Longitudinal polarization in general relativity: Can quasi-inertial oscillations impact the gravitational and microwave backgrounds?

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A novel general relativistic plane-wave metric is presented with a proposal for possible detection. Called a *quasi-inertial oscillation* (QIO), the metric is an exact solution to Einstein's vacuum field equations, yet carries non-Einsteinian longitudinal polarization, permitted by its quasi-inertial status. Although QIOs are metric waves in spacetime, they differ fundamentally from the standard gravitational waves assumed to be detected by LIGO and Virgo. QIOs are an oscillating type of *quasi-inertial disturbance* (QID), a broader class of exact solutions to Einstein's equations with varying features that travel at the speed of light. The observable properties of QIDs, and of QIOs in particular, have rarely if ever been studied in the literature. Yet it is shown here that if general relativity accurately models gravity, QIOs should produce test particle acceleration and are thus in principle observable by space-based detectors. Due to Riemann flatness, QIOs were historically dismissed as unphysical, and hence unobservable, by authors such as Taub, McVittie, Weber and others. However, these authors were seeking gravitational waves capable of forming gravitons, a far more stringent requirement than mere observability. Moreover, the claim that Riemann flatness precludes detection does not apply to metrics for which the coordinate system is fixed by a physical structure, such as a space-based platform, an accelerating rocket, or the cosmic microwave background, nor to metrics associated with frame-dependent quantum processes such as the Unruh effect. Nevertheless, subsequent authors continued to dismiss exact longitudinal plane-wave solutions and thus overlooked a real possibility of detection. With the benefit of hindsight, it is proposed here that QIOs may not only be detectable, but that Riemann flatness does not rule out the potential influence of QIOs on astrophysical or cosmological backgrounds, including the stochastic gravitational wave and photon backgrounds. QIOs are presented first in rectangular coordinates, then in Brinkmann coordinates for comparison with pp-waves. It is also shown that the Riemann-flat metric of a uniformly accelerating Rindler frame, theoretically detectable by Unruh radiation, can be constructed as a product of (rectangular) advanced and retarded QIDs, offering further argument for observability through an Unruh-type effect. Astrophysical and cosmological mechanisms are proposed as physical sources of QIOs. It is further suggested that, since QIOs are energy-free exact solutions to Einstein's equations, there is nothing to obstruct their generation by accidental alignments of matter, and thus nothing to prevent the vacuum from being filled with weak transient random spacetime fluctuations. Spacetime fluctuations are of course predicted in theories of quantum gravity; however QIOs would constitute a *classical* source of fluctuations. Finally, the role of Riemann curvature in pseudo-Riemannian spacetime is challenged in view of categorical differences between space and time. It is proposed that Riemann flatness does not preclude detection for spatially curved metrics in which the Riemann tensor vanishes due to cancellation of space versus time components. Examples include the Milne and Rindler metrics. Overall, questions are raised about space, time, and the foundations of gravitational wave theory, leading to the suggestion that historical assumptions may have been misapplied in the standard approaches of today.

I. INTRODUCTION

Interest in gravitational waves (GWs) of both astrophysical and cosmological origin has flourished since the advent of the Advanced LIGO and Virgo detectors, whose observations have unleashed a new era of multi-messenger astronomy [1]. GWs are believed to interact only weakly with matter and may thus provide a powerful tool for investigating inflation models and the high-energy fields of the early universe [2-6]. It is anticipated that GW detectors in the near future will offer precise tests of quantum gravity and of general relativity (GR) in the strong-field regime [1,7]. Unresolved GWs generated by a variety of mechanisms are assumed to form a possibly anisotropic stochastic gravitational wave background (SGWB) [8-11], observable by current or near-future detectors. The SGWB is predicted to arise from the superposition of unresolved GWs emanating from a broad array of astrophysical and cosmological sources [12-15], including close binary systems, supermassive black hole mergers [16], galactic millisecond pulsars [17], primordial black holes [18-25], quasars [26], cosmic strings [27-30], inflation [2,31-33], primordial GWs [34-36], cosmological phase transitions [37-40], photon graviton conversion from black hole photon spheres [41], superradiant instability of spinning black holes in the presence of massive bosons [42], and magnetohydrodynamic turbulence due to high conductivity at early epochs [43,44].

The SGWB is in some ways a gravitational analog of electromagnetic radiation backgrounds such as the cosmic microwave background (CMB) and the *stochastic photon background* (SPB). The SPB is defined here as a composite of photon backgrounds arising from or influenced by non-CMB sources. These backgrounds include secondary photons from relatively cold CMB photons scattered by the inverse Compton process off of free hot electrons in the intracluster medium of galaxy groups and clusters, causing a distortion of the CMB spectrum known as the thermal Sunyaev-Zel'dovich effect [45,46]. Other SPB components include the cosmic radio background (CRB), believed to be of extra-galactic or cosmological origin and displaying temperatures substantially greater than those observed in radio-emitting galaxies [47]; and the cosmic infrared background (CIB), comprising photons arising from thermal dust emission in star-forming galaxies [45,46].

The SGWB promises to provide one of the best windows we have into the physics of the early universe [22,48,49], and is a topic of growing importance in the literature [9,50-56]. Indeed, there is evidence the SGWB may have already been detected in pulsar timing array (PTA) data from the North American Nanoherz Observatory for Gravitational Waves (NANOGrav) [15,16,21,43]. However, the NANOGrav data implies a GW polarization that departs from that expected in GR, suggesting a nonstandard form of gravitational oscillation or a modified theory of gravity [15]. This question remains open.

Given the relevance of the SGWB and other radiation backgrounds to this and other fundamental questions in gravitation and cosmology, it is worthwhile to reexamine seldom-explored general relativistic phenomena related to nonstandard spacetime disturbances. The purpose of this paper is to derive and investigate a novel spacetime disturbance, to be called the *quasi-inertial disturbance* (QID), which could conceivably impact both the SGWB and SPB, and potentially shed light on the NANOGrav question. Defined as a plane-symmetric general relativistic spacetime variation traveling at the speed of light c, the QID is represented by a dynamic metric $g_{\mu\nu}(t,x)$ in rectangular coordinates that exactly solves the vacuum Einstein equations $R_{\mu\nu}=0$. QIDs may be either periodic or nonperiodic. The periodic variety investigated here are called *quasi-inertial oscillations* (QIOs).

QIOs possess neither energy nor Riemann curvature and are best described as *quasi-inertial* rather than as *gravitational* phenomena. QIDs, of which QIOs form an oscillating subclass, are perhaps the simplest dynamic metrics that exactly solve Einstein's vacuum field equations (EVFE), and one might wonder why such solutions are not routinely studied. The historical reason for this omission is precisely their curvature-free nature, a property that led earlier researchers such as Taub and McVittie to dismiss previously known QIO solutions as *spurious* or *unphysical*, and hence unviable as GW candidates [57-59]. The claim that curvature-free waves are unphysical will be referred to here as *Taub's rule*. However the above authors were seeking GWs capable of forming gravitons, a unique requirement beyond mere observability. Hence in Taub's usage, the term *unphysical* does not automatically mean *unobservable*. Indeed, that QIOs are observable forms a central theme of the present work. Here and throughout this paper, a metric is defined as *observable* if the *varying features* of the metric can in principle be measured in some physical reference frame, where the term *varying features* denotes explicit space or time dependent quantities.

It is well known that *Riemann flat* metrics, i.e. metrics with a vanishing Riemann tensor $R_{\mu\nu\alpha\beta}=0$, can be transformed into the Minkowski metric by a suitable change of coordinates. Such a transformation property is often assumed to mean that the metric is also undetectable. However, this assumption does not hold for metrics whose coordinates are defined by a physical platform or structure, such as the surface of a planet, a rotating space station, an accelerating rocket, or the CMB, nor for metrics associated with frame-dependent quantum effects, such as the Unruh effect [60]. Indeed, physical structures defining *de facto* preferred coordinate frames, a setup unavoidable in practice, offer detection capabilities that are often overlooked in pure mathematical contexts, in which all frames are held to be equally viable even if based on null [61] or other unrealizable coordinates. The existence of a *de facto* preferred frame, of course, does not violate covariance, but arises simply as an artifact of the indispensable role of the observer. Indeed, when observational aspects are paramount, *de facto* preferred frames are sometimes employed in the literature to provide deeper insight into GR solutions. An informative example is offered by Hobson [62], who constructs preferred frames for Schwarzschild and Reissner-Nordstrom metrics based on trajectories of massive particles. These frames define coordinates that are well-behaved at the event horizon and useful in clarifying many observable phenomena, including Hawking radiation. Thus specific coordinate frames can be essential for detecting or describing the varying features of a metric. With this understanding, it is shown in Section IV that QIOs are in principle observable by suitable detectors aboard *any* solid space-based platform, where the platform must be larger than the QIO wavelength to hold the detectors rigid. It follows that, contrary to the historical interpretation of Taub's rule, QIOs are nontrivial from an observational standpoint.

It is further conjectured in Section VI that Riemann flatness does not rule out detection for metrics with the property that the Riemann tensor vanishes due to cancellation between space and time components. Such a cancellation may be questionable in any case by virtue of manifest differences between space and time, as will be argued. Flat yet observable metrics with the above property include the Milne and Rindler metrics. Note that although these metrics have zero *spacetime* curvature, their *spatial* curvature is nonzero, supporting the additional conjecture that spatial curvature alone offers a sufficient criterion for detection. The Milne and Rindler metrics will be discussed briefly in the next paragraph. The Rindler metric will be explored more thoroughly in Section IV.

The idea that Riemann flatness does not preclude detection can be made intuitive by the following three examples. In the first example, we consider the *Rindler frame*, as defined by Sugiyama et al. in [63], which corresponds to an observation platform undergoing constant acceleration. The Rindler metric can be made Minkowskian by a coordinate transformation, meaning the metric is Riemann flat. Nevertheless an observer in a Rindler frame can detect the varying features of the metric both by inertial forces and theoretically also by Unruh radiation, the latter a prediction of quantum field theory for non-inertial or curved spacetimes [60,64-66]. The Rindler metric is particularly relevant to QID observability in that Rindler metrics can be expressed as a product of rectangular advanced and retarded QIDs. This result is derived in Section IV. The close relationship between Rindler metrics and QIDs suggests that QIDs might also be detectable through a form of Unruh-type radiation. This is a topic for future research.

The second example involves the metric for a constantly expanding Milne universe, given by

$$ds^2 = dt^2 - b^2 t^2 (dr^2 + r^2 \, d\Omega^2)$$

where b is a constant coefficient. Milne originally derived this metric in a special relativistic framework [67]. However, the metric may also be derived in the GR formalism as a solution to Einstein's field equations. In the latter case, the Milne metric constitutes a special case of the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 \, d\Omega^2)$$

for an empty universe, where the comoving coordinates expand at a constant rate with scale factor $a(t)=bt$ [68-72]. The Milne metric can be transformed into a patch of Minkowski space and is therefore Riemann flat. However a hypothetical Milne universe, whose spatial coordinates are defined by small comoving test stars, would be detectable by a linear Hubble redshift relation, where in GR, the redshift is assumed to arise from spatial expansion, a behavior incompatible with Minkowski spacetime. It is significant that the Milne universe, despite its zero *spacetime* curvature, has nonzero *spatial* curvature, again supporting the conjecture that spatial curvature suffices for detection. As an interesting aside, the Milne metric, though a solution to Einstein's field equations for empty space only, is not unrealistic when compared to modern astronomical data on cosmic acceleration. Indeed, some authors maintain that SNIa data, as of 2020, do not convincingly demonstrate acceleration, but instead constitute a fair, or even excellent, fit to the Milne universe [69,71,72].

The third example involves the metric of a rotating coordinate frame. This metric can also be made constant by a change of coordinates, meaning it too is curvature free. Yet such a metric is observable through for example the Coriolis force, an inertial force measurable in any earth-based laboratory. In addition, rotating frames are believed detectable by Unruh radiation [73]. The above three examples support the present postulate that, contrary to a *prima facie* interpretation of Taub's rule, the vanishing of the Riemann tensor does not make detection impossible with respect to ordinary rigid observation platforms.

It is important to emphasize that the wave features of QIOs are arguably *more* real the inertial forces found in Rindler or rotating frames, since the latter forces can be measured only on specific accelerating platforms, while QIOs can in principle be measured on *all* sufficiently large rigid space-based platforms. Moreover, Rindler and rotating metrics are thought to describe spacetime distortions arising from the motion of the platform, while a QIO metric describes spacetime distortions arising from the *motion of distant matter*, a phenomenon somewhat reminiscent of Mach's principle. Due to these distinctions, a QIO is defined here as *quasi-inertial* rather than *inertial*. With regard to observation, it therefore seems plausible that broad-spectrum, low-amplitude QIOs populate the universe and exert a hidden influence on the dynamic properties of the vacuum, particularly in the sparse realms of the intergalactic medium. Higher-frequency QIOs would be expected to penetrate galaxies, and could conceivably pierce solid matter in the ultraviolet limit, causing elementary particle fluctuations, perhaps by a process akin to the ponderomotive effect discussed by Deepen Garg et al. in [74].

In the vacuum region of a QIO, the energy-momentum tensor $T_{\mu\nu}$ on the rhs of Einsteins field equations vanishes by definition, where Einstein's equations are given by

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

for $R_{\mu\nu}$ the Ricci tensor, R the curvature scalar, and $\kappa = -8\pi G/c^2$. Thus a QIO does not contribute to its own background energy-momentum field and constitutes an energy-free plane wave. The physical interpretation of a QIO therefore requires an understanding of gravitational field energy which, while common in many GR applications [58], differs from that in the literature on standard GWs. There is of course nothing in the GR formalism that prohibits energy-free wave solutions [74]. Energy-free GWs, sometimes associated with "soft gravitons", are an active topic of research, and methods of detection are now under discussion [75,76]. Historically, the energy relation $E=h\nu$ for GWs arose from analogies with electromagnetism or quantum mechanical wave functions. More sophisticated definitions of effective GW energy density have also emerged [9]. However, these definitions are approximate and often ambiguous [58,77]. Moreover, introducing field energy into GR frequently involves *ad hoc* definitions of quasi-local energy density, which is problematic due to gauge dependence [9,78-80]. These issues are avoided by assuming the gravitational field does not contribute to the energy-momentum tensor in any case, and indeed that the field itself contains no energy. It might be objected that waves cannot propagate without energy, implying spacetime somehow impedes wave motion. However according to the present interpretation, the gravitational field is synonymous with the geometry of spacetime, to which energy, impediments, or other material properties cannot be attributed.

Gravitational radiation in the form of standard GWs is believed to have two possible polarizations in GR, both of them transverse spin-2 tensor modes. These are called *Einsteinian polarizations*. Modified gravity theories, such as *f(R)* and other metric theories, permit up to four additional modes, including two spin-1 vector modes and two spin-0 scalar modes, where the latter are the *breathing mode* and the *longitudinal mode* [81-92]. These are referred to as *non-Einsteinian polarizations*. QIOs, although consistent with GR, carry non-Einsteinian longitudinal polarization, allowable due to their quasi-inertial nature. It is often claimed that the detection of longitudinal GWs would prove some form of modified gravity prevails in our present universe [81,89,93,94]. However unless the detector were able to distinguish longitudinal GWs from QIOs, the latter might conceivably be misindentified as a GW carrying non-Einsteinian polarization. Modified gravity theories that permit longitudinal GWs include tensor-vector-scalar theories such as TeVeS, bimetric gravity theories, the Einstein-Ether theory [93], the Lightman-Lee theory [90], *f(R)* and Horndeski theories [95], higher-order R^2 scalar gravity [96], and massive gravity theories [50,89], including massive teleparallel Horndeski gravity [97].

Some authors propose that the high-energy regime of the early universe may have been governed by a type of modified gravity that allows emission of non-Einsteinian longitudinal GWs [4]. These primordial longitudinal GWs (PLGWs), should they exist, are expected to traverse the cosmos and be observable by future longitudinal GW detectors. However, if GR accurately describes the universe today, and if non-Einsteinian modes are impossible in GR, it seems evident that PLGWs would be unable to propagate through the regions of our present cosmic neighborhood. PLGWs emitted under a modified gravity regime would therefore decay or change form on their way to our observatories. It is postulated here that PLGWs might in fact change form, losing both energy and curvature as they propagate over cosmic distances, finally evolving into a wavelike imprint whose features are consistent with GR. According to this postulate, QIOs may arise as an end state of PLGW evolution, a scenario to be discussed in Section V.

Should provable sources exist and a realistic interpretation be established, QIOs may lead to new physics and exert measurable effects on the SGWB. It is also conceivable that QIOs could induce oscillations in the free electrons of the intracluster medium, which would in turn emit photons as a component of the SPB. This process would create an additional photon background masquerading as a component of the CMB, but with spectral and isotropy signatures distinct from that of the blackbody radiation emitted at recombination. QIOs might in addition be directly observable as a component of the SGWB by future longitudinal mode gravitational wave detectors [98]. Such effects would be particularly relevant today, in view of the recent emergence of tantalizing evidence for SGWB detection [18,99-101].

Traditionally, GWs were derived based on long-debated conventions about what constitutes a viable wave solution to Einstein's field equations [57,58,60,109,110]. One such convention requires the waves be described by perturbations on a background metric, where the perturbation term $h_{\mu\nu}$ in the metric $g_{\mu\nu}=f_{\mu\nu}+h_{\mu\nu}$ must be small compared to the background term $f_{\mu\nu}$ [111-115]. Standard GWs are therefore approximations valid only in the weak-field regime, (see however [76,116-118]), *if they are valid at all*, a position debated in [119]. The background metric is usually chosen to have a high degree of symmetry, and may for example be the Minkowski metric, the Schwarzschild metric [120], the Friedman-Robertson-Walker (FRW) metric [3,18,121-123] or the Newtonian gauge representation of the FRW metric [2,43].

The weak-field approximation to GR is often referred to as *linearized gravity* [111-115,124]. However, the motivation for linearizing GR to derive GWs is somewhat obscure in the literature. Weinberg drew an analogy with the wave equations of particle physics to conclude that GWs carry energy $E=h\nu$,[1] which in turn contributes to the energy-momentum tensor $T_{\mu\nu}$ and prevents solutions to the exact Einstein equations, a dilemma supposedly avoided by linearization [125]. Some authors adopt linearized GR to make the field equations tractable [126], while others work in the weak-field regime without explicit justification [112,114,115]. Still others claim that exact plane wave solutions are unphysical, perhaps due to Riemann flatness or to the energy-free nature of the waves [104], although the reason is not always stated [79]. These claims led to the belief that only linearized GWs are viable [57,58,79,110]. (See however [108] for discussion of exact plane wave solutions.) It thus appears linearized gravity is adopted due to any of three motivating factors: to simplify Einstein's equations, to compensate for assumed GW energy, or to avoid supposedly unphysical exact plane wave solutions. These three factors, while potentially relevant to standard GWs, are irrelevant in the context of exact plane waves, a viewpoint highlighted in the next paragraph.

First, regarding simplification of Einstein's equations, it is important to recognize that in rectangular coordinates, the field equations can be reduced to two elementary wave equations that are easily solved in exact form. Indeed, exact plane wave derivations use fewer assumptions and are often far simpler than linearized GW solutions. Second, with regard to Weinberg's claim that GW energy feeds back into $T_{\mu\nu}$, requiring perturbations to make the energy negligible, it should be noted that in other GR applications, gravitational field energy is assumed to make no contribution to the energy-momentum tensor in the first place [114,126], rendering moot any need for perturbations. Moreover, some authors point out that metric perturbations are unphysical and inherently incorrect due to gauge dependence, casting doubt on the validity of all supposed GW detections to date [119]. Third, regarding the viability of exact plane wave solutions, there are contexts in which exact plane waves are considered more viable than linearized waves. In such cases, perturbation methods may be inadequate, again due to gauge dependence [3]. In particular, the *GW memory effect*, defined as a distortion in a GW detector that persists after the wave has passed [127], involves exact plane wave solutions [105] and is described by Christodoulou as "an inherently nonlinear phenomenon that cannot be captured by perturbation theory [128]." (Interestingly, QIDs may also produce a memory effect, as will be shown in Section II.) In any event, fundamental logic calls into question how a physical process could even exist for which approximate solutions are more viable than exact ones, an enigma not addressed this paper.

Another convention adopted in traditional derivations of GWs is that of the traceless transverse (TT) gauge. This stems from the consensus, based on Taub's rule, that real GWs must have nonzero curvature, and therefore that the Riemann tensor $R_{\mu\nu\alpha\beta}$ must not vanish [129]. Accordingly, Taub, Weber, and others obtained GW solutions by identifying nonzero components of the Riemann tensor and showing that these correspond to transverse polarizations [57,79,110]. Standard GWs were thus assumed to be transverse waves much like electromagnetic waves [12,130], a similarity often touted as confirmation of Taub's method. Transverse polarization of course means that a GW with x-directed flow will cause space in the y,z directions to be alternately stretched and contracted, while space in the x direction remains unchanged. The QIO, in contrast, is a longitudinal plane wave. Hence a QIO with x-directed flow will cause space in the x direction to be stretched and contracted, leaving space in the y,z directions unchanged. Notably, a QIO causes time to be stretched and contracted with magnitude and phase identical to that of space.

There appears an intriguing possibility that colliding QIOs might manifest nonzero Riemann curvature. The idea of colliding gravitational wave fronts was explored historically in [102], and is discussed in [103] in the context of *Kundt waves*, of which QIOs are perhaps a special case [104]. Kundt waves are defined in [103] as exact wave solutions to Einstein's field equations for the vacuum with shearfree null hypersurface wave fronts that may or may not be hyperplanes; the QIO would be a case of a hyperplane wave front. However, available references to prove a QIO is a Kundt wave are currently unknown to this author. QIOs, when expressed in lightcone or *Brinkmann* coordinates [75,104], also resemble the exact gravitational plane waves known as *pp-waves* [106,107], defined in [108] as plane-fronted GWs with parallel rays. However the pp-wave and QIO classes of metric share only a trivial common subclass. QIOs will be transformed into Brinkmann coordinates and compared with pp-waves in Section III.

This paper is organized as follows. The elementary wave nature of Einstein's equations will be shown in Section II, where the vacuum field equations are solved for a QIO metric in rectangular coordinates. In Section III, lightcone coordinates are used to derive a transformation that renders the QIO metric Minkowskian. QIOs will also be presented in Brinkmann coordinates and compared with pp-waves. In Section IV it will be shown that the particle Lagrangian $L=mds/dt$ predicts test mass acceleration in the field of a QIO, indicating QIOs can in principle be observed in any non-freely falling frame. In addition, QIO observability will be argued by analogy with the Rindler metric, which can be constructed as a product of rectangular advanced and retarded QIDs. Section V is devoted to generation, propagation and detection of QIOs, offering the hypothesis that QIOs may be an end product of PLGW evolution, and featuring a skeletal design for a space-based QIO detector. Section VI highlights categorical differences between space and time, leading to the conjecture that in *pseudo*-Riemannian spacetime, Riemann curvature is not a necessary criterion for observability.

Greek indices run from 0 to 3 throughout this paper. Units $G=c=1$ will be used, although these constants are sometimes inserted for clarity. We will work in the signature (+−−−), using Dirac's sign convention [114]. The notation ∂_μ designates the partial derivative $\partial/\partial x^\mu$.

II. DERIVING THE QIO FROM EINSTEIN'S FIELD EQUATIONS

It is of course true that a curvature-free metric exactly solves EVFE, since if the metric obeys $R_{\mu\nu\alpha\beta}=0$, it also obeys the contracted equation $R_{\mu\nu}=0$. However, it will be instructive to solve EVFE explicitly in order to highlight the simple wave nature of Einstein's equations, especially as this contrasts with the comparative complexity of standard linearized GWs. Accordingly, it is shown in this section that QIDs, and hence QIOs, are exact solutions to Einstein's field equations. Physical and intuitive properties of QIDs will also be discussed below Eqs. (2).

To proceed, we substitute into EVFE a diagonal metric $g_{\mu\nu}(t,x)$ in rectangular coordinates obeying the relations $g_{11}(t,x)=-g_{00}(t,x)$ and $g_{22}=g_{33}=-1$. This class of metric may be represented by the line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a(t,x)dt^2 - a(t,x)dx^2 - dy^2 - dz^2 \quad (1)$$

where $a(t,x)$ is any positive function of t and x. When this metric is substituted into EVFE, the field equations simplify, after some calculation (given below), to a pair of homogeneous wave equations

$$\partial_0^2 a(x,t) - \partial_1^2 a(x,t) = 0, \quad (2a)$$

$$[\partial_0\, a(x,t)]^2 - [\partial_1\, a(x,t)]^2 = 0. \quad (2b)$$

It is clear from Eqs. (2) that the metric of Eq. (1) reduces EVFE to two elementary equations with exact plane-wave solutions. That Einstein's equations in rectangular coordinates can be reduced to this simple form is apparently not well known and rarely if ever mentioned in the literature. One might expect these equations to be occasionally discussed, if only to illustrate the simple wave structure of EVFE, and their omission may seem surprising. As previously noted, earlier authors in search of the graviton dismissed such solutions as spurious, causing subsequent authors to ignore them [118], an omission that can now be addressed with the benefit of hindsight.

Examining Eqs. (2), it is easy to see that if $a(t,x)$ can be expressed in a form $a(t,x)=\beta(u)$, where β is any function of a null variable u such that $u\equiv ct-x$, with c the speed of light, the metric $g_{\mu\nu}$ of Eq. (1) exactly solves Eqs. (2) and hence solves EVFE. Such a solution, expressed in terms of u, is defined, in the context of rectangular coordinates, as a *forward-propagating* or *retarded* QID. QID solutions are also possible for $a(t,x)=\gamma(v)$, where γ is any function of variable v with $v\equiv ct+x$. These solutions are defined as *backward-propagating* or *advanced* QIDs. Retarded and advanced QIDs will be shown in Section IV to be mathematically related to the Rindler metric. We will focus on retarded QIDs in this section.

A QID can be viewed as a ripple in spacetime that may take any shape, periodic or non-periodic. As the ripple passes a freely floating test clock or ring of test masses, a space-based observer on a nearby non-freely falling platform in coordinate frame (t,x,y,z) could in principle detect dilation and contraction of both the rate of the test clock and the diameter of the test ring, as will be demonstrated in Section III. The effect on test clocks and test rings can be read directly from the line element,

$$ds^2 = \beta(ct-x)dt^2 - \beta(ct-x)dx^2 - dy^2 - dz^2$$

where the clock rate $d\tau=\beta^{½}dt$ and diameter $dD=\beta^{½}dx$ are functions of observer time t. This class of metric is shown by Taub to be Riemann flat [57]. Despite its flatness, QID observability will be substantiated in Section III both by invoking the particle Lagrangian and by examining the closely related Rindler metric.

As an interesting aside, QIDs are capable of producing memory effects, a scenario mentioned in Section I. Memory effects would arise when the value of the function $\beta(ct-x)$ differs between negative and positive asymptotic regions, so that the passage of the QID permanently changes clock rates and test ring diameters. In another curious application, QIDs could conceivably take the shape of Dirac delta or impulse functions, and thus may constitute energy-free counterparts of the exact plane-wave Lorentz-boosted shockwaves discussed in [132].

In the present work, we are mainly interested in periodic QIDs, or QIOs. The line element for a QIO plane wave with x-directed flow is a special case of Eq. (1) and can be represented by

$$ds^2=(1+be^{i(vt-kx)})dt^2-(1+be^{i(vt-kx)})dx^2-dy^2-dz^2 \quad (3)$$

where b is a constant amplitude, v is a constant frequency, k is the wave number with $k\equiv 1/\lambda$ for constant wavelength λ, and phase velocity w is given by $w=v/k$. If the amplitude b is such that $0\leq|b|<1$, the metric is singularity-free. Our investigation will focus on nonsingular QIOs.

That the phase velocity w must equal the speed of light c is not assumed *a priori*, but is derived by solving Einstein's field equations. We begin the procedure by stating *Einstein's law of gravity* for the vacuum

$$R_{\mu\nu}=0. \quad (4)$$

Here $R_{\mu\nu}$ is the *Ricci tensor* given by

$$R_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\alpha,\nu} - \Gamma^{\alpha}{}_{\mu\nu,\alpha} - \Gamma^{\alpha}{}_{\mu\nu}\Gamma^{\beta}{}_{\alpha\beta} + \Gamma^{\alpha}{}_{\mu\beta}\Gamma^{\beta}{}_{\mu\alpha} \quad (5)$$

where a comma denotes partial derivative, and the Christoffel symbols Γ are defined by the Levi-Civita connection

$$\Gamma_{\alpha\mu\nu} = ½(g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha}).$$

To prove the metric $g_{\mu\nu}$ of Eq. (3) exactly solves Eq.(4), we first assume a general diagonal metric of the form $g_{\mu\nu}=diag[g_{00}(t,x),g_{11}(t,x),-1,-1]$. It is sufficient to show that R_{00} and R_{11} vanish for phase velocity $w=c$, noting that R_{22} and R_{33} are automatically zero due to the vanishing of all Christoffel symbols with indices 2 or 3. We evaluate R_{00} and R_{11} by taking the rhs of Eq. (5) and summing separately the first pair of terms, denoted $R_{\mu\mu}(1+2)$, and the second pair of terms, denoted $R_{\mu\mu}(3+4)$. We then show that each of these sums vanishes for metrics of the form $diag[g_{00}(ct-x),-g_{00}(ct-x),-1,-1]$, of which QIOs are a special case. The calculation does not require explicit evaluation of the non-zero Christoffel symbols.

For R_{00}, the first pair of terms on the rhs of Eq. (5) are:

$$R_{00}(1+2) \equiv \Gamma^{\alpha}{}_{0\alpha,0} - \Gamma^{\alpha}{}_{00,\alpha}$$

$$= \partial_0 (\Gamma^0{}_{00} + \Gamma^1{}_{01}) - (\partial_0\Gamma^0{}_{00} + \partial_1 \Gamma^1{}_{00}).$$

Cancelling and lowering indices gives:

$$R_{00}(1+2) = \partial_0(g^{11}\, \Gamma_{101}) - \partial_1(g^{11}\, \Gamma_{100}).$$

Since $\Gamma_{101}=½\partial_0\, g_{11}$ and $\Gamma_{100}=-½\partial_1\, g_{00}$, this becomes

$$R_{00}(1+2) = ½\partial_0(g^{11}\, \partial_0\, g_{11}) + ½\partial_1(g^{11}\partial_1 g_{00})$$

$$=½[(\partial_0 g^{11})(\partial_0 g_{11})+(\partial_1 g^{11})(\partial_1 g_{00})+g^{11}(\partial_0{}^2 g_{11}+\partial_1{}^2 g_{00})].$$

Now since $g^{11}=1/g_{11}$ we have that

$$\partial_0\, g^{11} = -(g^{11})^2\, \partial_0\, g_{11}$$

$$\partial_1 g^{11} = -(g^{11})^2\, \partial_1 g_{11}$$

and hence

$$R_{00}(1+2) = -\tfrac{1}{2}(g^{11})^{2}\,[(\partial_0 g_{11})^{2} + (\partial_1 g_{11})(\partial_1 g_{00})]$$

$$+\ \tfrac{1}{2}\, g^{11}\,(\partial_0^2 g_{11} + \partial_1^2 g_{00}).$$

We next evaluate the last two terms on the rhs of Eq. (5):

$$R_{00}(3+4) = -\Gamma^{\alpha}{}_{00}\Gamma^{\beta}{}_{\alpha\beta} + \Gamma^{\alpha}{}_{0\beta}\Gamma^{\beta}{}_{0\alpha}$$

Using the relevant Christoffel symbols in their general form

$$\Gamma_{000} = \tfrac{1}{2}\partial_0 g_{00},\ \ \Gamma_{100} = -\tfrac{1}{2}\partial_1 g_{00},\ \ \Gamma_{101} = \tfrac{1}{2}\partial_0 g_{11},$$

$$\Gamma_{001} = \tfrac{1}{2}\partial_1 g_{00},\ \ \Gamma_{111} = \tfrac{1}{2}\partial_1 g_{11},$$

we obtain after some calculation

$$R_{00}(3+4) = -(1/4)\, g^{00}g^{11}(\partial_0 g_{00})\,(\partial_0 g_{11})$$

$$+\ (1/4)(g^{11})^{2}(\partial_1 g_{00})(\partial_1 g_{11}) - (1/4)g^{11}g^{00}(\partial_1 g_{00})^{2}$$

$$+\ (1/4)(g^{11})^{2}(\partial_0 g_{11})^{2}$$

Combining $R_{00}(1+2)$ and $R_{00}(3+4)$ and cancelling terms gives:

$$R_{00} = -\,(1/4)(g^{11})^{2}\,[(\partial_0 g_{11})^{2} + (\partial_1 g_{11})(\partial_1 g_{00})]$$

$$-\ (1/4)g^{00}g^{11}\,[(\partial_0 g_{00})(\partial_0 g_{11}) + (\partial_1 g_{00})^{2}]$$

$$+\ \tfrac{1}{2}\, g^{11}\,(\partial_0^2 g_{11} - \partial_1^2 g_{00})$$

By the same procedure, we obtain for R_{11}:

$$R_{11} = -\,(1/4)(g^{00})^{2}\,[(\partial_1 g_{00})^{2} - (\partial_0 g_{00})(\partial_0 g_{11})]$$

$$-\ (1/4)g^{00}g^{11}\,[(\partial_1 g_{11})(\partial_1 g_{00}) + (\partial_0 g_{11})^{2}]$$

$$+\ \tfrac{1}{2}\, g^{00}\,(\partial_1^2 g_{00} - \partial_0^2 g_{11})$$

The results so far apply to metrics of the type

$$ds^2 = g_{00}(t,x)dt^2 + g_{11}(t,x)dx^2 - dy^2 - dz^2$$

where g_{00} and g_{11} may be different functions of t and x. Next, we consider a metric for which these components are related by $g_{00}=-g_{11}$. This metric may be given by the line element

$$ds^2 = -g_{11}(t,x)dt^2 + g_{11}(t,x)dx^2 - dy^2 - dz^2.$$

Now we can express both R_{00} and R_{11} in terms of g_{11} alone, obtaining

$$R_{00} = \tfrac{1}{2}(g^{11})^{2}[(\partial_1 g_{11})^2-(\partial_0 g_{11})^2] + \tfrac{1}{2}g^{11}(\partial_0^2 g_{11}-\partial_1^2\, g_{11})$$

$$R_{11} = \tfrac{1}{2}(g^{11})^{2}[(\partial_0 g_{11})^2-(\partial_1 g_{11})^2] + \tfrac{1}{2}g^{11}(\partial_0^2 g_{11}-\partial_1^2\, g_{11})$$

If each of the two terms on the rhs of the above equations vanishes separately, then $R_{00}=R_{11}=0$ as required. This condition is equivalent to two simultaneous wave equations

$$(\partial_0 g_{11})^2 - (\partial_1 g_{11})^2 = 0 \qquad (6a)$$

$$\partial_0^2 g_{11} - \partial_1^2\, g_{11} = 0. \qquad (6b)$$

These equations are identical to the wave equations given by Eqs. (2). If we now let $g_{11}(t,x)=\beta(a)$ for β any function of the variable a such that $a\equiv wt-x$ with w an unknown phase velocity, it is immediately clear, by taking the partial derivatives of g_{11}, that

$$\partial_0 g_{11} = (d\beta/da)\partial_0 a = w d\beta/da$$

$$\partial_1 g_{11} = (d\beta/da)\partial_1 a = -d\beta/da$$

and hence, for $w=c=1$, we have $\partial_0 g_{11}=-\partial_1 g_{11}$, which exactly solves Eq. (6a). Similarly, it is clear by taking the second derivative that

$$\partial_0^2 g_{11} = w^2(d^2\beta/da^2)$$

$$\partial_1^2 g_{11} = d^2\beta/da^2$$

and again, for $w=c=1$, we see that Eq. (6b) is solved. We have thus proven that any QID metric of the form

$$ds^2 = \beta(ct-x)dt^2 - \beta(ct-x)dx^2 - dy^2 - dz^2 \qquad (7)$$

exactly solves Einstein's equations for the vacuum. It follows that the QIO metric of Eq. (3), a special case of the QID, is also an exact solution to EVFE.

III. DETECTION OF QIOs: THE PARTICLE LAGRANGIAN AND RINDLER FRAME

There has been long-standing skepticism in the literature regarding detection of exact longitudinal gravitational plane waves. This skepticism arose from mathematical principles underlying Taub's rule. More explicitly, since exact longitudinal plane-wave metrics produce a vanishing Riemann tensor, they possess no intrinsic spacetime curvature, meaning the wavelike features of the metric can be removed by a coordinate transformation. Many researchers therefore concluded that exact longitudinal plane waves were unphysical [57,58], further supposing they were also unobservable. However, it will be demonstrated below that QIO metrics are not only observable, but can in principle be observed by non-freely-falling detectors aboard any space-based platform, where the platform must be larger than the QIO wavelength to ensure the detectors are rigid in the oscillating field.

We will argue the observability of QIOs using two approaches. First, we will demonstrate that the particle Lagrangian for a QIO predicts an effective acceleration of test masses, a condition for detectability ordinarily deemed sufficient. Second, detectability will be argued from a more abstract standpoint based on the idea, inspired by the Rindler metric, that nonzero Riemann curvature is not a necessary criterion for the existence of effective forces, and hence for detectability. This argument entails a far-reaching analysis of the Rindler frame, which is analogous to the frame of a planetary surface in the limit of extreme diameter [65,133].

To introduce the first approach, it must be emphasized that mathematical principles do not tell us how to interpret a metric physically. Interpretation requires intuition, observation, and well-tested deductive procedures. We now offer a well-tested procedure for determining whether a metric is detectable. The salient point is that the procedure requires no knowledge of the Riemann curvature. We will use the example of the Schwarzschild metric at the earth's surface. The Schwarzschild metric is expressed by the line element

$$ds^2=(1-2M/r)dt^2 - (1-2M/r)^{-1}dr^2 - r^2 d\Omega^2, \qquad (8)$$

where M is the central mass. From the particle action $S=m\int ds$ given by Dirac in Ref. [114], with ds the Schwarzschild line element and m the test mass, we use parameter t to construct a particle Lagrangian $L=mds/dt$, so that the force will be expressed in terms of observer time t. (The result of course does not depend the choice of timelike parameter.) Based on this Lagrangian, we can derive the effective force F_{eff}, as judged by an observer at infinity, on a freely falling test mass dropped from some small height above the earth's surface. The effective force will then be adjusted for an earth-based observer. The appropriate Euler-Lagrange equation is

$$(d/dt)\,[\partial L / \partial(dx^1/dt)] = \partial L / \partial x^1$$

with $x^1=r$. To make the calculation simpler, we note that the Euler-Lagrange equation greatly simplifies for diagonal metrics in cases where test particles are free to move in the x^1 direction only (i.e. $dg_{22}/dt=dg_{33}/dt=0$), and are initially at rest or have velocity $v<<c$. The *simplified Euler-Lagrange equation* is found to be

$$m\, d^2x^1/dt^2 = m\, \partial_1 g_{00} / 2g_{11}. \qquad (9)$$

The effective force at the earth's surface, as judged from infinity, is thus

$$F_{eff} \equiv ma = -GMm\,(1 - 2GM/R_e) / R_e^2$$

where R_e is the earth's radius, GMm/R_e^2 is the Newtonian force, and $(1-2GM/R_e)$ is the GR correction. Note that this correction, in the case of a black hole, would cause F_{eff} to vanish at the event horizon, corresponding to infinite time dilation for infalling matter as seen from infinity.

To transform F_{eff} to an earth-based frame, a factor k must be inserted to adjust for time dilation $d\tau=(1-2M/R_e)^{1/2}dt$ and length contraction $d\Lambda=(1-2M/R_e)^{-1/2}dr$. The factor k is given by

$$k = (dt/d\tau)^2\,(d\Lambda/dr) = (1-2M/R_e)^{-3/2}.$$

This yields an effective force at the earth's surface of

$$F_{earth} = kF_{eff} = -GMm / (1 - 2GM/R_e)^{1/2} R_e^2$$

Since F_{earth} is measurable by the acceleration of a test mass, the metric is clearly observable. Generalizing this procedure, the conditions for observability are: 1) that there exist a coordinate frame X^μ in which an observation takes place; 2) that the metric can be represented in terms of these coordinates by a line element $ds^2=g_{\mu\nu}dX^\mu dX^\nu$; and 3) that the particle Lagrangian $L=mds/dX^0$ for line element ds indicates an effective force on a test mass.

This procedure will now be applied to show that QIOs are in principle measurable by non-freely falling space-based detectors. We first express the QIO metric in the form

$$ds^2 = (1+be^{i\eta})dt^2 - (1+be^{i\eta})dx^2 - dy^2 - dz^2, \qquad (10)$$

where $\eta\equiv vt-kx$ for frequency v and wave number k, and where phase velocity is $w=v/k=c$. The detectors are assumed rigid in coordinate frame (t,x,y,z) on a nearby space-based observation platform larger than the QIO wavelength. A skeletal design of the platform and detector will be featured in Section V. Before calculating effective forces, note that at least two observable quantities can be read directly from the metric of Eq. (10):

1.) *Time dilation-contraction:* Proper time τ for a floating test clock is given by

$$d\tau = (1 + be^{i\eta})^{1/2}dt.$$

2.) *Length dilation-contraction:* Proper length Λ for the longitudinal diameter of a floating test particle ring is given by

$$d\Lambda = (1 + be^{i\eta})^{1/2}dx.$$

These two quantities are periodic and suggest wave behavior.

More critical is whether the QIO can exert an effective force on a test mass m. We begin by taking the real part of the QIO metric of Eq. (10):

$$ds^2 = (1+b\cos\eta)dt^2 - (1+b\cos\eta)dx^2 - dy^2 - dz^2. \qquad (11)$$

To calculate the effective force, we insert the particle Lagrangian $L=mds/dt$ into the simplified Euler-Lagrange equation of Eq. (9), where now $x^1=x$, obtaining

$$F_{eff} \equiv m\, d^2x/dt^2 = -kb\sin\eta / 2(1+b\cos\eta). \qquad (12)$$

Note that for $b=1$, the rhs of Eq. (12) is proportional to the well-known half-angle tangent formula, which displays periodic singularities. To avoid these unphysical divergences, we assume $|b|<1$, thereby ensuring a continuous oscillation. Hence it is evident that the QIO exerts, to first order, an effective periodic force on a test mass. Theoretically then, an observer would perceive the test mass as oscillating in some fashion. This effective force is akin to the ponderomotive force described in Ref.

[74], by which zero-energy gravitational plane waves excite free particle motion in a vacuum. If the particle were charged, e.g. an electron, the oscillation would produce electromagnetic waves as measured in the observer's frame due to the relativity of electromagnetic radiation. A stochastic background of QIOs might thus induce excitations in free electrons in the intracluster medium, which would in turn emit blackbody radiation in the observer's frame.[2] QIOs might thereby contribute to a photon background in the SPB, possibly causing distortion of the CMB spectrum.

The second approach to arguing QIO detectability involves the *Rindler frame*, often described as an observation platform undergoing uniform acceleration in Minkowski space. We first present the basic properties of the *Rindler metric* as defined in [63]. The Rindler metric may be expressed by the line element

$$ds^2 = e^{2ax}\,dt^2 - e^{2ax}\,dx^2 - dy^2 - dz^2, \qquad (13)$$

where a is a constant associated with platform acceleration. To show that this metric produces a uniform effective acceleration for a test mass initially at rest, we use the simplified Euler-Lagrange equation of Eq. (9) to obtain immediately

$$d^2x/dt^2 = -a.$$

Thus the effective acceleration is constant, i.e. independent of t and x. For clarity and to avoid confusion, it might be recalled that the Rindler metric is sometimes defined in the literature by line elements of the form [60,64]

$$ds^2 = (ax)^2\,dt^2 - dx^2 - dy^2 - dz^2.$$

However this type of metric produces an x-dependent acceleration. For simplicity therefore we will ignore the latter metric and define the Rindler metric as that of Eq. (13).

The Rindler metric is a special case of the *Taub metric*, defined here as a subclass of the metrics analyzed by A. H. Taub in his seminal work on GWs of 1951 [57] and also by McVittie [58]. The Taub metric can be expressed by the line element

$$ds^2 = e^{2U(x,t)}(dt^2 - dx^2) - dy^2 - dz^2,$$

for $U(x,t)$ any function of x and t. Taub proves the above metric solves EVFE only if the function U takes the form

$$U(x,t) = f(u) + h(v)$$

where f is any function of $u \equiv x-t$ and h is any function of $v \equiv x+t$. It is clear that the Rindler metric of Eq. (13) is a special case of the Taub metric for which $f(u)=au/2$ and $h(v)=av/2$. Taub furthermore proves that if the derivatives df/du and dh/dv are constant, then the metric is Riemann flat. Hence the Rindler metric, with $df/du=dh/dv=a/2$, is flat as expected. Interestingly, QIDs are also special cases of the Taub metric for which either $g_{00}=g_{00}^{+}=e^{2U}=e^{2f(u)}$ for $h=0$ and f an arbitrary function of u, or $g_{00}=g_{00}^{-}=e^{2U}=e^{2h(v)}$ for $f=0$ and h an arbitrary function of v. Here g_{00}^{+} denotes the tt-component of a retarded QID, while g_{00}^{-} denotes the tt-component of an advanced QID. Using these results, we may now express the Rindler metric as a product of retarded and advanced QIDs:

$$\begin{aligned} ds^2 &= e^{2ax}(dt^2 - dx^2) - dy^2 - dz^2 \\ &= e^{2f(u)}e^{2h(v)}(dt^2 - dx^2) - dy^2 - dz^2 \\ &= g_{00}^{+}g_{00}^{-}(dt^2 - dx^2) - dy^2 - dz^2, \end{aligned}$$

where $f(u)=au/2$, $h(v)=av/2$, and $2f(u)+2h(v)=au+av=2ax$. Thus we have found an intriguing relation between QIDs and Rindler metrics. As noted earlier, Rindler frames are believed to be observable by Unruh radiation, suggesting that QIDs may also give rise to a form of Unruh-type radiation. This is a topic for later study.

Intuitively, if we now envision a Rindler frame as any coordinate frame in which test masses undergo a constant acceleration from rest, then the surface of the earth constitutes an approximate Rindler frame, since test masses dropped from a small height accelerate at a near constant rate $-a=g\cong -M/R_e^2$. In the limit as a planet becomes infinitely large, the surface approaches an exact Rindler frame [65, 133].

Now it is well-known that the Riemann curvature of a Schwarzschild metric falls off as $1/r^3$ [148], while the effective force falls off as $1/r^2$. Therefore there exists an extreme-radius region in which the Riemann curvature effectively vanishes, while the force remains quite measurable. Such behavior is already approximated on the surface of the earth. This behavior at extreme radii would present a logical contradiction if Riemann curvature were the sole criterion for detectability, since a flat Rindler metric and a curved extreme-radius Schwarzschild metric are observationally identical; i.e. test particle acceleration appears the same in both frames. In other words, if the Rindler and extreme-radius Schwarzschild metrics are observationally indistinguishable, one metric cannot be observable while the other is unobservable. This again suggests that Riemann curvature does not dictate observability relative to rigid physical coordinate frames.

IV. RIEMANNIAN FLATNESS, LIGHTCONE COORDINATES AND PP-WAVES

As stated earlier, a vanishing Riemann tensor means a metric can be made constant, or *Minkowskian*, by a coordinate transformation. The appropriate transformation for QIO metrics will be derived in this section. Ordinarily, Brinkmann coordinates (U,V,y,z) are used for transformations of this type. Brinkmann coordinates may be expressed in terms of rectangular coordinates (t,x,y,z) as [72,105]

$$U=2^{-½}(x/c+t), \;\; V=2^{-½}(x/c-t), \;\; y=y, \;\; z=z.$$

However, we will work in the physically equivalent but simpler *lightcone* or *null coordinates* [131] given by

$$u=(ct-x), \;\; v=(ct+x), \;\; y=y, \;\; z=z,$$

where u and v are defined, in a rectangular context, as *retarded* and *advanced coordinates* respectively. For later reference, lightcone and Brinkmann coordinates may be interchanged by the relations

$$u=-2^{½}cV, \qquad v=2^{½}cU. \tag{14}$$

We begin by representing the QIO metric as

$$ds^2=(1+b\cos ku)dt^2-(1+b\cos ku)dx^2-dy^2-dz^2 \tag{15}$$

where $b \cos ku$ is the real part of the oscillating term in the metric of Eq. (3). Our goal is to find a transformation $(t,x,y,z)\!\!->\!\!(T,X,y,z)$ that renders the QIO metric Minkowskian, where the Minkowski metric is given by the line element

$$ds^2 = dT^2 - dX^2 - dy^2 - dz^2. \tag{16}$$

To accomplish this, we first transform the QIO metric of Eq. (15) into lightcone coordinates (u,v,y,z). Setting c to unity, we note that $du=dt-dx$, $dv=dt+dx$, and therefore $dudv=dt^2-dx^2$. Thus the QIO line element in lightcone coordinates is

$$ds^2 = (1+b\cos ku)\, dudv - dy^2 - dz^2. \tag{17}$$

Now since $1+b\cos ku$ is a function of u, we may introduce a coordinate W such that $dW=(1+b\cos ku)du$, obtaining

$$ds^2 = dWdv - dy^2 - dz^2.$$

To render this equivalent to the Minkowski metric of Eq. (16), we require that

$$dWdv = dT^2 - dX^2.$$

This can be accomplished by the transformation

$$dT = dt + ½\, b\cos ku\, du \tag{18a}$$

$$dX = dx - ½\, b\cos ku\, du. \tag{18b}$$

To check that this transformation produces a Minkowski metric, note first that the difference of Eqs. (18) gives

$$dT - dX = dt - dx + b\cos ku\, du$$

while the sum gives

$$dT+dX=dt+dx.$$

Hence

$$(dT - dX)(dT+dX) = (dt-dx + b\cos ku\, du)(dt + dx).$$

Since $du\equiv dt-dx$, we have immediately that

$$dT^2 - dX^2 = (1+ b\cos ku)\, du\, (dt+dx) = dWdv$$

as required. Thus we have proven that the transformation of Eqs. (18) renders the QIO metric of Eq. (15) Minkowskian. Integrating Eq. (18) we obtain the final transformation [58]

$$T=t+(b/2k)\sin ku, \quad X=x-(b/2k)\sin ku, \quad y=y, \quad z=z.$$

The coordinates (T,X,y,z) are those of a freely falling observer. To such an observer, the spacetime would appear Minkowskian. A complication arises, however, in that the coordinates (T,X,y,z) stretch and contract dynamically relative to the rigid frame (t,x,y,z), and indeed even relative to the CMB. Hence the coordinate system (T,X,y,z) cannot be affixed to an observation platform made of solid material. It is thus unlikely such an observer could exist in practice, rendering coordinate frame (T,X,y,z) an abstraction primarily of philosophical interest. Now it might be argued that if *any* observer, even an abstract one, could perceive a metric as Minkowskian, the metric is undetectable. However again, this argument does not hold for the *de facto* preferred frame of a rigid observation platform. In any event, the *perception* of a freely falling observer is insufficient to prove triviality, since all spacetimes appear Minkowskian, at least locally, to freely falling observers. One might object that QIOs are not just locally Minkowskian, but are Minkowskian everywhere, a property called *nonlocal flatness*. Yet in real scenarios, a freely falling observer cannot always perceive whether the spacetime is nonlocally flat, and thus cannot automatically determine whether a metric is Minkowskian.

It is informative to compare QIOs with the pp-waves commonly discussed in the literature. Pp-waves are exact plane-wave metrics often expressed in Brinkmann coordinates (U,V,y,z) by the line element [105,107]

$$ds^2 = - F(V,y,z)\, dU^2 - 2c^2dUdV - dy^2 - dz^2 \tag{19}$$

For comparison, the QIO line element of Eq. (17), when expressed in Brinkmann coordinates using Eqs. (14), becomes

$$ds^2 = - 2c^2(1+b\cos ku)dUdV - dy^2 - dz^2. \tag{20}$$

Comparing the pp-wave and QIO line elements, it is clear the two classes of metric share only a trivial common subclass, for which $F(V,x,y)$ in Eq. (19) and b in Eq. (20) both vanish. Thus QIOs are only trivially related to pp-waves.

V. QIO GENERATION, PROPAGATION, AND DETECTION

Theoretical analysis of waves in general and QIOs in particular may be divided into three stages: generation, propagation and detection, an idea put forth in [79]. Regarding generation, an intriguing source of QIO creation may reside in the early cosmos, a high-energy regime in which modified gravity might have prevailed. It is proposed, possibly for the first time, by Alsushi Nishizawa *et al.* in Ref.[4] that if GR did not strictly hold in the early universe, GWs carrying non-Einsteinian scalar and vector polarization modes could have been emitted during that era through mechanisms such as inflation [122], phase transition, and reheating. Scalar modes of course include the longitudinal mode, a signature of QIOs. Though longitudinal GWs are considered impossible in GR, they are permissible in modified gravities such as massive gravity [49] and $f(R)$ gravity, where the latter is a type of modified gravity in which the Ricci scalar R in the Einstein-Hilbert action is replaced by an arbitrary function $f(R)$. This type of modified gravity includes the Starobinsky model, for which $f(R)=aR+bR^2$ [96], a theory sometimes proposed to explain inflation. The implication in [4] is therefore that early modified gravity could have given rise to primordial longitudinal GWs, here called PLGWs, which persist to this day and are in principle observable by properly designed detectors. However it seems evident that if modified gravity prevailed the early universe, while GR prevails today, longitudinal GWs from the primordial era could not survive in today's cosmos, but instead must evolve into a form consistent with GR. According to this scenario, the evolution of a PLGW would coincide with the evolution of gravity itself. For example, if the early universe were governed by $f(R)$ gravity, then PLGWs would evolve synchronously as $f(R) \rightarrow R$, where $f(R)=R$ recovers GR exactly.

Since the end state of PLGW evolution must be consistent with gravity today, this end state should emulate a solution to EVFE, albeit with properties as yet undetermined. We may reasonably imagine the end state would have longitudinal rather than transverse polarization, would be periodic and plane-symmetric like the original PLGW, would travel at the speed of light, and would be described by a dynamic metric in the observer's coordinate frame. The QIO metric possesses all of these properties. It is therefore conceivable that QIOs might arise as remnants of scalar mode GWs generated during inflation. If QIOs are eventually observed by a future longitudinal GW detector [146], they may provide evidence for primordial modified gravity.

That QIOs are consistent with the end state of PLGW evolution leads to the postulate that PLGWs mutate or decay into QIOs during their transit of the cosmos, losing energy and curvature until they become energy- and curvature-free in the present era, while retaining a finite wavelength redshifted by universal expansion. QIOs may thus arise as an end-product of primordial GWs. This postulate does not rule out that QIOs may have other astrophysical or cosmological origins. For example, it is predicted in [79] that standard transverse GWs can acquire apparent longitudinal polarization after propagating across a pointlike gravitational lens, where the coupling of the transverse modes with the background curvature of the lens excites longitudinal modes. Such GWs might conceivably decay into remnant QIOs in asymptotic regions. Some authors also suggest that scalar perturbations may originate from quantum fluctuations of fields during inflation [2]. These perturbations could theoretically excite QIOs that persist as relics in the universe today. A third hypothetical source may arise from higher-dimensional gravity theories such as the braneworld model, in which longitudinal modes can propagate in the extra-dimensional bulk spacetime [4]. Additional origins of longitudinal modes are proposed in [89].

More fundamentally, however, since QIOs are energy-free exact solutions to EFVE, there is *nothing to obstruct* the generation of weak transient QIOs by accidentally aligned motions of matter, and thus nothing to prevent the vacuum from being filled with random spacetime fluctuations. Physicists have long predicted spacetime fluctuations in the context of quantum gravity. For example, authors Qingdi Wang and William G. Unruh [147] (2020) paraphrase John Archibald Wheeler (1955) as saying: "[O]ver sufficiently small distances and sufficiently small brief intervals of time, the *very geometry of spacetime fluctuates*. The spacetime would have a foamy, jittery nature and would consist of many small ever-changing regions [emphasis mine]." These authors of course attribute spacetime fluctuations to quantum processes, whereas QIOs would constitute a *classical* source of such fluctuations.

With regard to propagation, the idea that polarization modes or other properties of GWs evolve during cosmic transit is not new [36,79,89]. However, the evolution of longitudinal GWs into QIOs has not previously been considered in the literature. Longitudinal GWs, such as PLGWs, are said to stretch and contract space, exerting unspecified or independent effects on time [43,80]. QIOs, on the other hand, stretch and contract both space and time with equal magnitude and phase. As noted before, PLGWs are expected to lose curvature as they transit the cosmos, where curvature is defined by the Riemann tensor $R_{\mu\nu\alpha\beta}$. Curvature loss would coincide with a mutation of the PLGW time component to eventually cancel the space component in the Riemann tensor, producing $R_{\mu\nu\alpha\beta}=0$. Thus PLGW evolution would exhibit polarization that rotates between space and time, somewhat akin to a space-versus-time birefringence.

Since a QIO is a vacuum solution, it would disperse in unknown ways upon entering non-vacuum regions of the universe. To penetrate our galaxy and retain coherence, a QIO must have a wavelength small enough that the background appears as an effective vacuum. Locally detectable QIOs should thus have wavelengths smaller than the typical distance between stars and other galactic objects. Coincidentally, the wavelengths detected by current PTAs and the planned space-based Laser Interferometer Space Antenna (LISA) lie roughly between one astronomical unit (AU) and one parsec [79], well within the expected longer range of local QIO wavelengths.

It is also proposed here that QIOs with sufficiently high frequencies could pierce solid matter, transiting the empty space between elementary particles. Given an adequate presence of QIOs, such transmissions could cause random fluctuations in the positions of lighter particles in the atom such as electrons.

Regarding detection, it was shown in Section III that QIOs are in principle observable by any space-based non-freely-falling platform carrying suitably designed detectors. The platform must be rigid on a scale larger than the QIO wavelength to ensure the detectors are not freely oscillating in the field, as required by the proof of observability. A skeletal design for a detection apparatus might be as follows: Suspended in free space, at a distance greater than a QIO wavelength from the observation platform, a row of test clocks is allowed to float freely in the oscillating field of a QIO. The test clocks emit periodic low-energy photons both toward and away from the platform to prevent recoil. Due to the high frequencies of detectable QIOs, the photon emission rate could be some fraction of the expected QIO frequency. Detectors spaced along the rigid platfom would monitor photon reception, looking for in-phase correlations between length dilation-contraction of test clock positions, and time dilation-contraction of photon frequency and emission rate. Whether detection is possible using different designs is a future topic.

The detection apparatus must of course measure extremely minute effects, since QIOs are presumed to have amplitudes lower than those of standard GWs. Noise due to sources such as recoil of test clocks during emission are expected to make detection difficult, necessitating extraordinary sensitivity and precision. The range of detectable wavelengths is also highly constrained, being on the order of centimeters to 100 meters, corresponding roughly to ultrahigh frequencies in the range 10^{6} to 10^{10} Hz. Standard GW detectors in this range are only now being proposed [25,140].

As an alternative, PTAs may offer a means of detection for QIOs with wavelengths on the order of AUs to parsecs. In fact, there is a possibility QIOs have already been detected by PTAs. Researchers at NANOGrav reported in December 2020 strong evidence for a SGWB detection using PTAs, but added that the data lacks the spatial correlations given by the Hellings-and-Downs curve, whose functional form is a consequence of the quadrupole nature of standard GWs [21,141-144]. Some authors explain the lack of correlations by suggesting the observed GWs carry non-Einsteinian polarizations [100], possibly including the longitudinal mode [15], a distinct signature of QIOs. Others assume this lack is due to an unknown background [30], which might imaginably contain QIOs. Whether QIOs could in principle be detected by PTAs is a topic for further investigation.

VI. DISCUSSION AND CONCLUSION: TIME VERSUS SPACE

The physical meaning of wave solutions to Einstein's field equations was widely debated in the 1950s and 1960s [59,77,145]. A consensus, largely influenced by Taub, McVittie and Weber, emerged based on assumptions about the type of GWs these authors were seeking, namely GWs capable of forming gravitons. Since gravitons, if they exist, would be expected to persist as particles, it seems logical that the associated GWs could *not* be removed by a coordinate transformation. Hence the waves sought by these authors should be expected to possess curvature, as dictated by Taub's rule. However even this logic may be questionable, in that the particle concept poses ambiguities in curved Riemann space [62] and may appear ambiguous even in Minkowski space [124].

Nevertheless, subsequent authors assumed, without further debate [118], that Taub's rule applied to exact longitudinal plane waves, and consequently, that such plane waves possess no detectable effects. For example, Eardley, Lee and Lightman (ELL) claim, without explicit proof, that the Riemann tensor is *the only measurable field* for determining test particle acceleration and GW polarization modes [109],[3] a claim also put forth in [119]. However the ELL claim is in conflict with the result shown in Section III that exact longitudinal plane waves such a QIOs do in fact cause test particle acceleration. The ELL claim also implies a conflict with the Unruh effect, believed to be detectable in Rindler frames and rotating frames, both of which are Riemann flat.

To resolve these conflicts, the conjecture proposed here is that the Riemann tensor, while a valid measure for determining viable metrics in *Riemannian* geometry, is not a valid measure in *pseudo*-Riemannian geometry in cases where 4D Riemann flatness results from cancellation of space versus time components. In other words, if a pseudo-Riemannian metric has nonvanishing *3D spatial* curvature, the metric is nontrivial and potentially observable. This conjecture may not be new, as it possibly relates to the underlying concepts of what is called *spatially covariant gravity*, a type of modified gravity in which spatial curvature, rather than spacetime curvature, plays the fundamental role [149,150].

It is the assumption that space and time are dimensions on an equal footing that supposedly justifies mutual cancellation of time and space components in the Riemann tensor, thus allowing the notion that Riemann curvature defines the reality of a wave. Yet we have seen that metrics describing QIDs, Milne universes, Rindler frames and rotating frames possess a reality independent of Riemann curvature. Therefore the cancellation of space and time components in the Riemann tensor may not always be observationally meaningful. This in turn suggests that time is not just an extra dimension, but differs from space in categorical ways that transcend Riemannian geometry.

As a further point, the fact that QIDs have vanishing Riemann curvature does not mean they are *a priori* undetectable to human observers, any more than the vanishing in Kruskal coordinates [134] of the Schwarzschild event horizon renders it undetectable to human observers. Indeed, the Schwarzschild horizon may be quite real to human observers, who perceive space in a manner very different from time. (See [135] for methods of observing event horizons.) The underpinning of this idea is that physical space and physical time are not merely separate dimensions in 4D pseudo-Riemannian spacetime (the *geometric interpretation*), but that space and time differ in substantial ways that cannot be removed by geometrical operations.

Moreover, the geometric interpretation, while often tacitly assumed in the GR literature, is incompatible with theories of quantum gravity, where time is modeled completely differently from space [136,137]. Indeed, that time and space play distinct roles in quantum systems has posed an obstacle to the development of a complete theory of quantum gravity compatible with GR. An understanding of the distinctions between space and time may therefore reveal clues for reconciling the ELL claim with QIO observability, as well as for reconciling GR with quantum gravity.

Hence there arises the question: If time and space are categorically different, what exactly is different about them? To address this question, recall that in special relativity, the time and space axes undergo hyperbolic rotation due to the observer's velocity. Spacetime intervals, here to be called *vectors*, thus fall into three classes: timelike, spacelike, or null. To distinguish the properties of time and space, we are therefore tasked with identifying differences between timelike (or null) and spacelike vectors.

Causality marks one difference. Timelike or null vectors can carry information, meaning the endpoints of such vectors are causally connected [136]. Spacelike vectors cannot carry information and are causally disconnected. Causality further dictates that a diagonal metric expressed in spacelike and timelike coordinates has but one timelike component. The uniqueness of the time component means time acts more like a parameter of change than a dimension. Furthermore, time is irreversible and has an arrow or unambiguous ordering, while space does not. This distinction transcends the principles of pseudo-Riemannian geometry, to which time's arrow must be added as an independent postulate [136].

More concretely, electric charge, baryon number, lepton number and other quantum properties are conserved along timelike vectors but not along spacelike vectors. Due to these conservation laws, our universe is filled with very thin timelike fibers or threads of matter that extend indefinitely in time (elementary particles), while analogous spacelike fibers or threads (tachyons) have never been proven to exist, a profound manifestation of the uniqueness of time[4].

In a broader vein, the present moment of time defines a spacelike hypersheet that divides the 4D universe into past and future [138,139], where past events are determined, while future events are undeterminable due to quantum effects [136]. There is no such division of space. From these considerations, it is maintained here that, due to the distinct natures of space and time, Riemann flatness does not necessarily rule out the detectability of longitudinal metric plane waves, and furthermore that Taub's rule does not restrict quasi-inertial phenomena such as QIOs.

Strictly speaking, Taub's rule would introduce an added constraint on Einstein's law of gravity $R_{\mu\nu}=0$ for the vacuum, implying Einstein's law is incomplete as it stands, and should instead read:

$$R_{\mu\nu}= 0, \quad R_{\mu\nu\alpha\beta} \neq 0.$$

Yet Einstein's law is presented as complete in many contexts. This suggests that Taub's rule may not be fully integrated into the common understanding of GR. In view of the questions surrounding Taub's rule, it has been essential in this paper to reconsider the implications of a vanishing Riemann tensor for observations in real physical scenarios.

NOTES

1. (Section I) Steven Weinberg [125] assumes GWs should behave like the wave equations for elementary particles and photons, which carry energy $E=h\nu$. This assumption arguably defies logic. Elementary particles and photons, along with their attendant energy, are not intrinsic properties of spacetime, but are added to spacetime as *extrinsic* entities and reside locally within it. In contrast, the metric *defines* spacetime. It is not a localized entity residing within spacetime to which energy can be assigned. Thus the metric cannot be associated with energy, in turn implying energy is absent in the gravitational field and hence in gravitational waves.

2. (Section III) To first order, a test particle in the field of a QIO is subject to an effective periodic force in the x direction and will oscillate. For charged particles such as intergalactic electrons, the oscillation will cause emission of electromagnetic waves in directions y and z. This radiation can in principle be detected by a distant observer, as long as the observer’s frame does not follow the geodesic of the oscillating charge. (Were the observer to follow the same geodesic, no relative oscillation and hence no observed emission would occur.) It should be emphasized that QIO-induced photon emission has been derived here from an exact solution to EVFE, utilizing

only the particle Lagrangian, with no added assumptions. Indeed, photon emission induced by zero-energy waves is fully consistent with GR, given a proper understanding of gravitational field energy. That the gravitational field does not contribute to the energy-momentum tensor $T_{\mu\nu}$ is confirmed by many authors, including McVittie [58] and Dirac [114].

To visualize motion induced by an energy-free field, consider the Schwarzschild metric, an exact solution to Einstein's field equations for the vacuum. Despite the vanishing of the energy-momentum tensor, the field causes effective acceleration of freely falling bodies as measured in a laboratory frame. This effective acceleration does not imply potential energy is stored in the field, since no real force acts on a freely falling body. The observed gain in kinetic energy is actually due to a *pseudo-force* caused by *the observer's own deviation from a geodesic*. The fact that the Schwarzschild metric is static while the QIO metric is dynamic does not disturb the analogy.

3. (Section VI) According to Eardley *et al*. [86,109], whose calculations employ a linearized Riemann tensor and so are not exact, the Riemann tensor is all we can measure because, as the authors claim, forces are proportional to the Riemann tensor for freely falling reference frames. However, in the case of QIO detection, the reference frame is not freely falling, but remains rigid in the oscillating field of the QIO. Hence, the effective force is not proportional to the Riemann tensor, but is determined by the particle Lagrangian. Moreover, it is not clear which "forces" Eardley *et al.* are referring to, since gravitational forces do not play a formal role in GR, although *effective forces* can be calculated in a Hamiltonian framework via the Euler-Lagrange equation.

4. (Section VI) From this viewpoint, the universe is filled with timelike filaments or threads known as particles. Filaments such as protons and electrons, although having minute extent in space, may be infinitely extended in time, representing a profound asymmetry between space and time. Now quantum mechanics requires the universe to be divided into past and future, where the past has been determined, but the future is undetermined due to quantum uncertainty. Hence a filament such as an electron forms a more or less localized thread in the past, but smears out into an unlocalized swath in the future. As the spacelike hypersheet of the present moment progresses forward in time, it essentially combs the electron's swath into a localized thread. Thus, the present hypersheet may be envisioned as a comb that turns the swaths of unlocalized particles from the future universe into threads of localized particles filling the past universe. The roles of time and space are entirely asymmetric in this picture, leading to the question of how spacetime symmetry can even appear at all in well-proven theories such as special relativity.

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