Using Euler's Identity to Prove the Existence of Natural Logarithms of Numbers Approaching 0^+ on the Complex Plane

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Abstract

This paper provides an overview of using Euler's identity to prove that natural logarithms of numbers approaching zero exist on the complex plane.

$e^{i\pi} = -1$ (by Euler's identity)

Hence, $\ln(-1) = i\pi$,

Which means: $\ln(0-1) = i\pi$

We know that $\ln(a-b) = \ln(a(1-b/a) = \ln a + \ln(1-b/a)$

Hence,

$\ln(x(1-1/x)) = \ln x + \ln(1-1/x) = i\pi$

When $x \rightarrow 0$

Hence,

$$\ln x = i\pi - \ln(1-1/x) \text{ as } x \rightarrow 0+ \quad \ldots 1$$