$$

$$7168 = 4096 + 2048 + 1024$$
$$7168 = 2^{12} + 2^{11} + 2^{10}$$

Source of mass data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX A

Quark Content Possibilities by Factoring Unit Used

	Factoring Unit If……. Mass factors with		(π, r) Powers	Quark Content Possibilities Then……. Hadron has one of these Quark Contents								
u	S2h	=	(1, 1)									
d	S3h	=	(1 ,2)									
s	S4h	=	(2, 3)	du								
c	S5h	=	(2, 4)	dd								
b	S6h	=	(3, 5)	ddu	ds, uc							
t	S7h	=	(3, 6)	ddd	dc							
v	S8h	=	(4, 7)	dddu	dds, udc	sc, db, ut						
w	S9h	=	(4, 8)	dddd	ddc	cc, dt						
x	S10h	=	(5, 9)	ddddu	ddds	dsc, ddb	cb, st					
y	S11h	=	(5, 10)	ddddd	dddc	dcc	ct					
z	S12h	=	(6, 11)	dddddu	dddds	ddcs	ccs, dcb	bt, cv				
	S13h	=	(6, 12)	dddddd	ddddc	ddcc	ccc	t t				
	S14h	=	(7, 13)	ddddddu	ddddds	dddcs	dccs	ccb	tv			
	S15h	=	(7, 14)	ddddddd	dddddc	dddcc	dccc	cct	tw			
	S16h	=	(8, 15)	dddddddu	dddddds	ddddcs	ddccs	cccs	btc	vw		
	S17h	=	(8, 16)	dddddddd	ddddddc	ddddcc	ddccc	cccc	t t s	ww		
	S18h	=	(9, 17)	ddddddddu	ddddddds	dddddcs	dddccs	dcccs	cccb	t t b	wx	
	S19h	=	(9, 18)	ddddddddd	dddddddc	dddddcc	dddccc	dcccc	ccct	t t t	wy	
	S20h	=	(10, 19)	dddddddddu	dddddddds	ddddddcs	ddddccs	ddcccs	ccccs	cccv	t t w	xy
	S21h	=	(10, 20)	dddddddddd	ddddddddc	ddddddcc	ddddccc	ddcccc	ccccc	cccw	t t x	yy

Note: s=du c=dd b=ds=cu t=cd

The quark combinations shown above include only those quark combinations that result in an *n-sphere surface volume formula* similar equation. If two or more *even dimension* quark formulae are multiplied together, the result is not an *n-sphere surface volume formula* similar equation. Those are not listed. Examples are *uu, us, ub, ss, sb, bb,* etc.
All valid quark combinations for the factoring units from S4h to S9h are shown (I think). For the factoring units from S10h to S21h not all possible valid quark combinations are shown, especially for the triquarks (qqq, baryons) and the diquarks (qq, mesons). This was done so the table wouldn't look too complex and potentially confusing.

The parentheses enclosing two integers separated by a comma that is just to the right of the factoring units, such as the (1,2) in the line S3h = (1,2), means the surface volume formula of that factoring unit has the powers 1 and 2 for 'π' and 'r'. In the case of S3h, $S3 = 4\pi^1r^2$. 'π' is raised to the power 1, and 'r' is raised to the power 2, that's why it's written S3h = (1,2). Using this parentheses notation for surface volume formula representation makes it easy to determine which factoring unit will factor which quark combinations, or vice versa, which quark combinations can be factored by which factorung unit.

For instance, if you want to know which factoring unit will factor 'ddd', since 'd' = S3 = (1,2), just add the corresponding integers together of the product (1,2)(1,2)(1,2). You are multiplying numbers together ('π' and 'r') that are raised to integer powers, and, powers add, so you get (3, 6). Now find the line with (3, 6) in it. It is S7h = (3, 6). So the factoring unit needed to factor 'ddd' is S7h.

APPENDIX C

Examples of n-Sphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each

Subatomic Particle	ExpMass	Error	n-Sphere Factoring	ThrMass	Compatible QuarkContent
ρ(770)	775.02	0.35	**4.44444 S5h** =	775.071	dd
η	547.865	0.031	**2.66666 S6h** =	547.8660	ds
Δ(1232)	1232.9	1.2	**6.00000 S6h** =	1232.698	ddu
K(1430)	1438	8/4	**7.00000 S6h** =	1438.148	ds
Δ(1700)	1643	6/3	**8.00000 S6h** =	1643.598	ddu
Xi^0	1314.86	0.20	**6.00000 S7h** =	1314.878	ddd
Xi^-	1321.71	0.07	**6.03125 S7h** =	1321.727	ddd
a2(1700)	1721	11/44	**8.00000 S8h** =	1721.172	cs
Ds	1967.0	1.0/1.0	**64/7 S8h** =	1967.053	cs
Ds(2460)	2458.9	1.5	**80/7 S8h** =	2458.817	cs
B2(5747)	5737.2	0.7	**26.66666 S8h =**	5737.239	bd
Ds	1967.0	1.0/1.0	**10.00000 S9h** =	1967.053	cc
Ds(2460)	2458.9	1.5	**12.50000 S9h** =	2458.817	cc
Ds(2700)	2688	4	**13.66666 S9h** =	2688.307	cc
Ds(2700)	2710	2	**13.77777 S9h** =	2710.163	cc
Bj(5732)	5704	4/10	**29.00000 S9h** =	5704.455	cc
Ds(2212)	2112.2	0.4	**12.5000 S10h** =	2112.195	bc
Ω(2250)	2253	13	**13.3333 S10h** =	2253.008	dsc
Ds1(2536)	2534.6	0.3/0.7	**15.0000 S10h** =	2534.634	bc
Ds2(2572)	2572.2	0.3/1.0	**15.2222 S10h** =	2572.185	bc
Ds0(2590)	2591	13	**15.3333 S10h** =	2590.960	bc
Pc(4337)	4337	7/4	**25.6666 S10h** =	4337.041	ddddu
Pc(4457)	4449.8	1.7/2.5	**26.3333 S10h** =	4449.692	ddddu
Y(4500)	4506	11	**26.6666 S10h** =	4506.017	ddddu
b1(1235)	1236	16	**9.0000 S11h** =	1235.936	ddddd
X(2175)	2197.4	4.4	**16.0000 S11h** =	2197.219	ddddd
Z(3985)	3982.5	1.8	**29.0000 S11h** =	3982.461	ddddd
X(4660)	4669	21/3	**34.0000 S11h** =	4669.092	ddddd
Ds(2860)	2866.6(avg)		**27.0000 S12h** =	2866.605	bt
$D(3000)^0$	2971.8	8.7	**28.0000 S12h** =	2972.775	bt
$D(3000)^0$	3008.1	4.0	**28.3333 S12h** =	3008.165	bt
Dsj(3040)	3044	8	**28.6666 S12h** =	3043.555	bt
Ω	1673.4	1.7	**21.3333 S13h** =	1673.398	ccc
Xi(1950)	1952	11	**24.8888 S13h** =	1952.298	ccc
Xi(2500)	2505	10	**31.9375 S13h** =	2505.195	ccc
fj(2220)	2223.9	2.5	**40.0000 S14h** =	2223.630	vt
Xc0(1P)	3415.5	0.4/0.4	**61.4400 S14h** =	3415.496	ccsd
Xc2(1P)	3557.8	0.2/4	**64.0000 S14h** =	3557.808	ccsd
ηb(1S)	9394.8	2.7/3.1	**169.0000 S14h** =	9394.839	vt
f0(980)	977.3	0.9/3.7	**99.7500 S18h** =	977.298	cccb
f0(980)	982.2	1.0/8.1	**100.2500 S18h** =	982.197	cccb
f0(980)	984.7	0.4/2.4	**100.5000 S18h** =	984.646	cccb

Source of ExpMass and Error data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX D

n-Sphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

Sphere Dimension	Sn	Surface Volume Formula	(π, r) Powers
2	S2 =	2 $\pi^1 r^1$	(1, 1)
3	S3 =	4 $\pi^1 r^2$	(1, 2)
4	S4 =	2 $\pi^2 r^3$	(2, 3)
5	S5 =	8/3 $\pi^2 r^4$	(2, 4)
6	S6 =	$\pi^3 r^5$	(3, 5)
7	S7 =	16/15 $\pi^3 r^6$	(3, 6)
8	S8 =	1/3 $\pi^4 r^7$	(4, 7)
9	S9 =	32/105 $\pi^4 r^8$	(4, 8)
10	S10 =	1/12 $\pi^5 r^9$	(5, 9)
11	S11 =	64 / 945 $\pi^5 r^{10}$	(5, 10)
12	S12 =	1 / 60 $\pi^6 r^{11}$	(6, 11)
13	S13 =	128 / 10395 $\pi^6 r^{12}$	(6, 12)
14	S14 =	1 / 360 $\pi^7 r^{13}$	(7, 13)
15	S15 =	256 / 135135 $\pi^7 r^{14}$	(7, 14)
16	S16 =	1 / 2520 $\pi^8 r^{15}$	(8, 15)
17	S17 =	512 / 2027025 $\pi^8 r^{16}$	(8, 16)
18	S18 =	1 / 20160 $\pi^9 r^{17}$	(9, 17)
19	S19 =	1024 / 34459425 $\pi^9 r^{18}$	(9, 18)
20	S20 =	1 / 181440 $\pi^{10} r^{19}$	(10, 19)
21	S21 =	2048 / 654729075 $\pi^{10} r^{20}$	(10, 20)

APPENDIX E

Values of n-Sphere Surface Volume
Units of Factorization

(Dimension 2 - Dimension 21)

Sphere Dimension	Unit of Factorization		Formula		Value (MeV/c^2)
2	S2h	=	$2\ \pi^1 r^1 h$	=	41.63282661
3	S3h	=	$4\ \pi^1 r^2 h$	=	83.26565322
4	S4h	=	$2\ \pi^2 r^3 h$	=	130.7933822
5	S5h	=	$8/3\ \pi^2 r^4 h$	=	174.3911763
6	S6h	=	$\pi^3 r^5 h$	=	205.4497644
7	S7h	=	$16/15\ \pi^3 r^6 h$	=	219.1464153
8	S8h	=	$1/3\ \pi^4 r^7 h$	=	215.1464901
9	S9h	=	$32/105\ \pi^4 r^8 h$	=	196.7053624
10	S10h	=	$1/12\ \pi^5 r^9 h$	=	168.9756582
11	S11h	=	$64 / 945\ \pi^5 r^{10} h$	=	137.3262492
12	S12h	=	$1 / 60\ \pi^6 r^{11} h$	=	106.1705373
13	S13h	=	$128 / 10395\ \pi^6 r^{12} h$	=	78.44057013
14	S14h	=	$1 / 360\ \pi^7 r^{13} h$	=	55.59076334
15	S15h	=	$256 / 135135\ \pi^7 r^{14} h$	=	37.91204905
16	S16h	=	$1 / 2520\ \pi^8 r^{15} h$	=	24.94907624
17	S17h	=	$512 / 2027025\ \pi^8 r^{16} h$	=	15.88056197
18	S18h	=	$1 / 20160\ \pi^9 r^{17} h$	=	9.797479330
19	S19h	=	$1024 / 34459425\ \pi^9 r^{18} h$	=	5.869441980
20	S20h	=	$1 / 181440\ \pi^{10} r^{19} h$	=	3.419965454
21	S21h	=	$2048 / 654729075\ \pi^{10} r^{20} h$	=	1.940989032