CLOCKS IN QUANTUM SCALE

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APRIL 17, 2023

Abstract. In this paper I do present a model of quantum clock that follows a field equation. From that follows possible model of quantum gravity thus space-time in quantum scale.

Contents

1. Quantum Clock

On quantum scale time and space has to behave another way than they do in classical picture of space-time. Quantum clock is idea that measure of distance in space and time depends on rotation in complex plane of given complex tensor field. Full rotation goes back to same event, each clock carries how rotation changes compared to some units. So clock that needs one unit of distance to complete rotation it means that all events happen at one tick of a clock. If it's less than one it takes number of ticks to go back to starting point- and only some events happen at same time. This clock has to be quantized, it can only have a natural number of ticks to do full rotation. If n is number of ticks it takes to do a full rotation in complex plane it has to be a natural number. How long distance it takes for event to happen depends on it's energy. More energy complex tensor field has faster it rotates in complex plane so it reaches more events in one tick of a clock, each event can be mapped into complex plane as a point spanned by rotation of complex tensor field. It has to be a tensor field in order to be agreed with energy tensor that will have two indexes, vector field will only have one index so it wont match degrees of freedom of energy tensor that will have sixteen components in four dimensional complex space-time.

Clock defined this way has invariant part that is probability, let me denote a complex field as $\Psi_{\mu\nu}$ there is invariant probability that comes from it that can be written as a scalar:

$$\Psi_{\mu\nu}\Psi^{\mu\nu} = \Psi^2 \tag{1.1}$$

This field is a complex four dimensional field so it transforms by four matrix that rotate it, it can be easy shown that it leads to spin two:

$$U^{\alpha}_{\mu}\left(\frac{\varphi}{2}\right)U^{\beta}_{\nu}\left(\frac{\varphi}{2}\right)\Psi_{\alpha\beta}\Psi^{\alpha\beta}U^{\mu}_{\alpha}\left(\frac{\varphi}{2}\right)U^{\nu}_{\beta}\left(\frac{\varphi}{2}\right) \tag{1.2}$$

One part of field transforms as spin one and another part transforms as spin one. I will get a scalar field as final result that has sixteen parts that sum, each part transforms as particle with spin two, but field itself from fact that it's a scalar field does not transform.It means that field itself acts as spin one field but it has two possible states of spin one, depending on using covariant or contravariant form of that field. Probability of finding particle in the field is equal to integral over space of that scalar It has to be normalized so probability of finding particle is equal to one at whole space:

$$\int_V \Psi_{\mu\nu}\Psi^{\mu\nu} d^3x = \int_V \Psi^2 d^3x = 1 \tag{1.3}$$

2. Field Equation

There are two field equations for covariant and contravariant part of complex tensor field, they relate how covariant and contravariant derivative is connected to energy tensor that represents energy of matter field:

$$\left(\nabla_{\mu}\nabla_{\nu} + Q_{\mu\nu}\right) g_{\mu\nu}\Psi^{\mu\nu} = 0 \tag{2.1}$$

$$\left(\nabla^{\mu}\nabla^{\nu} + Q^{\mu\nu}\right) g^{\mu\nu}\Psi_{\mu\nu} = 0 \tag{2.2}$$

Where contravariant derivative is defined by using of metric tensor to raise an index:

$$\nabla^{\mu} = g^{\mu\alpha}\nabla_{\alpha} \tag{2.3}$$

Metric tensor is now a complex object that is defined in terms of complex numbers, otherwise it's same as normal metric tensor definition:

$$g_{\mu\nu} = \frac{\partial\xi^{\alpha}}{\partial z^{\mu}}\frac{\partial\xi^{\alpha}}{\partial z^{\mu}}\eta_{\alpha\beta} \tag{2.4}$$

Now I will add new scalar quality, that comes form field equation that scalar quality is number of clock ticks:

$$\nabla_{\mu}\nabla_{\nu}g_{\mu\nu}\Psi^{\mu\nu} = \Phi_{\mu\nu} \tag{2.5}$$

$$\nabla^{\mu}\nabla^{\nu}g^{\mu\nu}\Psi_{\mu\nu} = \Phi^{\mu\nu} \tag{2.6}$$

$$\Phi^{2} = \Phi_{\mu\nu}\Phi^{\mu\nu} = g_{\mu\nu}g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\Psi^{\mu\nu}\nabla^{\mu}\nabla^{\nu}\Psi_{\mu\nu} \tag{2.7}$$

This has also a probability build into it, so it's number of ticks time probability. Field equation reads that how field does change it's equal to it's energy tensor. This tensor can be understand as a tensor that says what is amount of energy stored in given rotation of on of tensor field in complex plane. Field equation itself is a tensor equation there parts $g^{\mu\nu}\Psi_{\mu\nu}$ and $g_{\mu\nu}\Psi^{\mu\nu}$ are parts of tensor field. Field equation can be rewritten:

$$g_{\mu\nu}\left(\nabla_{\mu}\nabla_{\nu} + Q_{\mu\nu}\right)\Psi^{\mu\nu} = 0 \tag{2.8}$$

$$g^{\mu\nu}\left(\nabla^{\mu}\nabla^{\nu} + Q^{\mu\nu}\right)\Psi_{\mu\nu} = 0 \tag{2.9}$$

Now it's clear that both terms in bracket act on a tensor complex field. Those two are complex field equations, when measured they turn into real fields. That turning into real fields gives a gravity effects and defines how distance is measured.

3. Gravity=Clock synchronization

Clock field can be understood as energy of event times it's probability, where both are scalars. First I define energy in scalar form:

$$Q^2 = Q_{\mu\nu}Q^{\mu\nu} \tag{3.1}$$

Now let me assume two clocks that synchronize, first clock will be denoted $\Phi_{\mu\nu}$ and second clock $\Phi'_{\mu\nu}$. I can write formula for clock synchronization that is equal to:

$$\Phi'_{\mu\nu} = N_{\mu\nu}N^{\mu\nu}\Phi_{\mu\nu} \tag{3.2}$$

Where both tensor $N_{\mu\nu}, N^{\mu\nu}$ consists only of natural numbers. So it's product with itself gives sum natural number where that number is sum of square numbers :

$$N = N_{\mu\nu}N^{\mu\nu} \tag{3.3}$$

This tensor says how much ticks of a clock is needed to reach another clock, if those two can't match there is no way to synchronize two clocks so they can't be positioned in same position in space-time. Now for scalar quantity I will get this equation for full clock:

$$\Phi'_{\mu\nu}\Phi'^{\mu\nu} = N_{\mu\nu}N^{\mu\nu}N_{\mu\nu}N^{\mu\nu}\Phi_{\mu\nu}\Phi^{\mu\nu} \tag{3.4}$$

$$\Phi'_{\mu\nu}\Phi'^{\mu\nu} = N^2\Phi_{\mu\nu}\Phi^{\mu\nu} \tag{3.5}$$

$$\frac{\Phi'_{\mu\nu}\Phi'^{\mu\nu}}{\Phi_{\mu\nu}\Phi^{\mu\nu}} = N^2 \tag{3.6}$$

$$\sqrt{\frac{\Phi'_{\mu\nu}\Phi'^{\mu\nu}}{\Phi_{\mu\nu}\Phi^{\mu\nu}}} = N \tag{3.7}$$

Gravity is synchronization of those clocks so i can vary function N to get shortest possible path:

$$\delta\int_P N = 0 \tag{3.8}$$

$$\delta\left(\int_P\sqrt{\frac{\Phi'_{\mu\nu}\Phi'^{\mu\nu}}{\Phi_{\mu\nu}\Phi^{\mu\nu}}}\right) = 0 \tag{3.9}$$

Now I can take any path that vary of that function is equal to zero so I can sum many paths, where probability of those paths is given by path integral over ratio:

$$\sum_i\delta\left(\int_{P_i}\sqrt{\frac{\Phi'_{\mu\nu}\Phi'^{\mu\nu}}{\Phi_{\mu\nu}\Phi^{\mu\nu}}}\right) = 0 \qquad \int_{P_i}\frac{\Psi'_{\mu\nu}\Psi'^{\mu\nu}}{\Psi_{\mu\nu}\Psi^{\mu\nu}} = \int_{P_i}\frac{\Psi'^2}{\Psi^2} \tag{3.10}$$

4. Energy tensor

Energy tensor $Q^{\mu\nu}$ says how much energy is stored in rotation on complex plane in direction $\mu\nu$. It has key one invariant property that is the mass of field or particle, it is just it's product with metric tensor:

$$g_{\mu\nu}Q^{\mu\nu} = g^{\mu\nu}Q_{\mu\nu} = m^2 \tag{4.1}$$

Energy of event is given by another scalar quantity:

$$Q^2 = Q_{\mu\nu}Q^{\mu\nu} \tag{4.2}$$

$$Q = \pm\sqrt{Q_{\mu\nu}Q^{\mu\nu}} \tag{4.3}$$

It means that energy of event without a square is always a square number. It comes from a fact that energy tensor has to give invariant property that is mass squared. It can represent both positive and negative rotation. Mass squared of a field has to be conserved. So if I take integral over whole space of that trace of energy tensor minus mass of a field I will get zero:

$$\int_V g_{\mu\nu}Q^{\mu\nu}d^3z = \int_V g^{\mu\nu}Q_{\mu\nu}d^3z = M^2 \tag{4.4}$$

$$\int_V g_{\mu\nu}Q^{\mu\nu}d^3z - M^2 = 0 \tag{4.5}$$

$$\int_V g^{\mu\nu}Q_{\mu\nu}d^3z - M^2 = 0 \tag{4.6}$$

Where I take integral over complex space, it means that mass can be real, imaginary or both so complex. Energy tensor is a complex object, it has real and imaginary part. In simplest case it's just a real tensor but in general it does not have to be. In general it represents energy in direction $\mu\nu$, that is equal to energy of rotation in complex plane spanned by directions $\mu\nu$. That tensor is not symmetric, if I assume that first plane of rotation is a real part and second one is imaginary part off diagonal elements are not equal. Direction for example 01 means real axis of time and imaginary axis of first space direction it do not always imply that it's equal to imaginary time axis and real first space direction axis. So first index of that tensor represents real axis and second one represents imaginary axis and it says how much energy in natural units unit of rotation in that plane has. There are sixteen total rotation planes that are represented by this tensor indexes.

5. Space-time

Space-time build this way is all about clock synchronization, distance between two point in space-time is spanned by state of clocks they have. If clocks can't be synchronized it means that particle can't be in some given point of space-time. Full rotation in complex plane means that we go to same event for given clock in given direction. Then energy tensor has to be multiply by a constant, that constant is just $4\pi^2$ that will give full rotation for energy tensor equal to one, so I can re-write field equation:

$$g_{\mu\nu}\left(\nabla_\mu\nabla_\nu + 4\pi^2 Q_{\mu\nu}\right)\Psi^{\mu\nu} = 0 \tag{5.1}$$

$$g^{\mu\nu}\left(\nabla^\mu\nabla^\nu + 4\pi^2 Q^{\mu\nu}\right)\Psi_{\mu\nu} = 0 \tag{5.2}$$

All space and time units are natural numbers. So space-time is spanned by rotation in complex plane, full rotation means that state of object stays same, when objected reaches full rotation and it has frequency lower than one, it will still rotate looping back to events. It means that space-time has a limit of of maximum number of events. Field equation can be reduced to a scalar equation:

$$g_{\mu\nu}g^{\mu\nu}\left(\nabla_\mu\nabla_\nu\nabla^\mu\nabla^\nu + 16\pi^4 Q_{\mu\nu}Q^{\mu\nu}\right)\Psi_{\mu\nu}\Psi^{\mu\nu} = 0 \tag{5.3}$$

That represents a field, this field has is wave of probability moving in space-time. That wave of probability will be responsible for all gravity effects. It's wave of clock of space and time that defines gravity field. Space-time is build from both tensor like qualities but in the end those tensor like qualities will lead to a scalar form of field equation. This is a real wave not a complex tensor field, so from complex tensor field I can finally reduce it to simpler form that is just a scalar field. It can be easy seen that this wave will have all properties set before. Simplest case solutions are just:

$$\Psi^{\mu\nu} = c_0 \exp\left(2\pi i E t\right)\hat{e}_0 \otimes \hat{e}_0 - \sum_a c_a \exp\left(\pm 2\pi i p_a x^a\right)\hat{e}_a \otimes \hat{e}_a \tag{5.4}$$

$$\Psi_{\mu\nu} = \overline{c_0} \exp\left(-2\pi i E t\right)\hat{e}^0 \otimes \hat{e}^0 - \sum_a \overline{c_a} \exp\left(\mp 2\pi i p_a x^a\right)\hat{e}^a \otimes \hat{e}^a \tag{5.5}$$

That gives probability function or just scalar field:

$$\Psi^2 = \frac{1}{\kappa}\int_V \left(c_0\left(z^a\right)\overline{c_0}\left(z^a\right) + \sum_a c_a\left(z^0\right)\overline{c_a}\left(z^0\right)\right) d^3x = 1 \tag{5.6}$$

Those solutions are simplest case scenario where field has constant energy everywhere and probability constants of time depend on space and of space depend on time, κ is normalization constant.

6. General form of solutions and anti-mater

By using only Minkowski space-time there can be created a general form of solutions, for flat space-time but still those are valid solution. Those solutions have matter and anti-matter parts, where contravariant form is matter, covariant is anti-matter. Trick is to sum infinite number of those simple solutions for each point of space time:

$$\Psi^{\mu\nu} = \sum_{n\in\mathbb{Z}} c_0(n)\exp\left(2\pi i E(n)\, n\right)\hat{e}_0\otimes\hat{e}_0 - \sum_a \sum_{m^a\in\mathbb{Z}} c_a(m^a)\exp\left(\pm 2\pi i p_a(m^a)\, m^a\right)\hat{e}_a\otimes\hat{e}_a \tag{6.1}$$

$$\Psi_{\mu\nu} = \sum_{n\in\mathbb{Z}} \overline{c_0}(n)\exp\left(-2\pi i E(n)\, n\right)\hat{e}^0\otimes\hat{e}^0 - \sum_a \sum_{m^a\in\mathbb{Z}} \overline{c_a}(m^a)\exp\left(\mp 2\pi i p_a(m^a)\, m^a\right)\hat{e}^a\otimes\hat{e}^a \tag{6.2}$$

Those solutions are discrete like all solutions in this model. One part can move only forward in time and positive and negative direction in space. Anti-matter part can move only backwards in time and in both direction in space but in opposite way. Space-time build this way has two way movement, one forward and one backward in all space-time directions. Scalar field is just combining both field to arrive at probability of particle being in some position. Where scalar part of field has only information about probability tensor part has information about space-time itself. There is need to always start form Minkowski flat space-time as a base space-time to be in agreement with mass of the field conservation. Anti-matter part of the field has still positive square of mass. Direction of time flow is given by rotation direction in complex plane, clock tensor will be equal to in both cases:

$$\Phi^{\mu\nu} = \sum_{n\in\mathbb{Z}} 4\pi^2 E^2(n)\, c_0(n)\exp\left(2\pi i E(n)\, n\right)\hat{e}_0\otimes\hat{e}_0 - \sum_a \sum_{m^a\in\mathbb{Z}} 4\pi^2 p_a^2(m^a)\, c_a(m^a)\exp\left(\pm 2\pi i p_a(m^a)\, m^a\right)\hat{e}_a\otimes\hat{e}_a \tag{6.3}$$

$$\Phi_{\mu\nu} = \sum_{n\in\mathbb{Z}} 4\pi^2 E^2(n)\, \overline{c_0}(n)\exp\left(-2\pi i E(n)\, n\right)\hat{e}^0\otimes\hat{e}^0 - \sum_a \sum_{m^a\in\mathbb{Z}} 4\pi^2 p_a^2(m^a)\, \overline{c_a}(m^a)\exp\left(\mp 2\pi i p_a(m^a)\, m^a\right)\hat{e}^a\otimes\hat{e}^a \tag{6.4}$$

So anti-matter part of field still rotates in opposite directions, otherwise it has same energy and momentum.

7. Rotation symmetry of field

Complex field rotation is given by $SU(n)$ rotation group. Here this rotation group adds to Minkowski flat space-time, so for flat Minkowski space-time I can define a symmetry group first by using $SU(3)$ matrix and then adding need to keep space-time distance constant. Field itself transforms as two fields with spin one, or it does not transform at all in scalar form. I can write two spin one field transformations:

$$U_\mu^\alpha U_\nu^\beta \Psi_{\alpha\beta} = \Psi_{\mu\nu} \tag{7.1}$$

$$U_\alpha^\mu U_\beta^\nu \Psi^{\alpha\beta} = \Psi^{\mu\nu} \tag{7.2}$$

Now in general I can add need to keep distance in space-time constant (remembering that it's a complex field):

$$\eta_{\mu\nu} U_\alpha^\mu dz^\alpha U_\beta^\nu dz^\beta = \eta_{\mu\nu} U_\alpha^\mu dz'^\alpha U_\beta^\nu dz'^\beta \tag{7.3}$$

Now there is not a clear approach how to move to any curved complex space-time, but this base symmetry rotations works in flat space-time. In flat space-time I have $SU(3)$ rotation symmetry in space and rotation symmetry in time and space that preserves space-time distance so complex analogy of Lorentz Transformation.

8. Field equation meaning and summary

There are two field equations that can be combined to one scalar equation. Those two equations say all about tensor parts of field, where scalar part says about part that is responsible for measurement. Field equation states that quantum system can be reduced to clock or ruler in given direction, that will say about how space-time distance is measured from information about rotation in complex plane. Field equation consists of sixteen separate equations to calculate each part of tensor field. Idea behind clock and ruler is that full rotation does not change state of quantum system and there is limit of how much in time and distance object can be apart. It uses only integers so it's quantized space and time in sense that there can be less than one unit of distance in space and time and both are measured in Planck Units.

Gravity is understood as synchronization of clocks between two parts of field. Full information about clock state is given by two tensor equations, scalar information is given only by one number, and that scalar part can have units of rotation given by $4\pi^2$ and for sixteen possible rotation directions it can lead to maximum clock number equal to $64\pi^2$. Scalar information drops information about position in space-time and only does compare state of clock in most general sense. Energy tensor has only one conserved property that is rest mass of field. Energy tensor informs how much energy is needed to get to event in given direction.

Connecting all this gives a pretty simple possible model of quantum gravity based on idea of quantum clock, I did present simplest solutions to field equation in general form in flat space-time, for not flat space-time equations become more complex from terms in covariant derivative. But still general rule holds that energy of event is given by energy tensor it's energy needed to in change state of field to be in given state. That given state is position in space-time.