ZFC Inconsistency Explained and Defended

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Abstract: There is a class of sets that can be constructed within ZFC that both have and do not have a largest element. Two contradictory arguments about these properties are developed and defended.

Introduction: This paper is a sequel to the three papers listed below[1]. For all rational numbers $\boldsymbol{a}$ in the closed interval $[0, 1]$ let the collection of all R_a sets be $\{ y$ is a rational number $| \ 0 \leq y \leq a \}$
The entire collection of R_a sets form a hierarchy of sets.
Each set contains all the elements in sets below it in the set hierarchy.
Each set contains a single element that is not in any set below it in the set hierarchy.
We take the largest element out of each set in the entire collection.
The set containing zero becomes the null set.
The previous largest element of every R_a set is now missing from each set. However, all R_a sets remain in the same position in the hierarchy of sets in descending order as $\{ y$ is a rational number $| \ 0 \leq y < a \}$

Argument #1: Each R_a contains a largest element.
1) Each R_a contains the former largest elements of the subsets below it in the set hierarchy.
2) Each R_a contains element(s) not in any subset below it in the set hierarchy. (See **Objection addressed** below Argument #2.)
3) Let $\boldsymbol{c}$ and $\boldsymbol{d}$ be two elements of a single R_a set with $\boldsymbol{c > d}$.
4) $\boldsymbol{d}$ is an element of R_c, which is a proper subset of R_a.
5) For any two elements in R_a the smaller element is contained in a proper subset of R_a.
6) By steps 2) and 5) each R_a set contains a single largest element not in any set below it in the hierarchy.

Argument #2: No R_a contains a largest element.
1) Suppose there is a largest element a' in some individual R_a.
2) $a' < (a' + a)/2 < a$.
3) Let $b = (a' + a)/2$.
4) Then b is in R_a and $a' < b$.
5) a' is in R_b a proper subset of R_a.

Objection addressed: When a largest element is assumed in Argument #2, it leads to a contradiction; so there is no largest element. Every R_a element is in one of the proper subsets below R_a in the set hierarchy. It is a valid proof by contradiction.

However, as explained below Step 2 in Argument #1 is true and each step within Argument #1 is either a true statement or a valid logical conclusion from true statements. No attempt is made to specify a largest element in Argument #1. It just proves that a largest element must exist.

The R_a sets are in descending hierarchical order. In every position below each individual R_a for every rational $\boldsymbol{x < a}$ is an R_x set$\{ y$ is a rational number $| \ 0 \leq y < x \}$.

Since each R_a and its collection of R_x subsets are part of the entire descending set hierarchy, **there exists at least one R_a set element $\boldsymbol{s} \geq$** (all values of) $\boldsymbol{x}$. Otherwise, there could be no descending set hierarchy.

As shown in Argument #1 steps 3), 4) and 5) for any two elements in R_a, the smaller element is contained in a proper subset of R_a. **There is at most one R_a set element** missing from all the R_x subsets.

Thus, **there must be a single largest R_a set element** missing from all the R_x subsets.

[1]https://vixra.org/pdf/2302.0145v1.pdf https://vixra.org/abs/2303.0105 https://vixra.org/pdf/2304.0008v1.pdf

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