Universal cosmological constant from holography

Matthew Stephenson
Stanford University[*]
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* matthewjstephenson@icloud.com

I. INTRODUCTION

The holographic gauge-string duality is a powerful tool for uncovering both qualitative and quantitative behaviour of certain classes of strongly-coupled gauge theories. Usually, a holographic calculation is performed either from a top-down or a bottom-up approach. The former, derived from consistent truncations of string theory, has the advantage of providing us with information about the microscopic details of the dual field theory. However, these theories are usually supersymmetric with a field content that has not been observed in nature. The bottom-up approach is easier to implement and relies, philosophically, on the Landau-Ginzburg-Wilson expansion of the bulk theory. Unfortunately, the dual field theories of such bulk models, if they exist, are unknown. Furthermore, computational control over both of these approaches relies on some form of a large-N limit, i.e. classical gravity approximation of the bulk dynamics.

Even thought we presently lack a holographic dual of any theory known to exist in nature, we can nevertheless extract physically useful information from holography. Qualitatively, the wealth of holographic setups can hopefully lead to a realisation of new strongly-coupled states of matter that could be reproduced, or noticed, in nature. Quantitatively, a wide variety of the top-down and bottom-up setups may agree on the behaviour of a dual observable, thus providing us with a universal statement of how field theories should behave at strong coupling. By far the most prominent such example is the universal ratio of shear viscosity to entropy density, $\eta/s = \hbar/4\pi k_B$, in theories with the gravitational Einstein-Hilbert action []. This quantity is not only universal holographically, but it also gives the correct order of magnitude of η/s as measured in nature, e.g. in quark-gluon plasmas []. Another universal relation regards three of the conformal second-order transport coefficients $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$, ...

It is easiest to expect universal holographic statements to be directly associated with either background geometry properties of with dynamical equations, which are very robust against coupling with other sectors. An example of this is the decoupling of the spin-2 perturbation of the background metric, which behaves as a minimally coupled massless scalar field. The proofs of the η/s universality are most commonly incarnations of this fact.

In this paper, we will neither take the top-down nor the bottom-up approach to exploring some holographic implications on cosmology.

II. COSMOLOGY WITH STRONGLY COUPLED MATTER

We begin this work by reviewing and generalising the work of Gubser [1], in which he showed that the dual of the AdS_5-Schwarzschild black brane, i.e. the $\mathcal{N} = 4$ super Yang-Mills theory at infinite 't Hooft coupling λ and an infinite number of colours N_c at non-zero temperature, can source a radiation dominated FRW cosmology on a four dimensional brane embedded into a five dimensional bulk.

III. UNIVERSAL COSMOLOGICAL CONSTANT

Having argued above for a wide landscape of potential gravitational backgrounds with dual field theories, coming either from top-down or bottom-up constructions, let us consider a general five dimensional bulk metric ansatz

$$ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + \left(\frac{r}{L}\right)^{2\beta} d\vec{x}^2, \tag{1}$$

parametrised by two functions of the radial coordinate λ and ν. In the spirit of standard holography, we adopt the view that the radial coordinate somehow encodes the RG flow of the dual field theory. Furthermore, we have assumed a spatially homogeneous boundary state.

By solving the Israel junction condition for the $t(r)$ foliation, the induced metric of the four-dimensional universe with a strongly-coupled matter sector is

$$ds^2 = -\left(\frac{\beta^2 L^2 e^{2\nu}}{r^2 e^{2\nu} - \beta^2 L^2}\right) dr^2 + \left(\frac{r}{L}\right)^{2\beta} d\vec{x}^2. \tag{2}$$

The boundary time can thus be defined as

$$\tau = \tau_0 \pm \int^r d\rho \frac{\beta L e^{\nu(\rho)}}{\sqrt{\rho^2 e^{2\nu(\rho)} - \beta^2 L^2}}, \tag{3}$$

and the FRW scale factor is given by

$$a(\tau) = \left(\frac{r(\tau)}{L}\right)^{\beta}. \tag{4}$$

Notice that this expression allows us to solve an algebraic equation to recover $r(\tau)$. The constant τ_0 can be chosen arbitrarily, depending on what radial coordinate r should correspond to the big bang singularity.

Our goal is now to work backwards compared to the usual approaches to holography and reconstruct the bulk metric from an arbitrary choice of the scale factor function $a(\tau)$. We assume that the foliation $\tau(r)$ is invertible, so that the entire metric can be conveniently written by τ replacing the radial component. We find

$$\frac{dr(\tau)}{d\tau} = \frac{\sqrt{r(\tau)^2 e^{2\nu(\tau)} - \beta^2 L^2}}{L\beta e^{\nu(\tau)}}, \tag{5}$$

and by using Eq. (4), i.e. $r(\tau) = La(\tau)^{1/\beta}$, the problem of finding the background metric reduces to solving a simple algebraic equation for $\nu(\tau)$,

$$\beta^2 e^{-2\nu(\tau)} = a(\tau)^{2/\beta}\left[1 - \left(\frac{L}{\beta}\frac{d\log a(\tau)}{d\tau}\right)^2\right]. \tag{6}$$

Following this procedure, we recover both $r(\tau)$ and $\nu(\tau)$ from a choice of $a(\tau)$. The only remaining function in the bulk metric, the function $\lambda(\tau)$, is left undetermined. The bulk metric can then be written in terms of new coordinates $(t, \tau, \vec{x})$,

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{\beta^2 L^2 \dot{a}(\tau)^2}{\beta^2 a(\tau)^2 - L^2\dot{a}(\tau)^2}d\tau^2 + a(\tau)^2 d\vec{x}^2, \tag{7}$$

where the overhead dot denotes a derivative w.r.t τ. In terms of the Hubble parameter,

$$H \equiv \frac{\dot{a}}{a}, \tag{8}$$

the bulk metric is

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{\beta^2 L^2 H(\tau)^2}{\beta^2 - L^2 H(\tau)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{9}$$

We can think of λ as parametrising a family of different metrics, which all give the same FRW universe on the boundary. The function will of course be important for the details of the field theory dynamics, but not the four dimensional metric, and hence not for the rate of the universe expansion.

Consider the structure of the bulk metric at late boundary times, (3), as $\tau \to \infty$. Assuming monotonicity of the bulk metric functions, the indefinite integral on the right-hand-side of Eq. (3) will contribute only at large values of the integrand variable ρ. Hence

$$\tau = \int^r d\rho \left[\lim_{\rho\to\infty} \frac{\beta L e^{\nu(\rho)}}{\sqrt{\rho^2 e^{2\nu(\rho)} - \beta^2 L^2}} \right], \tag{10}$$

where the integral on the right-hand-side is an indefinite integral.

[1] S. S. Gubser, Phys.Rev. **D63**, 084017 (2001), arXiv:hep-th/9912001 [hep-th].