Consideration of Collatz conjecture and its integer space

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Abstract

Investigation is tried about approach to Collatz conjecture and its integer space.

1. Introduction

Collatz conjecture operation is repeating following calculation regarding to positive odd integer (POI) n_i until $n_{i+1} = 1$.

$$n_{i+1} = \frac{3n_i+1}{2^{m_i}}, \qquad i = 1,2,3,\cdots \tag{1}$$

Here dividing by 2^{m_i} means dividing by possible power of 2 to make POI.

Meaning of this formula is;

Based on binary floating-point representation, calculation $3n_i + 1$ is done using integer mantissa n_i. Dividing by 2^{m_i} is to normalize the result and to get new integer mantissa n_{i+1}. Here integer mantissa means its mantissa part is integer.

2. Characteristic of Collatz Conjecture operation

In general, regarding to arithmetic procedure, operation might be done to variables and how to operate is represented using formula. But actually, sometimes operation could be different procedure depending on the value in the variable.

For example, m_i of (1) is repeating times of dividing by 2 and this procedure depends on the value of n_i.

In such case, operation is different depending on each value. Therefore, the difficulty to mention the result of Collatz Conjecture in general may be on such point.

Regarding to Collatz Conjecture, main theme to be resolved is following two issues.

1) All series of the POI is terminated at 1.
2) All series of the POI have no looping.

Each of two themes may be able to be approached independently. For the approach, there could be such methods as arithmetical, statistical, logical, topological and etc.

For example,

Report1 *1 is logical for 1) and arithmetically for 2).

Report2 *2 is statistically for 1).

Report3 *3 is logically for 2).

This report is statistically for 1).

3. Collatz Conjecture POI Space

Above (1) can be rewritten.

$$n' = \frac{3n+1}{2^m} \qquad (2)$$

2^m; possible maximum power of 2,

n; current POI of Collatz Conjecture POI series

About (2), relation between m *and* n is represented as following table1 listed for $1 \le m \le 6$ as a sample.

m	n
1	$4l+3$
2	$8l+1$
3	$16l+13$
4	$32l+5$
5	$64l+53$
6	$128l+21$

Table1

l is an arbitrary positive integer.

For example,

When l of first formula is 1, n is 7 $(4 \times 1 + 3)$. In this case $m = 1$.

This means $3n+1$ can be divided by 2^1.

When l of 6th formula is 0, n is 21 $(128 \times 0 + 21)$. In this case $m = 6$.

This means $3n+1$ can be divided by 2^6.

In general, m and n have relation (3).

$$n = 2^{m+1}l + c \qquad (3)$$

c is an odd integer constant calculated on (4) using minimum POI k which satisfies (4) without remain.

$$c = \frac{2^m k - 1}{3} \qquad (4)$$

Actually $k = 1$ when m is even, and $k = 5$ when m is odd.

(3) becomes (5) c being replaced by (4).

$$n = 2^{m+1}l + \frac{2^m k - 1}{3} \qquad (5)$$

(2) becomes using (5)

$$n' = \frac{3n+1}{2^m} = 3 \cdot 2l + k \qquad (6)$$

On (3), when l=0, n=c.
This means c is minimum POI which can be divided by 2^m at (2) calculation.
First term of (3) $2^{m+1}l$ is a part which depends on l, and can be divided by 2^m.
On (6), the term should be even in order that n' is odd because k is POI.
On these above, it is confirmed that (5) defines m and n relation in general because n defined at (5) can be divided by 2^m at (6) calculation and
using n defined at (5), n' becomes POI at (6) calculation.

Based on the m and n relation, Collatz Conjecture POI space shows that n which can be divided by 2^m at the calculation (2) are allocated periodically or averagely.

Based on (3), occurrence rate of n which are related with m is following.
Occurrence rate to positive integers; $\frac{1}{2^{m+1}}$ (7)
Occurrence rate to positive odd integers; $\frac{1}{2^m}$ (8)
because (number of POI)= $\frac{1}{2}(number\ of\ PI(positive\ intrger)$.
Therefore, expected value for m is

$$\frac{m}{2^m}. \tag{9}$$

Total expected value for whole POI space is

$$e = \sum_{m=1}^{\infty} \frac{m}{2^m}. \tag{10}$$

e from 1 to j is

$$\sum_{m=1}^{j} \frac{m}{2^m} = \frac{2}{2^0} - \frac{j+2}{2^j}. \tag{11}$$

When j = ∞,

$$e = \sum_{m=1}^{\infty} \frac{m}{2^m} = 2. \tag{12}$$

4. Collatz Conjecture operation

Collatz Conjecture operation is doing (13) (*1, 6. (4) page6.)

$$n_i = (3((3((3 \times n + 1)/2^{m_1}) + 1)/2^{m_2}) + 1)/2^{m_3} \cdots$$

$$= \frac{3^i n}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-2}}{2^{m_2+\cdots+m_i}} + \cdots + \frac{3^1}{2^{m_{i-1}+m_i}} + \frac{3^0}{2^{m_i}} \tag{13}$$

n: initial value (POI)

One of the space characteristics is that POI n which can be divided by 2^m on (2) is distributed periodically in the space based on (5). (a)
Selection of n_1 of (1) is arbitrary or at random. (b)
Based on (a)(b), according to increasing of i, $2^{m_1+m_2+\cdots+m_i}$ is reaching to 2^{ei}. Therefore, in order to calculate targeted value, m_i is replaced by expected value e and α is

defined.

$$\alpha = \frac{3}{2^e} \tag{14}$$

Then (13) is

$$n_i = \alpha^i n + \frac{1}{2^e}(\alpha^{i-1} + \alpha^{i-2} + \cdots + \alpha^1 + \alpha^0).$$

$$=\alpha^i n + \frac{1}{2^e}\frac{1-\alpha^i}{1-\alpha} \tag{15}$$

Based on (15), according to increasing of i, α^i is reaching to 0, then n_i is reaching to 1.

This is estimated target value and Collatz Conjecture operation is reaching toward it, or never stop until reaching to it.

5. Conclusion

Investigation of the characteristic of Collatz Conjecture POI space shows its every POI series is reaching to 1 on the assumption that sufficient times unique trial (=no looping) can be done.

References

*1 https://vixra.org/abs/2302.0015

*2 https://vixra.org/abs/2204.0151

*3 https://vixra.org/abs/2304.0070