The Dirac large number hypothesis and a system of evolving fundamental constants.

Andrew Holster. Oct. 2021.

ABSTRACT

In his [1937, 1938], Paul Dirac proposed his *"Large Number Hypothesis"* (LNH), as a speculative law, based upon what we will call the *"Large Number Coincidences"* (LNC's), which are essentially "coincidences" in the ratios of about six large dimensionless numbers in physics. Dirac's LNH postulates that these numerical coincidences reflect a deeper set of law-like relations, pointing to a revolutionary theory of cosmology. This led to substantial work, including the development of Dirac's later [1969/74] cosmology, and other alternative cosmologies, such as Brans-Dicke modifications of GTR, and to extensive empirical tests. We may refer to the generic hypothesis of *"Large Number Relations"* (LNR's), as the proposal that there are lawlike relations of some kind between the dimensionless numbers, not necessarily those proposed in Dirac's early LNH.

Such relations would have a profound effect on our concepts of physics, but they remain shrouded in mystery. Although Dirac's specific proposals for LNR theories have been largely rejected, the subject retains interest, especially among cosmologists seeking to test possible variations in fundamental constants, and to explain dark energy or the cosmological constant. In the first two sections here we briefly summarise the basic concepts of LNR's. We then introduce an alternative LNR theory, using a systematic formalism to express variable transformations between conventional measurement variables and the true variables of the theory. We demonstrate some consistency results and review the evidence for changes in the gravitational constant *G.*

The theory adopted in the strongest tests of $\dot{G}/G$, by the Lunar Laser Ranging (LLR) experiments, assumes: $\frac{\dot{G}}{G} = \frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m}$ as a fundamental relationship. Experimental measurements show the RHS to be close to zero, so it is inferred that significant changes in *G* are ruled out. However when the relation is derived in our alternative theory it gives: $\frac{\dot{G}}{G} = \frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m} - \frac{d\hat{R}}{dt}$.The extra final term is not taken into account in conventional derivations. This means the LLR experiments are consistent with our LNR theory (and others), and they do not really test for a *changing value of G* at all. This failure to transform predictions of LNR theories correctly is a serious conceptual flaw in current experiment and theory.

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Section 1. Dirac and Large Number Relations.

Introduction.

In his [1937, 1938], Paul Dirac proposed his *"Large Number Hypothesis"* (LNH), as a speculative law, based upon what we will call the *"Large Number Coincidences"* (LNC's), which are essentially "coincidences" in the ratios of about six large dimensionless numbers in physics. Dirac's LNH postulates that these numerical coincidences reflect a deeper set of law-like relations, pointing to a revolutionary theory of cosmology. This led to substantial work, including the development of Dirac's later [1969/74] cosmology and other alternative cosmologies, modifications of GTR, and empirical tests. See [Ray, 2007/2019], [Solà, 2015], [Barrow, 1986] for good reviews, that capture the main historical development, up to their point in time.[1] In fact several physicists had speculated about the concept of law-like relations before Dirac, notably [Weyl, 1917, 1919], [Eddington 1931], [Milne, 1935]. We may refer to the generic hypothesis of *"Large Number Relations"* (LNR's), as the proposal that there are lawlike relations of some kind between the dimensionless numbers, not necessarily those in Dirac's early LNH, but that was what drew the most attention to them.

"Large Number Coincidences" (LNC)	Several coincidences in values among dimensionless numbers that can be constructed in fundamental physics and cosmology. (Noticed by Heaviside? Planck?)
"Large Number Hypothesis" (LNH)	A speculative hypothesis by Dirac [1937] that dimensionless numbers in physics-cosmology reflect a scale pattern, separated by powers of about 10^{40N}, *with N = 0, 1, 2.*
"Large Number Relations" (LNR)	General hypotheses of law-like relations among the dimensionless numbers. Weyl 1917, Eddington 1931, Milne 1935, Dirac 1937, others.

A number of LNR theories have been proposed, but it is fair to say they are regarded as speculative, and none is regarded as successful in mainstream physics. There is no LNR-type theory taught in physics courses, and it is not part of conventional cosmology. Dirac's own theory/s was abandoned in mainstream cosmology because of conflicts with the experimental data, and the majority opinion among most physicists is probably dismissive of LNR's, if they are aware of the concept at all.

Nonetheless it involves the question of whether the fundamental constants of nature are changing, and what fixes them, and how they are related, and this is a very fundamental question that many

[1] [Ray, 2019] gives an extensive review and bibliography of the subject, with about 138 references, the earlier (unpublished) 2007 version has 121 references. This appears to be the broadest literature review in 2007, and probably up to 2019. There are not a lot of previous reviews. The articles introduced in the volume by [Sola, 2015] give detail on a range of special topics. [Barrow and Tipler 1986] gave the best short summary of the main research up to the 1980's.

leading physicists and cosmologists continue to work on. Substantial interest in the concept remains among specialists, and we support the case that *prospects for a LNR theory remain open and the concept should be investigated further*.

Here we propose a novel type of *LNR theory*, presented as a theory of *metric transformations of the units of all the dimensional quantities*, along with the *dynamic equations of motion.* This emphasises the need to represent the *transformations of dimensional unit quantities* in LNR theories correctly. This theory makes strong predictions, and has good consistency with the known empirical data. But regardless of its empirical success, it demonstrates that the predictions made by LNR theories are not necessarily being calculated correctly in present studies. The notable example is the LLR study of changing *G* [Turyshev, 2007]. The analysts assume GTR to be the correct background theory, and carefully make calculations and models for the GTR-based theory. But when this is claimed to show that LNR theories that predict changing *G* are wrong, *the interpretation of this for the LNR theory is not done*. We will see in our example that it requires solving the system equations in the new theory. We propose, generalising Dirac's [1969, 1974] example, that the theories should be systematically stated, with explicit metric transformations, boundary conditions, and equations of motion, as we set out in Section 3, so that we can check the equations and calculations.

This feature, that we must transform the equations too, is something Dirac realised in his second attempt to develop a cosmological theory based upon LNRs. Although Weyl, Eddington and Milne all proposed LNRs earlier, Dirac's [1937] theory subsequently led to the most interest, but this broke down empirically. His second cosmological theory [Dirac, 1969, 1974] was a vital step forward conceptually. It appears to be empirically disproved, but our argument is that another variation works.

The *primary tests* of these theories are from tests of temporal variations of fundamental constants that they predict. They generally predict *G* weakening with time, and this leads to changes in predictions of rates of various natural processes, like formation of galaxies, stars and planets, geological processes, etc, as well as changing *G* in direct tests of gravity. Predictions from Dirac's and other LNR theories were tested in several empirical-theoretical stages, through the 1940's – 1970's – 1990s, as observational data from cosmology and precision tests of gravity improved.

Popular opinion in physics for the last few decades is that LNR's have failed to be verified, and most physicists probably believe they have been empirically abandoned on fairly conclusive evidence, or at least have become an unlikely prospect that has little interest. This is primarily because no temporal variations of constants $(G, c, h, \varepsilon_0, m_e, q_e, m_p, q_p)$ have been found, within what appear to be increasingly strong limits.

However the story does not stop there. First there remain alternative LNR theories that are compatible with current measurements, and the question has not yet been empirically decided with any degree of conclusiveness. Second because the *a priori chance* of the LNC's occurring by accident is significantly unlikely, so that the coincidences demand serious attention. Below we estimate a chance: $p < 1/100$ as a conservative *a priori* chance of the LNC's occurring in a purely *accidental* way. In the real world. $1/100^{th}$ chances do not happen very often. A third reason is the monumental effect changing constants would have on the foundations of physics. The present theories are based on a tiny set of equations, with the constants: $(G, c, h, \varepsilon_0, m_e, q_e, m_p, q_p)$ treated as if they are carved in stone, assumed as absolutely constant fixed values. But if one of them varies with time at all – even by a tiny amount – or if it is even possible for them to vary - then some deeper mechanism is working in nature, to relate them. It is mysterious how they were set for our universe in the first place, and it is quite plausible within the bounds of normal physics that they are dynamic quantities.

This would force us to change the very expression of our laws of nature, because we would then have to take into account the variation of constants in our equations. E.g. when we differentiate quantities like: mG/r, or $q/4\pi\varepsilon_0$, or mc^2, or hf, we will now have to take the terms: dG/dt, dc/dt, dh/dt, dm/dt, dq/dt, $d\varepsilon_0/dt$, into account (using the chain rule). What happens if all of these become real terms in the equations of GTR, cosmology, or QM? It means the differential equations in fundamental physics would get tiny mysterious extra terms from the dynamics of the constants. The rates of change expected are very small, so we may only notice the effects over longer periods, but nonetheless there would have to be a deeper rationalisation of fundamental physics to accommodate this. It would force a change of fundamental models. Because of this primary importance to the project of fundamental physics, therefore, we think that:

- Physicists should check the possibility of LNR's carefully, as long as they remain physically plausible, since the existence of such relations would dramatically alter our future theories of cosmology, gravity and fundamental particles.

- If the *a priori* chance of the LNC's is only 1/100, then we should make about 0.999 certain that LNR theories have been independently *disproved* before we dismiss their possibility. This is to give us a 90% chance that LNC's are not real.

- The problem of testing this to 0.999 probability lies in making the *systematic error* of underestimating, overlooking or ignoring the range of possible models that could account for LNRs. I.e. how thoroughly have *alternative theories* have been checked?

We give a quick estimate of these epistemic likelihoods below. *A priori chance* here means *before tests of various theoretical explanations are known.* Now since the 1940's, many experimental studies have been done, and these have caused a number of LNR theories to be rejected; in fact all the original ones. So with this additional evidence, an LNR theory becomes less likely, and attributing the LNC's to chance becomes more likely. That is how Bayesian probability works. As evidence accumulates, you change your probability estimates.

But the question is: have we really tried or tested all the plausible theories?

There are still many proponents of LNR theories, and we quote from the extensive review by [Ray *at al*, 2007/19] who highlight reasons for continuing interest.

> "Although so many years have passed after the inception of LNH, it has not lost its significance. Instead, perhaps it has gained a new momentum after the discovery of accelerating universe. The present cosmological picture emerging out of SN Ia observations [40,41] reveals that the present universe is accelerating. Some kind of exotic repulsive force in the form of vacuum energy is supposed to be responsible for this acceleration which started about 7 Gyr ago. This repelling force is termed as dark energy and is designated by Λ.

On the way of investigating dark energy, many variants of Λ have been proposed including a constant Λ. But, compatibility with other areas of physics demands that Λ should be slowly time decreasing. Moreover, the currently observed 42,43 small (≈ 10−35s −2) value of Λ suggests that it has decreased slowly from a very high value to its present nearly zero value. This type of time dependency of Λ has a similarity, so far as the main spirit of the idea is concerned, with that of the gravitational constant G as proposed by Dirac in his LNH 44. Both Λ and G have descended from a very high initial value to its present small value because the universe is so old." [Ray, 2019]

[Ray, 2019] reviews the development from Dirac in substantial detail, including how adaptations of GTR developed, and gives an extensive set of references for this, so we will not repeat this history of adaptations of Dirac's theory here.

Rather, we want to introduce an alternative type of LNR theory to Dirac's, with constants varying as functions of the *radius of an expanding universe,* rather with time or age from the Big Bang. Dirac's theory is based on constants varying with the time or *age of the universe*. But it turns out much successful in our view to set up a model based on relationships through the spatial expansion parameter *R,* which is a function of time: $R = R(t)$.

The apparent LNR relationships with time are then really due to the approximately constant expansion speed since the Big Bang, along with the coincidence that the Hubble constant matches: $1/H \approx T$. The true relationships in the alternative model are between the spatial variables, with the age of the universe *T* or *t* merely a parameter.

To state our theory, we need to set up a formalism for *differential transformations* between the basis units for *physical quantities (space, time, mass, charge)* in two coordinate systems. The need for this is the new fundamental concept Dirac essentially introduced in his [1969, 1974], but he only gave one example, and did not generalise it, or consider other possibilities. Here we provide an alternative system of transformations, and introduce a simpler and more systematic formalism.

We emphasise that we need such a formalism to express any LNR theory adequately. After specifying a set of formal relationships, we will briefly explain how they arise from a natural *geometric model.* This model helps ensure the mathematical consistency of the system, and suggests a physical interpretation.

However such a model is not needed to make predictions from the theory. We show this LNR theory makes surprisingly accurate predictions of the LNC's. The most peculiar result, however, is that it is consistent with the results of the Lunar Laser Ranging and other tests of changing *G.* The LLR tests of changing *G* are now thought to rule out changes in *G* to a tiny amount, and to rule out LNR theories that require significant changes in *G.* But in our theory, the opposite is the case: the LLR experiments are not a genuine test of changing *G* at all! The cosmological-based tests, on the other hand, are genuine tests of changing *G*, but their limits of accuracy are much weaker, and conform to our model predictions. It is argued this theory also explains the anomalous recessional speed of the moon.

This result gets over the main empirical objections to LNR theories. The main insight of the theory in this respect is that we must transform the equations and observations in a systematic way to get the right predictions out for a LNR theory.

Our novel theory is represented by the equations in Section 3. This choice of evolution and transformation equations may seem a little mysterious at first sight, but it is really based on a dimensional analysis. The constraints on LNR theories for dimensional consistency is the fundamental concept of the subject. Getting a clear picture of this is important as a starting point, so in the first two sections, we review the basic concepts, and also give an estimate of the chances of the pattern of LNC's happening by accident.

The physical constants and dimensionless numbers.

Dirac's LNH might be taken as a (rare) example of a *simple inductive hypothesis* made in modern physics, because it draws a theory from a pattern of coincidences perceived among a small number of empirical observations. In fact, the main theory refers to only about 13 independent empirical observations in total. These are:

- The 4 independent fundamental constants c, h, G, ε_0, which are all the constants required to state the usual laws of electrodynamics and gravity.
- The fundamental masses: m_e, m_p and charges: q_e, q_p of the electron and proton.
- Five global parameters: R_u, T_u, M_u, Q_u, H, describing the universe as a whole.[2]

H is the Hubble parameter, and is the normalised *speed of expansion of space*: $H = (dR/dt)/R$.

These are the only independent quantities available in the first instance at least to construct the Dirac-type "coincidences" or LNR's. The possibilities for *relations* are therefore strictly constrained by the number of *dimensionless numbers* we can construct from these *13 quantities* with 4 *physical dimensions*, which is: 13-4 = 9, as we will see.

When we first encounter LNR theories, the number of dimensionless numbers or "ratios" and possible "coincidences" among constants and variables may seem open-ended, but in fact it is strictly limited. We now review the quantities, and see that there are four main coincidences, which involve approximately matching values of *pairs of dimensionless quantities*. Four LNC's may not sound like many, but actually it means the system of fundamental constants and cosmological quantities is *saturated with these coincidences,* as Dirac perceived.

2 Here we leave out the *acceleration/dark energy/cosmological constant parameter,* Λ, as referred to by [Ray, 2019] above. It was unknown to Dirac. It becomes relevant in the model we subsequently present.

__Table 1.__ The 13 main constants and variables involved in classic LNRs.

Id	Variable	Value	Units	Dimensions	Name
1	c	3.00E+08	m/s	X/T	Speed of light
2	h	6.63E-34	m^2kg/s	X^2M/T	Planck's constant
3	G	6.67E-11	m^3/s^2kg	X^3/T^2M	Newton's constant
4	ε_o	8.85E-12	$s^2C^2m^{-3}kg^{-1}$	T^2Q^2/X^3M	Electric permittivity
5	m_e	9.11E-31	kg	M	Electron mass
6	m_p	1.67E-27	kg	M	Proton mass
7	q_e	-1.60E-19	C	Q	Electron charge
8	q_p	**1.60E-19**	C	Q	Proton charge
9	R_u	**Unknown**	m	X	Radius of universe?
10	T_u	4.35E+17	s	T	Age of universe
11	M_u	10^{75} - 10^{80}	m_p	M	Mass of universe
12	Q_u	**0**	C	Q	Charge of universe
13	H_u	2.4E-18	1/s	1/T	Hubble parameter

Note these are all measured independently quantities, and are normally thought to have empirically independent values – except the identity of the elementary charges. Apart from this, none of the values is logically or theoretically inter-defined with the others. E.g. we do not include μ_0 , the magnetic force constant, because it is determined from *c*, ε_0, and the relation: $c^2 = 1/\varepsilon_0\mu_0$.

Also, we are limiting ourselves here to just the two stable fundamental mass particles, the electron and proton. But these determine all properties of charge and most properties of mass in atomic physics. We could add the *neutron,* which provides a further mass ratio, but it is so close to the proton mass that it adds nothing of significance to the LNRs (at least until we have more precise theory). Note the photon, the other essential particle required to do electromagnetic, particle, quantum and gravitational physics, is fully described by the constants: *c, h* and ε_0.[3]

The numbers in the table in black are all measured accurately, and represent empirical values, but the four numbers highlighted in red are more or less problematic or irrelevant for LNR's.

- First, the *charge of the proton is exactly -1 of the charge of the electron,* so while this is a *perfect coincidence,* it is taken as an independent law, and does not provide an extra test for

3 Note if we add more fundamental particle masses, e.g. neutrons and neutrinos, this leads to further possible patterns among *particle masses in the standard model.* This is another topic where *ratios of fundamental constants, viz. particle rest masses,* are significant. But the LNR's are relations between just the small group of most essential fundamental constants, and a few cosmological variables.

a LNR theory. It also only combines two quantities of the same kind, i.e. elementary electric charges, while the main LNC's combine multiple quantities of different types.[4]

- Second, the actual *radius of the universe* is not measured directly – it is estimated theoretically from the age of the universe, the Hubble parameter, and other highly theoretical assumptions, and conventional estimates vary from about 30 b.l.y. to 300 b.l.y. This does not (yet) provide a meaningful independent quantity to define a LNC.

- Third, the *mass of the universe* is here really referring to the *count of protons and neutrons (or fermionic particles),* and this is only estimated in the range of about 10^{75} - 10^{80} for the "visible universe". This gives an important LNC, but it must be recognised that the value has quite a large uncertainty, and provides a correspondingly weak "coincidence".

- Fourth, the total charge of the universe is assumed to be zero, as a logical consequence of the cancelling of proton and electron charges, so this empirical value gives no additional ratio to test LNR theories either.

Hence of our 13 fundamental quantities, there are only ten with independent values, and one, M_U or *N,* is quite approximate, in the range of about *10^{75} – 10^{80} proton masses.* From these ten constants, we can construct: *10-4 = 6 independent dimensionless numbers* (DN's). The most important point to start with is that:

- *Only equalities among the six DN's can provide LN coincidences.*

We discuss this next.

Dimensionless Numbers and LNC's.

The first three DNs are determined by the first seven constants above: these are the *proton-electron mass ratio, ρ, the Dirac constant, D, and the Fine Structure constant, α.* The second three DNs are determined by the three extra global variables we introduce for the universe: *T_U (or age), M_U (or particle number, N),* and *H_U,* the *Hubble constant.* We state the definition and approximate values of these six ratios, which defines the *complete set of LNCs,* and then explain this choice.

4 So it is assumed this has a separate explanation, in terms of conservation of charge and quantum physics, but no explanation is known in the Standard Model yet, where the coincidence is the fractional charges of quarks.

Table 2. The six classic dimensionless numbers.

Id	Dimensionless Number	Value	Name
14	$\rho = m_p/m_e$	1.8E+03	Mass ratio
15	$\alpha = q^2/4\pi\varepsilon_0\hbar c$	7.3E-03	Fine Structure Constant
16	$D_{PP} = hc/m_p^2 G$	1.1E+39	Dirac constant (proton mass)
17	T_u/T_p	9.9E+40	Universe Time/Proton Time
18	M_u/M_p	**1E+80**	Universe Mass/Proton Mass
19	$T_u H_U$	1.03E+00	Universe Time X Hubble Parameter

There are the six *independent DN's* that we have available. Note the *fine structure constant* is defined as: $\alpha = q^2/4\pi\varepsilon_0\hbar c = q^2/2\varepsilon_0 hc \approx 1/137$. We call the term *D* the *Dirac constant,* but there are variations depending on whether we use the electron or proton mass.

- We can construct further dimensionless quantities only by multiplying powers of these *six independent DN's* together (e.g. $D\alpha$).
- Alternatively, any other dimensionless quantities can be decomposed uniquely as powers of these six DNs.

The set is defined here in the simplest way possible to bring out the LNC's. Note that we have used the *mass of the proton* (rather than the electron, which is more common, see below), to define the quantity: D_{PP} in (16). We will discuss this choice later, but we can now observe the main point: the LNC's are no more or less than the approximate coincidence in these *pairs of values:*

The coincidences. **(1.1)**

$\rho \approx 1/\alpha \approx 10^2$ (14) ≈ (15)

$D_{PP} \approx T_U/T_p \approx 10^{40}$ (16) ≈ (17)

$D_{PP}^2 \approx M_U/M_p \approx 10^{80}$ (16) ≈ √(18)

$T_U H_U \approx 1$ (19) ≈ 1

$q_e/q_P \approx -1$ A DN coincidence but not a LNR.

We have included the *charge ratio* here as the fifth "coincidence" for completeness. These five "coincidences" are all the *primary empirical evidence* for the LNH or LNR theories. As we have noted, the fifth one plays no direct role in the LNR theories. The fourth: *Hubble ≈ 1/Age* plays a role in some later LNR theories, but not in the original Dirac theories. It does not suggest relations between fundamental constants and cosmological variables, only between two simple cosmological quantities. This coincidence is not generally thought to reflect any direct law-like relation (like Dirac's other LNCs) because it is thought the expansion rate *H* is determined by other physical factors, and varies with time. In fact it is now widely believed that *H* started *increasing* about 7 billion years ago, after decreasing for most of the previous 7 billion years. This accelerating expansion is interpreted as "dark energy", or Λ. So the fourth coincidence is commonly seen as accidental. So we set this (along with other possible cosmological "coincidences" based on *dynamic cosmological variables,* relating quantities like Λ (acceleration of *H),* or the average background temperature of the MBR), aside for now, as involving dynamics, and focus on the primary LNC's, as recognised by Dirac.[5]

So we can take the primary evidence for the LNH to lie in just the first three main coincidences.[6] And the third is somewhat approximate. Seen in this way, the evidence may start to appear rather flimsy!

But in fact the intuition of Dirac, Eddington, Weyl, Milne and others, that these should be taken seriously was correct. The system of quantities is *saturated with coincidences* – there are five evident among ten quantities, even if the last two may be irrelevant to LNR theories. The first three coincidences appear unlikely by accident, essentially because the values span such a large scale, from $1 – 10^{40} – 10^{80}$. We may ask: *how unlikely are they exactly*? If the values were purely random w.r.t. each other, what is the chance of three pairs from just six quantities coinciding over such a scale? We can do a quick analysis to reassure ourselves.

Estimating the chance of coincidence.

We will do a quick epistemic probability analysis. Suppose that two dimensionless numbers are randomly chosen first (e.g. ρ and *D),* and they turn out to be 10^3 and 10^{40}. This gives a range of expectations for further numbers we "randomly" select. What is the chance that another randomly selected number will be "close to" either 10^3 or 10^{40}? Creating the equivalent of a random LNC? We take the next number to be chosen randomly, in the same range as the first, with probabilities linear with the logs.[7]

So what is the chance of the next number selected having a value either in the range of: 10^1-10^5 or: 10^{38}-10^{42}, which may be considered "close to" the first values in LNH terms? These ranges around the two values represents a total span of 10^8 over the full range, which should be taken as spanning 10^{80}, since values could be from: $10^{-40} – 10^{40}$. (Values and their inverses must be given equal probabilities).

Hence we get about: $p = 8/80 = 1/10$ as roughly the probability of *one coincidence happening by chance*.

This is perhaps more *likely* than our first intuition, and it is certainly plausible that a 1/10 chance occurs at least *once*. (The $H = 1/T$ coincidence may be taken as such a 1/10 accident.) But the chance of this happening twice is only about 1/100. And the chance of three coincidences is 1/1,000.8

5 We do not dismiss these additional cosmological LNC's, but we will see when we introduce our own model, their explanation is likely to derive from solutions to dynamics *within a core LNR theory,* rather than being direct or fundamental relationships.
6 In fact there is one more: that $ln(D) \approx 1/\alpha$. But this is too approximate to be a meaningful "coincidence", because e.g. $ln(D^2) = 2ln(D) \approx 2/\alpha$, so a *logarithmic relation* could hold approximately between quantities on quite different scales. But a good LNR theory might predict or explain such a relation more precisely, so it is ultimately relevant to evaluating LNR theories, just not useful evidence for LNC's.
7 So e.g. there is equal probability of selecting a value in the range: 10^{-5}-10^{+5} or: 10^{30}-10^{40} or: 10^{-40}-10^{-30}.
8 The third coincidence, with: $N \approx D^2 \approx 10^{80}$, falls within a span of about 10^8 around 10^{80}, so we can give it roughly the same 1/10 probability.

This is only an approximate way to estimate the chances, but it appears realistic.9 It removes the sense that the coincidences are extremely improbable. But it is also consistent with common sense or intuition that they are somewhat improbable.

- The chance of each *individual* LN *coincidence* is plausibly as large as 1/10, because of the quite large *approximations* in the *equalities.*

- But they are still significantly improbable, because each requires a combination of several constants with values from about $10^{-34} – 10^{10}$ to multiply out to close to 1.

So let us provisionally take the *a priori* probability of all three main coincidences occurring randomly as somewhere around 1/100 (maximum; conservative) to 1/1,000 (medium; perhaps more realistic). These are quite small chances, and such chances do not come true very often. And note that we are not dealing with a repeated event: the values of the fundamental constants are a singular and unique feature of our universe, and any relations between them imposes profound constraints on the fundamental laws. (See Appendix 1 for another example.)

There is little point trying to determine "epistemic probabilities" of the LNC's more precisely than this estimate, without considering tests of specific LNR theories. These are only starting estimates of *a priori probabilities,* and a more careful evaluation of the LNH must depend on how well a specific LNR theory succeeds.

We can put the inference *from the observation of LNC's* to the *probability of LNR's* in a Bayesian framework like this:

Bayesian probabilities. **(1.2)**

	prob(LNC|LNR) = 1	Prob of the coincidences given LNR theory - certain.
	prob(LNC|~LNR) = 1/100	Prob of the coincidences given non-LNT theory - weak.
	prob(LNR)/prob(~LNR) = 1 (??)	Equal prior probs of *LNR versus non-LNR* ?
Then:	*prob(LNR|LNC)) = (0.5 x 1)/(0.5 x 1 + 0.5 x 1/100) = 0.99*	Prob of LNR given LNC.

- The first probability is the *conditional probability of the LNC's given an LNR theory is true.* We define LNR's for this purpose as theories that imply LNC's, so this probability is taken as 1.

- The second probability is the *conditional probability of the LNC's given no LNR theory is true.* This is the chance of the LNC's happening by accident, which is what we estimated as 1/100.

- To estimate the probability of LNR's from LNC's we have to invert the first conditional probability, as follows.

9 It can be verified by taking random combinations of half a dozen constants at a time from the set of *c, h, G, m, q, etc,* to estimate the standard deviation of randomly constructed quantities, and it turns about 10^{20}.

Derivation. By definition: *p(A|B) = p(A&B)/p(B)* and: *p(B|A) = p(A&B)/p(A),* so: *p(A|B) = p(B|A)p(A)/p(B).* Given we set a *prior probability*: *p(A)/p(~A) = q,* and: *p(~A) = 1-p(A),* then: *p(A) = q/(1+q).* So: *p(B) = p(B|A)p(A) + p(B|~A)p(~A) = p(B|A)q/(1+q) + p(B|~A)/(1+q).*

The "inversion theorem" is therefore obtained: $p(A|B) = \frac{p(B|A)}{p(B|A)+ p(B|{\sim}A)/q}$.

From this, we obtain the epistemic probability: *prob(LNR|LNC) = 0.99.*

But note that this depends on our choice of the *prior chance, q, of a LNR theory versus a non-LNR theory.* Given equal *prior chances,* the low *1/100-th* chance of a coincidence points us quite strongly towards the *LNR,* and to suspecting a *lawlike pattern underlying the LNC's.* This was the epistemological starting point of Dirac and others in the 1930-50's. They were quite correct to insist on investigating LNR theories.

But the subsequent epistemic situation has changed of course, because some LNR theories have been tested and rejected, while other proposals have proved difficult to complete theoretically. But given our analysis, we may ask:

- Has it been ruled out with a 0.999 probability that there are no realistic LNR theories?

I.e. can we set: *q = p(LNR)/p(~LNR) < 1/1000?* This would be required on our analysis to provide reasonably strong evidence against LNR theories. Our conclusion is that there is no strong evidence against LNR's yet, and the prospect remains open.

Discussion.

LNR theories are still quite an obscure branch of cosmology, and appear to have become quite unpopular by the 1990's[10], but a dedicated interest has remained. The question, far from being closed, is perhaps now at its most exciting stage, where we should expect some further progress.

[Solà 2015] summarises the origin of the subject, and the reasons for renewed interest in the new era of cosmology – with the problems of dark matter, dark energy and the cosmological constant becoming prominent in cosmology in the 2000's.

> "The history of this subject traces back mainly to Dirac's pioneering work in the thirties on the "large number hypothesis" [1] , from which a time evolution of the gravitational constant G was suggested, and later on the first discussions on new forms of the principle of

[10] LNR theories also have an association with static universe theories, spontaneous particle creation, fading light and no Big Bang. This is regarded sceptically today, given multiple lines of evidence for an explosive origin of the universe, and the development of galactic and stellar and radiation structures, that we can increasingly see in past time, with more powerful telescopes and gravity wave detectors. Nonetheless static theories remain a serious topic, theoretically and epistemically, that interest many LNR theorists. In this respect, an eternal cyclic universe in a cycle of expansion and contraction is surely the main alternative to the present standard cosmology, in which the universe appeared from a "singularity" 14 billion years ago out of … equations. See [Lopez-Corridora, 2014] for a wider survey of some "alternative cosmological theories", which covers static theories, but does not include any discussion of the Dirac or LNR type theories.

equivalence emerged [2] and finally triggered the Jordan, Fierz and Brans-Dicke approach to gravity [3], in which General Relativity was extended to accommodate variations in G. It also triggered subsequent speculations by Gamow [4] and others on the possible variation of the fine structure constant.

> "Despite the initial difficulties, these seminal works were a real spur to start changing the mentality and the strong prejudice on the supposedly imperturbable and "sacrosanct" rigid status of the constants of Nature. It is amazing to realize nowadays how much we have opened our minds to the new horizons that these ideas offered since those early times. Modern investigations on this subject are performed not only at the theoretical but also at the experimental level, both in the lab (through high precision quantum optic techniques) and in the astrophysical domain (using absorption systems in the spectra of distant quasars)." [Joan Solà, 2015]

This is from the *Preface to the Special Issue of Modern Physics Letters A on Fundamental Constants in Physics and Their Time Variation*, by Guest Editor of the Special Issue, Joan Sol`a. This contains seven excellent specialist articles. Solà summarises:

> "Quite obviously this is a very active field of research. Exciting new results are expected soon which could significantly modify our current scientific paradigms. If we attend to what we know about the energy budget of the Universe, which is believed to consists roughly of only 5% of baryonic matter (atoms), 25% of dark matter (DM) and 70% of dark energy (DE), it is pretty obvious that our knowledge on its composition is more than limited and hence leaves much to be desired. It is not surprising that many researchers, spurred by the positive observational hints and the unsatisfactory theoretical situation, have seriously adhered to the possibility that the so-called fundamental "constants" of nature can be, in reality, slowly varying quantities possibly related to underlying fundamental theories of the elementary interactions. These theories are unknown at present, but they might help explaining the origin of the hidden components of the DM and the nature of the DE, which dominate by far the structure and fate of our Universe." [Joan Solà, 2015]

Like [Ray, 2007/2019], this emphasises the modern problems of dark energy and the cosmological constant, and the explanation of dark matter, as prime reasons for interest in LNR theories today.

So we may identify two main attitudes.

- **The orthodox attitude** is that the LNC's are really just coincidences, and the "fundamental laws of nature" (meaning QM and GTR to most physicists) are essentially correct as we know them, and should not be revised to allow for time variations in constants, and that there are no deeper connections between constants and cosmological parameters.

- **The heterodox attitude** is that the "fundamental laws of nature" are *not* correct as we know them, and the LNC's point to this, and they provide a major clue to a revision of laws that will be required in a more complete unified theory, which should incorporate LNRs.

We may agree that the LNC's by themselves are by no means conclusive evidence, or perhaps not even very strong evidence at this stage (opinions will be divided). But they are certainly very significant evidence. If LNC's were absent, there would be little prospect of a LNR theory. If a LNR theory is correct, the LNC's will be seen as highly significant.

Also, we should recognise that primary evidence *against* LNR's is theoretical: it is the apparent difficulty of combining LNR's with our fundamental theories, QM and GTR, as we know them in their present form. To support LNR's, we must reconstruct GTR and/or QM to some extent. If we have high confidence in QM and GTR in their present form, and we do not feel like trying to "reconstruct" them, then LNR's will appear unlikely. However if we think fundamental physics needs substantial revision in any case (to obtain a unified theory), then the LNR's take on a higher significance – indeed, they may be expected to provide essential clues to a new theory.

Our attitude depends on how we judge this larger theoretical context – including our awareness of the kinds of alternative theories that could be possible, and the kinds of tests that have been conducted. If the laws of physics were properly unified, and there were no major anomalies in fundamental physics and cosmology, there would be much less motivation to explore alternatives. But in fact physics is stuck in the opposite situation: QM and GTR are inconsistent, there is no unified theory, and there are multiple perplexing anomalies. So we take the second view-point seriously.

But Dirac's type of theory (and variations of it) has weaknesses, empirical and theoretical, and we propose another type of LNR theory is more realistic, and that it conforms to the empirical data, as well as being consistent with known physics. In the main body of this paper, we present a version of this alternative LNR theory. There will be other variations of this type of theory. The most important step is to introduce a systematic way to formalise such theories, by specifying a system of differential transformations, required to relate alternative *coordinate bases.* This lets us systematically work out transformations of fundamental laws and measurements on the alternative theory.

The need for this was recognised by Dirac [1969, 1974], but he gave only one example. This is an essential concept that has had very little recognition or development after Dirac, but unless it is properly addressed, empirical tests of LNRs cannot be reliably calculated. We will see it can give results that at first appear counter-intuitive.

Before going on to this, we continue with some further discussion of the role of the dimensionless quantities and the system of constants we are dealing with.

Section 2. Background concepts of LNR theories and DNs.

The basis of LNR theories.

The primary problem is: what type of LNR theory could explain the LNCs? The most obvious way forward was first taken systematically by Dirac: *propose the simplest form of LNC's as law-like* *relationships.* Doing this, and replacing the definitions of quantities with the fundamental constants, the first three coincidences above would give three distinct "laws":

$$m_e/m_p \approx q^2/4\pi\varepsilon_0 hc \approx 10^{-3}$$

$$D_{pp} = hc/m_P{}^2G \approx T_U m_P c^2/h \approx 10^{40}$$

$$D_{PP}{}^2 = h^2c^2/m_P{}^4G^2 \approx N_p \approx 10^{80}$$

- The first relation poses no conflict with current physics, as there is no implication that any of these constants is *changing with time*. If there is an underlying relationship between the *mass ratio* and the *electric force,* a fully unified theory may be expected to provide it. This is an open question, and does not conflict with any laws of dynamics currently known.

- The second relation caused the major consternation, because it requires that at least one of the constants varies with time. We see this by rearranging it as: $D = h^2/cm^3G \approx T_U$. The age of the universe is increasing, so if this is a *law-like relation,* at least one constant must vary with time. The common choice, adopted by Dirac and most others, is to take *c, h* and *m* as fixed, and take: $G \propto \frac{1}{T_U}$. This is the monumental conclusion that Dirac reached: gravity is weakening with time! This is a special problem mainly because ordinary GTR does not allow a variable *G*.

- The third relation is equally radical. By itself, it poses a mysterious connection between the local constants: *c, h, G, m* and the number of particles, *N,* in the cosmos, which is odd enough. But if *D* is also increasing (linearly) with time, *N* must be increasing (quadratically) with time. This means *matter is being continuously created.*

Attempts to explain and test these relationships have led to the major work in the subject. This includes tests of time variations of *G,* α and ρ, and looking for evidence in astronomical processes, including evidence for spontaneous particle creation. This is done in the context of various LNR theories, including Dirac's [1974] and subsequent attempts to generalise GTR, such as Brans-Dicke theory, and steady state cosmologies.

Now a conundrum for all such theories is to relate the alternative theories to the *interpretation of measurements.* Measurements in cosmology are theory-dependant. Changing fundamental assumptions (e.g. particle creation, weakening gravity, decreasing light-energy) requires us to recalculate predictions and interpretations of astronomical data, and this is quite involved. It is not approached very systematically, and this is a primary issue we address.

Physical ratios and dimensionless constants.

Many discussions of LNCs start by drawing attention to coincidences in a number of possible ratios, such as the strength of the *electric force to the gravitational force,* and the *age of the universe to the characteristic quantum time of a fundamental particle (electron or proton)* (or similar ratios involving *distances, energies, masses, charges, etc*). It may seem at first that there is an open-ended choice of possible “ratios” to compare – but when we examine them, we find they reduce to variations of just the three coincidences in dimensionless ratios we have seen above.

Take the ratio of *forces* for example. We may take two fundamental particles, say protons, separated by a distance *r.* They experience mutual gravitational and EM forces (ignoring signs):

$F_G = m_P^2G/r^2$	Gravity force
$F_E = q^2/4\pi\varepsilon_0 r^2$	Electric force

With *q* the elementary charge. Dividing these, the r^2’s cancel, and the ratio is an invariant*:*

$F_E/F_G = q^2/4\pi\varepsilon_0\, Gm_P^2 \approx 10^{40}$	Force ratio

This conforms to Dirac’s “magic large number” 10^{40} above, but this is not an independent “coincidence” in additional to what we have already seen. Rearranging we find the force ratio is just:

$F_E/F_G = (q^2/4\pi\varepsilon_0\, hc)(hc/Gm_P^2) = \alpha D_{PP}$	Force ratio

This is the ratio observed above – except it has an extra factor of α. But the value is still approximately 10^{40} because α is approximately 1. Hence this *force ratio* does not define any additional dimensionless number or coincidence in addition to those we have already seen above – it just presents a slightly different combination.

The *potential energy ratio* is exactly the same:

$E_G = m_P^2G/r$	Gravity potential
$E_E = q^2/4\pi\varepsilon_0 r$	Electric potential
$E_E/E_G = (q_e^2/4\pi\varepsilon_0\, hc)(hc/Gm_P^2) = \alpha D_{PP}$	Energy ratio

Similarly with ratios of “self-energy” of charges and masses, and other ratios we can construct. Because *unless we introduce some further quantities (constants or variables),* we cannot construct any further independent relationships.

The key example in most presentations is the ratio of the “quantum time” of a fundamental particle with the age of the universe. The particle time is taken as the Compton wavelength divided by *c.* So for the proton:

$R_P = h/cm_P$	Proton wave-length (Compton)
$T_P = h/c^2m_P$	Proton time (divide R_P by *c*)

Then taking the ratio of this with the age of the universe:

$T_u/T_P = T_uc^2m_P/h \approx 10^{40}$	Time ratio

The “coincidence” most commonly observed is that this is close to the *force ratio:*

$T_u/T_P \approx F_E/F_G \approx 10^{40}$ The force-time LNC

But this just means: $T_u/T_P \approx \alpha D_{PP} \approx 10^{40}$, which is essentially the second "coincidence" observed above. And we cannot really distinguish the LN relation: $T_u/T_P \approx \alpha D_{PP}$ from: $T_u/T_P \approx D_{PP}$, because $\alpha \approx 1$ (in the approximations).

So the general point is that we cannot invent or discover any more independent LNCs *from our limited set of 13 constants and parameters*.

This example also raises the essential question whether the relationship should really be taken as: $T_u/T_P \approx D_{PP}$ or: $T_u/T_P \approx \alpha D_{PP}$. Or more generally, which of several possible alternatives we should use for the dimensionless quantity D in: $T_u/T_P \approx D$, such as:

$D_{pp} = hc/m_P^2G$ Dirac constant – proton-proton mass

Or: $D_{ep} = hc/m_e m_P G = D_{pp}\rho$ Dirac constant – electron-proton mass

Or: $D_{ee} = hc/m_e^2G = D_{pp}\rho^2$ Dirac constant – electron-electron mass

Without some reason for this choice, we cannot get a LNR precise within about 10^3. And since: $\rho \approx 1/\alpha$, the same approximate relations with $1/\alpha$ for ρ are equally possible. And there is also similar choice of small logical constants, such as *2, π, 4π, etc,* that commonly occur in relations. E.g. we have so far generally used *h,* but more often: $\hbar = h/2\pi$ is used instead to define *D* or *T.* Without some principle for choosing a specific combination of these *small constants*, we cannot determine a relation precisely within a factor of 10 or 100. The theory we will subsequently propose does determine this choice more precisely, which makes it much more strongly testable.

Note also that there is a scale of "fundamental lengths" that we can define:

Table 3. Length scale for electron.

	Name	**Length *m***	**Definition**
	R_e *Electron Radius*	*3.86E-13*	$R_e = \hbar/m_e c$
Smaller	L_p *Planck length*	*1.62E-35*	$\surd(G\hbar/c^3) = R_e/\surd D_{ee}$
Smaller	R_B *Electron black hole*	*6.76E-58*	$m_e G/c^2 = R_e/D_{ee}$
	Spacing = $\surd D_{ee}$	2.39E+22	Spacing = $\surd D_{ee} = \surd(hc/m_e^2G)$
Larger	L_e *Large Electron Radius*	*9.23E+09*	$L_e = R_e\surd(D_{ee})$
Larger	U_e *Universe Electron Radius*	*2.20E+32*	$U_e = R_e D_{ee}$

Taking the *electron radius,* $R_e = \hbar/m_e c$, as a fundamental unit of length, four more lengths arise from multiples of $\surd D_{ee}$. These may appear as four lengths related to the electron that are available to find coincidences in, but in fact there is only one fundamental length (R_e) multiplied by one dimensionless constant D_{ee}, or rather its square root, $\sqrt{D_{ee}}$. So for instance, any coincidences involving the *Planck length* are really coincidences in $R_e/\sqrt{D_{ee}}$.

We cannot construct any dimensionless numbers from these fundamental four quantities: *h, c,* m_e, *G,* except powers of the Dirac constant, D_{ee}. This includes polynomial series, such as exponentials or logarithms of D_{ee}. It is interesting to note in this respect that *DNs* occur in two archetypal forms in fundamental physics.

- First in GTR, in the dimensionless factor: MG/c^2r, which is at the heart of all GTR solutions, e.g. in the dimensionless term: $k = 1/\sqrt{(1-2MG/c^2r)}$ in the Schwarzschild metric. If MG/c^2r was not dimensionless, we could not subtract it from the number 1! This means it can occur in polynomial series, e.g. $K = exp(MG/c^2r)$. In fact, *k* is defined as a polynomial series too, its Taylor series.

- Second in QM, in dimensionless factors like: $mv_xx/\hbar = p_xx/\hbar$ and: $mc^2t/\hbar = Et/\hbar$, found in (complex) terms: $(i/\hbar)(px - Et)$ in solutions to the (non-relativistic) Schrodinger or (relativistic) Klein-Gordon equations: $\Psi(w,x,t) = Ae^{(\frac{i}{\hbar})(p_w w + p_x x - E_w t - E_x t)}$, which is a general form of K-G solution, with: $p_w = m_ec$ and $E_w = m_ec^2$ for a particle with mass m_e. These are solutions for a K-G equation: $\pm(\nabla_w^2\Psi + \nabla_x^2)\Psi = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi$. The solutions are exponential functions, and the factor in any exponential function must be dimensionless, because the exponential is a polynomial series.

A third dimensionless form is found in electrodynamics, with the dimensionless fine structure constant, α. (Another is in the Dirac delta function.)

Dimensional constraints.

In constructing LNC's, why are we constrained to relations between *dimensionless numbers?* It stems from the fact that relations between physical quantities are only meaningful if they are between two identical types of quantities. E.g. we can equate (or compare) two *lengths,* or two *energies,* or two *times,* or two *masses,* etc, but we cannot equate or compare a time with a length, or a speed with a mass. We can state a relationship, e.g. like: $F_E = DF_G$, where F_E and F_G are both forces, and *D* is a dimensionless number. But we cannot even meaningfully state a relation for example like: $t = Dx$, with dimensionless *D,* or just: $t = x$, because this is equating or identifying *a quantity of time* with *a quantity of space*. There are no such identities.

As all physicists know: anywhere you see *time* related to *space* in physics, the *physical units* are "translated" using a velocity variable, e.g. $x = ct$, or: $x = vt$. Velocity or speed is a dimensional quantity: $[c] \equiv [X/T]$.

For another example, consider: $m_e = \rho h$, where m_e is a mass, *h* is an angular momentum, and ρ is our dimensionless mass ratio: *1,836.* This equation does not state a meaningful proposition in physics as it stands, because a *quantity of mass* cannot be identified with *a quantity of angular momentum.*

However, this nonetheless appears *numerically accurate in SI units,* with: $m_e = 9.11E\text{-}31$ and: $\rho h = 1.22E\text{-}30$, so that: $\rho h/m_e = 1.34$. So we might write: $m_e \approx \rho h$. But this depends on SI units of course: if we change our "conventional units" for length or time, the value of *h* will change, and this "coincidence" of values will disappear. Let us try by rescaling the units for *time.*

Physicists often adopt an alternative time variable with units defined: $t' = ct$, so that the speed of light in the new units is unity by definition: $c' = dr/dt' = 1$. Because: $dr/dt = c$, and: $dt/dt' = 1/c$, so: $dr/dt' = (dr/dt)(dt/dt') = c/c = 1$. This is done when adopting "natural units".

With this change in the units for *t',* the value *h'* for Planck's constant *h* changes, because *h* is a physical quantity, involving *space* and *mass* combined with *time*. Its dimensional analysis is: $[h] \equiv [X^2M/T]$. So given: $m_e = \rho h$, the relation: $m_e = \rho h'$ cannot hold in the new system of units.

Instead *h'* transforms to: $h' = h/c$. Because: $dt' = cdt$ and: $h'/h \equiv [T/T'] \equiv dt/dt' = 1/c$.

This means that expressions like: $m_e \approx \rho h$ *with unmatched physical quantities* cannot be taken as stating lawlike relations. If a purely *numerical relation* is intended, we should strictly write it like this:

$|m_e| \approx |\rho||h|$ A relation between magnitudes.

This uses the "magnitude" function: |.|, to make it explicit that we are *extracting numbers from the physical quantities.* The measurement units enter through the "numerical magnitude" function: *|.|,* through which we assign a (coordinate) number to a physical quantity.

It may have been noticed above that we used the constant "*c*" in two different ways, in the two statements: $dr/dt = c$, and: $dt'/dt = c$. The term "*c*" cannot have the same meaning in both of these, since in the first it is a *dimensional quantity,* while in the second it is a *dimensionless number.* What we should really write in the second case is: $dt'/dt = |c|$, denoting the specific numerical value of: $|c|$ in a system of units, e.g. the SI system of units.11

Any *physically unbalanced equality* that gives a numerical coincidence, like: $|m_e| \approx |\rho||h|$, is dependant on the choice of units. Indeed this goes to the logical heart of the *vector calculus (*generalised to the *tensor calculus)*, which is a system for writing meaningful propositions about geometric objects in formal equations. The vector or tensor formalism does two main things:

(1) First it separates the *spatial (or space-time) coordinate functions* from the *spatial basis vectors* (so e.g. we write: $\mathbf{z} = z\hat{\mathbf{z}}$, with *z* as the *magnitude* and $\hat{\mathbf{z}}$ as the *basis vector*), making it explicit that both *coordinates and physical quantities are combined,* and

(2) Second it distinguishes the inverse operations of "*multiplication*" and *"division" by basis vectors,* or what we know as *covariant and contravariant tensor behaviour.* (The prototypes being the dual vectors: ***dr*** and: ***d/dr,*** equivalent to *gradient* and *tangent vectors).*

Dimensional analysis captures a kind of simplified aspect of tensors – it can be thought of as an image of the "functional skeleton" of equations. Unlike tensors, it does not distinguish the *multiple*

11 This reminds us that the formalism of ordinary physics is not always functionally explicit.

dimensions of space, but it extends from the space-time construction alone to the functional construction in all four physical quantities: [*X, T, M, Q]*.

- The general principle is that *law-like relations in physics must be expressed through dimensionally balanced equations.*
- This is a logical requirement that comes back to the logical construction of propositions as stating identities between pairs of quantities.

Rules for simple scale transformations.

There is a simple formalism for *scale transformations,* which we can work out through examples and generalise. Note the value of *h'* under the transformation of the time units: *t' = ct* is completely determined by its dimensional analysis, and this holds generally for all variables of the same type:

- Any two variables with the same dimensional analysis have the same transformations, on a given transformations of units.

E.g. we can write the dimensional analysis for *h* and *h'* like: $[h] \equiv [X^2M/T]$ and: $[h'] \equiv [X^2M/T']$. In this example: $[T'] = c[T]$, so: $[1/T'] = [1/cT]$, where *c* is here just the number: *dt'/dt.* Substituting in our dimensional equation: $[h'] \equiv [X^2M/T'] = [X^2M/cT] \equiv [h]/c$. We can see the pattern is:

- $h' = h(dt/dt') = h/c$.

Generalising to include all the dimensional quantities, if *h'* is in units of: $[X'^2M'/T']$, and all three units are transformed: $[X,T,M] \rightarrow [X',T',M']$, we must transform:

- $h' = h(dx'/dx)^2(dm'/dm)^1(dt'/dt)^{-1}$.

Or generally, using *A, B, C, D* as the powers of dimensions of a quantity η:

- For any variable η with the dimensional analysis: $[\eta] \equiv [X^AT^BM^CQ^D]$,
- A scale transformation to alternative variables: $[\eta'] \equiv [X'^AT'^BM'^CQ'^D]$
- Entails that η transforms to: $\eta' = \eta(dx'/dx)^A(dt'/dt)^B(dm'/dm)^C(dq'/dq)^D$

For a practical system, we can represent the dimensional form of a quantity with the four exponents as a vector, like: *[A,B,C,D].* E.g. a speed: *c ≡ [1,-1,0,0].* A mass: *m ≡ [0,0,1,0].* Etc. Then when we multiple or divide quantities (including integrating and differentiating), we add or subtract the dimensional vectors to get the dimensional analysis of the new quantity. This is what we normally do in physics, but it is usually simple enough that is not formalised. But in some problems we need to solve transformations more systematically. Dimensional analysis corresponds to linear equations. For an example, let us verify that *D* is dimensionless.

$D = hc/m^2G$	$+h$	$+ [2,-1,1,0]$
	$+ c$	$+ [1,-1,0,0]$
	$- m^2$	$- 2[0,0,-1,0]$
	$- G$	$- [3,-2,-1,0]$
	$= D$	$= [0,0,0,0]$

To show *there are only 3 independent dimensionless quantities that can be defined from seven variables,* we can refer to theorems in linear algebra, and observe that there are only three solutions to seven independent linear equations in four variables. This becomes more obvious in *natural units.*

Converting to Natural Units.

In "natural units" the quantities: *c, h (*or sometimes *ħ), m_e* and *q* are set to 1, and this is useful to simplify relations. We give a second list of our key quantities below in these units. Remarkably, this list contains as much real information about fundamental physics as the previous list with values in SI units.

Rather than starting with *c* and *h* as usual, we define our system by starting with four independent "basis units", for *space, time, mass and charge,* based on the *electron.* We define the basis unit for *space* as the Compton wavelength for the electron: $W_e = h/m_e c$, (using *W's* to distinguish from the *reduced Compton wavelength:* $R_e = \hbar/m_e c = W_e/2\pi$.) The basis unit for *time* is similarly defined: $T_e = W_e/c$. Note this gives units where: *c = 1* and *h = 1* as usual, so we can see that adopting natural units in the normal way forces this same set of *base units.*

Table 4. Fundamental constants in natural units.

Id	Variable	Natural units	Inverse	Dimensions	SI Value	SI units	Name
1	$W_e = h/cm_e$	1		(1,0,0,0)	2.43E-12	m	Electron length
2	$T_e = h/cm_e$	1		(0,1,0,0)	8.09E-21	s	Electron time
3	m_e	1		(0,0,1,0)	9.11E-31	kg	Electron mass
4	q_e	1		(0,0,0,1)	-1.60E-19	C	Electron charge
5	c	1		(1,-1,0,0)	3.00E+08	m/s	Speed of light
6	h	1		(2,-1,1,0)	6.63E-34	$m^2 kg/s$	Planck's constant
7	$\boldsymbol{\rho = m_p/m_e}$	**1836.2**	**5.45E-04**	**(0,0,0,0)**	**1836.2**	**1**	**Mass ratio**
8	m_p	1836.2		(0,0,1,0)	1.67E-27	kg	Proton mass
9	$\boldsymbol{D_{ee} = hc/m_e^2 G}$	**1.06E+39**	**9.40E-40**	**(0,0,0,0)**	**1.06E+39**	**1**	**Dirac constant (electron)**
10	$G = hc/m_e^2 D_{ee}$	9.40E-40		(3,-2,-1,0)	6.67E-11	$m^2/kg\,s^2$	Newton's constant
11	$\boldsymbol{\alpha = q^2/2\varepsilon_0 hc}$	**0.0073**	**137.1**	**(0,0,0,0)**	**0.0073**	**1**	**Fine Structure Constant**
12	$\varepsilon_0 = q^2/2\alpha hc$	68.5		(-3,2,-1,-2)	8.85E-12	$s^2 C^2 m^{-3} kg^{-1}$	Electric permittivity
13	q_p	-1		(0,0,0,1)	1.60E-19	C	Proton charge
14	$W_p = h/cm_p$	5.45E-04	1836.2	(1,0,0,0)	1.32E-15	m	Proton length
15	$T_p = h/cm_p$	5.45E-04	1836.2	(0,1,0,0)	4.41E-24	s	Proton time
16	R_u			(1,0,0,0)	**Unknown**	m	Radius of universe?
17	T_u	5.38E+37		(0,1,0,0)	4.35E+17	s	Age of universe
18	M_u	**1E+80**		(0,0,1,0)	$\boldsymbol{10^{75}\text{-}10^{80}}$	N (kg)	Proton-count of universe
19	Q_u	0		(0,0,0,1)	0	C	Charge of universe
20	H_u	1.9E-38	5.22E+37	(0,-1,0,0)	2.4E-18	1/s	Hubble parameter

(Note the universe mass, (18), is here represented as the estimated count of protons, not the mass in kg.) Note that (1-4) fully determines the choice of *units.* They form an orthogonal basis set, seen in their "vectorial" representation. They entail the values of (5,6).

In these “natural units”, we can easily see that there are only seven contingent empirical quantities: the three dimensionless quantities (7, 9, 11), and the five cosmological quantities (16-20).[12]

- The four LNC’s are the equalities: (7) ≈ 1/(11), (9) ≈ (17), $(9)^2$ ≈ (18), and (17) ≈ 1/(20).

- Note the dimensionless numbers have the same values in any system of units.

The other three independent empirical quantities of particle physics come in mutually dependant pairs: (7,8), (9,10), (11,12). We can take the three DNs: *(7,9,11)* to represent them all. In a certain sense, these are the only three “absolute quantities” that have been measured in the fundamental physics of gravity and electromagnetism! (Apart from the perfect equality of the two elementary charges, a fourth absolute quantity: $q_e/q_p = -1$).

The five cosmological quantities (16-20) are independently measured properties (… from observations and theoretical assumptions …) for the universe as a whole.

So this is a system of natural units based on taking the *electron* as the most fundamental particle. Changing the system of units makes no difference to the value of dimensionless numbers, or the numerical relationships present. However:

- The LNC’s are still open to interpretation as LNR’s, depending on the relations we posit.

In the system of relations we introduce next, the natural units are based around a different choice, and we will see that this imposes stronger relations and improves the accuracy of the LHC. This choice is based on an underlying model of the relations. However the main feature of this system is that it forces us to work in two different systems of units or variables: the “true variables” in which the model relations are correctly expressed, and our “ordinary variables”, as defined instrumentally in ordinary physics (and represented in the table of values above). Dirac [1969, 1974] introduced this distinction, as presented in [Ray, 2019, Section 3] as the distinction between the “Einstein metric” and the “Atomic metric”. They explain the need to distinguish the metrics, but only in the context of Dirac’s theory (or some derivatives of it). In our view, this point is not generalised adequately in any discussions, and remains the central point of incompleteness for systematically constructing LNR theories.

12 Again, the charge of the proton, now -1, is an empirically measured result, but because of its precise relation to the electron charge, it is not taken as an additional *contingent quantity.*

Section 3. A system of evolving constants.

Introduction.

We now introduce an alternative system of time-varying constants and LNR's. Dirac started by first observing the LNC's, and then proposing a cosmological theory with fundamental constants changing in time to force these LNC's as lawlike relations. Our theory started from the opposite direction: we started with a fundamental model (a proposal for a unified theory), which then quite independently turned out to require LNR's, between three quantities of length, the *universe radius,* R or R_U*, the electron radius,* $R_e = \hbar/m_e c$*, and the proton radius,* $R_P = \hbar/m_P c$. As a result the last two quantities must *decrease* as the universe radius increases (in an expanding universe). See Appendix 2 for a little more detail on this theory, but we do not need to introduce it in any detail here, except to illustrate why it forces a set of LNR's on us. It entails LNR's, and poses the problem of how to represent and calculate the time variations in the fundamental constants. We highlight three differences with Dirac's theory:

- *(A)* The evolution equations for the local constants, *c, h, G, m, q, etc,* will be functions of the *universe radius,* not the *universe age.*
- *(B)* A systematic method of representing formal transformations between alternative physical units for all the physical quantities, *[X, T, M, Q],* is needed to calculate *measurements,* since the time variations in local constants now also affects our ordinary measuring instruments.
- *(C)* The LNR's are determined precisely by the underlying model, so we obtain a more precise theory, which makes more precise predictions.

The major constraint added to conventional cosmology and physics is that the following quantity, a *six-D volume,* is constant over time:

$$(\pi/2)R_U^3 R_p^2 R_e = \text{Constant (6-D vol)} \qquad (1)$$

This is a strong constraint that more or less determines the rest of the theory. We can see the key implications fairly quickly. First we define this special "fundamental length" as a key variable:

$$R_W^3 = R_p^2 R_e \qquad \text{or:} \qquad R_W = (R_p^2 R_e)^{1/3} \qquad \text{Definition of } R_W.$$

This is a kind of "average" of the electron and proton radii. It is called their *volume-average,* because in the underlying model, R_e and R_P are the major and minor radii of a torus, and the volume of a torus is: $(2\pi^2)(R_p^2 R_e) = (2\pi^2)(R_W^3)$. In the physical model, this *volume* (combined into the *hyper-volume* in (1)*)* is what the *conservation law* (1) really applies to.

(1) therefore means we can define an *invariant length, L*.

$$L^6 = R_U^3 R_W^3 = \text{Constant (6-D vol)} \qquad \text{From (1).}$$

Or: $$L^2 = R_U R_W = \text{Constant (area)}$$

The quantity L becomes the (only) fundamental *length* in the model, being invariant with time. So this is the key fundamental postulate. This invariance principle, *with this precise combination of* R_{or}

and R_p*,* is the extra key constraint on the system that our underlying model determines, which makes it quite different to Dirac's theory or other LNR theories.

Because $R(t) = R_u(t)$ is changing with time, and remembering L is a fixed constant length, we get the inverse dependence between $R_W(t)$ and $R_U(t)$:

$R_W(t) = L^2/R_U(t)$ Equation of motion from (1).

And: $R_U(t) = L^2/R_W(t)$

The theory is then primarily determined by a second law identifying the quantity *L.* The solution in our model is simply:

$$R_U/R_W = D_W = \hbar c/m_w^2 G \qquad (2)$$

This is the key LNR relation in the new theory. It is essentially similar to Dirac's main LNC, but whereas Dirac's relation is open to different choices of mass in D, in the context of our underlying model, this specific form is forced on us. From (1), we see this is equivalent to identifying L as:

$L = R_W \sqrt{D_W} = R_U/\sqrt{D_W}$ From (1) and (2).

We can substitute for R_W to get:

$R_U = R_W D_W = \hbar^2/m_w^3 G = \hbar^2/m_e m_P^2 G$ From (1) and (2).

Putting in the values, the result predicted for the present universe radius then turns out to be:

R_U *= 6.91 billion l.y.* Prediction.

Remarkably this is almost *exactly half* the measured age of the universe (13.8 b.l.y) times the speed of light.

In normal cosmology, this is the radius of the *co-moving light sphere* around a point in the early universe. In the full interpretation of the model, this is exactly what it turns out to be (modified by a small integration constant), but this also turns out to be predict a full universe "radius". Before we work it out more precisely, this already appears much more accurate as a LNC than the Dirac-type theories, where the choice for the *mass term* in D is undetermined, and can vary by a factor of 10^6. It is now quite strictly determined, and the attempt gets it within ½ of an exact value.

These quantities are functions of time, so we may write this as:

$$R_U(t) = R_W(t)D_W(t) = \hbar(t)^2/m_w(t)^3 G(t) = \hbar(t)^2/m_e(t)m_P(t)^2 G(t)$$

But solving: $R_U(t)$*,* $\hbar(t)$*, etc,* directly as functions of time is not possible without a stronger relationship. The key step is to take *h, c, m, G* as direct functions of R_U*,* so we can leave time out of the equation. This is required by our underlying model, and distinguishes it from Dirac's theory.

So this gives us a third postulate, to the effect that the quantities are simple functions of the *normalised radius,* defined as[13]: $\hat{R} = R/R_0$, where R_0 is the *present universe radius.* There are simple polynomial functions in $\hat{R}$: $c = c_0\hat{R}^A$, $h = h_0\hat{R}^B$, $m_w = m_{w0}\hat{R}^C$, $G = G_0\hat{R}^D$, where c_0, h_0, m_{W0}, G_0 are the *present values* of these constants, and *A, B, C, D* will be whole numbers. This gives a simple system of relationships.

Now in general there could be a large number of possibilities for these functions. But our choice is again forced by our underlying model, and it is very simple. We can state the main solution (without the electric constants) as essentially:

Dynamic relations proposed for the constants.	(3)
$c(\hat{R}) = c_0\hat{R}$	Speed of light increases linearly with $\hat{R}$.
$h(\hat{R}) = h_0/\hat{R}$	Planck's constant decreases linearly with $\hat{R}$.
$m(\hat{R}) = m_0/\hat{R}$	Mass decreases linearly with $\hat{R}$.
$G(\hat{R}) = G_0$	G is truly constant.

These give temporal functions if we can solve the expansion function: $\hat{R}(t) = R(t)/R_0$. But the physical relationships are proposed as simple direct functions of $\hat{R}$. The time evolution is derived in a second step. As a consistency check, note these relations determine the evolution of *D* correctly as:

$$D(\hat{R}) = h(\hat{R})c(\hat{R})/m_W(\hat{R})G(\hat{R}) = (h_0c_0/m_{W0}^2G_0)\hat{R}^2 = D_0\hat{R}^2 = D_0(R/R_0)^2$$

This is consistent with (1) and (2), which state that: $D = (R/L)^2$ and: $L = constant = (R_0/\surd D_0)$ at the present time, so: $D = (R^2)(\surd D_0/R_0)^2 = D_0(R/R_0)^2$.[14] The system is internally consistent.

This is the essential idea, and it looks simple, but the presentation above is not adequate, because it does not systematically distinguish between two different systems of *basis units* for physical quantities that are required. We normally assume *basis units for the quantities {X,T,M,Q]* are fixed or static for measurements of our normal constants, *c, h, G, m, q,* etc. But when we introduce dynamic constants, we find that our ordinary basis units must be changing relative to the "true" basis units, in which the constants are changing. If we do not make this distinction properly, we will be confused about whether variables are referring to the normal *instrumental system of basis units,* and the *dynamic basis units.*

To represent this unambiguously, we will retain our normal constants and variables in the normal (undashed) system, as the *static constants:* $c = c_0$, $h = h_0$, $m_e = m_{e0}$, etc. Their values are defined instrumentally in the normal way, so they do not change. E.g. in this system, the mass of the electron, m_e, is *defined* as a unit mass, against which other masses can be compared, so *even if the electron mass really decreases,* the numerical value of m_e does not change.

Indeed, we can take the natural units we saw earlier, where: R_e, *q, c, h,* are defined with the value 1 *at all times*. But this does not mean these quantities are truly constant, and in our dynamic system

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[13] We now use the variable: $R = R_u$ for the universe radius to simplify the notation slightly.

[14] The relations can be seen to be forced by: $c(\hat{R}) = c_0\hat{R}$ which is intrinsic to the model, and: $m(\hat{R}) = m_0/\hat{R}$ which is forced so that momentum: $mc = m_oc_0$ is *constant,* and then: $mcR_w = \hbar = h_0/\hat{R}$ which is forced by angular momentum in the model, and then (2) which forces the evolution of *G*. See later.

they change. Note also that *G* depends on *D* in these units, so *G* and *D* can change in tandem. Similarly, m_P and ρ would have to change together, and likewise ε_0 and α.

We define the dynamic constants using a second set of dashed variables: $c'(\hat{R}) = c_0\hat{R}$, $h'(\hat{R}) = h_0/\hat{R}$, etc. These are the quantities that vary with time – and in fact we should be using these quantities in the relation (3) above, not the undashed variable. From now on, we will systematically use the dashed and undashed variables in this way, so we do not get confused between the two.

Note also that only dimensionless quantities are identical in both systems of units – because they have no physical units, and there is no transformation of these quantities between the two system. So we always have: $D = D'$, $\alpha = \alpha'$, $\rho = \rho'$.

As Dirac [1969] realised, this requires a transformation between *the basis units used in our normal physical quantities [X,T,M,Q]* and *the basis units for the evolving physical quantities, [X',T',M',Q'].* We will define these through differential transformations, relating the differential quantities: *dr, dt, dm, dq* (normal system) to: *dr', dt', dm', dq'* (dynamic system). So in the following sections we strictly distinguish these two systems of variables. (Dirac distinguished them in his specific theory as "atomic metric" and "Einstein metric".)

If we do not work out the transformations explicitly, in a suitable formalism, our theoretical predictions, relating the theory to the measurement variables in experiments, cannot be trusted. This holds for theories like Dirac's [1974] as well. We may wonder why we do not see any such system of (scale) transformations used in conventional cosmology in experimental tests of LNR's? How do the analysts work out the *alternative hypothesis predictions* if they do not transform to the *alternative hypothesis metric?* Because these *scale transformations* are not a normal part of GTR.

Dirac recognised this need for *transformations between physical basis units whenever we have evolving constants,* and provided an example of how to work it out in his [1969,1974]. But because he only considered one example, and chose it as simple as possible, his approach is not understood or introduced as a systematic method yet. Dirac's theory makes *G* change with time, and other constants remain constant, so physicists try to test this by measuring for changes in the strength of *G.* (E.g. the Lunar Laser Ranging experiments.) We will reanalyse these experiments on our alternative LNR theory.

Now we have stated the basic relations we will introduce, without explaining why. To conclude this section, we briefly sketch the physical model underlying our theory. This may be useful to help visualise the relations in a concrete way.

The geometry of the ring.

We now need to work out how to represent our models and make predictions. To illustrate the essential feature, we start with the following simplified example. Suppose we have a *ring,* in two dimensions, which has a certain area. The ring can expand (or contract), with the law is that *its area is constant*.

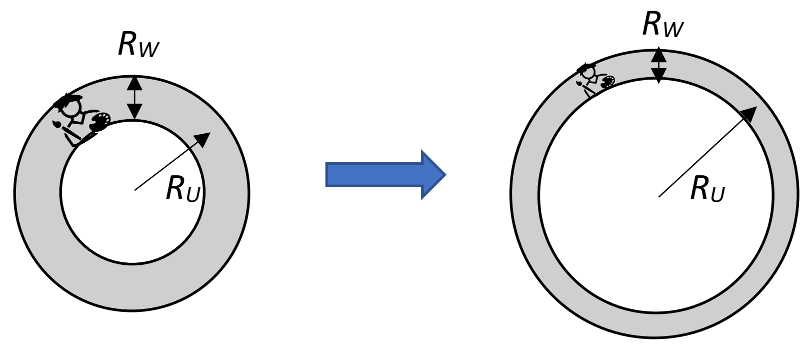

Fig. 1. An expanding ring, with a constant area. Things in the ring shrink.

We characterise the ring with two radii, R_U and R_W. The area is: $L^2 = 2\pi R_U R_W$. The law is that this area is constant as the ring expands. Hence we have the same relation as stated in (1) above. There is an invariant length, L defined by: $L^2 = R_U R_W$. As the large radius expands, the small radius contracts: $R_U = L^2/R_W$ or equivalently: $R_W = L^2/R_U$.

Now we suppose that there are natural "objects" on the ring, each with a natural unit size of R_W. We can call these "atoms". We assume that *all local physical objects* are changing in size exactly as R_W. These atoms can be used as measuring rods, to measure distances around the circumference. From the point of view of someone living on the ring, the natural *instrumental definition* of a unit length at any moment is R_W. In "natural units" we set this value as 1 (constant at all times).

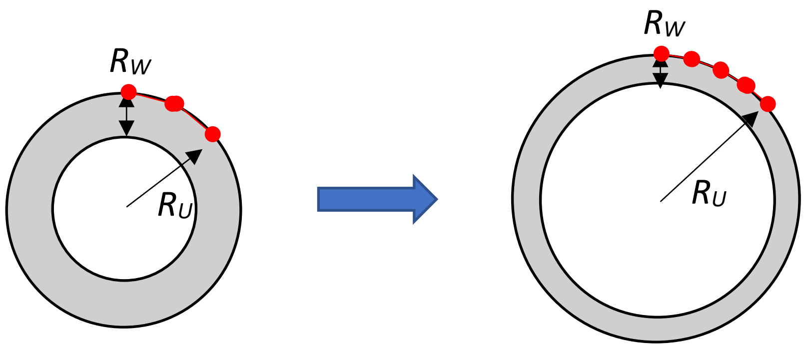

Fig. 2. Measuring distance between two fixed points around the circumference, using atomic units. As the ring expands, the atomic units shrink. This diagram itself shows the process of expansion in true units of space. The distance in "atomic units" doubles – but in fact the true distance has only increased by √2.

The ring is expanding, and the distance between two fixed points around the circumference is increasing. But the expansion rate in true variables is not the same as in the conventional *instrumental atomic measuring units.* If: $R = f(t)$ for some function $f(t)$, in atomic variables R for space, we will find that: $R' = \sqrt{f(t)}$ in true variables for space. So we have two different measurement units, and we can transform between them.

In true units, the ring is expanding at only the rate of $\sqrt{f}$, which is about ½ the expansion rate that appears in the atomic units, measured from within the ring-world.

Atomic Variables.	**True Variables.**
$R_W = constant = 1$	$R_W' = L^2/R_U'$
$L = \sqrt{(R_U R_W)} = \sqrt{R_U} = Increasing$	$L' = \sqrt{(R_U' R_W')} = constant = 1$
$\mathbf{R}_U = f(t)\,\mathbf{R}_0$	$\mathbf{R}_U' = \sqrt{f(t)}\,\mathbf{R}_0'$
$R_U R_W = L^2 = f(t)$ (because $R_W = 1$)	$R_U' R_W' = L'^2 = constant$
	$R_W' = L^2/R_U' = L^2/\sqrt{f(t)}$

In our model the solution is:

$f(t) = \check{D}(t) = D(t)/D_0$ $\qquad\qquad$ $f(t)' = \check{D}(t) = D(t)/D_0$

Here we make $\mathbf{R}_U$ a vector, and *f(t)* must be a dimensionless function of time. The fourth line illustrates a key feature: in atomic variables, *L* increases, while in true variables, *L* is constant. Hence this conservation law only holds correctly in dashed variables.

Two examples of *f(t)*.

To get the idea, let us first just try the quadratic function: $f(t) = (t^2/t_0^2) = \check{T}^2$. Then R_U appears to be expanding quadratically in time, with an accelerating expansion. But in true variables: $\mathbf{R}_U' = \sqrt{f(t)}\,\mathbf{R}_0' = (t/t_0)\,\mathbf{R}_0'$, with only a linear expansion in time.

Or we could make *f* a cyclic function. For example: $\mathbf{R}_U' = sin(A\check{T})\mathbf{R}_0'$. Where the $sin(A\check{T})$ is in the first quadrant, i.e. $A\check{T}\ \varepsilon\ (0,\pi/2)$, for the expanding phase. Then: $\mathbf{R}_U = sin^2(A\check{T})\mathbf{R}_0$. Then differentiating by time, the speed in R_U' is: $Acos(A\check{T})$. The speed in R_U is: $2Acos(A\check{T})sin(A\check{T})$.

Then the acceleration in R_U' is: $-A^2cos(A\check{T})$. This is always negative in our quadrant. The acceleration in R_U is: $(2A^2)(cos^2(A\check{T}) - sin^2(A\check{T}))$. This starts positive and switches to negative. Now this is quite surprising. Our universe currently looks like its expansion rate began increasing, perhaps 7 billion years ago. But if we are in a cyclic universe, in our type of model, we might be in a stage quite early in the cycle, when the expansion rate appears to be accelerating, but is not really.

What will appear to happen in the conventional variables? Sin-squared is steeper than sin.

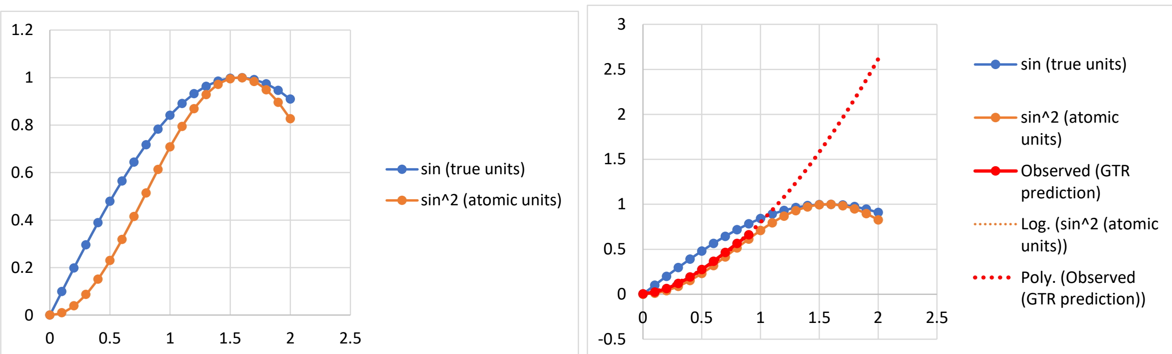

Fig. 3. Left. In the ((brown) atomic units, the expansion rate is indeed increasing, until half-way through the quadrant. Then it slows. The true expansion rate (blue) is always slowing. Right. The present GTR-based theories extrapolate (red dots) from the observed rate, and predict R will keep expanding forever. A cyclic model predicts the brown and blue lines.

Physicists now measure that the rate of expansion is increasing (red lines). This is *in conventional units,* and as far as observations of past expansion goes, it appears consistent with our model – while we are still in the first part of the expansion. But it is predicted behaviour in the second part of the cycle, when the contraction begins, where the conventional GTR models will go wrong.

Note that in the atomic (instrumental) units, the law we assumed at the start, that *the area is constant,* does not hold. It only holds in true variables. The underlying *model* only makes sense in true variables, because it is based on conservation principles applying to physical-geometric objects existing through time with certain constant properties.

Time translation invariance.

Instrumentalists may ask: "*how do we know what the 'true' variables are?* Why not assume the *instrumentally defined variables are the true variables?* We can *transform between the two variable systems,* and write the laws and description of the physical system in either way. If we do this correctly, we will always confirm the same empirical predictions."

This is not really in issue for us, but in any case our answer is simply that *the laws of physics are only time translation invariant* in *the true variables for time* (and similarly for space). The laws of our system above – and more important, the laws of real physics – are based on conservation laws. Quantities like *momentum and energy* are taken to be constant in time. If we transform to alternative time coordinates that alter the relative time intervals (e.g. using $\sqrt{t}$ instead of *t)* conservation laws no longer hold. The laws of physics are simple and coherent in variables that make conservation laws work, and these are what we adopt as *true variables*. In our example above, this applies to a conservation for space (area). The true variables give the laws an especially simple form. If we transform the physical units for space or time to a non-linear system, the laws of physics look very messy – in fact, incomprehensible.

To continue with the model, the ring model above illustrates the need to transform between two systems of variables. The real model takes space to be *six dimensional.* We take the three ordinary spatial dimensions, (x, y, z), as usual, and assume they are curved, as in the usual type of standard Friedmann-GTR cosmology. This defines a finite closed 3-space, which is expanding. The expansion is represented by our universe radius, R. We take this space to have a spherical symmetry (on the large scale of the universe). See Appendix 2 for some more details.

The novel conjecture of our model is to add three additional dimensions, which are curled up on a microscopic scale into a *torus.* The size of this torus is defined by the *electron and proton radii.* The electron radius is the large torus radius, and the proton radius is the small torus radius. (Actually strictly speaking they should be half the Compton wavelengths, but we worry about small factors of two or pi later, when the physical relations are clear).

This special topology is what gives the QM particle mechanics. There seems little choice in a six dimensional space except to adopt a torus, because this is what gives the precise properties of the electron (Dirac wave functions) and ultimately the electromagnetic force dynamics. In any case, this is the proposition.

Hence the full spatial manifold is this 6D hyper-volume, with a finite, closed 5D surface. The fundamental law (1) applies, and it means that the six dimensional volume of this *Torus X 3-Sphere* manifold is conserved. So as *R* increases with the universe expansion, the torus radii decreases – meaning the *electron and proton radii decrease* – to keep the total volume constant, just as in the example.

The total hyper-volume is given by (1), and it is easy to see that it must be proportional to the product of: $R_U^3 R_P^2 R_e$, with a small geometric constant of integration ($\pi/2$). Hence we have conservation of this volume, so we must define the volume-average radius as: $R_W^3 = R_P^2 R_e$. This forces the dynamics.

The other laws (2) and (3) are also required by this model, although they require a little more effort to show. We not do this in detail here, but the reasons are briefly as follows.

- First, the speed of light evolution (3), which we write in the dashed variable: $c' = c_0\hat{R}$, is forced by the gravitational model (or GTR) because this law must correspond to how the speed of light changes locally, through the Schwarzschild solution (or rather the model variation of it, called *K-gravity.*[15]) Physically, we think of the speed of light as dependant on the *surface tension,* and in the model this increases with expansion – which changes the *Area/Volume ratio* for the manifold.
- Second, *conservation of momentum in each spatial dimension independently* remains the most fundamental law, and since: $p = c'm' = constant$, and this forces: $m' = m_0\hat{R}$. This holds for all masses. (Energy is slightly more complicated – we do maintain energy conservation in the model, but energy is now exchanged between the manifold and the particle-waves. But we do not have to go into this here).
- Third: $h' = h_0/\hat{R}$, because the angular momentum is: $m_W'c'R_W' = \hbar' m_W'c'/c'm_W' = \hbar' = \hbar_0/\hat{R}$.
- Fourth, the mass ratio: $\rho = m_p/m_e$ must be constant given the mass evolution is the same for all masses.
- Fifth, the Dirac constant: $D' = h'c'/m'^2G'$ must evolve as: $D' = D_0\hat{R}^2$ to maintain invariance of *L,* and this means: $G' = h'c'/D'm'^2 = (h_0/\hat{R})(c_0\hat{R})/(D_0\hat{R}^2)(m_0/\hat{R}^2) = G_0$, so the gravitational constant must be constant.
- Sixth, the electric constants are determined in a similar way, which we will not go into, except to observe that the model now identifies the *unit mass* as m_W, and the mass ratio as: $\rho' = m_e'/m_W' = 1/149.9$ and we identify this with the fine structure constant: $\rho' \approx \alpha = 1/137.$ This is accurate to about 9%. The reason it is not perfectly exact is one of the main features still to be explained.[16]

[15] [Holster, 2017].

[16] It is thought it is because we are in a galaxy with a substantial mass and not a truly "flat" region of space, but we will not worry about this. A LNR prediction accurate to 9% is quite good enough at this point.

The shrinking atoms phenomenon.

To illustrate again, in this model the universe expansion makes physical objects like atoms and rulers shrink in length, and atomic-based clocks run faster, while the distances between galaxies increases as the universe expands.

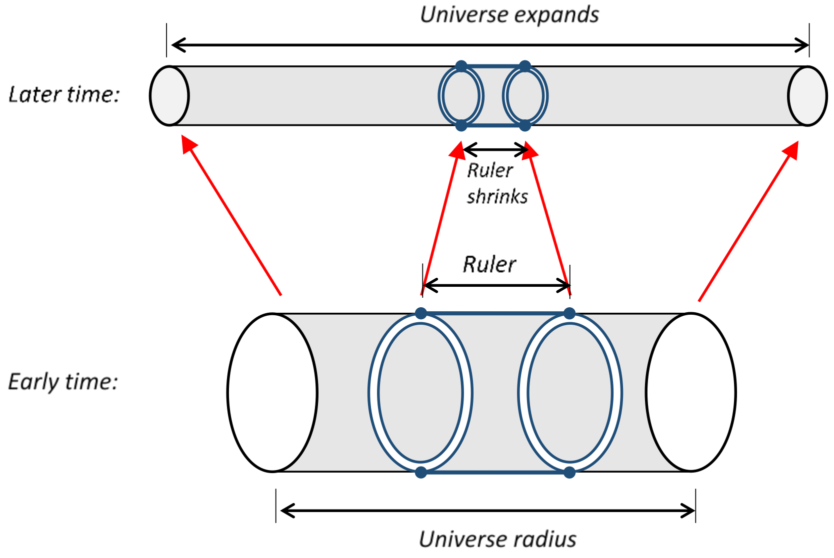

Fig. 4. The expanding universe has two opposite effects. Local objects bound together by EM forces, e.g. atoms, molecules, rulers, etc, shrink – as the torus dimension shrinks. The universe as a whole expands, and unbound objects like galaxies move apart.

As the universe expands and the torus shrinks, the *real dimensions* of atoms shrink, the *real speed of light* increases, the *real particle masses* decrease, the *fine structure constant* is constant, the real linear momentum is constant. Because of the scale symmetry, as the torus shrinks, all physical objects shrink with it. Rulers, clocks, our bodies, etc. We are actually shrinking people! Yikes! But we don't notice because everything is shrinking around us. Locally, all the material objects that are locked together in strong (meaning EM) bonds are shrinking together.

But the opposite effect is happening on the astronomical-intergalactic scale: the distance between galaxies (or objects not bound by local forces) increases, as they are separated by the global expansion of space. This is the Hubble expansion. There is a question about gravitationally bound objects, like planets in a solar system, or stars in a galaxy, however. This is more difficult to figure out in a theory where space is stretching and the gravitational force is decreasing.

The upshot is that we need to find transformations from conventional *atomic or instrumental variables*, which are changing, into *true model variables,* which are not changing. The latter are the dashed variables. This applies to the *base units for: r′, t′, m′, q′,* or in our representation, the *differential quantities: dr′, dt′, dm′, dq′*. These units are *not changing*. The instrumentally-defined quantities or coordinates that use the mass of atoms and periods of atomic processes, etc, to define space and time measurements, are changing.

Transformations and evolution equations.

We now specify the system of variables and dynamics. We will give functions for the *values of constants like c, h, G, etc,* in terms of a *normalised radius of the universe, R-hat:*

$\hat{R}(t) = R_U(t)/R_0$ Normalised radius.

Here: $R_0 \equiv R_U(t_0) \equiv R_U(Now)$ is the present radius, and $t_0 = Now$ is the present time for us. We must recognise in the formalism that our measurements and observations are made *at our present time,* when the universe radius and the constants have their particular values *Now*.

We use the special time variable: *T* with the origin: *T = 0* set at the minimal expansion in the past (the Big Bang). So *T* effectively represents *time since the Big Bang,* which is what physicists generally call the *age of the universe.* It has the same metric as ordinary time, *t,* i.e. $dT = dt$.

We can imagine physicists at a different cosmological time to us, say a billion years ago, who measure quantities *instrumentally* in the same way as we do. Some of the constants will have changed between the two times, and their *basis units for time, space, mass and charge* will be in a different system of physical units to ours. E.g.

- If we both define mass using the *mass of the electron or proton* as a standard unit, the *basis unit for mass* is reducing.
- If we both define length using the *Compton radius of the electron:* $R_e = \hbar/m_e c$ as a standard unit, the *basis unit for length* is reducing.

This means there is a difference between *true variables,* and *measured variables.*

- The laws of physics must be written in the *true variables* to have their correct form to match the model, which makes them *time translation invariant.*

Only in the true variables will the laws be simple and invariant and correspond to the model.

So we assume that there is a system of *true variables for space, time, mass, charge* in which the laws have their simplest form, and are *time translation invariant*.

- We will take: (x,t,m,q) as our conventional *measured variables,* and (x',t',m',q') as the *true variables,* and establish transformations: $(x,t,m,q) \Leftrightarrow (x',t',m',q')$.
- Similarly we take: $(c,h,G,m_e,m_p,q,\varepsilon_0)$ as our *measured constants,* and $(c',h',G',m_e',m_p',q',\varepsilon_0')$ as the true constants, and establish transformations between them.

The same issue arises for our *radius* variable: R_u and "age of the universe" *variable: T.* We must use the true variables, R_u', and *T',* to define the fundamental laws, and subsequently transform them back into our conventional variables, *R* and *T* to interpret ordinary measurements. So actually we must use *normalised variables* in these true variables as the key parameters in evolution functions.

Normalised true and conventional variables.

$\hat{R}' = R'/R_0'$ and: $\hat{R} = R/R_0$	Definition of normalised radius universe.
$\check{T}' = T'/T_0'$ and: $\check{T} = T/T_0$	Definition of normalised age.

The *true normalised radius variable*: $\hat{R}'(T') = R_U'(T')/R_0'$ must be used as the parameter in the functions for the laws of nature.

Note we use *R* or *R'* for the universe radius now, instead of R_U or R_U', as long as it is unambiguous. We generally use a subscripted *0* to represent *present values.*

The basis unit transformations are defined by differential transformations like: $dt = f'dt'$, where *f's* are all *simple functions of the normalised radius,* $\hat{R}' = R'/R_0'$*.* The solutions for variables and constants in our system are as follows.

The variable transformations. (3.1)

$dx = \hat{R}'dx'$	Space metric transformation
$dt = \hat{R}'^2 dt'$	Time metric transformation
$dm = \hat{R}'dm'$	Mass metric transformation
$dq = \hat{R}'dq'$	Electric charge metric transformation

The evolution equations for the constants in true variables. (3.2)

$c' = c_0\hat{R}'$	Evolution of speed of light constant.
$h' = h_0/\hat{R}'$	Evolution of Planck constant.
$G' = G_0$	**True gravitational constant is constant.**
$m_e' = m_{e0}/\hat{R}'$	Evolution of electron mass.
$m_p' = m_{p0}/\hat{R}'$	Evolution of proton mass.
$q_e' = q_{e0}/\hat{R}'$	Evolution of elementary electron charge.
$\varepsilon' = \varepsilon_0/\hat{R}'^2$	Evolution of electric force constant.
$\mu' = \mu_0$	Evolution of magnetic force constant.

Note that *the gravitational constant G* is the only constant that is truly constant in true variables, while all the others change. We will see that the reverse applies in the instrumental variables!

The transformations are completed by specifying boundary conditions. First at the origin:

Boundary conditions. (3.3)

$R \rightarrow 0$ as $R' \rightarrow 0$			Zero radius of the universe.	
$T \rightarrow 0$ as $T' \rightarrow 0$			Zero age of the universe.	
$dx_0' = dx_0$	$dt_0' = dt_0$	$dm_0' = dm_0$	$dq_0' = dq_0$	
$c_0' = c_0$	$h_0' = h_0$	$G_0' = G_0$		
$m_{e0}' = m_{e0}$	$m_{p0}' = m_{p0}$	$q_{e0}' = q_{e0}$	$\varepsilon_0' = \varepsilon_0$	$\mu_0' = \mu_0$

The second set are the BCs matching true and conventional values at the present time.

This defines the general system of dynamic constants. We can now solve the evolution of conventional variables in terms of true variables.

Inverse evolution equations. **(3.4)**

$c = c'/\hat{R}' = c_0$	c is constant.
$h = h'\hat{R}' = h_0$	h is constant.
$G = G'/\hat{R}'^2 = G_0/\hat{R}'^2$	**G is decreasing.**
$m_e = m_e'\hat{R}' = m_{e0}$	m_e is constant.
$m_p = m_p'\hat{R}' = m_{p0}$	m_p is constant.
$q_e = q_e'\hat{R}' = q_{e0}$	q_e is constant.
$\varepsilon = \varepsilon'\hat{R}'^2 = \varepsilon_0$	ε_0 is constant.
$\mu = \mu' = \mu_0$	μ_0 is constant.

Note that all the conventional variables *except G* are constant. This is expected as the "atomic units" are all defined instrumentally so that they must be constant.

Evolution of special quantities.

Note that the *conservation of 6D volume* holds in *true variables,* by substitution from above:

$$R_U'R_A' = R_U'\hbar'/c'(m_e'm_p'^2)^{1/3} = R_U\hbar/\hat{R}'c(m_em_p^2)^{1/3} = R_0R_{A0} = constant$$

But the 6D volume in conventional variables is not constant:

$R_UR_A = (R_0R_{A0})\hat{R}$ Volume in conventional variables varies with R.

This is an example of the fact that *the laws of nature only appear to be invariant in true variables.* In true variables, constants change, to reflect properties of space, e.g. the *speed of light* increases with expansion, and the speed of atomic processes increases, including clocks. Physical objects shrink, including rulers. In conventional variables, these appear constant, *by their instrumental definitions* – because instrumentally, we are only comparing one physical object with another, and there is nothing to tell us what the true quantities are. But the laws of nature are only *time translation invariant* in the true system. Conservation laws mean certain quantities are constant in time, but this only appears to hold in true variables. It fails in conventional variables.

Dimensionless quantities.

However the *dimensionless quantities* have the same values in either system. There are four independent dimensionless quantities the characterise our model.

- The *fine structure constant,* α*,* is predicted to be constant:

  $\alpha = q^2/4\pi\varepsilon_0hc = q_0^2/4\pi\varepsilon_0h_0c_0 = constant = 1/137$ Fine structure is constant.

This holds in either set of variables (because it is *dimensionless)*, and it follows by substituting from the equations above. Similarly:

- The dimensionless ratio of electron to proton mass is constant:

$m_e/m_p = m_{e0}/m_{p0} = constant = 1/1836$ Mass ratio is constant.

In our system, the *average mass ratio* is really the fundamental quantity, and it is constant:

$m_e/m_A = m_{e0}/m_{A0} = (1/1836)^{2/3} = 1/150$ Average mass ratio is constant.

This is why it is possible to identify: $\alpha \approx m_e/m_A$ in our model.

The dimensionless Dirac constant defined from *c, h, G, m,* increases with expansion.

- The dimensionless *Dirac constant, D,* changes with $\hat{R}'^2$.

$D = \hbar c/m_A^2 G$ Definition Dirac constant.

Substituting values from above:

$$D = \hbar c/(m_e m_p^2)^{2/3} G = D_0 \hat{R}'^2$$

We must use the mass combination: $(m_e m_p^2)^{2/3}$ in the theory, because this is the *invariant average mass* corresponding to volume. Because *D* is dimensionless it has the same value in any variables:

$D' = D$ Identity.

And there is one more: the dimensionless cosmological ratio:

- The universe – torus ratio changes with $\hat{R}'^2$.

$R_U/R_A = (R_{U0}/R_{A0})\hat{R}'^2$ Universe/Torus ratio.

We then state the fundamental relation, that was predicted in a different form by Dirac, by equating these two dimensionless constants:

$R_U'/R_A' = D' = D$ Dirac-TAU relation.

So this predicts the *radius of the universe* from purely local constants as:

$$R_U' = R_A' D = \hbar'^2/(m_e' m_p'^2)G'$$

This relation holds at all times, so we can substitute the present values: $\hbar_0^2/(m_{e0} m_{p0}^2)G_0$, and we get the value as (the simplest possible combination, without geometric factors, except in $\hbar^2$):

$R_{U0}' = \hbar^2/(m_e m_p^2)G = 6.91$ *billion light years.*

- This is exactly half the measured *age: 13.82 billion years times the speed of light.*

It seems quite a striking coincidence that is *precisely half the measured age of the universe times the speed of light.* There are two main current methods used to estimate the age (time since the Big Bang), which give measured values of about 13.77 - 13.82 billion years.

The age predicted by these methods is really giving us estimates in some way or other of the time it has taken light to travel across the universe from events shortly after the Big Bang. So we can calculate *co-moving coordinates* of the light source relative to us now (receiving it), and this is what the result appears to represent: the age times the speed of light. We are measuring how far the light has come, and this seems to be what the radius in our model means. We see this is consistent when we subsequently solve for the actual time evolution of $R'(t')$. The measurement of age from the CMBR also measures how far the light has come from its production in the primordial universe, divided by c to get the time.

This prediction looks eerily like it is pointing to an *exact relationship with the age of the universe.* In which case, we may ask: where does the factor of ½ come from? But factors of ½ and 2π, etc, are floating around in our geometry, and in the choice of h or $\hbar$. (In fact, we should strictly define R_w as half the value above to conform to our real electron-proton model.) And we expect the relation to predict a comoving-distance around the *circumference,* the comoving distance from the light source, not the radius or total circumference of the universe. (The latter will turn out to be more like 40 bly.) We should not worry about the factor of ½ initially, nor conclude that the coincidence above points to any exact relation yet, because before we can interpret it there are several more questions.

First, our prediction is for the *present "radius" of the universe,* in the *true distance variable,* R_{U0}'. But:

- How does this prediction of a value for R_{U0}' relate to the *conventional variable,* R_{U0}?

Second, the "coincidence" is that the *predicted radius* is related to the measured *age,* T_0, by the speed of light: $R_{U0}' = (½)T_0c$.

- But how does this predicted radius relate to the conventionally measured age in our theory? And what is the relation between T_0 in conventional variables and the true age, T_0'?

We will next determine the transformations: $R' \rightarrow R$, which is necessary to determine the predicted rate of change of *G in conventional variables.* We then return to the second question.

Section 4. Predictions and tests of gravity.

Note that our evolution equations above predict that:

- All the constants appear *invariant* in conventional variables *except G* (which actually just conforms to the instrumentalist definitions of conventional variables), but:
- All the constants actually *change* in true variables *except G*.

Hence the primary measurable prediction is that:

- G in conventional variables is decreasing with $1/\hat{R}'^2$ (in true variables).

This predicts the rate of change of G w.r.t. the *true variable,* $\hat{R}'$. But what we measure in experiments to determine the rate of expansion (the Hubble constant) is the rate of change w.r.t. the *conventional variable,* $\hat{R}$. And this is determined from measurements made *at the present time* – but referring back (through light sources from various types of stars) to distant events in the past. We have to translate the prediction for G into $\hat{R}$.

The space integration.

We can relate R and R', using the differential transformations above. We integrate using: $dR/dR' = dx/dx' = \hat{R}'$, and use the boundary condition that: $R \rightarrow 0$ *when* $R' \rightarrow 0$. (We will not assume a singularity in R' or R at the temporal origin, just a very small minimal value of R' and R close to the time origin we set for T.) For an arbitrary radius, R_1, we take the definite integral:

$$R_1 = \int_{0,R1} dR = \int_{0,R1'} (dR/dR')\, dR'$$
$$= \int_{0,R1'} \hat{R}'\, dR' = \int_{0,R1'} R'/R_0'\, dR'$$
$$= [R'^2/2R_0']_{0,R1'} = R_1'^2/2R_0'$$

This gives the key relationships, for an arbitrary radius R:

Solution for *R*. **(4.1)**

$R = R'^2/2R_0' = R'\hat{R}'/2$	General solution for R.
$\hat{R} = R/R_0 = \hat{R}'^2$	Solution for normalised radii.
$R_0 = R_0'/2$	Solution for present radius.

And the corresponding inverse relations:

Solution for *R'*. **(4.2)**

$R' = 2R/\sqrt{\hat{R}}$

$\hat{R}' = \sqrt{\hat{R}}$

$R_0' = 2R_0$

The relation between R and R' is quadratic, not linear, but at the *present moment* the values are given by the simple linear relationship: $R_0 = R_0'/2$ or: $R_0' = 2R_0$.

Now substituting in the relation: $G = G_0/\hat{R}'^2$

$G(R) = G_0/\hat{R}$ Change of G in conventional variables.

So the rate of change is only the *linear inverse of* $\hat{R}$, not the squared-inverse, as on Dirac's original (1937) theory. Hence: $dG/dt = d(G_0/\hat{R})/dt$ at the present time, when: $R = R_0$. So:

$$\dot{G} = dG/dt = d(G_0/\hat{R})/dt$$
$$= -(G_0R_0/R^2)(dR/dt) = -G_0(dR/dt)/R_0$$
$$= -G_0(d\hat{R}/dt)$$

Hence the *normalised rate of change of G* is predicted to be just:

Normalised rate of change of *G*. **(4.3)**

$\dot{G}/G = -d\hat{R}/dt$ Normalised rate of change of G.

Hence TAU predicts that the normalised rate of decrease of G at the present time is precisely the normalised rate of change of *R* in conventional variables.

- This escapes the flaw of theories that make the rate of change of *G* the squared-inverse of the rate of change of *R.*

Hence the predicted rate of change in *G* is simply determined by the *present rate of expansion.* This is measured by the Hubble constant. But estimates of the Hubble constant derive from cosmological periods in the past, the rate is changing, and the interpretation of data has to be checked in the context of the new model.

Strictly we need to know the radius expansion function, and our point in the expansion cycle (next section). But the simplest assumption is that the Hubble constant is fairly stable, and its measured value is about: $H = 70\ (km/sec)/Mps.$ (Or about 67 – 73, depending on the method). In units of sec^{-1} this is:

$$d\hat{R}/dt = H = (70\ km/sec)/(3.09 \times 10^{19}\ km) = 2.3 \times 10^{-18}\ s^{-1}$$

We convert this to a rate per year, and the normalised rate of change in *G per year* over the last few billion years should be about:

$$\dot{G}/G_0 \approx -7.14 \times 10^{-11}\ year^{-1}$$ Simplest predicted change of *G.*

Note the magnitude may be smaller, depending on the expansion function, and the point we are at, but it is probably no less than about half this value.

Note the coincidence that the inverse of the Hubble constant is the *Hubble time,* which would represent the time for *R* to increase from 0 to its present size at this rate of change, and this is very close to the measured age of the universe: $1/H \approx 14$ *billion years.*

This prediction for the change in *G* depends on the present expansion rate, and as we will see subsequently when we solve the expansion function, it is slowing in our model. But we may take the value above as the higher realistic limit, and fairly accurate over the past few billion years.

Empirical studies of rate of change of G.

So we must now examine whether this is consistent with measured values. Various attempts have been made to measure the value of $\dot{G}/G_0$ directly. But we will start with something of a curiosity, reflecting the fact that the direct measurement of *G* is very problematic, and measurements have been found to vary between different studies at different times, well outside the standard errors estimated for the studies. Anderson *et al* (2015) finds a cyclic pattern of variation, with a period of about 5.9 years (which coincides quite strikingly with a variation in the Length of Day).[17]

17 Anderson *et al* 2015. Note that variations in *G* are possible in TAU if the solar system passes through clumps of dark matter. But that is not proposed as the cause of these cyclic variations.

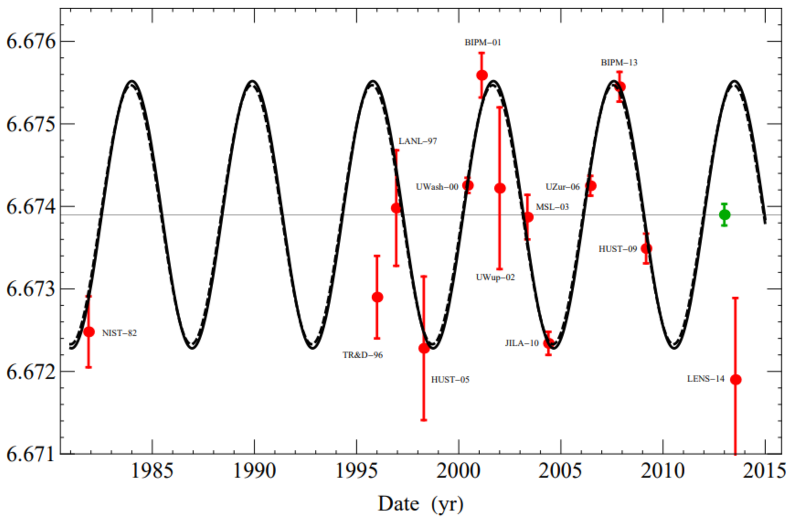

Fig. 5. From: Anderson *et al* 2015, *p.2*. "*Comparison of the CODATA set of G measurements with a fitted sine wave (solid curve) and the 5.9 year oscillation in LOD daily measurements (dashed curve)*".

These are essentially the best (laboratory-based) estimates of *G* over the three decades up to 2015. This is a good illustration of how poorly we really know the value of *G.* The estimated standard errors are much smaller than cyclic variations, and measurements of *G* appear to vary dramatically, in a 5.9 year cycle. These would represent $\dot{G}/G_0$ varying cyclically by about 10^{-4} parts per year! This would swamp our predicted variation on the order of 10^{-11}.

But since it is thought that *G* itself cannot possibly vary so much, it is expected that there is some local physical (e.g. mass flows in the Earth) variation, associated by Anderson *et al* with a similar oscillation in the terrestrial Length of Day. This is an interesting anomaly in itself, and it raises some questions over experiments. However although this is very curious, it not so important for us, because the *average rate of change:* $\dot{G}/G_0$ over longer periods can be measured much more precisely than *G* itself. There are two main types of measurements:

- Estimates from cosmological data, mainly supernovae and pulsars, determining *average change over billions of years*, and:
- Estimates from Lunar Laser Ranging (LLR) measurements, determining *current rates of change*, from data over several decades, since the 1970's.

A robust set of estimates from our point of view (i.e. in the context of our theory) are from studies of supernovae and pulsars[18]. The simplest (and most robust) cosmological studies limit the rate of change of *G* to less than about $\pm 10^{-10}$ *parts per year,* as an average over about the last 9 billion years. These limits should be valid when interpreted in our theoretical framework, because they do not make further fine-grained modelling assumptions, based on the correctness of standard cosmology.

18 Mould et al 2014. Zhao et al 2019.

Researchers have then tried to fit better models, to take account of various cosmological factors, to improve precisions. E.g. a recent analysis [Zhao et al 2018] claims the strongest precision so far from this kind of study, of around 3×10^{-12} year^{-1}. But this is the result from some complicated modelling assumptions, including in this case an assumption about the dependence of *G on t*: $G \propto t^{-\alpha}$. But this assumption and others do not hold in our theory. E.g.

> "As examples to compare our results with other constraints, we adopt z = 0.4 (z = 0.9) and assume a power-law cosmic time dependence, $G \propto t^{-\alpha}$, then the constraint $\Delta G(z)/G < 0.015$ is equivalent to a constraint on the index of $|\alpha|$. 0.04 (0.02), which can be translated into $|(dG/dt)/G|_{t=t0}$. 3×10^{-12} year^{-1} (1.5×10−12 year−1). This is of the same order as constraints from pulsars [4], lunar laser ranging [3] and BBN [5] (|(dG/dt)/G|t=t0 . 10^{-12} year−1). Most importantly, the new method offers a novel and independent way to constrain Newton's constant G over a wide redshift range 0 < z < 1.3, which could also be extended to 0 < z < 2 by future SNIa observations [37]." Zhao et al 2018.

If we look at the modelling in this paper and others that claim these very high precisions, we get a sense of how complicated it is to get from about the precision of about 10^{-10} (which seems quite robust), then to 10^{-11} (which is probably the reliable limit within *systematic errors,* and is approximately the Hubble constant, and the background rate of expansion of space) then to 10^{-12} …10^{-15} (which requires high levels of modelling, to take into account a raft of small background effects.) The latter represent *models optimised to conform to GTR*. But they do not test the prediction of the alternative LNR theory. The strongest limit (e.g. [Mould, 2014]) is:

- Robust cosmological studies show $\dot{G}/G_0$ is changing by less than $\pm10^{-10}$ y^{-1}.
- This is consistent with the variation our theory predicts, but it is close to the predicted variation.

These cosmological estimates are important, because in our theory, the change in *G* should show up certain differences, in the formation of galaxies, stars, supernova, pulsars, etc, in the distant past.

A second and quite different type of measurement has been done, using lasers bouncing off mirrors on the moon, in the *Lunar Laser Ranging* (LLR) studies, which claim to set phenomenally accurate limits on *the current average rate of change in G ,* measuring its change over the last few decades (with data from the 1970s – 2010s). The precisions that are claimed improved by three orders of magnitude over 10 years, from around $<10^{-12}$ *parts per year* (Turyshev *et al* 2007, Müller *et al* 2007), to around <[-5 x 10^{-15} to 1.5 x 10^{-13}] (Hofmann, F. and Müller, J. 2018).

This precision involves measuring the distance to the moon within a millimeter and estimating multiple parameters, to fit the models. (Pavlov et al, 2016; Boggs et al, 2016).

- These studies may be taken to rule out a significantly changing value of *G* <u>in the context of conventional *GTR*</u>.

At first glance, this appear appears to rule out the predictions of our LNR theory. But when we examine the LLR measurement carefully, we find it predicts the same empirical result. This because both *laws and measurements transform.*

The transformation of Kepler's law.

We now look at how our transformations affect laws. We examine Kepler's third law, which is a law for the rotation of planets. It relates the period, *P,* to *r, m, G.* It is valid in both GTR and LNR. The LLR analysis is based on it.

$P^2 = 4\pi^2 r^3/Gm$ Kepler's Third Law.

Taking the time derivative and rearranging gives the normalised rate of change in *G*:

$$\frac{\dot{G}}{G} = \frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m}$$ Time derivative of Kepler's Law.

The *LLR* method then depends on determining the values on the RHS empirically – primarily the *distance term* and the *period term, as* the mass term is regarded as effectively constant. If *G* is changing, it is then inferred that this will be reflected in changes of: $\frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m}$. This is the term they are now claiming to have determined to be less than about 10^{-14}.

This relation or law holds in conventional GTR. Our LNR theory means that, in conventional units, *G* should be changing. But how is this related to the other variables? I.e. what law holds in the *alternative theory?*

The gravitational theory is very similar to GTR and predicts the same relationships, and we may assume the theory conforms to Kepler's law. But it is the law *in the true variables of our theory, not in conventional variables of GTR.*

We can work this out by considering an idealised experiment, with an orbiting mass, *m,* subject to the expansion of space. In the true variables, we obtain these relationships in our theory:

Relations for Kepler's Law in True Variables **(4.4)**

$$G' = G_0; \qquad \frac{dG'}{dt'} = 0; \qquad \frac{\dot{G'}}{G'} = 0$$

$$m' = \frac{m_0}{\widehat{R'}}; \qquad \frac{dm'}{dt'} = -\frac{m_0}{\widehat{R'}^2}\frac{d\widehat{R'}}{dt'}; \qquad \frac{\dot{m'}}{m'} = -\frac{1}{\widehat{R'}}\frac{d\widehat{R'}}{dt'}$$

$$r' = r_0\widehat{R}'; \qquad \frac{dr'}{dt'} = r_0\frac{d\widehat{R'}}{dt'}; \qquad \frac{\dot{r'}}{r'} = -\frac{1}{\widehat{R'}}\frac{d\widehat{R'}}{dt'}$$

$$P' = P_0\widehat{R}'^2; \qquad \frac{dP'}{dt'} = 2P_0\widehat{R'}\frac{d\widehat{R'}}{dt'}; \qquad \frac{\dot{P'}}{P'} = \frac{2}{\widehat{R'}}\frac{d\widehat{R'}}{dt'}$$

Setting the boundary condition: $P_0{}^2 = \left(\frac{4\pi^2 r_0{}^3}{G_0 m_0}\right)$ we have:

Kepler's Law in True Variables **(4.5)**

$$P'^2 = P_0{}^2\widehat{R}'^4 = \frac{4\pi^2 r'^3}{G'm'} = \left(\frac{4\pi^2 r_0{}^3}{G_0 m_0}\right)\widehat{R}'^4$$ Kepler's Law in True Variables

$$\frac{\dot{G'}}{G'} = \frac{3\dot{r'}}{r'} - \frac{2\dot{P'}}{P'} - \frac{\dot{m'}}{m'} = 0$$ Differential in True Variables

Kepler's law holds in the true variables of our theory because G does not change in true variables. Now we must transform this relationship into *conventional variables,* to see what it will predict for the LLR experiments. The transformed equations are:

Relations for Kepler's Law in Conventional Variables **(4.6)**

$$\frac{\dot{G}}{G} = -\frac{d\hat{R}}{dt}$$

$$m = m_0; \qquad \frac{\dot{m\prime}}{m\prime} = 0$$

$$r = r_0\hat{R}^2; \qquad \frac{dr}{dt} = 2r_0\hat{R}\frac{d\hat{R}}{dt}; \qquad \frac{\dot{r}}{r} = \frac{2}{\hat{R}}\frac{d\hat{R}}{dt}$$

$$P = P_0\hat{R}^3; \qquad \frac{dP}{dt} = 3P_0\hat{R}^2\frac{d\hat{R}}{dt} \qquad \frac{\dot{P}}{P} = \frac{3}{\hat{R}}\frac{d\hat{R}}{dt}$$

And adding these up we get:

Kepler's Law in Conventional Variables **(4.7)**

$$\frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m} = 0$$ Our prediction in conventional variables for LLR.

$$\frac{\dot{G}}{G} = \frac{3\dot{r}}{r} - \frac{2\dot{P}}{P} - \frac{\dot{m}}{m} - \frac{d\hat{R}}{dt}$$ Our Kepler law in conventional variables.

- The first equation predicts the result actually measured in the LLR experiments. There should appear to be no change in this measurement.
- The second equation gives our Kepler relation in conventional variables. It has an extra term.

This means *our prediction for the LLR experiment* is exactly the same as for GTR.

But our theory relates *G* the expansion, *R.*

The LLR experiment does not test our theory against GTR.

- The assumption that the conventional Kepler relation holds in our system is wrong.
- This means the

To test theories you normally have to design experiments that distinguish their predictions from each other. This has not been done, for cases like our LNR, because the alternative hypothesis has not been defined.

Prediction for distance to the moon.

This is not the only prediction of our theory for the moon. We get another empirical prediction using the relation:

$$\frac{dr}{dt} = 2r_0\hat{R}\frac{d\hat{R}}{dt}$$ Recession speed of the moon from weakening G and expansion.

using the distance to the moon, which is also very accurately measured and predicted on standard GTR theory. This integrates to: $r = r_0 \hat{R}^2$. This should correspond to the moon's present radial motion away from the earth, *due to the effects of both cosmic expansion and weakening G combined* (but not including the effect of tidal forces, which are non-conservative, and presently transferring rotational energy from the Earth to the Moon). The factor of *2* adds two equal components: the *speed from cosmic expansion* and *the speed from changing basis vectors*. Putting in the numbers:

$$r_0 = 380 \; x \; 10^6 \qquad \hat{R} = 1 \qquad \frac{d\hat{R}}{dt} = 7.1 \; x \; 10^{-11}$$

We get: $\frac{dr}{dt} = 2 \; x \; 380 \; x \; 10^6 \;\; x \; 7.1 \; x \; 10^{-11}$ = 5.3 cm/year

This is very close to *twice* the rate of separation of two *unbound objects* due to the present Hubble expansion of the universe:

$\frac{dr}{dt} = 380 \; x \; 10^6 \;\; x \; 7.1 \; x \; 10^{-11}$ = 2.7 cm/year

However, in *gravitationally bound systems* like the sun, earth, moon, it is normally assumed that orbital distances are locked, and the cosmological expansion does not come into play. I.e. the expansion is not usually added in the analysis. (Although it is 1,000 times larger than precisions being claimed in LLR studies!) The same reasons for this appear to apply in both theories, so in our theory we may expect the same locking of gravitationally bound systems against the effects of cosmological expansion. We should therefore subtract this expansion effect for the bound orbits, as in the normal analysis.

The remainder is a component of 2.6 *cm/year* increase in the moon orbit. This is the change from the weakening G, independent of the expansion effect, and before other effects like tidal friction.

Tidal friction effects then only need to account for 1.1 cm/yr to make up the observed value of 3.8 cm/yr. This allows a consistent history for the moon's orbit without any special hypotheses.

This reconciles the mystery of how the moon can be moving away so fast, at 3.8 cm/yr. When this speed is projected back into the past, we find the moon would have been close to the Earth only about 1 billion years ago, whereas we know it formed (close to Earth) about 4.5 billion years ago. So it is believed that the moon's speed away from earth must have increased recently – within the last few hundreds of million years. The main suggestion is that this is due to changing tidal patterns, but that is still entirely hypothetical. In our theory, the distance to the moon appears realistic, back to 4.5 billion years ago, without this special hypothesis.

Discussion.

We find that the LLR measurements do not test whether gravity is static or weakening in the context of our *alternative to GTR.* The failure to find temporal variations in G is taken as the strongest positive evidence for standard GTR and against LNR theories. But this does not take the alternative predictions into account correctly. So although the LLR experiments are claimed as strong confirmation of GTR, they confirm the predictions of our theory just as well!

These experiments appear as strong tests of GTR, but in fact they are weak tests of GTR *against* our type of LNR theory. An important principle of experimental testing is that we must test *alternative hypotheses or theories* against each other – not simply accumulated data on how accurate a favoured theory is. For a well-known example in the philosophy of science, the predictions of planetary motions in C16th Ptolemaic astronomy were very accurate – and they could be made more accurate, with *ad hoc* additions of more epicycles, etc. But the data confirming their accuracy does not really test the Ptolemaic theory against the Copernican theory. For this, you must do experiments in a domain that specifically distinguishes the behaviour of the two theories.

Equally, solar system tests of our alternative LNR theory versus static GTR requires experiments specifically designed to *differentiate the theories*, and this has not been done. We should also beware in this respect that the interpretation of the LLR data uses models for making fine adjustments for multiple effects, such as tidal friction of the moon, solar perturbations, seasonal variations in orbits, changed speed of light in gravity and in the atmosphere, flow of molten cores within the Earth or moon, thermal expansions, etc. To get a sense of the sensitivity, the claimed LLR precision requires measuring the distance from the earth to the moon to within a *millimetre!* There is some intricate modelling involved, with calculations of lunar flow boundaries and other properties.

We may be sceptical that these really set precisions of $\pm 10^{-15}$ for $\dot{G}/G_0$. Rather, they show that models can be fitted to match this idealised accuracy, but that is different from testing $\dot{G}/G_0$ at this precision.[19]

In any case, the LLR does not test TAU directly, but improved cosmological data can. Besides these tests, the general prediction is that gravity should appear to be substantially stronger in the very early universe. And this should result in some substantially different dynamics to conventional cosmology – e.g. galaxies or stars forming faster than expected. A variety of such anomalies is known. They have not yet been analysed in the context of TAU.

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19 Another question is why these results do not reflect anything like the large variations found in G measured directly in other studies, as illustrated above? There may be real cyclic variations in G locally, due to unknown factors, such as mass flows with the earth. But then it seems strange that no sign shows up in the LLR experiments – or are they compatible with the claimed precisions?

Section 5. Competing the solution.

The time integration.

We saw the space integration is quite easy, but now we have to do the same kind of integration for time, to relate the true time variable to the conventional time variable. We must integrate using: $dT/dT' = dt/dt' = \hat{R}'^2$, and the boundary condition: $T \rightarrow 0$ *when* $T' \rightarrow 0$.

The time integral. **(5.1)**

$$T = \int_{0,T} dt = \int_{0,T'} (dt/dt')\, dt' = \int_{0,T'} \hat{R}'^2\, dt' = \int_{0,T'} (R'^2/R_0'^2)\, dt' \qquad \text{Time integral.}$$

However (unlike the spatial integration) we cannot do the time integration until we have an evolution equation for R' in terms of T', i.e. the function for $R'(T')$. We propose a full solution next. But to illustrate what happens generally, an approximate solution can be obtained, for a period of approximately linear expansion in the recent past. For this, we can take R' as a linear function of time, e.g. $dR'/dt' = H_0$, between the present time: $t_0' = T_0'$ and an earlier time, $t_1' = t_0' - \Delta t'$. We get:

Linear expansion (not a realistic solution for long term).

$$\Delta T = T_0 - T_1 = \int_{[T_0'-\Delta t, T_0']} (H_0^2 t'^2/R_0'^2) dt' = H_0^2 T_0'^3/3R_0'^2 - H_0^2(T_0'-\Delta T')^3/3R_0'^2$$
$$= 3H_0^2 T_0'^2 \Delta T'/3R_0'^2 - 3H_0^2 T_0' \Delta T'^2/3R_0'^2 + H_0^2 \Delta T'^3/3R_0'^2$$
$$= \Delta T' - \Delta T'^2/T_0' + \Delta T'^3/3T_0'^2$$

If $\Delta T' << T_0'$, the third term is small, and we have: $\Delta T \approx \Delta T' - \Delta T'^2/T_0'$. So an expansion may appear quadratic or cubic in T while it is really linear in T'.

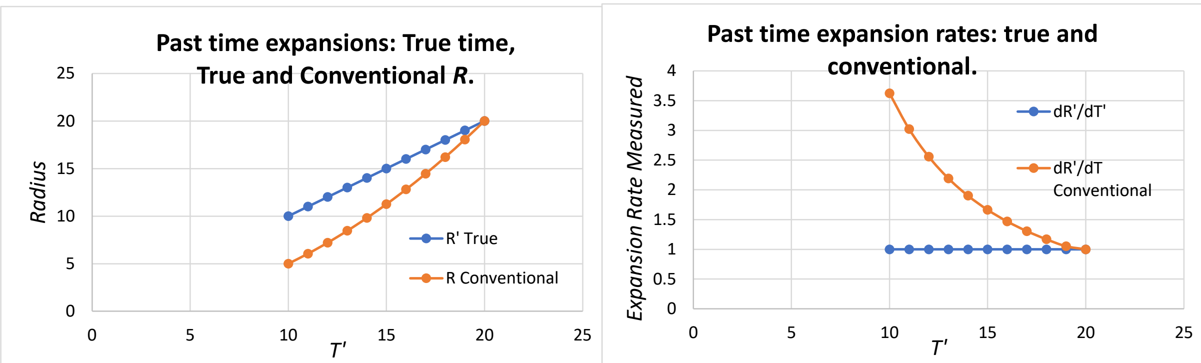

Fig. 6. Looking back from a present time (at $T' = 20$ here), we see the linear expansion in true variables is distorted in conventional variables. On the RHS, the conventional expansion rate appears to be reducing.

This shows a general phenomenon, which we may call the *perspectival effect.* If we mix up true and conventional variables, we will get the appearance of changing rates of expansion and non-linear time intervals, due to variable transformations, not real physics.

However a purely linear expansion is of limited use, and not realistic over several billion years (the period in which *dark energy* appears). The more realistic solution is a smooth cyclic one, as shown next. This demonstrates the more essential effects of changing expansion speeds.

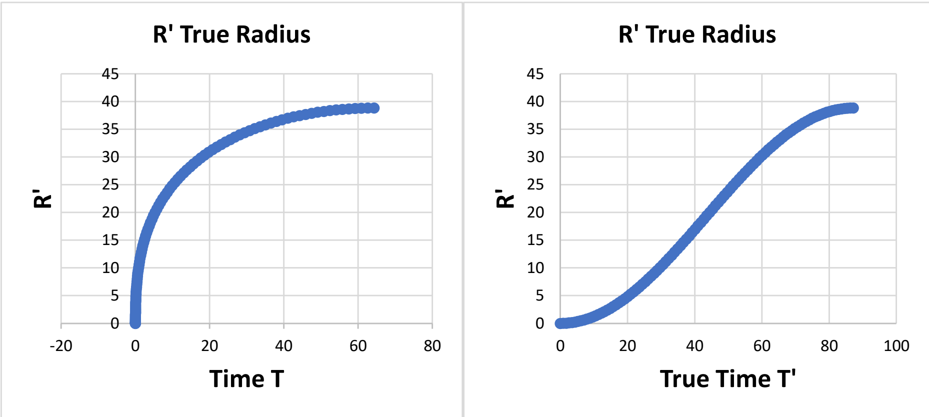

Fig. 7. Cycloid solution to the expansion *R′* against conventional time and true time. The expansion in true time matches observations of the Hubble constants in the medium-past.

These are graphs of the *cardiod solution,* obtained next as a real solution to TAU. The solution looks totally different in the two different variables. What do the measured expansions corrospond to? We cannot assume that the cosmological measurements we actually make correspond to either diagram. Instead we must work out the measurement proceedures in detail for different types of meaurements. Different empirical methods determine these through their measurement assumptions. And they can be mixed up. However the RHS diagram is what we think the key measurements really reflect.

Note that if measurements of the rate of expansion reflect true expansion against true time (RHS graph above), then in the period in the early-mid expansion (where we appear to be), we will see the expansion accelerating. But it will eventually slow. This shows how a *dark energy effect* may arise, so the universe appears to have an accelerating expansion although it is really in a cycle.

This also illustrates how the so-called *Hubble tension* may arise: different measurement procedures that have the same expected results in GTR have different results expected in TAU.

We now obtain the precise solution graphed above. It is not known if it is accurate when analysed against cosmological data in full detail, but it is at least broadly realistic and plausible, and it has such simple and powerful properties that it is ideal as one type of prototype solution.

The Cardioid solution for expansion.

We get the *cardioid solution* by imposing a simple conservation of energy principle. The evolution of our constants already means that the *local momentum of particles in the manifold* is conserved:

$$mc = (m_0/\hat{R}')(c_0\hat{R}') = m_0c_0 = constant \qquad \text{Momentum conservation.} \qquad \textbf{(5.2)}$$

Momentum is conserved on any expansion function. But the mass-energy is different:

$mc^2 = (m_0/\hat{R}')(c_0^2\hat{R}'^2) = m_0c_0^2\hat{R}$ Mass-energy increases. **(5.3)**

This is *increasing with expansion!* Particles are increasing their mass-energy as the universe expands! Where does this energy come from? It must come from the spatial manifold, one way or another. We propose the simple hypothesis that the total kinetic energy in the global frame of the universe is conserved. The mass has a *surface speed c,* but also an "expansion speed" in *R',* with a total energy:

6D Energy Conservation Principle **(5.4)**

$$E' = m'V'^2 = m'c'^2 + m'(dR'/dt')^2 = constant$$

(This is on the assumption that the speed *c* is orthogonal to the expansion, *dR'/dt*. Particles have a cyclic component in *R'*, but this is true on average). Note also that this takes: $E = mV^2$ generally as energy. This is consistent with STR. When we add up momentum diagrams, *momenta* in the different directions add up as vectors, and their squares add up like energies.

$p = mc = \gamma m_0c$ Relativistic momentum. **(5.5)**

When we add the momenta-squared, it gives the sum of velocity-components-squared, and we get the energy times mass. Using: $u = dw/dt = c/\gamma$, and: $v = dr/dt$:

$$P^2 = m^2c^2 = \gamma^2m_0^2(u^2+v^2) = Em$$

$$P^2/m = mc^2 = (\gamma m_0)(u^2+v^2) = mu^2+mv^2 = mc^2 = E$$

And this is generalised to all component velocities, including speed of expansion (which is really in the direction w_1), as well as the normal surface speed, *c*. Note that these two components are not the *rest mass energy and kinetic energy* in STR, which add up like this:

$\gamma m_0c^2 = m_0c^2 + E_{kinetic}$ Total Energy = Rest Mass + Kinetic.

This is a different way of separating the energy components. $m_0c^2 + E_{kinetic} = m(u^2+v^2)$. The $E_{kinetic}$ is the extra energy we have to add to stationary m_0 to give it a speed *v.* But the *orthogonal energy components* in the resulting system are: $mu^2 + mv^2$, and do not match $m_0c^2 + E_{kinetic}$.

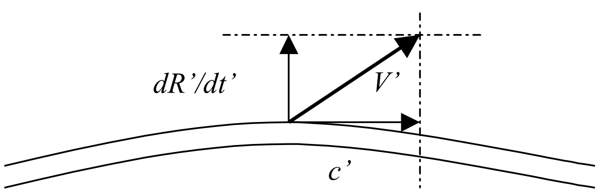

Fig. 8. Total resultant velocity, *V'*, for a free particle in the expanding manifold.

So this takes the kinetic energy from both the motion in the space manifold and the motion of the space manifold into account. The present total kinetic energy is then:

$$m_0'c_0'^2 + m_0'(dR_0'/dt')^2 = m_0'V_0'^2 = E_0' \qquad \text{Present 6D kinetic energy.} \qquad \textbf{(5.6)}$$

From the conservation principle and the evolution of *m'* we have:

$$V'^2 = E_0'/(m_0'/\hat{R}') = V_0'^2\hat{R}' \qquad \textbf{(5.7)}$$

And then from the evolution of c' we have:

$$(dR'(t')/dt')^2 = V_0'^2\hat{R}' - c_0'^2\hat{R}'^2$$

$$= (V_0'^2/R_0')R'(t') - (c_0'^2/R_0'^2)\, R'(t')^2 \qquad \text{Equation of motion.} \qquad \textbf{(5.8)}$$

This is an equation of motion for *R'*, and it has a simple solution:

$$R'(t) = (R'_{MAX}/2)(1-\cos(\pi T'/T'_{MAX})) \qquad \text{Evolution equation.} \qquad \textbf{(5.9)}$$

Where R'_{MAX} is the maximum expansion (in true variables). This is a simple cardioid function.

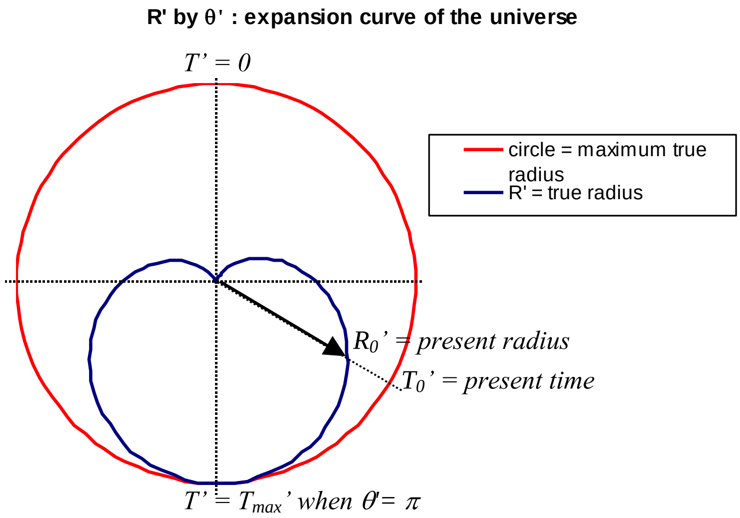

Fig. 9. The cardioid solution for the universe expansion, blue. Time *T'* is the angle around the circle. The radius *R'* is a cardioid function, the length (black arrow) from the centre.

This is in *true (dashed) variables,* so must be converted to conventional variables.

The result converted into the conventional time variable, *T,* is:

$$T = (R'_{MAX}/R_0')^2(3T'/8 - \sin(AT')\cos(AT')/2A + \sin(2AT')\cos(2AT')/16A) \qquad \textbf{(5.10)}$$

where: $A = (\pi/2T_{MAX}')$ and: $T_{MAX}' = \pi R_0'/c_0'$, so that: $A = (c_0'/2R_0')$.

There is one more essential result for interpreting the cycloid model, the *comoving distance* for photons. This tells us how far apart the origin and detections points for a photon are.

The co-moving distance of a photon moving between two moments of time is easily defined in terms of the radial angle θ', as defined in the diagram above. A change in the radial angle of $\Delta\theta'$ in a period $\Delta T'$ corresponds to a co-moving distance $\Delta L'$:

$$\Delta L' = \Delta\theta' R' \qquad \text{Comoving distance.} \qquad \textbf{(5.11)}$$

where R' is the radius at the final time (time of detection). This gives:

$$\Delta L' = \Delta T' c_0 R'/R_0' = \Delta T' c_0 \hat{R}' = \Delta T' c' \qquad \text{Comoving distance.} \qquad \textbf{(5.12)}$$

where c' is the speed of light at end of the period (time of detection). In the special case where we detect light at the present time from the origin of the universe (approximately the last scattering surface for the CMBR), we define $\Delta L' = L_0'$, and $\Delta T' = T_0'$, and $c' = c_0'$, giving:

$$L_0' = T_0' c_0' \qquad \textbf{(5.13)}$$

Hence the present *co-moving distance of light from the origin* (the Big Bang, or shortly thereafter) is simply the *present age of the universe times the present speed of light!*

- This is what appears to be reflected in the TAU relationship: $DR_A = R_u = L_0' = T_0' c_0'$.

The peculiar problem with interpreting the Dirac Large Number relationship we obtained earlier was that it predicts a *distance, Ru,* that should reflect the radius of the universe; but it actually corresponds to the *measured age, T,* times the present speed of light, *c.* The reason these are closely connected now turns out to be through the special form of the radial expansion function in time.

Note in the special case of light from the origin of the universe detected at the time of maximum expansion, L_{MAX}' is half the maximum circumference:

$$L_{MAX}' = \pi R_{MAX}' = T_{MAX}' c_{MAX}' = T_{MAX}' c_0 \hat{R}_{MAX}' = T_{MAX}' c_0 R_{MAX}'/R_0' \qquad \textbf{(5.14)}$$

This means that: $T_{MAX}' = \pi R_0'/c_0$. Thus we can determine the eventual expansion, and subsequently our position in the cycle, from R_0' and c_0. This solution obviously makes the cosmology highly deterministic. We now return to the problem of interpreting the predictions.

Interpreting the cosmological predictions.

The main general feature is that expansion rates can look very different in different variables. E.g.

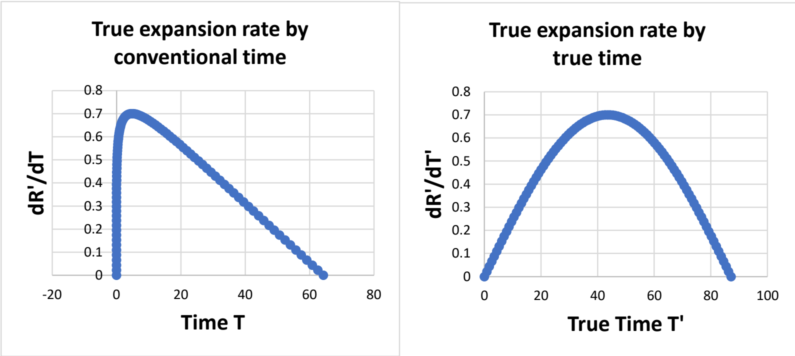

Fig. 10. The cycloid expansion rates against conventional and true time.

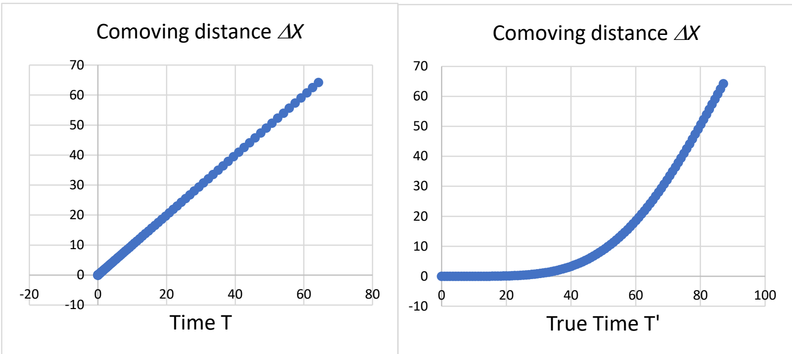

Fig. 11. Co-moving distance between light origin point (Big Bang) and detection point (CMBR), against conventional time and true time.

Two distinctive effects we find from the model are:

- In the early-middle period the measured expansion should be accelerating – but it will slow, and is really cyclic. The inference of *dark energy* in the conventional model is just a way of reconciling it with GTR, but it is probably mistaken.
- Conventional measurements of the Hubble constant estimated from different periods in the past, and by using different methods, should be inconsistent.

The first predicts a "dark energy" effect should appear but the cause is not "energy", while the second predicts that a "Hubble tension" should appear. These effects have been confirmed over the last 10-30 years, and represent serious anomalies in conventional cosmology.

However the application of this model depends upon a re-analysis of measurements of the Hubble constant, expansion acceleration rates, etc, to determine what they are measuring *in the context of TAU*. Hence our problem comes back to determining the TAU analysis of measurements. Note there are two main methods for determining the Hubble constant: the *distance ladder,* and the *CMBR*.

- The *distance ladder* at its simplest involves identifying "standard" types of stars, supernovae, pulsars, etc, and determining (i) their distance from their brightness (luminosity), (ii) their recessional velocity from their red-shift.
- The *CMBR* method involves measuring the CMBR very precisely, which comes from a period quite soon after the Big Bang, and determining wave-length expansions or red shifts.[20]

Both methods are theory-laden.

> "The figure astronomers derive for the Hubble Constant using a wide variety of cutting-edge observations, including some from Hubble's namesake observatory, the NASA/ESA Hubble Space Telescope, and most recently from ESA's Gaia mission, is 73.5 km/s/Mpc, with an uncertainty of only two percent. … A second way to estimate the Hubble Constant is to use the cosmological model that fits the cosmic microwave background image, which represents the very young Universe, and calculate a prediction for what the Hubble Constant should be today. When applied to Planck data, this method gives a lower value of 67.4 km/s/Mpc, with a tiny uncertainty of less than a percent." Jan Tauber, Markus Bauer 2019. European Space Agency.

Hubble Constant	Estimate
Distance ladder	73.5
CMBR (Planck)	67.4
Ratio:	109%

> " 'The CMB is a portrait of the young Universe, and the picture delivered by Planck is so precise that we can use it to scrutinise in painstaking detail all possible models for the origin and evolution of the cosmos,' comments Jan Tauber, Planck Project Scientist at ESA. 'After this close examination, the standard model of cosmology is still standing tall, but at the same time evidence of anomalous features in the CMB is more serious than previously thought, suggesting that something fundamental may be missing from the standard framework,' he adds. *ESA Science & Technology - Simple but challenging: the Universe according to Planck.*

The Planck project is a great experimental achievement, as are other leading experiments. But the comment that "we can use it (Planck data) to scrutinise in painstaking detail all possible models for the origin and evolution of the cosmos" is an overstatement. We cannot test all the "possible models" *unless we know the range of possible models*, and this is the difficulty. No one has worked out the interpretation of the Planck or LLR data on the LNR model we have just introduced.

The difficulty should be recognised that to interpret cosmological measurements, we must interpret the physical processes underlying the measurements, and their subsequent modelling. There are rates of stellar processes and formation of stars, and radioactive and chemical radiation processes affecting formation and brightness of stars and their expected spectra. These are affected by changes in fundamental constants and gravity. Then in the conventional modelling, there are estimates of different material components of the universe, which includes ordinary matter (fermions/baryons, photons, neutrinos, etc), and dark matter (a large component), and finally dark energy (or cosmological constant) – another large component that has become essential to make

20 A third method was found in 2018, "gravitational lensing" of merging neutron stars using gravitational wave detectors. There is little data yet, but this should give an independent method over the next few years.

the conventional models work. These ingredients are normally combined using GTR and the Friedmann model. Modelling the acceleration of the Hubble constant is a good example of how theory-laden the measurements are.

> “The accelerated expansion of the universe is thought to have begun since the universe entered its dark-energy-dominated era roughly 4 billion years ago. Within the framework of general relativity, an accelerated expansion can be accounted for by a positive value of the cosmological constant Λ, equivalent to the presence of a positive vacuum energy, dubbed "dark energy". While there are alternative possible explanations, the description assuming dark energy (positive Λ) is used in the current standard model of cosmology, which also includes cold dark matter (CDM) and is known as the Lambda-CDM model.” Accelerating Expansion of the Universe. WIKIPEDIA.

The cosmological constant (or dark energy) in the conventional model may duplicate some effects of our model, but they will not match in the end. It appears that cosmologists have measured a real Hubble acceleration effect, but our explanation is not “dark energy”. It is that the simplest type of *cyclic solution* to our LNR theory predicts this apparent effect, in the middle stages of expansion. It fits just as well as the conventional *dark energy* hypothesis, and gives alternative explanations of several major effects, including Hubble tension and some other large number relations. This type of alternative theory should be considered. It reveals physical mechanisms and explanations that cannot be represented in conventional cosmological theory. It emphasises the fact that the *entire class of theories with changing constants* is largely ignored in cosmology, and there are no realistic tests of such theories in cosmology or gravitational studies.

Conclusion.

To conclude here, our model now needs to be applied to several other phenomenon in detail: dark matter, dark energy, the number of particles, and tests of gravity. These have been worked out in varying degrees of detail, and so far appear consistent and highly promising. The background gravitational theory is worked out in some detail in [Holster, 2017]. But this take us beyond the LNR theory we specifically want to introduce here and into the larger world of cosmology and gravity. This is the point at which our proposal connects up with the lines of investigation in cosmology suggested in the quotations by Ray and Solà we started with. So we leave that for another place, where we can more closely examine the solution against cosmological data.

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Appendix 1. Example of a random coincidence.

This is a set of numbers produced by a search for "Moon radius" while I was writing this paper. I just happened to notice the unusual pattern of numbers. Does it have any significance?

Fig. 12. These numbers appear to have a lot of 6's and to display a regular scale.

Is this a meaningful pattern, or is it just coincidence? Is the repeated starting numeral "*6*", along with *9, 3, 1,* significant?

Table 5. Table of moon numbers.

Moon:	**1,737**	x 2 =	**3,474**	x 2 =	**6,948**	x 10 =	**69,480**	x 10 =	**694,800**
Mars			**3,390**	x 2 =	**6,780**				
Venus					**6,052**				
Earth					**6,371**	x 10 =	**63,710**		
Jupiter							**69,911**	X 10 =	**699,110**
Sun									**695,842**

This means the moon-sun radius ratio is close to: 2 x 2 x 10 x 10 = 400. This is close to the Rtio of their distances from Earth, resulting in the coincidence that the moon appears the same size as the sun in eclipses.

Digits in the radii in km: 0, 0, 1, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8, 9, 9, 9, 9.

Is this pattern of numerals an unlikely coincidence? Is it significant? Well: yes and no. It is a coincidence, and it does have some kind of significance; but there is no significance for *physics* in these apparent numerical relationships.[21]

First, the data does represent a genuine *scale pattern,* in a very generic sense. Different types of objects in the solar system form at certain natural scales – *asteroids and comets, moons, small rocky planets, large gas planets, stars.* And their sizes appear to be typically separated by about an order

[21] Special numeric patterns may have symbolic significance in mystical or mythological or magical systems of belief. That has its own interest, but it is a different realm to physics.

of magnitude of radius (which is cubed for volume). We understand fairly well, in general terms, why they form at these scales. This scale pattern is real enough, and we can generalise from it, and predict that *moon – small planet – large planet – sun-sized* objects will appear on these same general scales in other solar systems too.

But can we extrapolate from the specific pattern in the moon ratios above, and predict precise ratios of 2 and 10 between objects in solar systems generally? No. Our data is only a small sample. When we look at the full range of planets, moons, asteroids and stars in own our solar system, we see a continuous variation of sizes. The matching of the *Moon radius* in a neat scale with the five objects: *Mars, Venus, Earth, Jupiter, Sun,* is a coincidence, due to a special selection of planets. The apparent scale pattern disappears if *Mercury, Saturn, Neptune, Uranus, Pluto*, or the large moons of Jupiter, Saturn, Uranus, are included.

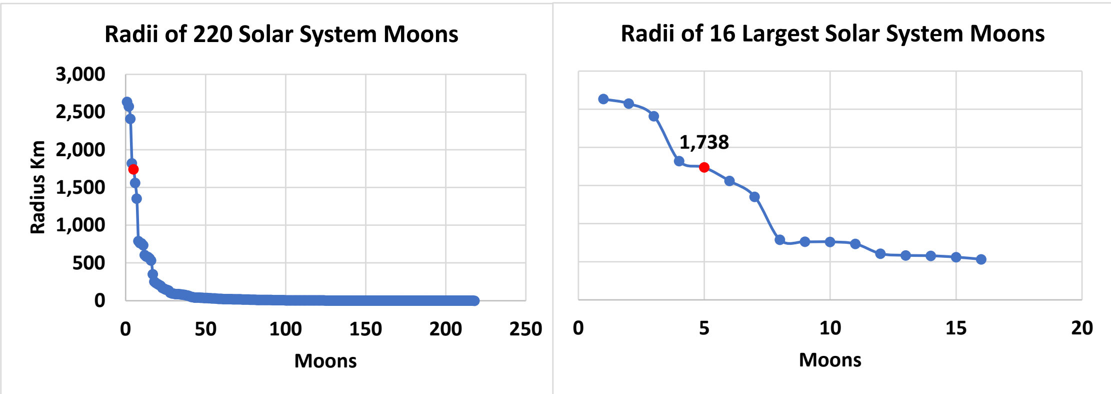

Fig. 13. Earth's Moon (red) is the fifth largest moon in the solar system. It lies in a continuous range of scales of other solar system moons. Any precise ratios with other bodies appear as random coincidences.

Our moon is exceptionally large for the size of our planet – this is its really unusual feature. Normally a small planet could not capture such a large moon, but it happened because the moon was formed from the collision of Earth with another planet, Thea, some 4.5 billion years ago. But the further "coincidences" involving the radius of the moon are random.

The point of this example is to contrast this kind of natural scale pattern, and the "accidental" numerical coincidences that naturally arise, with the LNC's, which are precisely defined, have low *a priori* probabilities, and have strong implications for the fundamental laws of nature.

Appendix 2. Note on the model that determines the proposed dynamics.

The LNR system presented here was not found by starting from the LNC's, but arose from an independent proposal for an alternative unified theory.[22] This introduces a 6D spatial manifold, instead of our usual 3D space, and it models quantum particles as wave motions in the three extra dimensions, which are curled up on a microscopic scale, into a torus. We do not go into detail of this theory here, but the upshot is that it is *designed* to reproduce the quantum mechanical laws for the main long-lasting particles: *photons, electrons, protons,* and in a simple extension, *neutrons and neutrinos.* (It gives STR in the local limit). This works well, and promises to be a good model for duplicating the properties of the full set of Standard Model particles. It may be considered somewhat like a *Grand Unified Theory (GUT),* but with a different higher-dimensional geometric space, with a special topology.

Now this *particle theory* is difficult to test directly, because it gives back almost exactly the same predictions as ordinary QM. It is selected to do this. It appears as a coincidence that the simple model can achieve this, but it may be suspected it is just a mathematical coincidence. After all, if we are *looking for a geometry* that can reproduce the behaviour of two fundamental types of QM particles (fermions and bosons), and we find a certain 6D topological structure that can do this, this may be significant, but it is only verified if it leads to further testable implications. It is similar in a way to Grand Unified Theories (GUT's), which interpret particles in abstract geometric spaces (SU(2), etc). GUTs at first appeared very promising, in 1970's, but eventually they encountered problems with reproducing the full range of the (empirically confirmed) particles of Standard Model, and they now seem to have broken down. So it seems quite probable now that the coincidence of structures that first made GUTs appear viable is really just a mathematical coincidence.

So we have not been able to show whether our particle model will ultimately be successful or not *against the Standard Model*. It duplicates standard particle physics so closely (partly by design, partly because the coincidence of structure), that it is not easy to find tests to distinguish them.

However it has stronger implications for gravity, and the big difference with other unification theories is that it gives a definite theory, similar in form to GTR, *but not identical.* It has a distinct difference with ordinary GTR, predicting a slightly different version of the *Schwarzschild solution*.[23] So we can test this (gravitational) part of the theory separately.

Our analysis of this so far shows that it is consistent with the empirical data from gravitational studies, which is still not quite precise enough to distinguish the two theories, although it is close and could be tested. So this remains an open empirical question. The fact that it gives a good theory of gravity and unifies the forces means this background theory has good explanatory power. We think it should be considered as a serious candidate for a unified theory. Until it can be tested empirically, there is no conclusive evidence for or against it.

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[22] There are several preprints of this theory in development over the years. A recent summary is given in [Holster, 2021].

[23] An analysis of this solution, called K-gravity, is given in [Holster, 2017].

However it has a third set of implications, for cosmology. It predicts the fundamental constants must change as the universe expands. By precisely the relations above in (1), (2) and (3) (rewritten properly in *true variables*). It predicts a set of LNR's, giving precise LNC's. We can compare the main predictions of this LNR theory immediately against experimental data.

So ignoring the particle and gravitational parts of the theory as presently inconclusive, this serves as the strongest immediate empirical test. Note that the theory was not *designed* to predict these LNR's – they just appear out of the model. There may also be several alternative choices or variations possible for this type of theory. But once we recognise Dirac's LNC's, the probability that they are lawlike dramatically increases in the context of our theory, and our subsequent choice of cosmological model is strongly determined by this.

This also changes our problem somewhat from conventional approaches, that start by trying to *reformulate GTR to match LNR theories* (such as Brans-Dicke theory). We do not have to do this, because our background theory already gives an alternative to GTR. Its predictions are too close to GTR at present to be tested. This is why we do not discuss modifications to GTR here.