An example of super determinism

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April 2023

Abstract

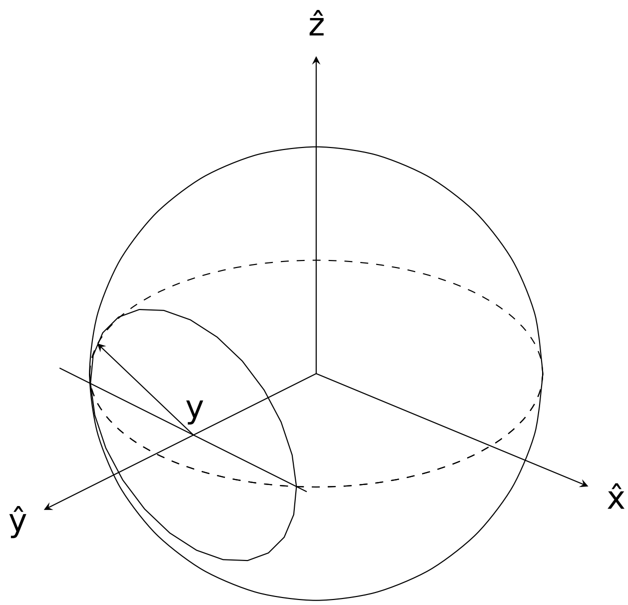
One of the interpretations of the result of the famous Aspect experiment and the modern variants of it are that we must go to the notion of super determinism in order to avoid a nondeterministic interpretation or a non local theory. To find a super determenistic example the task is to avoid a probabilistic independance assumption and construct a joint probability distribution to yield something close to what have been measured. There is a lack of such examples in the litterature and as I was curious about this and I have the nessesary probabilistic background, I took the task to present a more hands on example to illustrate what this notion really can look like. It is not a claim that this is a correct model and is plainly a theoretical exercise.

If we consider Aspects experiment, an experiment that got the Nobel Price 2022 in physics, we are e.g. measuring the spin up and spin down of two entangled particles, or in finer words, they have e.g. opposite spin in order to preserve a conserved quantity, like spin (the addition of them). Consider the view that there is an underlying parameter that the detector chooses from in order to show a spin measurement. The tricky thing is that this internal structure can be lost when looking at the experiment from the outside. We have in this case super-determinism or proven that,

$$\rho(X|x) \neq \rho(X),$$

where X represent the outcome and x is the internal state. This is what we shall explore.

We shall discuss the case of two entangled electrons where spin ads up to 0 in order to conserve the spin. This system we produce will also break Bell's inequality. The setup is schematically as,

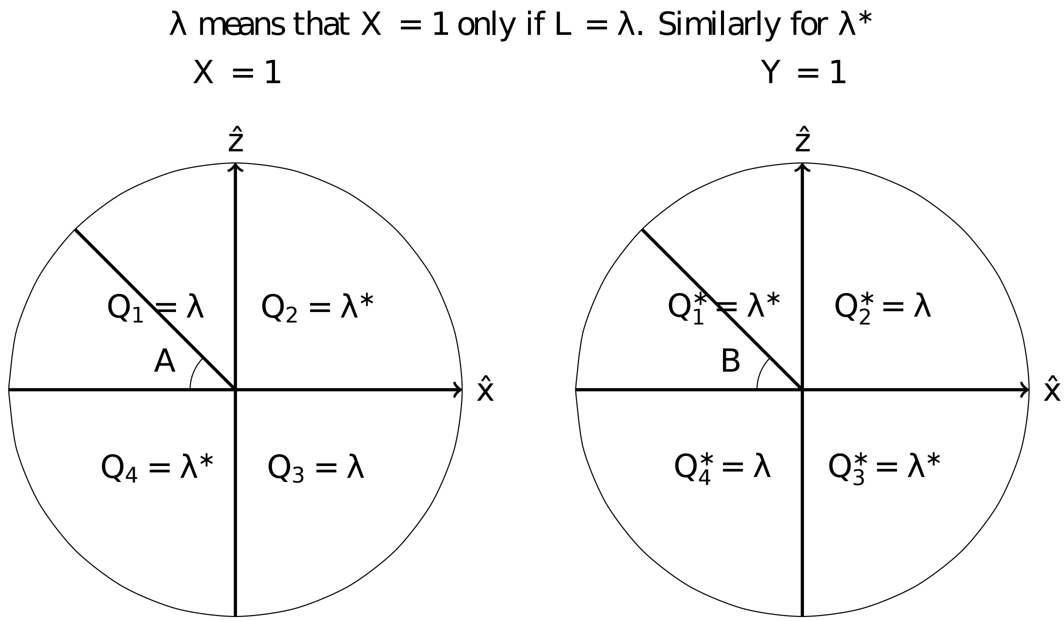


The picture above show a measurement slice for a fixed y value e.g. the slice will be a disk of possible measurements, The maximal disc is scaled by a factor of

$$\sqrt{1-y^2}.$$

To get this slice.

The setup is that we have two measurement devices, X, Y. Where X measures particle 1's spin and Y particle 2's spin. E.g the measuremnt will produce a judgment if teh spin is up or down or ($X \in \{1, -1\}$) and similarly for Y. We shall consider that the the measurement X, Y depends on a hidden variable $L \in \lambda, \lambda^*$ according to the diagram below (we indicate λ when $L = \lambda$ is selected and λ^* when $L = \lambda^*$ is selected). We will also assume that the true spin A, B is a hidden variables with $A \in [0, 360]$ is Particle's 1 true spin angle with the z-component and $B \in [0, 360]$ particle's 2 spin angle with the z-component.



Note how the selection of λ relates the two measurements. The spin down is symmetrically constrained.

The target is to measure

$$P(X = 1, Y = 1 | T = \alpha),$$

where T is the tilted angle $T = \alpha$ of the angle α in the xz-plane (without loss of generality we can assume no tilt in the y direction) that measurement device 2 is tilted. Due to the standard entanglement we have that $B = A+180$ is always true due to conservation of momentum, hence we can drop B in the probabilities below. Furthermore if we reflect along the horizontal axis, we see that particle 1 goes into particle 2 and vice versa and also the angle between them is the same but of opposite angle. Also we will assume that for a spin with angle $A = \beta$ we assume we have the probability distribuition

$$P(X = 1, L = \lambda, A = \beta \in Q_1) = P(X = 1, L = \lambda^*, A = \beta \in Q_2),$$

which is the same as,

$$P(X = 1, L = \lambda, A = \beta \in Q_1) = C|\sin(\beta)\cos(\beta)|,$$

and note that Y is assumed to follow from the condition in this probability density. Also,

$$P(X = 1, L = \lambda, A = \beta \in Q_2 \cup Q_3 \cup Q_4),$$

is the same as,

$$P(X = 1, L = \lambda^*, A = \beta \in Q_1 \cup Q_3 \cup Q_4) = 0.$$

Further more we shall assume that (spherical symmetry in upper half plane)

$$P(X = 1, A = \beta) = P(X = 1, L = \lambda, A = \beta) \propto \cos(\beta).$$

Assume that we are measuring a spin up in a quadrant. Then we will have an associated L that will decide the corresponding L in the second measurement.

Note, the probability measure is essentially the area of the underlying spin vector with the xy-plane projection of it, hence circular symmetric around the z-axis. Let $A = \beta$ be defined as the angle between the x-axis and z-axis in the plane intersecting the point y. Note how the disc at y is scaled by length factor of (we assume without loss of generally a unit sphere),

$$\sqrt{1 - y^2}.$$

And hence the Area scale is scaled by

$$1 - y^2$$

And that by combining all results for discs with varying y we see that the result of the probabilities is invariant of β below (another constant though) and hence we will just consider one such disc.

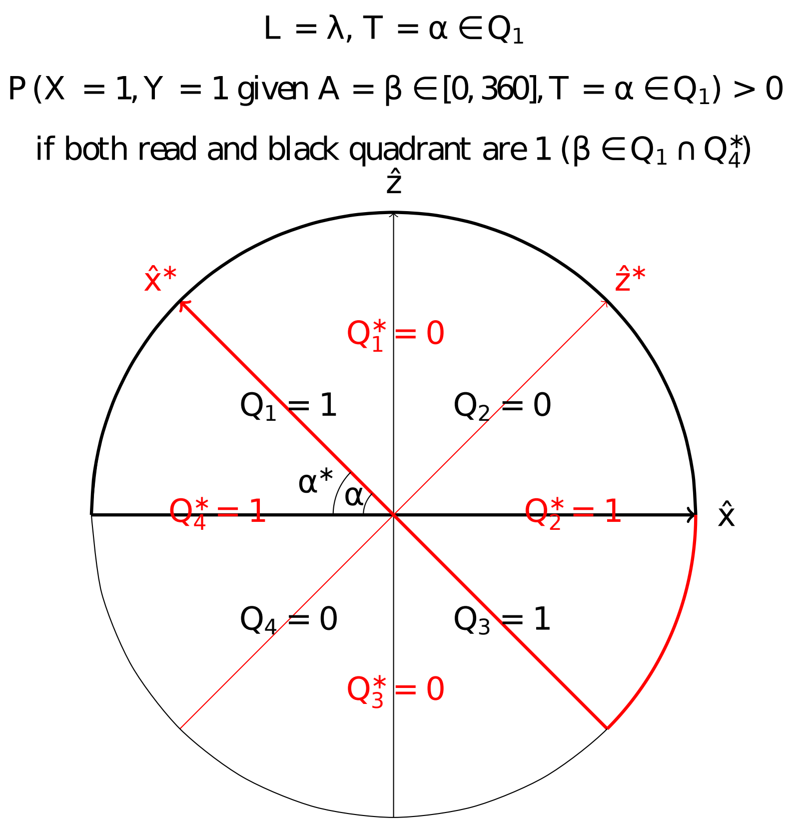
As the model is symmetric of the transform $A \to A + 180$, we will only concentrate ourselves on the region $A \in [0, 180]$. Also as the transform $\lambda \to \lambda^*$ doesn't change the model, we see that if we starting with particle 1 and analyze the result will lead to the same as if we started with particle 2. An obvious symmetry that must holds. Also we see that the system is symmetric to changes in the rotation in the xy plane of the setup.

The model defines an underlying hidden state A, L selected via a deterministic entanglement and the result how the outcome of the measurement depends on this selected hidden variable. As A is the physical spin of the particle, L is the only innovation here.

So in order to analyze the requested probabilities we will first analyze the probability for a tilt T in quadrants Q_1, Q_2 (the other quadrants will follow through the flip symmetry).

We will divide the calculation of the probability through 4 cases.

1 Case 1, $A \in Q_2 \quad L = \lambda$



Assume we tilt the second device and angle of $T = \alpha \in Q_1$. Then we get the diagram above and we note that the only place we have 1 in both particles is if $A = \beta \in Q_1 \cap Q_4^*$. And this is with probability

$$P(X = 1, Y = 1, A = \beta \in Q_1, L = \lambda) = C \sin(\beta)\cos(\beta).$$

This means that,

$$P(X=1,Y=1,A\in Q_1,L=\lambda)=C\int_0^{\alpha}\sin(\beta))\cos(\beta)\,d\beta=C\sin^2(\alpha)/2.$$

To see this note that,

$$\int_0^{\alpha}\sin(x)\cos(x)\,dx=\frac{1}{2}\int_0^{\alpha}\sin(2x)\,dx=\frac{1}{4}(1-\cos(2x)).$$

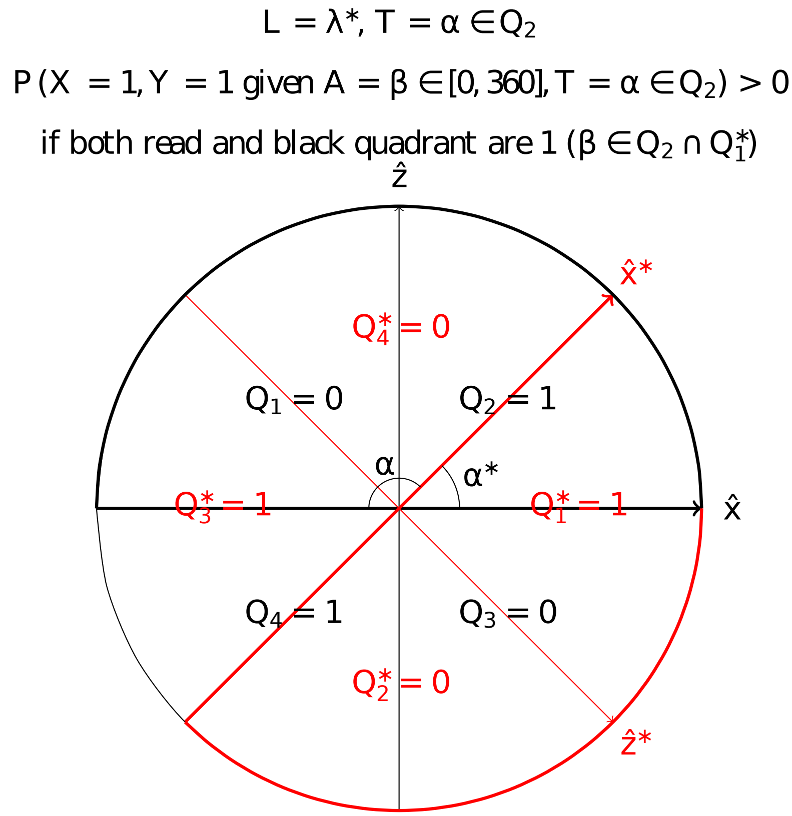
This is equal to,

$$\frac{1}{4}(1+(\sin^2(x)-\cos^2(x))\,dx=\frac{1}{4}(1+(2\sin^2(x)-1)\,dx=\frac{1}{2}\sin^2(x).$$

Similarly we see that,

$$P(X=1,Y=-1|A\in Q_1,L=\lambda)=C\cos^2(\alpha)/2.$$

2 case 2, $A \in Q_2, L = \lambda$



For $\alpha \in Q_2$ we get the situation in the above figure. We see that the intersecting region is again $Q_1 \cap Q_4^*$ and we also have

$$\bar{\alpha} = \alpha^* = 180 - \alpha.$$

Then the probability is,

$$P(X = 1, Y = 1, A = \beta \in Q_2, L = \lambda) = C \sin(180 - \beta) \cos(180 - \beta).$$

So the integral becomes,

$$P(X = 1, Y = 1, A \in Q_2, L = \lambda) = C \int_0^{180-\alpha} \sin(\beta) \cos(\beta)\, d\beta = C \sin^2(180-\alpha)/2.$$

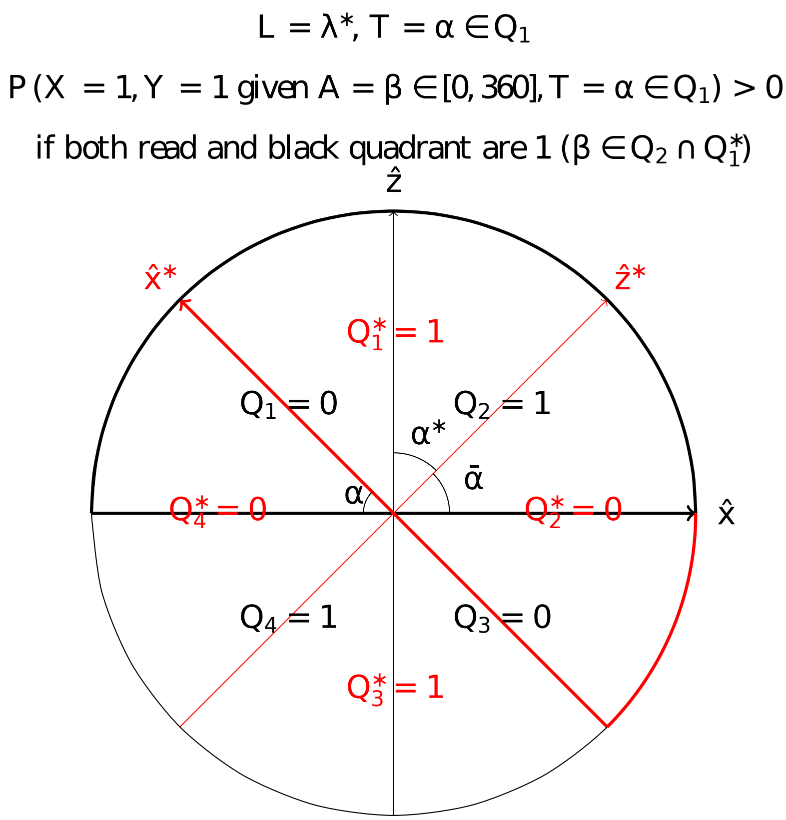
But this is the same as,

$$C \sin^2(\alpha)/2.$$

Similarly we see that,

$$P(X = 1, Y = -1 | A \in Q_2, L = \lambda) = C \cos^2(\alpha)/2.$$

3 Case 3, $A \in Q_1, L = \lambda^*$



For $\alpha \in Q_1$ we get the above situation and we see that the interesting region is $A = \beta \in Q_2 \cap Q_1^*$ and $\alpha^* = \alpha$ and hence $\bar{\alpha} = 90 - \alpha$. We read the probability as,

$$P(X = 1, Y = 1, A = \beta \in Q_1, L = \lambda^*) = C \sin(90 - \beta)) \cos(90 - \beta).$$

But,

$$\sin(90-\beta)\cos(90-\beta) = \sin(\beta)\cos(\beta).$$

And hence integrating the probability of $A \in Q_1$ is,

$$P(X=1, Y=1, A \in Q_1, L=\lambda^*) = C\int_0^\alpha \sin(\beta)\cos(\beta)\,d\beta$$

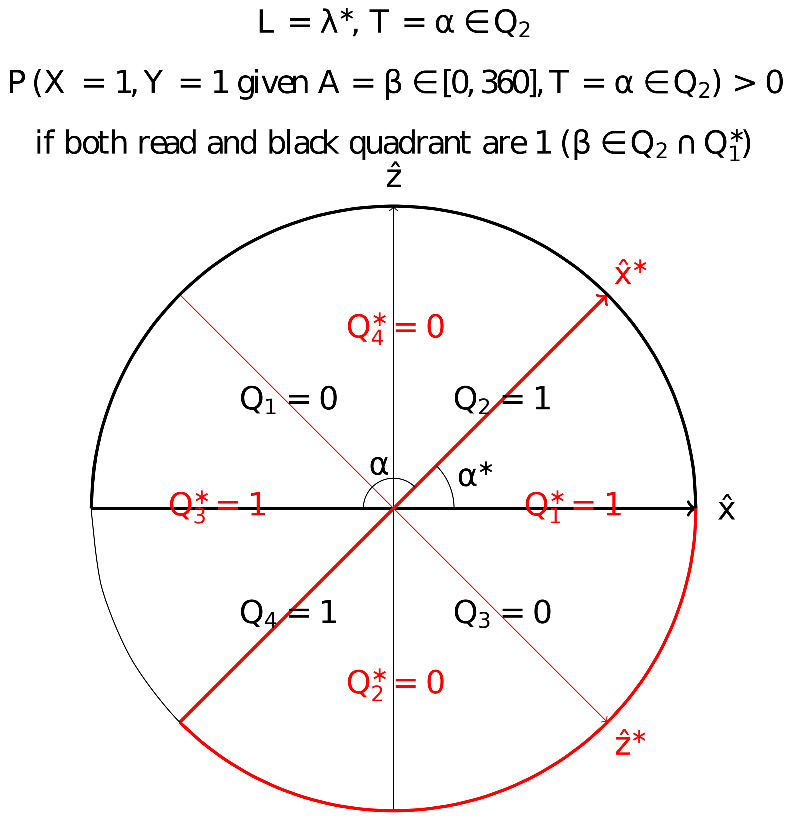
which we calculated before as,

$$P(X=1, Y=1, A \in Q_1, L=\lambda^*) = C\sin^2(\beta)/2.$$

Similarly we see that,

$$P(X=1, Y=-1|A \in Q_1, L=\lambda^*) = C\cos^2(\alpha)/2.$$

4 Case 4, $A \in Q_2, L = \lambda^*$



The final situation is for the case $L = \lambda^*, \alpha \in Q_2$ according to the figure above. Here, for $A = \beta \in Q_2 \cap Q_1^*$, $\alpha^* = 180 - \alpha$ and we get the probability,

$$P(X = 1, Y = 1, A = \beta \in Q_2, L = \lambda^*) = C \sin(180 - \beta) \cos(180 - \beta).$$

And hence integrating the probability of $A \in Q_2$ is,

$$P(X = 1, Y = 1, A \in Q_2, L = \lambda^*) = C \int_0^{180-\alpha} \sin(\beta)) \cos(\beta)\, d\beta.$$

Which integrated means,

$$P(X = 1, Y = 1, A \in Q_2, L = \lambda^*) = C \sin^2(180 - \alpha)/2 = C \sin^2(\alpha)/2.$$

Similarly we see that

$$P(X=1,Y=-1|A\in Q_2,L=\lambda^*)=C\cos^2(\alpha)/2.$$

5 Wrap up

We shall show that indeed we reproduce the quantum mechanical results. Note that combining both λ,λ^* we get,

$$P(X=1,Y=1|T=\alpha)=P(X=1,Y=1,L=\lambda|T=\alpha)+P(X=1,Y=1,L=\lambda^*|T=\alpha),$$

is the same as,

$$P(X=1,Y=1|T=\alpha,A\in Q1\cup Q_2)=C\sin^2(\alpha).$$

Hence, symmetry gives,

$$P(X=1,Y=1|T=\alpha)=2C\sin^2(\alpha)=\sin^2(\alpha)/2,$$

$$P(X=1,Y=-1,T=\alpha)=\cos^2(\alpha)/2.$$

We see that this is the same as in the Aspects Experiment using QM Therefore, the correlation is,

$$\mathrm{Cor}(\alpha)=E(XY|T=\alpha)=-\cos^2(\alpha)+\sin^2(\alpha)=-\cos(2\alpha).$$

We know that for $\alpha=45/2$, we get essentially the same situation as in the classical QM theory for this experiment.

As mentioned, uper determinism or a violation of the statistical independence condition meaning,

$$P(A,L|XY^*)\neq P(A,L).$$

For $\beta,\alpha\in Q_1$ we know that we can only select $\lambda=1$ if we know that $X==Y$. This means,

$$P(A=\beta\in Q_1,L=\lambda^*|XY=1)=0$$

But

$$P(A \in Q_1, L = \lambda^*) = P(A \in Q_1, L = \lambda^* | XY = -1) > 0.$$

Hence the statistical independence is broken and the model we describe, as it produces the same results as we get in the Aspect experiment, we know it must be super deterministic as the model defined here is deterministic and this proves that indeed it is.