Vortons Revisited

M. I. Aksman

Abstract

Reasons for introduction of vortons. Answers to vortons critiques, “modifications” and “improvements”. New interpretations of vorton results.
The simplest solenoid vortex singularity is a vorton Ref.[1-5]. Finite core vortex tubes may be represented as superposition of vortons Ref.[1- 5]. Experimentally observed vortex structures topological metamorphoses could be seen in numerical vortons simulations without any additional assumptions. Vortons in plasma have magnetic dipole moments. Magnetic vorton tubes reconnect Ref.[3-5,14]. The 3D Euler equations of incompressible inviscid fluid mechanics develop singularities on timescales on the order of vortex rotations. Solutions to the 3D Navier-Stokes equations for incompressible viscous flow are smooth for all time, but they bifurcate and are not unique Ref.[6-8].
The 3D Navier-Stokes equations probably have an inviscid attractor that exhibits inviscid turbulence and dissipation Ref.[1- 5, 10].

SECTION 1: Navier-Stokes solutions and vorton dynamics.

Original reasons for introduction of vortons were to resolve the most important structures for description of fully developed 3D turbulence, 3D vortex tubes Ref.(1-5), Eq.(1-5),

$$\upsilon_i^{(\alpha)}(t,x) = -\frac{\epsilon_{ijk}\left(x_j - x_j^{(\alpha)}\right)\gamma_k^{(\alpha)}}{4\pi\left|x - x^{(\alpha)}\right|^3} \ ,$$

where ϵ_{ijk} is the unit antisymmetric tensor, and $x_j^{(\alpha)}$ (t) and $\gamma_k^{(\alpha)}$ (t) are the components of position and intensity, respectively, of the vorton labeled α.

(1)

Velocity field generated by individual vorton.

$$\Omega_i^{(\alpha)}(t,x) = \gamma_i^{(\alpha)}\delta\left(r^{(\alpha)}\right) + (4\pi)^{-1}\left(3r_i^{(\alpha)}r_k^{(\alpha)}\gamma_k^{(\alpha)} - \left|r^{(\alpha)}\right|^2\gamma_i^{(\alpha)}\right)\left|r^{(\alpha)}\right|^{-5} \quad (2)$$

Vorticity field of individual vorton. See solenoid dipole field picture fig.(19).

$$\dot{\gamma}_i^{(\alpha)} = \frac{\epsilon_{ijk}}{4\pi}\sum_\beta\left(r_{\alpha\beta}^{-3}\gamma_j^{(\alpha)} - 3r_{\alpha\beta}^{-5}r_j^{(\alpha\beta)}r_m^{(\alpha,\beta)}\gamma_m^{(\alpha)}\right)\gamma_k^{(\beta)}, \quad (3)$$

$$r_i^{(\alpha,\beta)} \equiv x_i^{(\alpha)} - x_i^{(\beta)}, r_{\alpha\beta} \equiv \left|r^{(\alpha,\beta)}\right|.$$

Vorton intensity change due to interaction with other vortons.

$$\dot{x}_i^{(\alpha)} = -\frac{1}{4\pi}\epsilon_{ijk}\sum_{\beta=1}^{N}{}' \, r_{\alpha\beta}^{-1}\, n_j^{(\alpha,\beta)}\gamma_k^{(\beta)}, \quad (4)$$

$$r_i^{(\alpha,\beta)} = x_i^{(\alpha)} - x_i^{(\beta)},$$

$$r_{\alpha\beta} = \left|r^{(\alpha,\beta)}\right|,$$
$$n_i^{(\alpha,\beta)} = r_i^{(\alpha,\beta)} r_{\alpha\beta}^{-1}$$

Vorton position change due to interaction with other vortons.

(5)

Fig.(1,2,19).

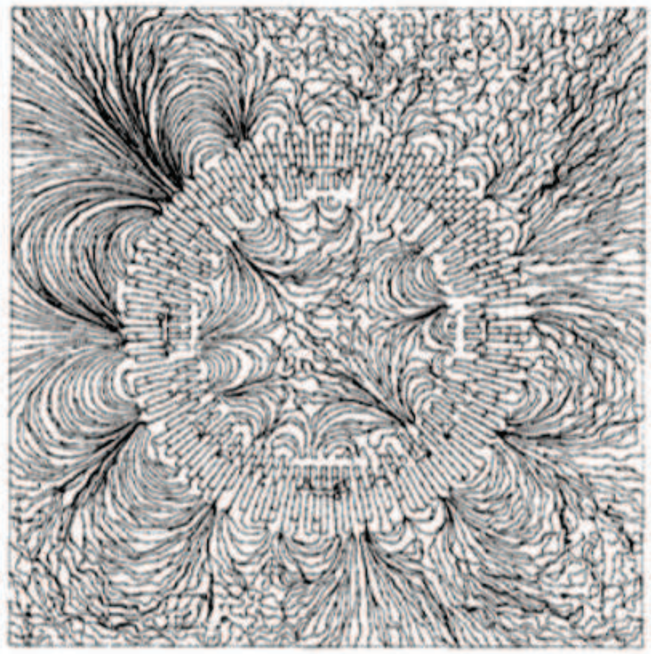


Fig. 1. Distribution of vorticity (vortex lines) in the plane of 80-vorton ring.

Vortex tube clearly seen in a vorton ring. With vorticity cancelling out beyond it ref.(3-5).

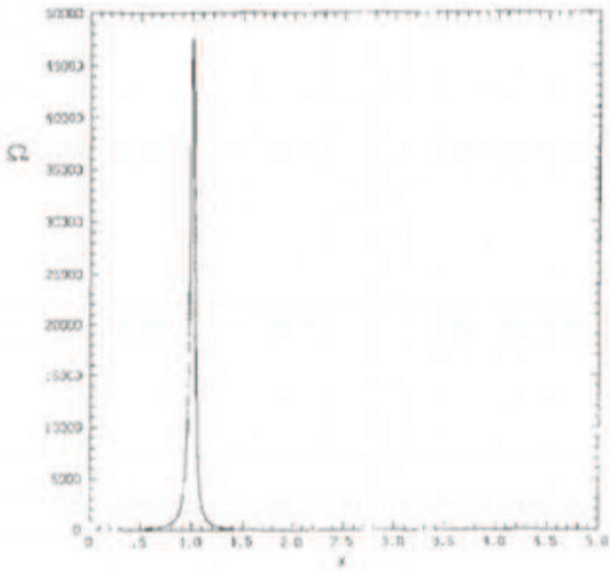
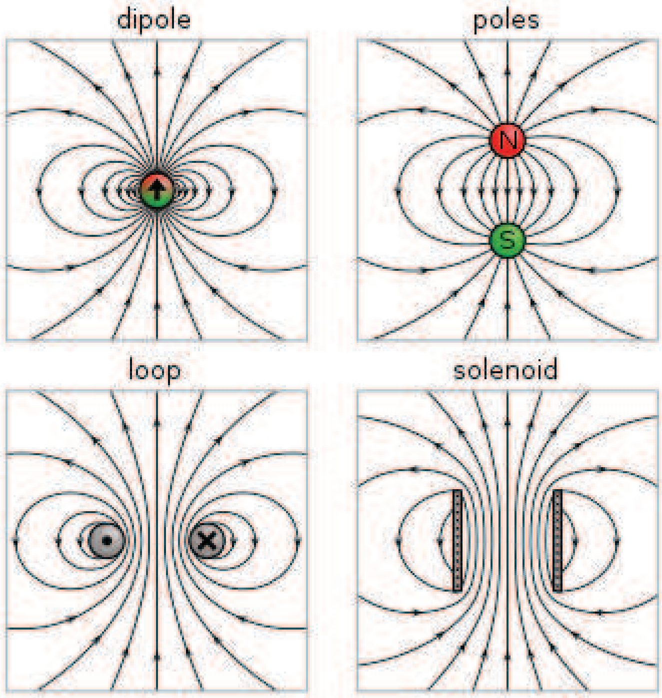


Fig. 2. Cross section of vorticity distribution in 80-vorton ring.

Non zero vorticity around radius of the ring ref.(3-5).



Field of solenoid dipole. Vortex and magnetic dipole Ref.(1-5,14).
Fig. 19.

In fact we showed both analytically and numerically that chain of vortons approximates finite core vortex tubes with number of vortons corresponding to Reynolds number at hand Ref.(1-5), Fig.(1,2,19).

Another hope was that by introducing simplest solenoid vortex singularities which are analytically tractable we would be able to pass through some unknown NS "singular events" without any additional assumptions. Those events turn out to be vortex reconnections corresponding to bifurcations and branching of NS solutions Ref.(1-5), Fig.(3-18). Both reasons appear to work out despite later vorton critiques, "modifications" and "improvements" Ref.(21-23).
Thus bifurcation and branching of Navier-Stokes solutions Ref.[6-9] can be interpreted as inviscid vorton reconnections Ref.(3-5). Very likely vortons describe dynamics on the Navier-Stokes attractor Ref.[9]. From a physical point of view vortons resolve the entire 3D vorticity field, which is superposition of 3D vortex tubes and is the only important aspect for 3D fully developed turbulence dynamics representation. And vortons showed ability to pass through NS "singular events" without any additional assumptions Ref.(3-5).
We know from analysis Ref.[10] that the limit of a smooth sequence of functions may not be smooth. So even if Navier-Stokes solutions are all smooth, their inviscid limit could be singular. These singular inviscid solutions express the bifurcation of viscous solutions and finite dissipation rates. Rudin's math analysis book Ref.[10] says that limit of smooth sequence could be singular. So even if NS solutions are smooth their limit/attractor could be singular/inviscid, especially if it explains all NS solutions dynamics without any additional assumptions and provide most economical description of dynamics of 3D NS solutions for fully developed turbulence.

SECTION 2: Vortex singularities and inviscid energy dissipation

Singular vortex dipoles, i.e. vortons, interact according to formulas Eq.(3,4),Ref.[1- 5]. As a consequence their amplitudes may self-amplify, a reflection of vortex stretching in three dimensions.

The interaction energy of vortons is given by Eq.(6),Ref.[1-5].

$$\mathcal{E}_{\mathrm{int}} = \frac{1}{8\pi}\sum_{\alpha<\beta} r_{\alpha\beta}^{-1}\left(\gamma_i^{(\alpha)}\gamma_i^{(\beta)} + \gamma_i^{(\alpha)} n_i^{(\alpha,\beta)}\gamma_j^{(\beta)} n_j^{(\alpha,\beta)}\right) \tag{6}$$

Interaction energy of vortons ref.(2-5).

The self-energy of a single vorton is infinite, therefore it is not included in that formula. Due to the three dimensional stretching of vortons and self amplification, the interaction energy is not conserved. We may imagine that it goes into or comes out of the infinite self-energy of individual vortons Ref.[1- 5].

In the real world this corresponds to the fact that internal rotational degrees of freedom of 3D vortices are not resolved in the self-energy formula and interaction energy formula. The stretching energy of vortex interactions may go into or out of unresolved rotational motions. Physically energy is dissipated at small scales by viscosity, but the precise amount is determined by inviscid dynamics.

The same phenomenon occurs in shock waves in compressible fluids, and also in magnetic field reconnection in the solar atmosphere. Dissipation provides the mechanism, but inviscid dynamics governs the behavior while viscosity, diffusivity, and resistivity play mop-up roles Ref.[16-18].

We performed numerical experiments with two vorton rings executing a "leapfrogging" cycle. The rings always self-destructed after 5 periods, regardless of the number of vortons in the rings.

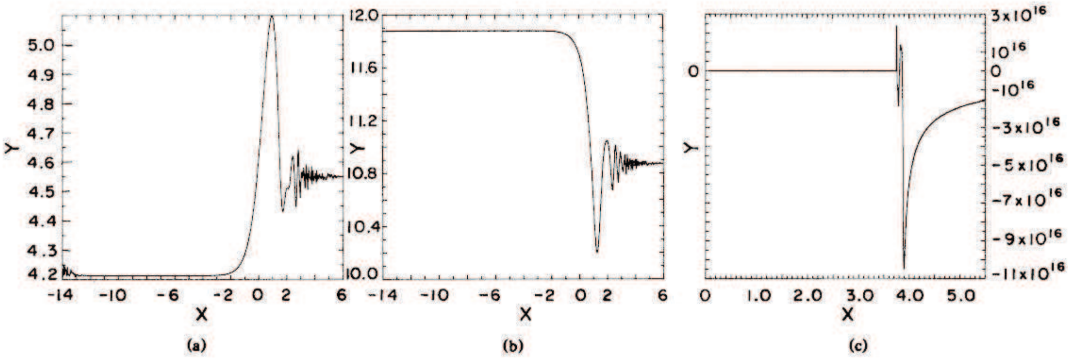
Multiple reconnections then happened, with small rings appearing. During the destruction of the original rings we observed a negative spike in the vorton interaction energy, and a Kolmogorov-like energy power spectrum developed. This agrees with the physical phenomena of energy transfer to small scales and inviscid dissipation of vorton interaction energy Eq.(5),

$$E_{int}(k) = \frac{1}{2\pi^2}\sum_{\alpha,\beta}\left[\phi_1(kr_{\alpha\beta})\gamma_i^{(\alpha)}\gamma_i^{(\beta)} + \phi_2(kr_{\alpha\beta})\gamma_i^{(\alpha)}n_i^{(\alpha,\beta)}\gamma_j^{(\beta)}n_j^{(\alpha,\beta)}\right], \tag{5}$$

$$\phi_1(z) = z^{-3}[(z^2-1)\sin z + z\cos z], \qquad \phi_2(z) = z^{-3}[(3-z^2)\sin z - 3z\cos z]$$

Vortons interaction energy as a function of wave number ref.(2-5).

Fig.(20).



Energy spectrum of two leapfrogging vortex rings originally and at the moment of self distraction. Slope in log-log graph around -1.7. Plus negative spike in time derivative of interaction energy during distraction of the rings ref.(2,3,5).

Recent experiments on colliding vortex rings show strikingly similar behavior, with large numbers of vortex reconnections and the appearance of multiple small rings Ref.[19].

SECTION 3: 3D Navier-Stokes attractor dimension

A chain of vortons can approximate a vortex tube with a finite core size that is roughly equal to the distance between vortons Ref.[1-5], see Fig.(5) for a ring with 80 vortons. This observation allows us to estimate the dimension of the attractor of the 3D Navier-Stokes equations in fully-developed turbulence.
We start by assuming that the vorticity consists of one-dimensional tubes whose core size is the Kolmogorov microscale **Re^-3/4** Ref.[1-5,13] and a distance between neighbour vortons in a tube. Therefore the number of vortons needed to resolve such tubes is on the order of **Re^3/4**. We deduce that the dimension of the Navier-Stokes attractor is **Re^3/4**, which is the cube root of the usual Kolmogorov estimate of **Re^9/4** Ref.[1,13] based on resolving the 3D velocity field.

SECTION 4: Magnetic vortons

The vorticity field of a vorton is identical in structure to the magnetic field of a magnetic dipole Fig.(19). In fact, in a turbulent plasma every vorton becomes magnetic due to the rotation of electrically charged plasma. So magnetic vortons interact both as a vorticity and magnetic dipoles Eq.(3,4,7).
Also, the magnetic amplitude and the vorticity amplitude will remain in a constant ratio, as they are stretched by the same fluid strain field. Formulas Eq.(3,4,7) describe the evolution of the magnetic vorton amplitude and their dipole-dipole interaction; note that there is no self-amplification of magnetic dipole momentum since the Maxwell equations are linear Ref.[14].
In section (3) we saw that chains of vortons provide an approximation of finite-core vortex tubes. Similarly, chains of magnetic dipoles provide an approximation of finite core magnetic flux tubes, as may be observed in the Sun or in nuclear fusion experiments. Inviscid dissipation associated with reconnection of magnetic vorton tubes may be the energy source for the very high temperatures on the surface of the Sun.
Our vorton horseshoe numerical experiments show reconnection and the expulsion of vortex rings, see Fig. [1,4,5].

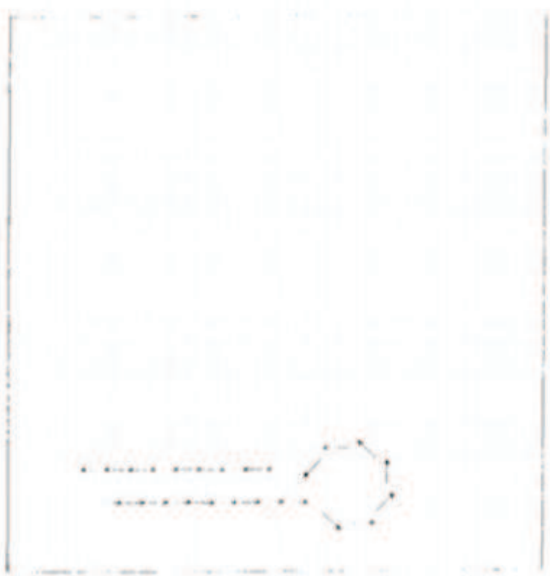


Fig. 7.

Fig. 8.

Perturbed horseshoe vortex and it expelling vortex ring ref.(3-5).

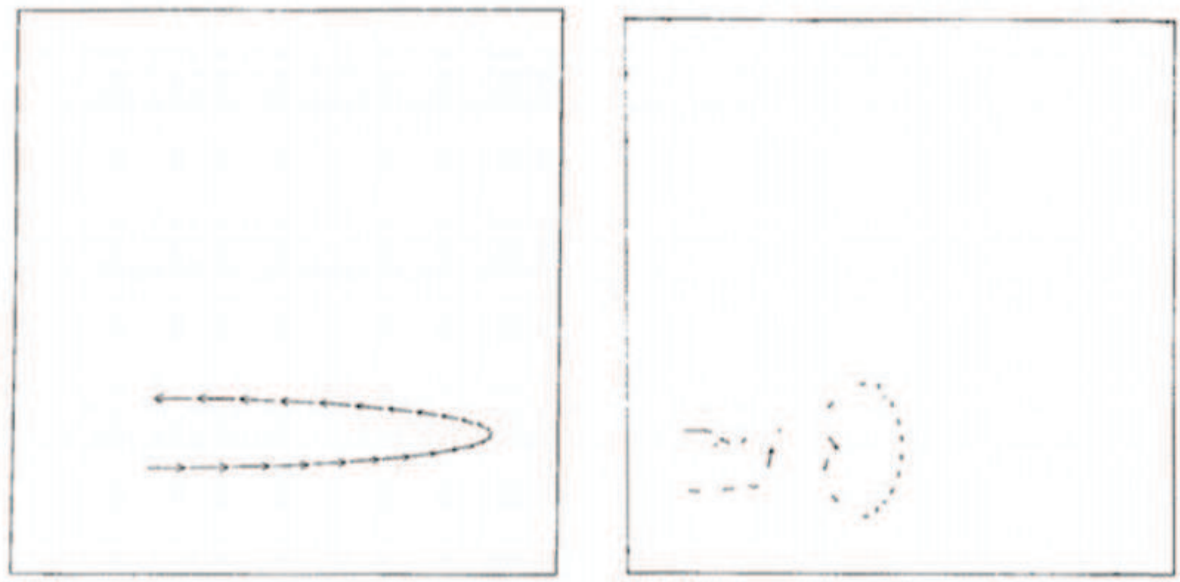


Fig. 9. Fig. 10.

Unperturbed horseshoe vortex and it expelling vortex ring ref.(3-5).

These resemble the magnetic vortices that produce magnetic storms on Earth and disrupts magnetic confinement in plasma fusion experiments Ref.[16,17].

SECTION 5: Instability of vorton collapse.

As we saw in articles Ref.[3-5] the system of 3 slightly non-parallel vortons almost perpendicular to 3 vortons plane can start to collapse toward a point. Just before that collapse, however, vorton self-amplification commences and leads to explosive vortons amplitudes and distances growth.

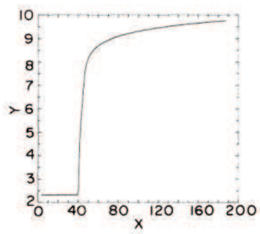


Fig.21

Jump in vorticity during 3 vorton collapse, log vorticity intensity against time ref.(2,3).

This could be a “turbulence” mechanism for the Big Bang initial stage of the Universe Ref.[15].

More precisely, imagine that the Universe starts as a point singularity fluctuation, and becomes a 2D or 3D turbulent fluid with gravity. In the 2D case gravity will collapse the Universe back to a singularity. However in 3D or quasi-3D the vorton amplitudes and distances increase exponentially, see Fig.(21) in Ref.[3-5]. This may explain why our world is 3 dimensional!
A quasi-3D frisbee-like world has 2 fully-developed dimensions and 1 short dimension; it might better be described as 2D+ or “frisbee-like”. In 2+D frisbee-like space the Universe may be slightly less complex, and have slightly more chances to fluctuate from the original singularity than a fully 3D Universe. And once gravitational collapse explodes in quasi 3D universe it will never get to any more complex spaces.
So vorton collapse followed by explosive self-amplification can also occur in 2D+. However in 2D+ gravitational force decays proportionally to **1/r** instead of **1/r^2** as in 3D.
A universe with two-plus dimensional structure could provide an alternative to “dark matter” as an explanation for anomalous rotation curves in galaxies. It has been observed in the edges of frisbee-like galaxies that star velocities do not depend on distance from the center. Explicitly, setting centripetal acceleration **v^2/r** equal to gravitational acceleration **(constant)*M/r**, we deduce that **v** must be constant in such a situation. This is a necessary condition for galactic disk stability Ref.[12].

SECTION 6 Simulation of vortex structures dynamics and no slip boundary layer. Ref.[3-5]

In this section we show how vorton simulations reproduce vortex structures typically observed in experiments Fig.(1-20).


Fig. 13.


Fig. 14.

Crow instability of vortex tubes behind airplanes Ref. (3-5). Antiparallel perturbed vortex tubes reconnect into vortex rings.


Fig. 15.


Fig. 16.


Fig. 17.


Fig. 18.

Two parallel moving vortex rings merging and splitting in perpendicular direction ref.(3-5).

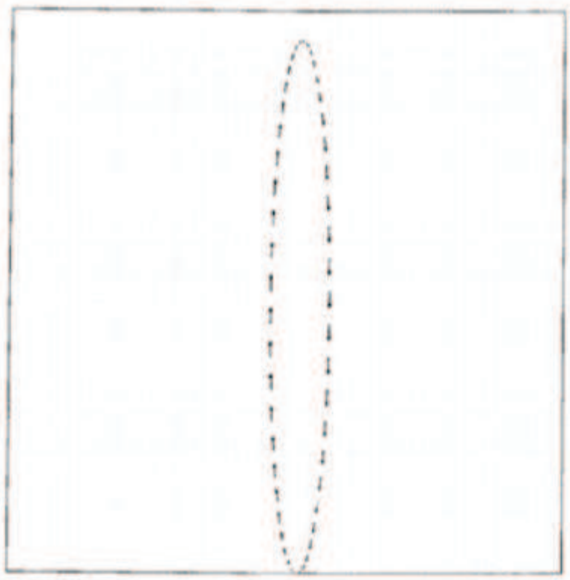


Fig. 11.

Fig. 12.

Elliptic vorton ring splits into 2 rings in perpendicular direction ref.(3-5).

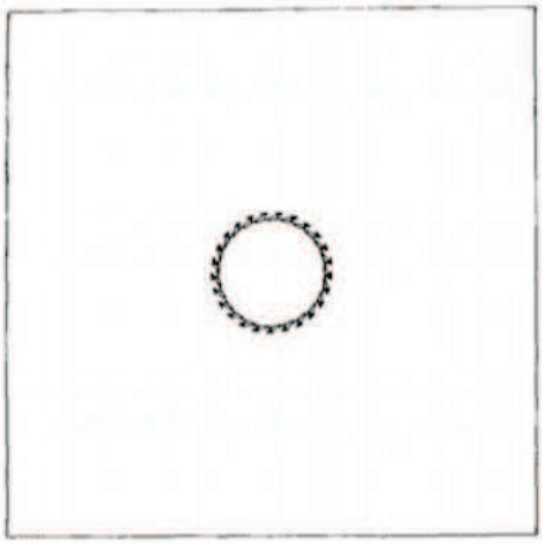


Fig. 3. Reconnections of vortex filaments (see text). The scale of vortex intensities adjusts to the maximal intensity, which increases continuously. The scale of positions does not change.

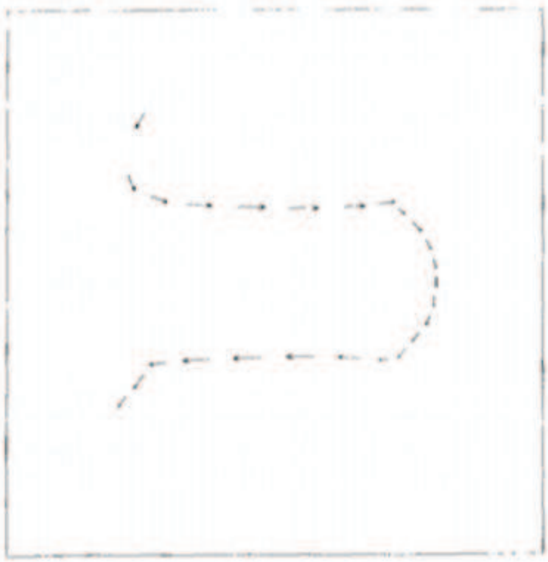


Fig. 4.

Vortex ring approaches boundary under 45 degrees angle and turns into horseshoe vortex ref.(3-5).

We performed numerical experiments with two vorton rings executing a “leapfrogging” cycle. The rings always self-destructed after 5 periods, regardless of the number of vortons in the rings. Multiple reconnections then happened, with small rings appearing. During the destruction of the original rings we observed a negative spike in the vorton interaction energy, and a Kolmogorov-like power spectrum developed Eq.(5,6),Fig.(20). This agrees with the physical phenomena of energy transfer to small scales and inviscid dissipation of vorton interaction energy Ref.(3-5).Recent experiments on colliding vortex rings show strikingly similar behavior Ref.[19], with large numbers of vortex reconnections and the appearance of multiple small rings ref.[19].

In wall-bounded flows vorticity is created via no-slip boundary conditions. We represent this process by placing arrays of vorton tubes at a distance from the wall equal to the tube core size and perpendicular to velocity and mirror image vortons to represent boundary Ref.(3-5). The numerical scheme continues to generate vorticity at the wall to keep enforcing the no-slip condition. The numerical experiments show that perturbations on the tubes then expel vortex rings into the flow away from the wall Ref.(3-5),Fig.(5,6).

Fig. 5.

Fig. 6.

Using modern fast multipole numerical schemes to simulate vortex dynamics we can come up with number of necessary calculations per time step for boundary layer simulation where it typically number of operations =n*ln(n)=(Re^3/4)*ln(Re), when in our case number of vortices is **Re^3/4** Ref.[1-5,11].

SECTION 7: CONCLUSIONS

Using “vortons”, i.e. discrete solenoid vortex dipole elements, helps elucidate many aspects of the 3D Navier Stokes equations and their solutions. Vortons most economically describe the physics of 3D vortex and magnetic vortex structures. Instability of vorton collapse in 3D and quasi 3D turbulence proposed as explanation of aspects of the origin of the Universe and phenomena attributed to dark matter and dark energy. Magnetic vortons describe dynamics of combined vortex and magnetic tubes in plasma including reconnections. No-slip boundary shown as a source of vorticity at the boundary and inside the flow. Plus in the process of self distraction of 2 leapfrogging vortex rings and subsequent reconnections we observed power law energy spectrum and energy cascade towards small scales.

Acknowledgments:

This work began in collaboration with Dr. E.A. Novikov and Dr. S.A. Orszag. Subsequently I enjoyed several productive discussions with Dr. B.J. Bayly. I am grateful to these colleagues for insights and suggestions.

Equations:

$$\upsilon_i^{(\alpha)}(t,x) = -\frac{\epsilon_{ijk}\left(x_j - x_j^{(\alpha)}\right)\gamma_k^{(\alpha)}}{4\pi\left|x - x^{(\alpha)}\right|^3} \qquad (1)$$

where ϵ_{ijk} is the unit antisymmetric tensor, and $x_j^{(\alpha)}$ (t) and $\gamma_k^{(\alpha)}$ (t) are the components of position and intensity, respectively, of the vorton labeled α.

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Velocity field generated by individual vorton.

$$\Omega_i^{(\alpha)}(t,x) = \gamma_i^{(\alpha)}\delta\left(r^{(\alpha)}\right) + (4\pi)^{-1}\left(3r_i^{(\alpha)}r_k^{(\alpha)}\gamma_k^{(\alpha)} - \left|r^{(\alpha)}\right|^2\gamma_i^{(\alpha)}\right)\left|r^{(\alpha)}\right|^{-5} \qquad (2)$$

Vorticity field of individual vorton. See solenoid dipole field picture fig.(19).

$$\dot{\gamma}_i^{(\alpha)} = \frac{\epsilon_{ijk}}{4\pi}\sum\nolimits_\beta \left(r_{\alpha\beta}^{-3}\gamma_j^{(\alpha)} - 3r_{\alpha\beta}^{-5}r_j^{(\alpha\beta)}r_m^{(\alpha,\beta)}\gamma_m^{(\alpha)}\right)\gamma_k^{(\beta)}, \qquad (3)$$

$$r_i^{(\alpha,\beta)} \equiv x_i^{(\alpha)} - x_i^{(\beta)}, r_{\alpha\beta} \equiv \left|r^{(\alpha,\beta)}\right|.$$

Vorton intensity change due to interaction with other vortons.

$$\dot{x}_i^{(\alpha)} = -\frac{1}{4\pi}\epsilon_{ijk}\sum\nolimits_{\beta=1}^{N}{}' r_{\alpha\beta}^{-1}\, n_j^{(\alpha,\beta)}\gamma_k^{(\beta)}, \qquad (4)$$

$$r_i^{(\alpha,\beta)} = x_i^{(\alpha)} - x_i^{(\beta)},$$
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$$n_i^{(\alpha,\beta)} = r_i^{(\alpha,\beta)}r_{\alpha\beta}^{-1}$$

Vorton position change due to interaction with other vortons.

(5)

$$E_{int}(k) = \frac{1}{2\pi^2}\sum_{\alpha,\beta}\left[\varphi_1\left(kr_{\alpha\beta}\right)\gamma_i^{(\alpha)}\gamma_i^{(\beta)} + \varphi_2\left(kr_{\alpha\beta}\right)\gamma_i^{(\alpha)}n_i^{(\alpha,\beta)}\gamma_j^{(\beta)}n_j^{(\alpha,\beta)}\right],$$

$$\varphi_1(z) = z^{-3}[(z^2 - 1)\sin z + z\cos z], \qquad \varphi_2(z) = z^{-3}[(3 - z^2)\sin z - 3z\cos z]$$

Vortons interaction energy as a function of wave number ref.(2-5).

$$\mathcal{E}_{\text{int}} = \frac{1}{8\pi}\sum_{\alpha<\beta} r_{\alpha\beta}^{-1}\left(\gamma_i^{(\alpha)}\gamma_i^{(\beta)} + \gamma_i^{(\alpha)} n_i^{(\alpha,\beta)}\gamma_j^{(\beta)} n_j^{(\alpha,\beta)}\right) \tag{6}$$

Interaction energy of vortons ref.(2-5).

Formula for force F between 2 magnetic dipoles m1 and m2 ref(14).

(7)

$$F = \frac{3\mu_0}{4\pi|r|^4}\{(\hat{r}\times m_1)\times m_2 + (\hat{r}\times m_2)\times m_1 - 2\hat{r}(m_1\cdot m_2) + 5\hat{r}[(\hat{r}\times m_1)\cdot(\hat{r}\times m_2)]\},$$

where μ_0 is the magnetic constant, $\hat{\mathbf{r}}$ is a unit vector parallel to the line joining the centers of the two dipoles, and $|\mathbf{r}|$ is the distance between the centers of $\mathbf{m}_1$ and $\mathbf{m}_2$.

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FIGURES:

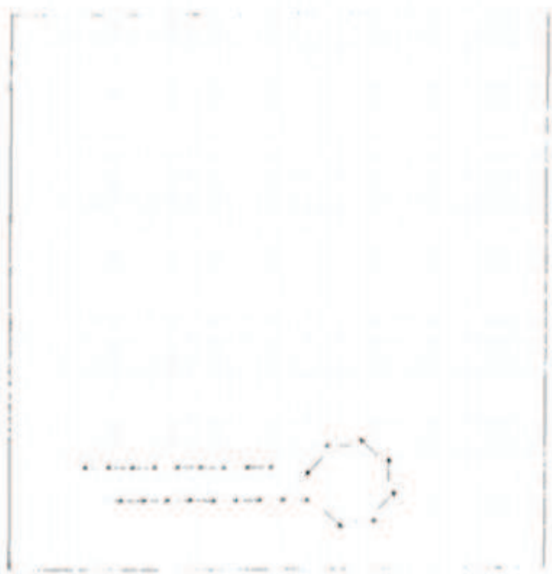


Fig. 7.

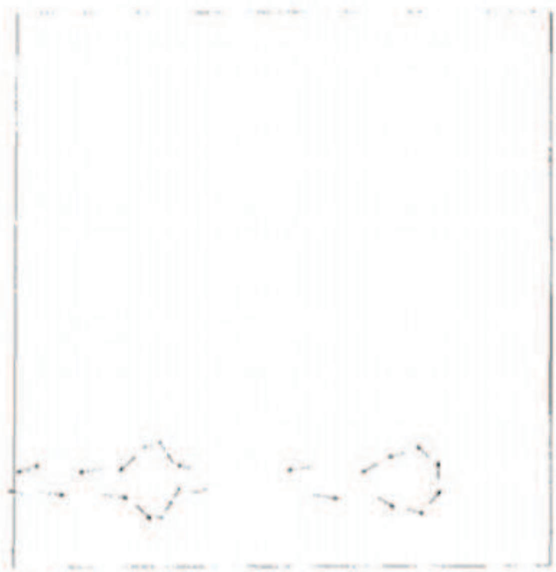


Fig. 8.

Perturbed horseshoe vortex and it expelling vortex ring ref.(3-5).

Fig. 9.

Fig. 10.

Unperturbed horseshoe vortex and it expelling vortex ring ref.(3-5).

Fig. 5.

Fig. 6.

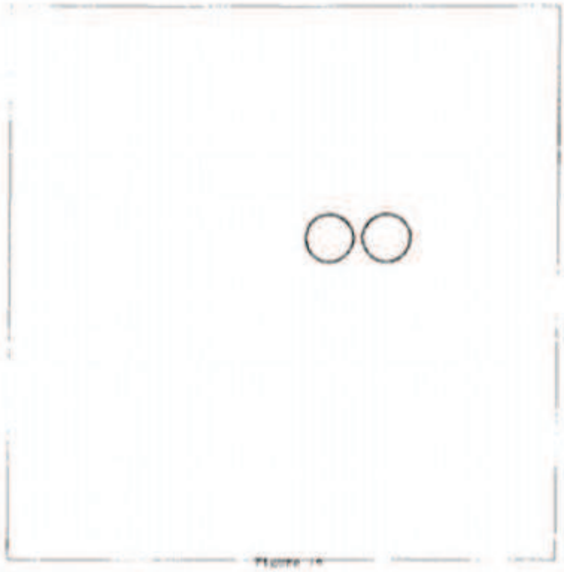
Perturbed vortex tube near the wall and it expelling vortex ring ref.(3-5).

Fig. 13.

Fig. 14.

Crow instability of vortex tubes behind airplanes Ref. (3-5). Antiparallel perturbed vortex tubes reconnect into vortex rings.

Fig. 15.



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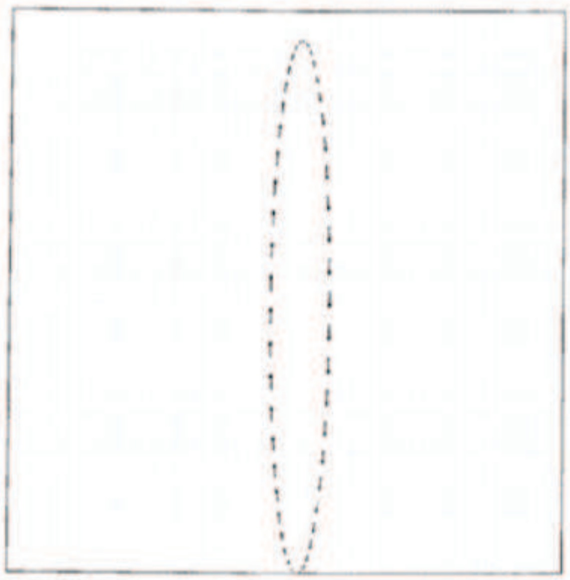
Two parallel moving vortex rings merging and splitting in perpendicular direction ref.(3-5).



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Vortex ring approaches boundary under 45 degrees angle and turns into horseshoe vortex ref.(3-5).

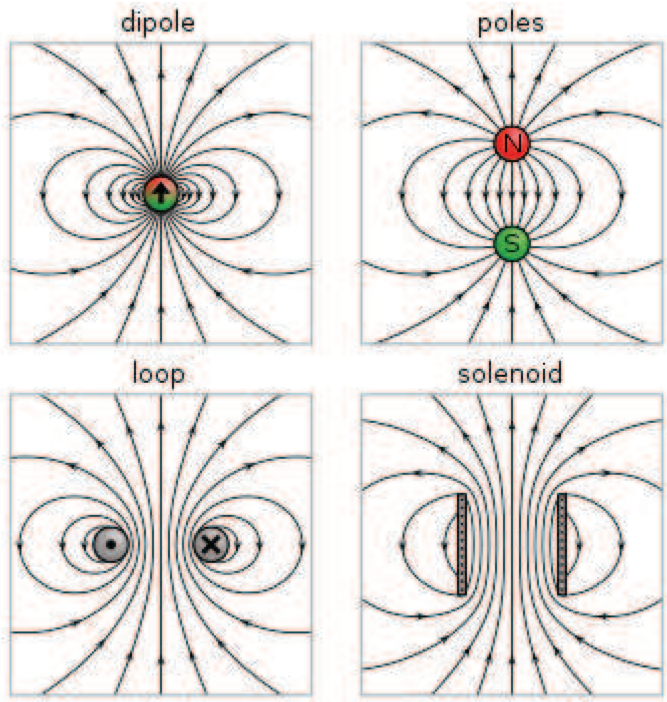


Fig.19.

Field of solenoid dipole. Vortex and magnetic dipole Ref.(1-5,14).

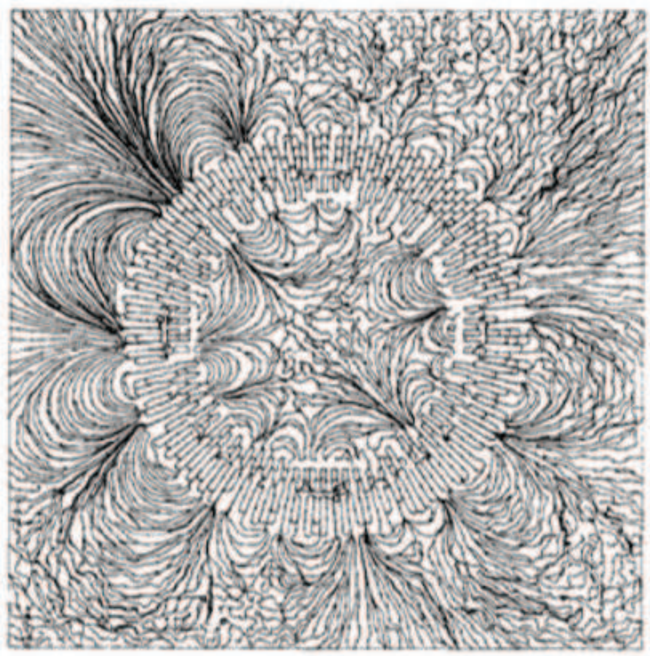
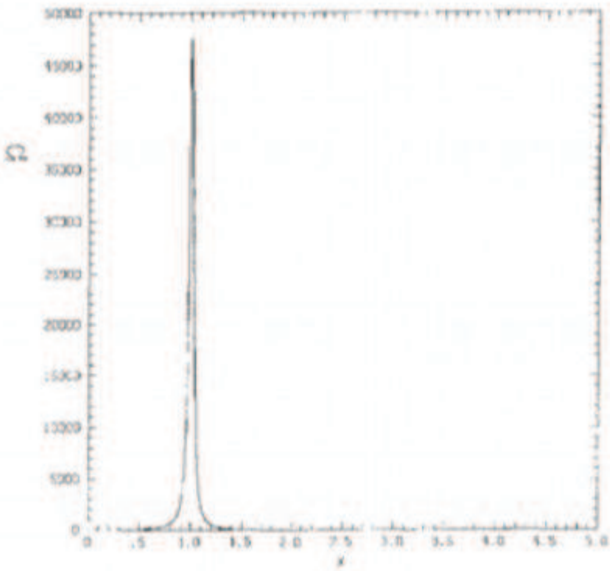


Fig. 1. Distribution of vorticity (vortex lines) in the plane of 80-vorton ring.

Vortex tube clearly seen in a vorton ring. With vorticity cancelling out beyond it ref.(3-5).



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Non zero vorticity around radius of the ring ref.(3-5).

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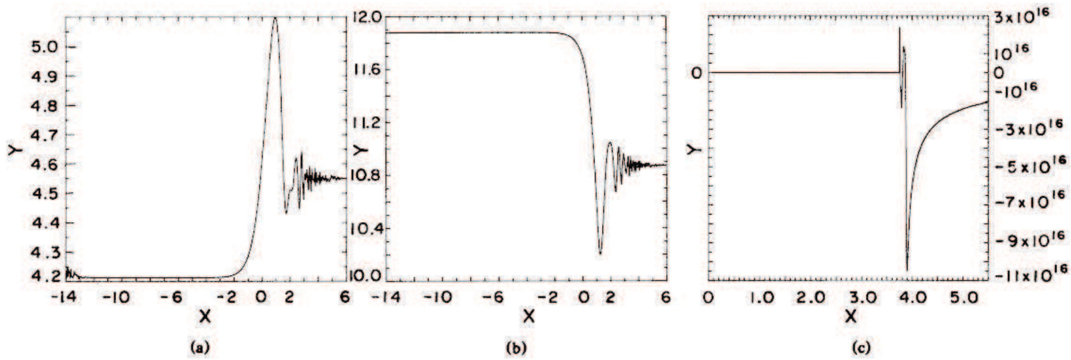

Fig.20.

Jump in vorticity during 3 vorton collapse, log vorticity intensity against time ref.(2,3).

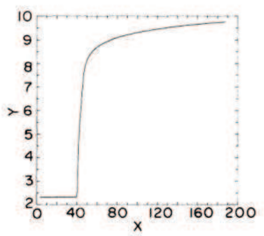

Fig.21.

SECTION8 : Appendix A: Dark energy and the expansion of the Universe

As we saw in the previous chapters, the instability of 3D vorton collapse leads to self-amplification and explosive expansion of vorton amplitudes Ref.[1- 5]. An interesting fact is that until 5 billion years ago the Universe's expansion was decelerating, but since then the expansion has accelerated ref.[15]. There is an antagonism between gravity pushing toward contraction and turbulent 3D vorticity interaction pushing toward expansion of Universe. More precisely, the gravitational field is consists of monopoles leading to a 1/r^2 strength falloff. Meanwhile vorticity is a dipole or quadrupole field with falloff proportional to 1/r^3 or faster. So in the early Universe gravity dominated vorticity interactions. But later vorticity self-amplification due to 3D stretching led to dominance of the vorticity interactions and accelerated the Universe's expansion, with gravity having no self amplification mechanism due to linearity of corresponding equations.The energy for vorticity/vorton interaction comes from the infinite self-energy of vortons being converted into vorton interaction energy. That is, vorton self-energy pushes the Universe to accelerated expansion. This observation is attributed to a mysterious "Dark Energy", which may simply be a manifestation of vorton self and interaction energy. In section 1 we interpreted the infinite self-energy as belonging to unresolved internal rotational degrees of freedom of individual vortons. So the dark energy responsible for the accelerating Universe expansion is simply the transfer of internal rotational energy into interaction energy of the system of vortons.
Gravity also by squeezing vorticity closer intensifies vortex interaction and stretching, self amplification and thus energy of interaction.

Energy is conserved.

As the vorton interaction energy, and thus dark energy, increases, we expect a corresponding decrease in gravitational interaction energy. In the 3D case the gravitational potential energy of universe is -(const)*(M^2)/r, while in the 2D it is (const)*(M^2)* ln(r), where M is universe mass and r is its radius ref.[20] and constants obviously are not the same. In the intermediate 2+D case we may be closer to the 2D or 3D formula depending on how much the third short dimension is developed compared to the two fully developed dimensions. We suspect that the

short dimension became more fully developed 5B years ago when the Universe expansion changed from decelerating to accelerating. The gravitational energy released from expansion regime change went into the interaction energy of vortexes/vortons associated with dark energy. The above mentioned potential energy formula then became closer to the 3D case.
Also in fully 3D physical space case as compared to 2+D case we have much stronger vorticity stretching with corresponding vorticity intensity self amplification and with commensurate vortexes interaction energy increase.

SECTION 9: Appendix B: Elementary particles as generalized solutions

The wave functions of elementary particles are often interpreted as probability amplitudes. When they take the form of Gaussian distributions for isolated particle, they look very much like test functions for generalized solutions of unknown nonlinear equations.
The strong interactions between protons and neutrons could be interpreted as reconnections of strings in some exotic space. William Thompson (Lord Kelvin) conjectured that fluid mechanics could provide examples for all other branches of mathematical physics.