

Title: A Resolution of Bell's Spaceship Paradox

(Revised Edition)

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Abstract:

The outcome of Bell's Spaceship Paradox has historically been very controversial. Some physicists said the thread breaks, and some physicists said it doesn't. More recently, most physicists say the thread breaks. But the correct answer is that the string does NOT break. That SAME answer is obtained by both of the two inertial observers who are at rest with the two spaceships immediately before acceleration begins, and also by both of the two observers who are forever stationary with respect to the two rockets themselves.

Two perpetually inertial observers (IO1 and IO2), perpetually mutually stationary with one another, are initially co-located with two separated observers (AO1 and AO2), with separation "L". AO1 and AO2 are about to begin a constant (according to them) acceleration "A" (with the separation in the direction of their acceleration). AO1 and AO2 KNOW that their acceleration is "A", because they each are carrying an accelerometer that confirms it. IO1 and IO2, based on their own measurements and calculations, AGREE with AO1 and AO2 about the acceleration. IO1 and IO2 will also agree with AO1 and AO2 that the separation between AO1 and AO2 always has the constant value "L" during the accelerations, and that the thread therefore DOESN'T break. (Two other inertial observers, IO3 and IO4, who are momentarily co-located with AO1 and AO2 at some time later in the trip, will NOT agree that the separation "L" is constant: they will say that it has increased since the start of the trip, and they will say that the two spaceships don't start accelerating at the same time. But the opinions of IO3 and IO4 are not required to determine whether or not the thread breaks, so their opinions will not be considered in this paper.

The separation of the two people undergoing the acceleration (AO1 and AO2), ACCORDING TO THOSE TWO PEOPLE THEMSELVES, can be referred to as the "PROPER" separation. But it is important to understand that IO1 and IO2 AGREE with AO1 and AO2 about the separation, and therefore the two groups agree that the thread DOESN'T break.

The REFERENCE FRAME of an accelerating observer, say AO1's reference frame, is constructed in the same manner as an inertial observer constructs his reference frame (as Einstein explained to us). AO1's helpers just lay out yardsticks, end-to-end, in the direction of the acceleration. To keep the yardsticks in place, they each are attached to an accelerometer-controlled rocket, so they each are accelerating at "A" lightyears/year/year. And between every pair of yardsticks, there is a clock. The clocks were initially synchronized before the constant acceleration started. Once the acceleration starts, the clocks don't remain synchronized. Clocks

located farther in the direction of the acceleration tic faster, by the rate ratio “R”. Einstein gave an exponential equation for “R” (<https://einsteinpapers.press.princeton.edu/vol2-trans/319>) but I have previously shown that his exponential equation is incorrect (<https://vixra.org/abs/2109.0076>). I later gave the corrected equation for the rate ratio “R” (<https://vixra.org/abs/2201.0015>).

Although Einstein’s exponential equation is incorrect, he was correct in his belief that the separation “L” is constant during the acceleration (according to the people actually undergoing the acceleration). Einstein clearly believed that "L" is constant, because he didn't give an equation for how "L" varies with time. If he had thought "L" varied with time, he would have needed to include an L(t) equation as part of his solution. He did NOT do that.

Each accelerometer directs its attached rocket to accelerate at exactly "A" lightyears per year per year. NEWTONIAN physics would say that the velocity of AO1 and AO2 would increase linearly with time, forever:

$$v = A * t. \quad (\text{incorrect})$$

That equation means that "v" would go to infinity as "t" goes to infinity, which we know can't be true in special relativity. So the above equation is clearly wrong. Special relativity says the quantity "(A * t)", which it calls the "rapidity" (denoted by the variable "theta"), is related to the velocity "v" by

$$\text{velocity} = v = \tanh(\text{rapidity}) = \tanh(\text{theta}) = \tanh(A * t). \quad (\text{correct})$$

That says that with a constant acceleration "A", "v" approaches (but never equals) the speed of light, "c", as "t" goes to infinity.

The distance "D" each rocket moves, according to AO1 and AO2, is

$$\begin{aligned} D &= \text{integral} \{ \text{from } 0 \text{ to } \tau \} [v \, dt] . \\ &= \text{integral} \{ \text{from } 0 \text{ to } \tau \} [\tanh(A*t) \, dt] . \end{aligned}$$

The integral of tanh(x) is equal to log[cosh(x)], so

$$D = \log[\cosh(\tau)] / A - \log[\cosh(0)] / A .$$

$$D = \log[\cosh(\tau)] / A .$$

So "D" grows forever, but it's RATE of growth decreases as tau increases.

The distance "D" each rocket moves during the acceleration is EXACTLY the same, so the separation "L" between AO1 and AO2, according to THEM, can't change during the acceleration. The INERTIAL observers (IO1 and IO2) also conclude that the separation "L" between AO1 and AO2 stays constant during the acceleration.

Note that the above description is relevant to the well-known (and much misunderstood) Bell's Spaceship Paradox:

https://en.wikipedia.org/wiki/Bell%27s_spaceship_paradox .

The above Wiki article should be read in its entirety, including its list of references. There have been MANY papers written on Bell's paradox over the years, and different conclusions have been drawn. There has definitely NOT been a unanimous conclusion.

Does the string break or not? It does NOT!