Dedicated to Heinrich Rudolph Hertz

on the occasion of the 150th anniversary of his birth

DERIVATION OF QUANTUM CONDITIONS OUT OF PROVISIONS OF THE SPECIAL THEORY OF RELATIVITY

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Equations, consistent with the kinematic relativistic relations and electro dynamics' equations, are obtained in the tetrameric Euclidean space. Under the transition to a unitary tetrameric space, Born-Jordan permutable relations are obtained.

The relativistic quantum theory, formed in the last century, was methodically developed based on provisions of the quantum (wave or matrix) mechanics. The covariance in relation to Lorenz transformations [1] was traditionally the criterion of applicability of equations or operators. Thus, the acceptability of the Maxwell-Dirac equations in Majorana form [2], Feynman-Dyson S-matrix [3]and many others was defined. Numerous experiments confirmed the majority of positions of both quantum electrodynamics (QED) and quantum field theory [4,5]. Nevertheless, the undisputable validity of the basics of the relativistic quantum theory does not always lead to the equally successful practical application of physical-chemical researches [6]. The reason, as it seems, lies not in the lack of computational capabilities, but in some overload of fundamental equations (for instance, relativistic Hamiltonian [7]), which eliminates analytical solutions without using approximations [8]. Owing to works of Pauli, Dirac, Schwinger and their followers, no one doubts the intimate connection between quantum phenomena and relativistic effects. Therefore, it seems appropriate to formulate the solved problem in a different way. Do the provisions of the special theory of relativity (STR) contain the data on the possible discreteness of the quantity observed? The clarification of this issue could have led to a simplification of the apparatus of the relativistic quantum theory. In its turn, this would allow to get rid of a number of approximations (for instance, quantum chemistry abounds with them [9]), which do not allow to take the full advantage of achievements of the quantum mechanical description.

In general, it is not necessary to introduce imaginary axes to achieve the Lorentz invariance. To prove this statement, let us consider the orthonormal basis in the tetrameric Euclidean space. The position of a particle is defined by four linearly independent coordinates:

$$\mathbf{s} = \mathbf{w} + \mathbf{r}(x, y, z) = \mathbf{w} + \mathbf{x} + \mathbf{y} + \mathbf{z}, \tag{1}$$

three of which are space coordinated (**x**, **y**, **z**), and **w** is hidden, possessing properties of the local time of a particle. The Michelson–Morley experiments, which are reflected in the postulate on the lightspeed consistency, can be laid down as the consistency principle of the change of position (module of the full differential of the coordinate).

$$d\mathbf{s}^2 = d\mathbf{w}^2 + d\mathbf{x}^2 + d\mathbf{y}^2 + d\mathbf{z}^2 = const. \tag{2}$$

The specified scalar (square of the change of the position) is accepted as an invariant for any particle considered in the indicated reference frame. The uniform movement of a particle can be defined as the constancy of the position change (ds vector). The choice of the reference frame associated with a uniformly moving material point is an orthogonal transformation of the *WXYZ* coordinate axes into such *W'X'Y'Z'* , that the *W'* axis is collinear with the $d\mathbf{s}$ vector. Introducing momenta, conjugated by coordinates as quantities proportional to their changes (full differentials), in virtue of the postulate on the equivalency of the uniformly moving reference frames, we can express

$$\mathbf{P}^2 = \mathbf{P}_0^2 + \mathbf{P}_x^2 + \mathbf{P}_y^2 + \mathbf{P}_z^2, \tag{3}$$

where **P** is a full (relativistic) particle momentum, $\mathbf{P}_0$ is a latent momentum orthogonal to all of three components of the $\mathbf{P}_x$, $\mathbf{P}_y$ and $\mathbf{P}_z$ kinetic momentum. Obviously, the formula (3) corresponds to the basic relativistic equation

$$\frac{E^2}{c^2} = (mc)^2 + \mathbf{P}_x^2 + \mathbf{P}_y^2 + \mathbf{P}_z^2, \tag{4}$$

where m is a stable particle mass, shall its total energy E be set equal to Pc, and $P_0 = mc$. From the postulate on the consistency on the stable mass, it follows that

$$\mathbf{P}_0^2 = const. \tag{5}$$

In this case, **w** coordinate, conjugated to the $\mathbf{P}_0$ momentum, is cyclic one, which is true in the absence of the interactions. Equations (1)-(3) and (5) are completely consistent with the kinematic equations, arising from the special theory of relativity. It should be noted, that the metric tensor of the space, where the indicated equations are observed, corresponds to the Kronecker symbol

$$g_{ij} = \delta_{ij}. \tag{6}$$

To define the first *f* integrals in this basis it is sufficient to comply with the equality:

$$\frac{\partial f}{\partial w}\mathbf{P}_0 + \frac{\partial f}{\partial x}\mathbf{P}_x + \frac{\partial f}{\partial y}\mathbf{P}_y + \frac{\partial f}{\partial z}\mathbf{P}_z = 0 . \tag{7}$$

One of the first integrals is straightforward. For this purpose, we can use the Lanczos statement [10], made on the basis of the variational principles as applied to the relativistic mechanics. “The presence of a scalar potential energy is equivalent to the particle mass increase by the value of the potential energy per the square of the light-speed”. Thus, shall *U* be the potential energy of the particle, the formula (3) is transformed into

$$\mathbf{P}_x^2 + \mathbf{P}_y^2 + \mathbf{P}_z^2 + 2mU + \frac{U^2}{c^2} = f , \tag{8}$$

it, in the classic limit (c is sufficiently large), corresponds the law of conservation of the energy. The equation (3) gives one more important consequence. The total differential of the $d\mathbf{P}$ relativistic momentum, assigned to the *ds* module of the change of the position (which is an invariant), is related to the **P** value itself by the **J** linear antisymmetric operator

$$\frac{d\mathbf{P}}{ds} = \mathbf{JP} , \tag{9}$$

where the matrix of **J** operator is represented as follows:

$$\mathbf{J} = \begin{pmatrix} 0 & -j_{10} & -j_{20} & -j_{30} \\ j_{10} & 0 & -j_{21} & -j_{31} \\ j_{20} & j_{21} & 0 & -j_{32} \\ j_{30} & j_{31} & j_{32} & 0 \end{pmatrix} . \tag{10}$$

As applied to the electrodynamics problem, the three spatial components of the indicated value in (9) are proportional to the force acting on the *e* charge in the electromagnetic field

$$\frac{\partial \mathbf{P}_x}{\partial s} + \frac{\partial \mathbf{P}_y}{\partial s} + \frac{\partial \mathbf{P}_z}{\partial s} = \frac{e}{c}\mathbf{E} + \frac{e}{c}\left[\frac{d\mathbf{r}(x, y, z)}{dw}, \mathbf{H}\right], \tag{11}$$

where **E** and **H** are experimentally observed values of the intensity of electric and magnetic fields, *dw* is a latent differential module. Considering the proportionality of momenta to changes in the conjugate coordinate, formulas for the matrix elements of **J** operator are as follows:

$$j_{10} = \frac{eE_x}{cP_0} \qquad j_{20} = \frac{eE_y}{cP_0} \qquad j_{30} = \frac{eE_z}{cP_0}$$
$$j_{21} = -\frac{eH_z}{cP_0} \qquad j_{31} = \frac{eH_y}{cP_0} \qquad j_{32} = -\frac{eH_x}{cP_0}, \tag{12}$$

where P_0 is the module of the $\mathbf{P}_0$ momentum. The existence condition of the operator, opposite to $\mathbf{J}$ one, consists in the inequality to zero of the determinant of a matrix of $\mathbf{J}$ operator

$$\det \mathbf{J} = \frac{e^2 (\mathbf{E}, \mathbf{H})^2}{c^2 P_0^2} \neq 0 . \tag{13}$$

Subject to this condition, the following equation holds

$$\mathbf{P} = \frac{cP_0}{e(\mathbf{E}, \mathbf{H})} \mathbf{V} \frac{d\mathbf{P}}{ds}, \tag{14}$$

where $\mathbf{V}$ is a linear anti-symmetric operator, with its elements of a matrix be expressed as follows

$$v_{10} = H_x \qquad v_{20} = H_y \qquad v_{30} = H_z$$
$$v_{21} = -E_z \qquad v_{31} = E_y \qquad v_{32} = -E_x . \tag{15}$$

According to the assumption on the presence of $\mathbf{A}$ (A_0, A_{x}, A_{y}, A_{z}) tetra-vector of the electromagnetic field [11], the expression (14) can be presented as follows:

$$\mathbf{P}\frac{e(\mathbf{E}, \mathbf{H})}{cP_0} = \left[\frac{d\mathbf{P}}{ds}, \nabla, \mathbf{A} \right] = \begin{vmatrix} \mathbf{h} & \mathbf{i} & \mathbf{j} & \mathbf{k} \\ dP_0/ds & dP_x/ds & dP_y/ds & dP_z/ds \\ \partial/\partial w & \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_0 & A_x & A_y & A_z \end{vmatrix} . \tag{16}$$

The “nabla” sign indicates the tetrameric differential operator

$$\nabla = \frac{\partial}{\partial w}\mathbf{h} + \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}, \tag{17}$$

where **h**, **i**, **j** and **k** are single unit vectors of **w**, **x**, **y** and **z** axes respectively. Components of $\mathbf{A}$ vector are related to $\mathbf{E}$ and $\mathbf{H}$ strengths contingent on the (14)-(16) relations:

$$\mathbf{H}(H_x, H_y, H_z) = rot\left(\mathbf{A}_r (A_x, A_y, A_z)\right)$$
$$\mathbf{E}(E_x, E_y, E_z) = gradA_0 - \frac{\partial}{\partial w}\mathbf{A}_r (A_x, A_y, A_z) . \tag{18}$$

The difference from the generally accepted formula [11] lies in the + sign in front of the gradient of the scalar potential of $\mathbf{A}$ tetra-vector. However, considering the gauge invariance of equations of the electromagnetic field, this difference is not a contradiction. The fundamentally important consequence out of the equation (16) is the fact of the orthogonality of the total momentum and differential operator (17)

$$\nabla \mathbf{P} = div\mathbf{P}_r(P_x, P_y, P_z) + \frac{\partial P_0}{\partial w} = 0 . \tag{19}$$

This equality is nothing but the condition of flow continuity [11]. Thus, the proposed dynamic model of the STR in the tetrameric Euclidean space in a number of fundamental moments is consistent with the presentation in a form of Minkowski space-time. To find quantum conditions in the basis (1) it is advisable to consider **N** linear operator such that

$$\mathbf{s} = \mathbf{NP} . \tag{20}$$

It is easy to show that the matrix of the indicated operator is as follows

$$\mathbf{N} = \frac{1}{\mathbf{P}^2}\begin{pmatrix} Q & -L_{10} & -L_{20} & -L_{30} \\ L_{10} & Q & -L_{21} & -L_{31} \\ L_{20} & L_{21} & Q & -L_{32} \\ L_{30} & L_{31} & L_{32} & Q \end{pmatrix}, \tag{21}$$

where Q is a scalar product (**s**,**P**), and L_{ij} are determinants composed of cyclic changes of the components of coordinates and momenta, which according to the condition (7), are the first integrals given that momenta are clearly independent of coordinates.

$$\begin{matrix} L_{10} = xP_0 - wP_x & L_{20} = yP_0 - wP_y & L_{30} = zP_0 - wP_z \\ L_{21} = yP_x - xP_y & L_{31} = zP_x - xP_z & L_{32} = zP_y - yP_z \end{matrix} . \tag{22}$$

Under real-valued and at least one non-zero L_{ij} (hereinafter referred to as the general case) it is possible to define roots of the characteristic polynominal of **N** operator – complex ones. It is a necessary and sufficient condition for the self-adjoint **N** operator. By itself, this fact is of no importance in the Euclidian spaces, but it enables us to justify the use of a unitary space instead of the Euclidean one in order to find quantum conditions. It should be noted that below this replacement is made for convenience only. The study of **N** operator in the Euclidean space in the same way leads to the principle, named after Werner Heisenberg. However, the necessity to consider the bivectors instead of vectors and 2x2 matrix instead of eigenvalues deprives this way of some clarity.

The following are a number of suppositions and assumptions. For a **N** nonself-adjoint (non-Hermitian) operator in a unitary space as a whole

$$(\mathbf{NP}, \mathbf{P}) \neq (\mathbf{P}, \mathbf{NP}) , \tag{23}$$

in Euclidean spaces the equality always holds regardless the type of the operator. In addition, an imaginary component is a necessary and sufficient condition for the operator to be nonself-adjoint in a unitary space.

$$\operatorname{Im}(\mathbf{NP},\mathbf{P}) \neq 0 . \tag{24}$$

It can be shown that in a unitary space for a non-Hermitian operator, in the general case, the following equation holds

$$\operatorname{Re}\left((\mathbf{NP},\mathbf{P})-(\mathbf{P},\mathbf{NP})\right)=0 , \tag{25}$$

which coincides in form with Born-Jordan permutation relations

$$(\mathbf{s},\mathbf{P})-(\mathbf{P},\mathbf{s})=i\cdot const . \tag{26}$$

The complex axis in the tetrameric unitary space, according to Minkowski, can be determined by **w** axis. The selection of a hidden axis as a complex one is entirely justified, because there are no obvious reasons to use imaginary values for spatial coordinates. According to the scalar product rule, in a unitary space the commutator (26) is transformed to a form (a bar over a symbol denotes a complex conjunction)

$$(\mathbf{s},\mathbf{P})-(\mathbf{P},\mathbf{s})=2i\left(w_I P_{0R}-w_R P_{oi}\right) , \tag{27}$$

where w_R and w_I are real-valued and imaginary components of **w** coordinate, P_{0R} and P_{0I} are real-valued and imaginary components of $\mathbf{P}_0$ momentum. It should be noted that in the Minkowski space, where there is no real-valued component in the time coordinate, permutation relations will be identical to zero. The statement that the commutator (27) is a motion integral has the following argumentation. The orientation of the basis of the real-valued and imaginary axes in relation to the coordinate and momentum can be arbitrary. Thus, the real-valued and imaginary components of **w** and $\mathbf{P}_0$ vectors must be invariant in relation to the groups of coordinate rotations, which entails the constancy of the determinant in the right side (27). The established fact indicates the absence of the linear independence between real-valued and imaginary axes of **w** latent coordinate. In addition, for the commutator (27) the Cauchy-Riemann conditions in the theory of complex functions are not satisfied. Therefore, a function of the form (27) is no analytical. This means that the differentiation with respect to the latent coordinate is not defined.

To comply with the equation (20), in a unitary space the form of **N** operator will be changed due to the fact that the square of the relativistic momentum as an invariant in the equation (3) will be a product of the conjugate values

$$\mathbf{N} = \frac{1}{\left(\mathbf{P}, \bar{\mathbf{P}}\right)} \begin{pmatrix} Q & -L_{10} & -L_{20} & -L_{30} \\ \bar{L}_{10} & \bar{Q} & -L_{21} & -L_{31} \\ \bar{L}_{20} & \bar{L}_{21} & \bar{Q} & -L_{32} \\ \bar{L}_{30} & \bar{L}_{31} & \bar{L}_{32} & \bar{Q} \end{pmatrix}. \tag{28}$$

Based on the unique adjoint operator theorem, it is possible to make the following suggestion. Particles, which have operator of the momentum transformation into a coordinate out of (28) non-Hermitian ones, are fermions and those, having operators out of (28) Hermitian ones, are boson. It should be noted that the Hermitian character of **N** operator leads to the determinant of the right side (27) equality to zero. This can be achieved in two ways. By the absolute value of zero $\mathbf{P}_0$ (the photon is out of any interactions) and the linear dependence of **w** and $\mathbf{P}_0$ (neutral particles).

The proposed scheme to define quantum conditions on the special theory of relativity is a development of a Hertz concept on the kinematic origin of the potential energy [12]. In its present form, this concept takes a broadside approach. There are no quanta outside the fields. Hidden coordinates and momenta are either purely imaginary ones (Minkowski space) or strictly real-valued ones (Euclidean space). The operator of the momentum transformation to a coordinate according to (20) is always self-adjoint. The observed values can take on a continuous range of values. In the presence of other bodies, the potential energy takes a shape of the kinematic energy in the latent coordinate. The real-valued one in an explicit form reversibly replaces the material one in a latent form. Thus, both imaginary momentum and coordinate (already not always cyclical) are converted into complex ones. **N** operator can be non-Hermitian, permutation relations take place.

Perhaps, the given wording of quantum conditions will find its place in the interpretation of the EPR paradox. Free particles outside fields (outside measurements) do not obey quantum laws. The classical integrals of motion are preserved for them. The measurement process itself is the interaction. Consequently, the particle enters the field effect. Additional quantum conditions arise in the field of the measuring device, which does not cancel the compliance with conservation laws.

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