An arbitrary higher-derivative correction to Einstein-Hilbert action

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Abstract

We have made an observation that can be generalized to an arbitrary higher-derivative correction to Einstein-Hilbert action (to linear order in the corresponding couplings).

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I. NOTES

An ordinary second-order differential equation for a complex function $\phi(r)$ (momentum space),

$$\frac{\partial^2 \phi(r)}{\partial r^2} + P(r)\frac{\partial \phi(r)}{\partial r} + Q(r)\phi(r) = 0, \tag{1}$$

has an associated generalised Wronskian,

$$W(r) = \left[\phi^* \frac{\partial \phi}{\partial r} - \phi \frac{\partial \phi^*}{\partial r}\right] \exp\left\{\int^r P(r')dr'\right\}, \tag{2}$$

which is conserved, $\partial_r W(r) = 0$. Since the equation is linear, both $\Re\phi$ and $\Im\phi$ are independent solutions of the ODE.

II. UNIVERSALITY IN SECOND-ORDER HYDRODYNAMICS

In order to compute all second-order hydrodynamic transport coefficients, we need to compute the three-point functions of the stress-energy tensor and use the appropriate Kubo formulae. Imagine computing a fully retarded three-point function

$$\left\langle T_R^{\mu_1\nu_1}(0)\, T_A^{\mu_2\nu_2}(x_2)\, T_A^{\mu_3\nu_3}(x_3)\right\rangle . \tag{3}$$

In momentum space, we turn on the channels such that each of the first-order perturbations $h^{(1)}_{\mu\nu}$ sourcing $T^{\mu\nu}$ decouples from the other metric fluctuations and behaves as a scalar field. Let's define

$$\phi_i^{(1)} \equiv g^{\mu_i \lambda} h^{(1)}_{\lambda \nu_i}. \tag{4}$$

Then ϕ_i satisfy

$$\partial_r^2 \phi_i^{(1)} + \mathcal{A}_1(\omega, q)\partial_r \phi_i^{(1)} + \mathcal{A}_0(\omega, q)\phi_i^{(1)} = 0, \tag{5}$$

with the appropriate ω and q for each of the scalar fluctuations.

To compute the three-point function we need to perturb $g_{\mu_1\nu_1}$ to second order. The scalar mode is now governed by a non-homogeneous differential equation of the form

$$\partial_r^2 \phi_1^{(2)} + \mathcal{A}_1(\omega, q)\partial_r \phi_1^{(2)} + \mathcal{A}_0(\omega, q)\phi_1^{(2)} = \mathcal{B}(\omega, q). \tag{6}$$

$\mathcal{B}$ will be quadratic in ω and q and will depend on the product of the boundary values of $\phi_2^{(1)}(r \to \infty) \equiv \varphi_2^{(1)}$ and $\phi_3^{(1)}(r \to \infty) \equiv \varphi_3^{(1)}$.

Imagine now that we compute N independent three-point functions in different combinations of channels, but with each satisfying the same structure of equations as the one specified above.

$$\lim_{\omega \to 0} \lim_{p \to 0} \sum_{n=1}^{N} c_n \partial_n \partial_n \mathcal{B}_n = 0 \tag{7}$$