Serious Problems in Standard Complex Analysis Texts From The Viewpoint of Division by Zero Calculus

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Abstract: In this note, we shall refer to some serious problems for the standard complex analysis text books that may be considered as common facts for many years from the viewpoint of the division by zero calculus. We shall state clearly our opinions with the new book: V. Eiderman, An introduction to complex analysis and the Laplace transform (2022).

Key Words: Division by zero, division by zero calculus, $1/0 = 0/0 = z/0 = \tan\frac{\pi}{2} = 0$, complex analysis text, Laurent expansion, fractional linear transformation, discontinuity.

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1 Introduction

In this note, we shall refer to some serious problems for the standard complex analysis text books that may be considered as common facts for many years from the viewpoint of our division by zero calculus. We shall state clearly our opinions for the new book: V. Eiderman, An introduction to complex analysis and the Laplace transform, 2022([1]).

At first, our division by zero is given by the idea of the Yamada field axiom **Y** containing division by zero with generalized fractions. In its general fractions

$$\frac{a}{b},$$

we have

$$\frac{a}{0} = 0.$$

See [5] and [9] for the details.

Meanwhile, for the function case we need the concept of division by zero calculus.

The general definition of division by zero calculus is given by the following way:

For a function $y = f(x)$ which is n order differentiable at $x = a$, we will **define** the value of the function, for $n > 0$

$$\frac{f(x)}{(x-a)^n}$$

at the point $x = a$ by the value

$$\frac{f^{(n)}(a)}{n!}.$$

For the important case of $n = 1$,

$$\frac{f(x)}{x-a}|_{x=a} = f'(a). \tag{1.1}$$

In particular, the values of the functions $y = 1/x$ and $y = 0/x$ at the origin $x = 0$ are zero. **We write them as $1/0 = 0$ and $0/0 = 0$, respectively.** Of course, the definitions of $1/0 = 0$ and $0/0 = 0$ are not usual ones in the sense: $0 \cdot x = b$ and $x = b/0$ (however, when we consider its solution in the sense of the Moore-Penrose general inverse, our result is the same as the solution of $0 \cdot x = b$). Our division by zero is given in this sense and is not given by the usual sense as in stated in [7, 9, 10, 11] with many applications.

However, in complex analysis, we can introduce our division by zero calculus as follows, clearly and simply:

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z-a)^n \tag{1.2}$$

we will define

$$f(a) = C_0. \tag{1.3}$$

For the correspondence (1.3) for the function $f(z)$, we will call it **the division by zero calculus**. By considering derivatives in (1.2), we **can define** any order derivatives of the function f at the singular point a; that is,

$$f^{(n)}(a) = n!C_n.$$

In particular, note that for $a > 0$

$$\left[\frac{a^n}{n}\right]_{n=0} = \log a$$

and

$$\tan\frac{\pi}{2} = 0.$$

2 On the book by V. Eiderman

The following contents in the book [1] are very standard facts, but we would like to state our opinions clearly and concretely.

In page 55:

Linear and Moebius Transformations. Here are a few simple facts to note: if a point A approaches the circle, i.e. if OA $\to$ R, then OA also approaches R; every point on the circle is symmetric to itself; and if OA $\to$ 0 then OA $\to \infty$, and therefore the point O is symmetric with the point at infinity.

These statement and fact are classical ones, however, the point O is symmetric with the point at infinity is **wrong**; indeed, we can not say so, because from OA $\to$ 0, we can not say about O itself. We stated that O corresponds to O (not infinity point), because for $f(z) = 1/z, f(0) = 0$. We think the very classical result is wrong and the result will give a great impact to complex analysis and to our mathematics.

For its importance, we discussed this fact with one chapter of the book [9].

In page 31,

Let $f(z), z \in$ the closure of $\mathbf{C}$, be such that $f(z) = 1/z$ as z not zero and for $z = 0, f(0) = \infty$, and $f(\infty) = 0$. Prove that $f(z)$ is continuous on the closure of $\mathbf{C}$.

This statement is right. Contrary, we consider by continuity that we defined the point at infinity at $z = 0$, however, the fact was discontinuous at $z = 0$.

In 50 page:

Conformal Mappings. Moebius transformations. Now we move to the study of the Moebius transformation, or fractional linear transformation, defined by the equality: $w = (az + b)/(cz + d)$. The function which performs this transformation is a ratio of linear functions. Since $\lim_{z\to\infty}(az + b)/(cz + d) = a/c$ and $\lim_{z\to d/c}(az+b)/(cz+d) = \infty$, it is natural to define $w(\infty) = a/c$ and $w(d/c) = \infty$.

The result $w(d/c) = \infty$ is right, however, its value is the limiting value at $-d/c$ and practically the value of $z = -d/c$ is a/c, by the division by zero calculus. Note that in this case, the fractional linear transform maps from $\mathbf{C}$ onto $\mathbf{C}$ one to one, but not continuously at the point $z = -d/c$.

For example, for the typical linear mapping

$$W = \frac{z - i}{z + i},$$

it gives a conformal mapping on $\{\mathbf{C} \setminus \{-i\}\}$ onto $\{\mathbf{C} \setminus \{1\}\}$ in one to one and from

$$W = 1 + \frac{-2i}{z - (-i)},$$

we see that $-i$ corresponds to 1 and so the function maps the whole $\{\mathbf{C}\}$ onto $\{\mathbf{C}\}$ in one to one.

Meanwhile, note that for

$$W = (z - i) \cdot \frac{1}{z + i},$$

when we enter $z = -i$ in the way

$$[(z-i)]_{z=-i} \cdot \left[\frac{1}{z+i}\right|_{z=-i} = (-2i) \cdot 0 = 0,$$

we have the different value.

In many cases, the above two results will have practical meanings and so, we will need to consider many ways for the application of the division by zero to functions and we will need to check the results obtained, in some practical viewpoints. We referred to this delicate problem with many examples.

For the typical case z/z, it is 1 by the division by zero calculus at $z = 0$, however, $0/0 = 0$ by the division by zero.

In addition, in 33 page:

We will see that this "omnidirectionality" of the derivative imposes severe restrictions on functions for which the limit exists; and that in turn means that such functions of a complex variable have many surprising special properties.

Many people think so and say so for the mysterious deep property of analytic functions, however, its reason is not so, but it is caused for the total (the derivative as two variable function) differentiable and two directional derivatives are same. We have to see its reason with the algebraic property.

3 Remarks

These facts and opinions were informed to the author and Professor Edward Dunne Executive Editor, Mathematical Reviews American Mathematical Society, however, they are not accept our opinions at this moment. In particular, we stated to Professor Dunne as follows with our basic references:

We think that modern mathematics is still flawed. It is clear that there are basic defects in function theory, differential equations, geometry, and algebra, and it has been eight years since the discovery. This will be a stain on world history. I hope you can understand our new important mathematics (2023.1.23.20:55).

References

[1] V. Eiderman, *An introduction to complex analysis and the Laplace transform*, CRC Press, Boca Raton, FL, 2022.

[2] M. Kuroda, H. Michiwaki, S. Saitoh, and M. Yamane, New meanings of the division by zero and interpretations on $100/0 = 0$ and on $0/0 = 0$, Int. J. Appl. Math. **27** (2014), no 2, pp. 191-198, DOI: 10.12732/ijam.v27i2.9.

[3] T. Matsuura and S. Saitoh, Matrices and division by zero $z/0 = 0$, Advances in Linear Algebra & Matrix Theory, **6** (2016), 51-58. Published Online June (2016) in Sci. Res. http://www.scirp.org/journal/alamt, http://dx.doi.org/10.4236/alamt.2016.62007.

[4] T. Matsuura, H. Michiwaki and S. Saitoh, $\log 0 = \log \infty = 0$ and applications, Differential and Difference Equations with Applications, Springer Proceedings in Mathematics & Statistics. **230** (2018), 293-305.

[5] H. Michiwaki, S. Saitoh and M. Yamada, Reality of the division by zero $z/0 = 0$, IJAPM International J. of Applied Physics and Math. **6**(2015), 1–8. http://www.ijapm.org/show-63-504-1.html.

[6] H. Michiwaki, H. Okumura and S. Saitoh, Division by Zero $z/0 = 0$ in Euclidean Spaces, International Journal of Mathematics and Computation, **28** (2017), Issue 1, 1-16.

[7] H. Okumura, Geometry and division by zero calculus, International Journal of Division by Zero Calculus, **1** (2021), 1-36.

[8] S. Pinelas and S. Saitoh, Division by zero calculus and differential equations, Differential and Difference Equations with Applications. Springer Proceedings in Mathematics & Statistics, **230** (2018), 399–418.

[9] S. Saitoh, *Introduction to the Division by Zero Calculus*, Scientific Research Publishing, Inc. (2021), 202 pages.

[10] S. Saitoh, History of Division by Zero and Division by Zero Calculus, International Journal of Division by Zero Calculus, **1** (2021), 1-38.

[11] S. Saitoh, *Division by Zero Calculus - History and Development*, Scientific Research Publishing, Inc. (2021), 332 pages.

[12] S. Saitoh, *Introduction to the Division by Zero Calculus - an extended version*, 396 pages (draft).