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Number Theory

Part 1: Transcendental Equations:

Solving Transcendental Equations using the βw-convergence formula

Abstract

The main purpose of inventing this paper is based on the general idea that an equation of this form can't be $a^x + b^x = c$ algebraically. In this question derived, the formula $((\beta w - convergence)$ with mathematical proof can be used to solve such an equation with ease. Since the formula is purely invented with my own approach, the article lacks references

βw-convergence Formulae for Solving $a^x + b^x = c$

Say, n $\cong x$, then, n

$$(a^x - a^n) + (b^x - b^n) = c - a^n - b^n$$

Factorizing, then

$a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c - a^n - b^n$

Factorized, then

$(b^{x-n} - a^{x-n})(a^n b^n) = c - a^n - b^n$, this is true if n $\approx x$ or n $= x$

Dividing $a^n b^n$ both sides, then

$$(b^{x-n} - a^{x-n}) = \frac{c - a^n - b^n}{(a^n b^n)}$$

This can also be written as;

$$\frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c - a^n - b^n}{(a^n b^n)}$$

Back to the equation, $a^x + b^x = c$,

$$a^x = c - b^x$$

Therefore,

$$\frac{b^x}{b^n} - \frac{(c - b^x)}{a^n} = \frac{c - a^n - b^n}{(a^n b^n)}$$

$$\frac{a^n b^x - b^n c + b^n b^x}{a^n b^n} = \frac{c - a^n - b^n}{(a^n b^n)}$$

Multiplying both sides by $a^n b^n$

$$a^n b^x - b^n c + b^n b^x = c - a^n - b^n$$

This can also be written as;

$$a^n b^x + b^n b^x = c - a^n - b^n + b^n c$$

Factorizing,

$$b^x(a^n + b^n) = c - a^n - b^n + b^n c$$

Where b^x will be;

$$b^x = \frac{b^n c + (c - a^n - b^n)}{(a^n + b^n)}$$

Therefore,

$$x = \frac{\boldsymbol{log}(\frac{\boldsymbol{b^n c + (c - a^n - b^n)}}{\boldsymbol{(a^n + b^n)}})}{\boldsymbol{logb}}$$

Similarly, following the same procedure,

$$x = \frac{\boldsymbol{log}(\frac{\boldsymbol{a^n c + (c - a^n - b^n)}}{\boldsymbol{(a^n + b^n)}})}{\boldsymbol{loga}}$$

Theorem

I. When $n \to x$, the closer n approaches x, then accurate the answer until $n = x$

II. Meaning n_1 will be closer to answer than n_2, n_3 than n_2.... n_j

III. n_1 must be used to get n_2, n_2 to get n_3… until $n_j=x$

IV. Using $\beta w - convergence\ formula$, k can be used to calculate the first value of n

V. k can be any value assumed. as long the $c - a^n - b^n > 0, or\ equal\ to\ 0$

VI. Here in all calculations, I have taken $b^x > a^x$; however, whichever the case, it does not interfere with the calculations. One can also use $a^x > b^x$

VII. The larger the value of k, the more calculations would be needed, but the closer the value k to, n fewer calculations would be needed.

VIII. The same formula calculates the first value of n

$$\boldsymbol{n = \frac{log(\frac{b^k c + (c - a^k - b^k)}{(a^k + b^k)})}{logb}}$$

IX. $\boldsymbol{x = \frac{log(\frac{b^n c+(c-a^n-b^n)}{(a^n+b^n)})}{logb}}$ requires less calculation than $x = \frac{\boldsymbol{log(\frac{a^n c+(c-a^n-b^n)}{(a^n+b^n)})}}{\boldsymbol{loga}}$ to find the accurate answer.

βw-convergence formulae for Solving $\boldsymbol{b^x - a^x = c}$

Following the same rule & procedure

$n \to x$, then

$$(a^x - a^n) + (b^x - b^n) = c + a^n - b^n$$

Factorizing, then

$a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c + a^n - b^n$

However, if $n \to x$, where $n = x$, then $a^n(a^{x-n} - 1) + b^n(b^{x-n} - 1) = c + a^n - b^n$

Can be written (factorized) as;

$$(b^{x-n} - a^{x-n})(a^n b^n) = c + a^n - b^n$$

Dividing $a^n b^n$ both sides, then

$$(b^{x-n} - a^{x-n}) = \frac{c + a^n - b^n}{(a^n b^n)}$$

This can also be written as;

$$\frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c + a^n - b^n}{(a^n b^n)}$$

Back to the equation, $b^x - a^x = c$,

$$b^x = c + a^x$$

Therefore,

$$\frac{(c + a^x)}{b^n} - \frac{a^x}{a^n} = \frac{c + a^n - b^n}{(a^n b^n)}$$

$$\frac{a^n(a^x + c) - b^n a^x}{a^n b^n} = \frac{c + a^n - b^n}{(a^n b^n)}$$

Multiplying both sides by $a^n b^n$

$$a^n a^x + a^n c - b^n a^x = c + a^n - b^n$$

This can also be written as;

$$a^n a^x - b^n a^x = c + a^n - b^n - a^n c$$

Factorizing,

$$a^x(a^n - b^n) = c + a^n - b^n - a^n c$$

Where a^x will be;

$$a^x = \frac{-ca^n + (c + a^n - b^n)}{(a^n - b^n)}$$

Thus,

$$x = \frac{\boldsymbol{log}\left(\frac{\boldsymbol{-ca^n + (c + a^n - b^n)}}{\boldsymbol{(a^n - b^n)}}\right)}{\boldsymbol{loga}}$$

Similarly, following the same procedure,

$$x = \frac{\boldsymbol{log}(\frac{-\boldsymbol{cb^n} + (\boldsymbol{c} + \boldsymbol{a^n} - \boldsymbol{b^n})}{(\boldsymbol{a^n} - \boldsymbol{b^n})})}{\boldsymbol{logb}}$$

Theorem

I. k any value assumed. as long the $c - a^n - b^n < 0, or\ equal\ to\ 0$

II. This formula can also be used to find the value of x, in the equation

$$\boldsymbol{a^x} + \boldsymbol{b^x} = \boldsymbol{c^x}$$

III. However, to do so, the equation must be first changed into

$$({}^{a}/_{c})^x + ({}^{b}/_{c})^x = 1$$

Or

$$({}^{c}/_{a})^x - ({}^{b}/_{a})^x = 1$$

Example 1

Find the value of x

$3^x + 2^x = 14$

Solution

Let's take any value of k, say 8, and then

$$\boldsymbol{n1} = \frac{\boldsymbol{log}(\frac{\boldsymbol{b^k c} + \left(\boldsymbol{c} - \boldsymbol{a^k} - \boldsymbol{b^k}\right)}{(\boldsymbol{a^k} + \boldsymbol{b^k})})}{\boldsymbol{logb}}$$

So, applying the formula;

$$\boldsymbol{n1} = \frac{\boldsymbol{log}(\frac{(\boldsymbol{3}^8 \times \boldsymbol{14}) + (\boldsymbol{14} - \boldsymbol{2}^8 - \boldsymbol{3}^8)}{(\boldsymbol{2}^8 + \boldsymbol{3}^8)})}{\boldsymbol{log3}}$$

Thus,

$n1 = 2.29729050932$

So, using $n1 = 2.29729050932$, then,

$3^{2.29729050932} + 2^{2.29729050932}$=17.3916468296

Doing the second calculation, where now, n1=2.29729050932

The value of $n2$

$$\boldsymbol{n2} = \frac{\boldsymbol{log}\left(\frac{(\boldsymbol{3}^{2.29729050932} \times \boldsymbol{14}) + (\boldsymbol{14} - \boldsymbol{2}^{2.29729050932} - \boldsymbol{3}^{2.29729050932})}{(\boldsymbol{2}^{2.29729050932} + \boldsymbol{3}^{2.29729050932})}\right)}{\boldsymbol{log3}}$$

So, using $n2 = 2.08198130882$

$3^{2.08198130882} + 2^{2.08198130882}$=14.0820980231

Doing the third calculation, where now, n2=2.08198130882

The value of $n3$

$$\boldsymbol{n3} = \frac{\boldsymbol{log}\left(\frac{(\boldsymbol{3}^{2.08198130882} \times \boldsymbol{14}) + (\boldsymbol{14} - \boldsymbol{2}^{2.08198130882} - \boldsymbol{3}^{2.08198130882})}{(\boldsymbol{2}^{2.08198130882} + \boldsymbol{3}^{2.08198130882})}\right)}{\boldsymbol{log3}}$$

$n3 = 2.07611695862$,

So, using $n3 = 2.07611695862$

$3^{2.07611695862} + 2^{2.07611695862}$=14.0016781963

The value of $n4$

$$\boldsymbol{n4} = \frac{\boldsymbol{log}\left(\frac{(\boldsymbol{3}^{2.07611695862} \times \boldsymbol{14}) + (\boldsymbol{14} - \boldsymbol{2}^{2.07611695862} - \boldsymbol{3}^{2.07611695862})}{(\boldsymbol{2}^{2.07611695862} + \boldsymbol{3}^{2.07611695862})}\right)}{\boldsymbol{log3}}$$

$n4 = 2.07599670273$

So, using $n4 = 2.07599670273$

$3^{2.07599670273} + 2^{2.07599670273}$=14.0000340751

So, finding n5

$$\boldsymbol{n5} = \frac{\boldsymbol{log}\left(\frac{(\mathbf{3}^{2.07599670273} \times \mathbf{14}) + (\mathbf{14} - \mathbf{2}^{2.07599670273} - \mathbf{3}^{2.07599670273})}{(\mathbf{2}^{2.07599670273} + \mathbf{3}^{2.07599670273})}\right)}{\boldsymbol{log3}}$$

$n5 = 2.07599426082$

$3^{2.07599426082} + 2^{2.07599426082}$=14.0000006918

If the calculations are repeated, it will reach a point where the value of n_k, $(3^{nk} + 2^{nk} = 14)$, solution of exactly 14, meaning, $n_k = x$

Important Notice Based on Example 1

- If the value *k>n*, then the values of $C^n > C^x$. However, C^n will decrease with each calculation until it reaches, $C^n = C^x$

$$\boldsymbol{a^x + b^x = c^x}$$

- If *k<n*, then the value of $C^n < C^x$. However, C^n will increase with each calculation until it reaches $C^n = C^x$
- In the calculation have assumed *k=8,* though any number can be used if and only if $c - a^n - b^n > 0, or\ equal\ to\ 0$

Example 2

Find the value of x

$3^x - 2^x = 14$

$k = 0.86135311614$

Applying the formula

$$\boldsymbol{n1} = \frac{\boldsymbol{log}\left(\frac{-\boldsymbol{cb^n} + (\boldsymbol{c} + \boldsymbol{a^n} - \boldsymbol{b^n})}{(\boldsymbol{a^n} - \boldsymbol{b^n})}\right)}{\boldsymbol{logb}}$$

Then

$$n1 = \frac{log(\frac{(-3^{0.86135311614} \times 4) + (4 + 2^{0.86135311614} - 3^{0.86135311614})}{(2^{0.86135311614} - 3^{0.86135311614})})}{log}$$

$n1 = \ 2.03004521829$

So, using n1=2.03004521829, then,

$3^{2.03004521829} - 2^{2.03004521829}$=5.24193926067

*Doing the second calculation, where now, **n1= 2.03004521829***

The value of $n2$

$$n2 = \frac{log(\frac{(-3^{2.03004521829} \times 4) + (4 + 2^{2.03004521829} - 3^{2.03004521829})}{(2^{2.03004521829} - 3^{2.03004521829})})}{log3}$$

$\boldsymbol{n2 = 1.81742690735}$

$3^{1.81742690735} - 2^{1.81742690735} = 3.8398057357$

*Doing the third calculation, where now, **n2= 1.81742690735***

The value of x

$$n3 = \frac{log(\frac{(-3^{1.81742690735} \times 4) + (4 + 2^{1.56715353789} - 3^{1.56715353789})}{(2^{1.56715353789} - 3^{1.56715353789})})}{log3}$$

$n3 = \mathbf{1.84966718013}$

$3^{1.84966718013} - 2^{1.84966718013}$= 4.02567137244

*Doing the fourth calculation, where now, **n=1.84966718013***

The value of $n3$

$$n4 = \frac{log(\frac{(-3^{1.84966718013} \times 4) + (4 + 2^{1.84966718013} - 3^{1.84966718013})}{(2^{1.84966718013} - 3^{1.84966718013})})}{log3}$$

$n4 = \mathbf{1.84460939966}$

$3^{1.84460939966} - 2^{1.84460939966}$= 3.99600675004

Doing the fifth calculation, where $\boldsymbol{n4} = \mathbf{1.84460939966}$

$$x = \frac{log(\frac{(-3^{1.84460939966} \times 4) + (4 + 2^{1.84460939966} - 3^{1.84460939966})}{(2^{1.84460939966} - 3^{1.84460939966})})}{log3}$$

$n5 = 1.84539878668$

$3^{1.84539878668} - 2^{1.84539878668} = 4.00062407008$

Doing the sixth calculation, where $\boldsymbol{n5}$ =**1.84539878668**

$$x = \frac{log(\frac{(-3^{1.84539878668} \times 4) + (4 + 2^{1.84539878668} - 3^{1.84539878668})}{(2^{1.84539878668} - 3^{1.84539878668})})}{log3}$$

$n6 = \mathbf{1.84527548465}$

$3^{1.84527548465} - 2^{1.84527548465} = 3.99990254065$

If the calculations are repeated, it will reach a point where the value of x $(3^x - 2^x = 4)$, solution of exactly 4

Important Notice Based on Example 2

- As observed in the example, n1, n3, and n4, give $C^n > C^x$, but the value decrease with each calculation. While the value of n2, n4, n6, give $C^n < C^x$ but increases by each calculation
- In the calculation have assumed k= 0.86135311614*,* though any number can be used if and only if $c - a^n - b^n < 0$, *or equal to* 0

$\boldsymbol{\beta w - convergence}$ Formular for Solving $\mathbf{a^{x \pm e} + b^{x \pm d} = c}$

$n \rightarrow x$, then, and *where* (d, e) *are known numbers*

$$(a^{x+e} - a^{n+e}) + (b^{x+d} - b^{n+d}) = c - a^{n+e} - b^{n+e}$$

Factorizing, then

$$a^{n+e}(a^{x-n} - 1) + b^{n+d}(b^{x-n} - 1) = c - a^{n+e} - b^{n+d}$$

However, if $n \to x$, where $n = x$, then $a^{n+e}(a^{x-n} - 1) + b^{n+e}(b^{x-n} - 1) = c - a^{n+e} - b^{n+d}$

This be written (factorized) as;

$$(b^{x-n} - a^{x-n})(a^{n+e}b^{n+d}) = \; c - a^{n+e} - b^{n+d}$$

Dividing $a^n b^n$ both sides, then

$$(b^{x-n} - a^{x-n}) = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

This can also be written as;

$$\frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

Back to the equation, $a^{x+e} + b^{x+d} = c$,

$$a^x = \frac{c - b^{x+d}}{a^e}$$

Therefore,

$$\frac{b^x}{b^n} - \frac{(c - b^{x+d})}{a^e a^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

$$\frac{a^n b^x a^e - cb^n + b^{n+x+d}}{a^{e+n}b^n} = \frac{c - a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

Multiplying both sides by $a^{n+e}b^n$

$$a^n b^x a^e - cb^n + b^{n+x+d} = \frac{c - a^{n+e} - b^{n+d}}{(b^d)}$$

This can also be written as;

$$a^n b^x a^e + b^n b^x b^d = \frac{c - a^{n+e} - b^{n+d}}{(b^d)} + b^n c$$

$$a^n b^x a^e + b^n b^x b^d = \frac{cb^{n+d} + (c - a^{n+e} - b^{n+d})}{(b^d)}$$

Factorizing,

$$b^x(a^{n+e}+b^{n+d})=\frac{cb^{n+d}+(c-a^{n+e}-b^{n+d})}{(b^d)}$$

Where b^x will be;

$$b^x=\frac{cb^{n+d}+(c-a^{n+e}-b^{n+d})}{(a^{n+e}+b^{n+d})(b^d)}$$

Hence

$$x=\frac{\boldsymbol{log}(\frac{(\boldsymbol{b^{n\pm d}c})+\left(\boldsymbol{c}-\boldsymbol{a^{n\pm e}}-\boldsymbol{b^{n\pm d}}\right)}{(\boldsymbol{a^{n\pm e}}+\boldsymbol{b^{n\pm d}})(\boldsymbol{b^{\pm d}})})}{\boldsymbol{logb}}$$

Similarly, following the same procedure,

$$x=\frac{\boldsymbol{log}(\frac{(\boldsymbol{a^{n\pm e}c})+\left(\boldsymbol{c}-\boldsymbol{a^{n\pm e}}-\boldsymbol{b^{n\pm d}}\right)}{(\boldsymbol{a^{n\pm e}}+\boldsymbol{b^{n\pm d}})(\boldsymbol{a^{\pm e}})})}{\boldsymbol{loga}}$$

$\boldsymbol{\beta w-convergence}$ Formular for Solving $\mathbf{b^{x\pm d}-a^{x\pm e}=c}$

$n\rightarrow x$, then, and *where* (d,e) *are known numbers*

$$(a^{x+e}-a^{n+e})+(b^{x+d}-b^{n+d})=c+a^{n+e}-b^{n+e}$$

Factorizing, then

$a^{n+e}(a^{x-n}-1)+b^{n+d}(b^{x-n}-1)=c+a^{n+e}-b^{n+d}$

However, if $n\rightarrow x$, where $n=x$, then $a^{n+e}(a^{x-n}-1)+b^{n+e}(b^{x-n}-1)=c+a^{n+e}-b^{n+d}$

This be written (factorized) as;

$$(b^{x-n}-a^{x-n})(a^{n+e}b^{n+d})=\ c+a^{n+e}-b^{n+d}$$

Dividing a^nb^n both sides, then

$$(b^{x-n} - a^{x-n}) = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

This can also be written as;

$$\frac{b^x}{b^n} - \frac{a^x}{a^n} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

Back to the equation, $a^{x+e} + b^{x+d} = c$,

$$a^x = \frac{b^{x+d} - c}{a^e}$$

Therefore,

$$\frac{b^x}{b^n} - \frac{(b^{x+d} - c)}{a^e a^n} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

$$\frac{a^n b^x a^e + b^{n+x+d} - cb^n}{a^{e+n}b^n} = \frac{c + a^{n+e} - b^{n+d}}{(a^{n+e}b^{n+d})}$$

Multiplying both sides by $a^{n+e}b^n$

$$a^n b^x a^e + cb^n - b^{n+x+d} = \frac{c + a^{n+e} - b^{n+d}}{(b^d)}$$

This can also be written as;

$$a^n b^x a^e - b^n b^x b^d = \frac{c + a^{n+e} - b^{n+d}}{(b^d)} - b^n c$$

$$a^n b^x a^e + b^n b^x b^d = \frac{(-cb^{n+d}) + (c + a^{n+e} - b^{n+d})}{(b^d)}$$

Factorizing,

$$b^x(a^{n+e} - b^{n+d}) = \frac{(-cb^{n+d}) + (c + a^{n+e} - b^{n+d})}{(b^d)}$$

Where b^x will be;

$$b^x = \frac{(-cb^{n+d}) + (c + a^{n+e} - b^{n+d})}{(a^{n+e} - b^{n+d})(b^d)}$$

Thus

$$x = \frac{log(\frac{(-b^{n\pm d}c) + (c + a^{n\pm e} - b^{n\pm d})}{(a^{n\pm e} - b^{n\pm d})(b^{\pm d})})}{logb}$$

Similarly, following the same procedure,

$$x = \frac{log(\frac{(-a^{n\pm e}c) + (c + a^{n\pm e} - b^{n\pm d})}{(a^{n\pm e} - b^{n\pm d})(a^{\pm e})})}{loga}$$

βw-convergence Formular for Solving $\mathbf{b}^{x^d} + \mathbf{a}^{\mathrm{x}} = \mathbf{c}$

Using the same steps (procedure) as formula for solving $\mathbf{b}^{\mathrm{x}} + \mathbf{a}^{\mathrm{x}} = \mathbf{c}$

$$x = \frac{log(\frac{ca^n - (c - a^n - b^{n^d}}{(a^n + b^{n^d})})}{loga}$$

Or

$$x = \sqrt[d]{\frac{log(\frac{cb^{n^d} + (c - a^n - b^{n^d})}{(a^n + b^{n^d})})}{logb}}$$

However, this can be summarized as ($\mathbf{b}^{x^d} + a^{x^e} = \mathbf{c}$)

$$x = \sqrt[d]{\frac{log(\frac{cb^{n^d} + (c - b^{n^d} - a^{n^e})}{(b^{n^d} + a^{n^e})})}{logb}}$$

Or

$$x = \sqrt[e]{\frac{log(\frac{ca^{n^e} - (c - b^{n^d} - a^{n^e})}{(b^{n^d} + a^{n^e})})}{loga}}$$

General βw-convergence Formula

Suppose, $a^x \pm b^x \pm c^x \pm d^x \dots\dots\dots\dots. \pm z^x = \beta$

Then, applying the mathematical approach from the equation $\boldsymbol{a^x + b^x = c}$

The value x of any value selected, say

$$z^x = \frac{\beta z^n + (\beta \mp a^n \mp b^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)}{(a^n \pm b^n \pm c^n \pm d^n \dots\dots\dots\dots. \pm z^n)}$$

However, this is true if $\boldsymbol{z^x} + \boldsymbol{M} = \beta$

Where $\boldsymbol{M} = (a^x \pm b^x \pm c^x \pm d^x \dots\dots\dots\dots. \pm y^x)$

Thus

$$\boldsymbol{x = \frac{log(\frac{\beta z^n + (\beta \mp a^n \mp b^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)}{(a^n \pm b^n \pm c^n \pm d^n \dots\dots\dots\dots. \pm z^n)})}{logz}}$$

Where there is a subtraction in the equation,

Say, $\boldsymbol{a^x \mp b^x \pm c^x \pm d^x} \dots\dots\dots\dots. \pm \boldsymbol{z^x = \beta}$

Then, we apply the mathematical approach from the equation $\boldsymbol{b^x - a^x = c}$

This is true if $\boldsymbol{M - b^x = \beta}$

Where $M = (a^x \pm c^x \pm d^x \dots\dots\dots\dots. \pm z^x)$

$$b^x = \frac{-\beta b^n + (\beta \mp a^n \pm b^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)}{(b^n \mp a^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)}$$

Thus

$$\boldsymbol{x = \frac{log(\frac{-\beta z^n + (\beta \mp a^n \pm b^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)}{(b^n \mp a^n \mp c^n \mp d^n \dots\dots\dots\dots. \mp z^n)})}{logz}}$$