A New Interpretation of the Higgs Vacuum Potential Energy Based on a Planckion Composite Model for the Higgs

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ABSTRACT:

We present a new interpretation of the Higgs field as a composite particle made up of a positive, with, a *negative mass* Planck particle. According to the Winterberg hypothesis, space, i.e., the vacuum, consists of both positive and negative physical massive particles, which he called planckions, interacting through strong superfluid forces. In our composite model for the Higgs boson, there is an intrinsic length scale associated with the vacuum, different from the one introduced by Winterberg, where, when the vacuum is in a perfectly balanced state, the number density of positive Planck particles equals the number density of negative Planck particles. Due to the mass compensating effect, the vacuum thus appears massless, charge-less, without pressure, energy density, or entropy. However, a situation can arise where there is an effective mass density imbalance due to the two species of Planck particle not matching in terms of populations, within their respective excited energy states. This does not require the physical addition or removal of either positive or negative Planck particles, within a given region of space, as originally thought. Ordinary matter, dark matter, and dark energy can thus be given a new interpretation as *residual* vacuum energies within the context of a greater vacuum, where the populations of the positive and negative energy states exactly balance. In the present epoch, it is estimated that the dark energy number density imbalance amounts to, $(\overline{n_+} - \overline{n_-})_\Lambda = 8.52\,E-3$, per cubic meter, when cosmic distance scales in excess of, $100\ Mpc$, are considered. Compared to a strictly balanced vacuum, where we estimate that the positive, and the negative Planck number density, is of the order, $7.85\,E54$ particles per cubic meter, the above is a very small perturbation. This slight imbalance, we argue, would dramatically alleviate, if not altogether eliminate, the long standing cosmological constant problem.

Keywords: *Winterberg model, Planck particles, positive and negative mass Planck particles, planckions, quantum vacuum, space as a superfluid/ supersolid, extended models for space, cosmological constant, Higgs field as a composite particle, Higgs boson, inherent length scale for the vacuum, dark energy.*

I Introduction

Recently [1], we proposed a model for the Higgs field as a composite particle made up of a positive and a negative mass Planck particle pair. Planckion particles are material particles having, $\pm\, Planck\ mass$, and they were first introduced by Winterberg, in a series of papers, and in a book [2,3,4,5,6,7,8]. He proposed that these physical particles make up the vacuum, interact through very strong superfluid forces, and, due to their mass compensating effect, the vacuum (space) appears massless, without charge, energy density, pressure, or entropy. It is seemingly not there. Disturbances within that space can travel at the speed of light. His motivation was to explain the zero point energy, as well as provide a framework for quantum mechanics and the general theory of relativity. Both theories are derived as special limits within his more encompassing, and very ambitious model. His two component superfluid model was presented as an alternative to string theory.

The fundamental symmetry of nature, he argues, is, $SO(3)$ invariance, which our three dimensional space reflects, and not Lorentz, $SO(1,3)$ symmetry. The latter symmetry, and any higher symmetries, such as $SU(5)$, $SO(10)$, $SU(2,2/1)$, $SU(2,2/4)$, etc., if they exist, are *dynamical* symmetries, derivable from the more basic, $SO(3)$ invariance. The theory has been presented and developed by him, extensively, and we refer the reader to his original work for details. This author has also built upon his theory, and references [1,9,10,11] are also to be considered for further readings.

Our model for the Higgs boson is based on Winterberg's original thesis. We argued that the Higgs field, φ , is nothing else but a composite particle, consisting of a bound positive with a negative mass, Planck pair. Through very strong superfluid forces, the positive mass planckions are compelled to rub shoulders with the negative mass planckions, since both species of particle occupy the same space. They do not interact directly. Thus, their respective wave functions overlap. The Higgs potential energy, $U(\varphi)$, is the sum of the Planck mass potential energies, $U(\psi_+)$, with, $U(\psi_-)$, where, ψ_+, and, ψ_-, are the wave functions associated with the positive, and negative Planck particle, respectively. The equations of motion for all three fields, , ψ_+ , and, ψ_- , were shown to be consistent (compatible) with one another. Moreover, the continuity equations for all three fields were satisfied [1], when this identification was made.

In particular, we derived a specific equation where the Higgs field could increase or decrease its effective mass, above or below, $125.35\ GeV/c^2$, depending on whether the vacuum has a net positive or a negative energy density associated with it. It rested with the number density for each species, within a given region of space. Let, $n_+ = n_+(\vec{x})$, and $n_- = n_-(\vec{x})$, refer to the number density for the positive, and negative Planck particles, respectively. If the respective number densities are not perfectly balanced, the vacuum will have an inherent vacuum energy density (and pressure), associated with it, which will serve to either increase or decrease the rest mass of the Higgs, within a particular localized region of space.

The question naturally arise as to whether this is the only scheme possible for an increase, or decrease, in Higgs mass. The associated net vacuum energy density, and net vacuum pressure, positive or negative, will also depend on this imbalance in number density. The net vacuum energy density, which, in the Winterberg model, is also equal to the net vacuum pressure, is proportional to the difference in positive versus negative planckion number density. Quite literally, in order to get a net vacuum pressure, or, equivalently, a net vacuum energy density, the physical addition or removal of Planck particles was thought to be necessary [9,11]. Upon more careful analysis, however, we now find that this no longer holds true. What is equally possible is that the two competing planckion excited energy states, positive and negative, are not evenly populated. This can also create an effective imbalance within a given region of space, and create an effective net pressure or energy density for the vacuum within that region. This we will show explicitly in this work.

We also wish to reexamine the mass densities associated with ordinary matter, dark matter, and dark energy, within the Friedman equation. We proposed a specific model for dark matter, and dark energy, based on ordinary matter [9], and the positive/negative superfluid model of Winterberg. To explain dark matter, our thesis was that dipole moments were set up next to gravitating ordinary matter. In the space surrounding ordinary matter, we can literally have a slight physical separation between positive and negative Planck particles, which in turn, produces bound dipole matter. This is assuming that the region has cooled sufficiently, and that the aggregate gravitational fields produced by ordinary matter are also sufficiently strong, to counter the disruptive effects of temperature. The induced dipole matter distribution will follow the ordinary matter distribution which created it. For example, one might have spherical or cylindrical symmetry for ordinary matter, and the dark matter would reflect the same symmetry, in a static situation. This bound matter was identified as dark matter. Gravitational polarization, and gravitational susceptibility, could then be defined akin to electrostatics with one important caveat. In gravi-statics we have anti-screening, where the dipole matter will reinforce the original gravitation field set up by ordinary matter. In electrostatics, we have the opposite effect, screening, where the polarization of charge takes away from the original electric field. See reference [9] for details.

Dark energy, on the other hand, was associated with the gravitational fields produced (induced) by both ordinary matter, and dark matter. We argued that the dark energy mass density was, in effect, equal to, $\rho_\Lambda = 1/(2c^2)\, K\epsilon_0\, g^2 = 1/(2c^2)\, (K/(4\pi G))\, [g^{(0)}g^{(0)} + g^{(0)}g^{(1)}]$, by analogy to electro-statics. In this equation, K is the relative *gravitational* permittivity. In the present epoch, we estimate that, $K_0 = .158$, based on the current density parameter values in

Friedman's equation. The subscript "0" on a quantity will refer to the present epoch. The, g, is the gravitational field due to both ordinary matter, $g^{(0)}$, , and dark matter, $g^{(1)}$, i.e., $g = g^{(0)} + g^{(1)}$. The c is the speed of light, and, $\epsilon_0 \equiv 1/(4\pi G)$, is the *gravitational* permittivity of free space, where, G, is Newton's constant. We are assuming that we are considering distance scales in excess of $100\ Mpc$ for the calculation of dark energy, as is done in the Friedman equation. Thus the, $\overrightarrow{g^{(0)}}$, $\overrightarrow{g^{(1)}}, and\ \vec{g}$ values are smeared quantities, valid only when immense cosmic distances are considered. Using Gauss' law, these $\vec{g}$ fields would seem to permeate all of space, irrespective of where one is located within the universe. Again, we refer the reader to the above reference for specifics.

We originally claimed that, for the dark energy density, physical positive mass Planck particles were added, and a corresponding amount of negative mass Planck particles were removed [9,11], in order to create a net dark energy density within the vacuum. Due to the relative emptiness of space, when taken on a grand scale, the number density imbalance was extremely low. We now consider the possibility that no material particles need actually be added or subtracted. Rather if the excited positive and negative Planck particle energy states are unevenly (unequally) populated, then this also can create the necessary vacuum pressure, and energy density associated with dark energy. For dark matter, specifically, we will require equal numbers of positive and negative planckions, in a region of space. Polarization demands it. Dark energy, on the other hand, may be due to, unequal number densities, or unequally populated energy states, or both, between positive versus negative particles. This is a second goal behind this work. We wish to establish how it is possible to go beyond the addition or removal of physical Planck particles, and still obtain a net vacuum energy/mass density for the vacuum. A third goal is to see how ordinary matter, dark matter, and dark energy fit within the greater scheme of planckion mass density. They will turn out to be residual perturbations.

The outline of the paper is as follows. In section II we give a quick review of box quantization and show how this leads to an inherent length scale for space. In section III, we re-state the Higgs potential energy equation, derived in reference [1]. We then reinterpret (revise) this equation by allowing for differently populated energy states for positive versus negative Planck particles. If only one energy state were possible per species, then it would not be possible to extend, and generalize our definition of vacuum "imbalance". The physical addition or removal of physical particles would appear necessary in order to create a net vacuum pressure, or net energy density within space. Due to the results of section II, in section III, we also consider a very specific mass within the Higgs potential energy equation, different from the one that would have been introduced by Winterberg, had he considered a Higgs composite model.

In section IV, we highlight the fundamental difference between a strict Winterberg interpretation, where we have the addition and removal of Planck particles, and our new interpretation with differently populated energy states between positive and negative Planck particles, and apply it to dark energy. The vacuum will be shown to have a *residual* vacuum energy, which is nothing else but a perturbation, or anomaly, about a much greater balanced whole. This presents a solution to the cosmological constant problem. The greater symmetry of the vacuum will be shown, in a follow-up paper, to be broken, at lower CMB temperatures. Finally, in section V, we present our summary and conclusions.

II Review of Box Quantization for Planck Particles, and Derivation of an Intrinsic Length for the Vacuum

In this section we define an intrinsic length scale for the vacuum. Much of the development can be found in a previous paper, reference [11].

We start by considering a Planck radiator. According to quantum field theory, every particle in the vacuum radiates as a quantum mechanical oscillator (not just blackbody photons). At a particular frequency, ν , and temperature, T , we have as the energy emitted or absorbed,

$$\Delta E = h\nu/2 \ + \ h\nu/[e^{(h\nu/k_B T)} - 1] \qquad (2-1)$$

The, $h\nu/2$, is the zero point energy, added in 1912 by Planck, to the original 2nd term, which was discovered by him in 1901. The zero point energy (ZPE), is temperature independent, and is a consequence of the Heisenberg indeterminacy principle. The $2nd$ term on the right hand side of $equation$, $(2-1)$, vanishes in the limit of zero temperature.

To show that the $1st$ term on the right hand side of the above equation is related to the indeterminacy principle, we multiply both left and right hand sides of $equation$, $(2-1)$, by, Δt, the uncertainty in time, when taking a measurement. This gives for the $1st$ term,

$$\Delta E \ \Delta t = h\nu/2 \ (\Delta t)$$

$$= 1/2 \ \hbar \ (2\pi \ \Delta t/P) \geq \hbar/2 \qquad (2-2)$$

In this equation, P, is the period of field oscillation. By the right hand side, the time of observation, therefore, must be greater than, $\Delta t \geq P/2\pi$. Fixing the energy and time of a system, precisely and simultaneously, is impossible beyond a certain limit, according to this equation, which is Heisenberg's premise. When applied specifically, to oscillating fields, we see that the reduced relation, $\Delta t \geq P/2\pi$, is equivalent.

Now, according to work done by the author, the planckions undergo constant bombardment due to the CMB blackbody photons [9,10], which surround them. In the present epoch, we have a CMB temperature of, $T_0 = 2.726\ degrees\ Kelvin$. This is inherently what causes the Planck particles to vibrate or oscillate, about their equilibrium positions. These collisions allow for an exchange of energy and momentum, between the CMB photons, and the neighboring planckions. Originally, at extremely high blackbody temperatures, space is thought to consist exclusively, of blackbody radiation, and planckions. As the temperature cools, and the universe expands, material particles, such as electrons and protons, freeze out. Particles such as electrons and protons are treated as quasi-particle excitations within the vacuum. Particle formation occurs when the CMB temperature sinks below, $1\ TeV$, in various stages [12,13,14,15]. The planckions, which are, more or less, locked in position due to their very strong restoring fluid forces, rock to and fro about their equilibrium positions, due to individual blackbody photon bombardments. The severity of the bombardment is dictated by the CMB temperature. Once material particles, such as the electron, are formed, they too, as quasi-particle excitations, will start to experience this random chaotic motion. This would be analogous to a ship being placed upon the open ocean, where the waves can be dramatic. The Heisenberg uncertainty relation is the result, as demonstrated by Winterberg. He derives the Schroedinger equation from first principles using elastic collisions, and classical Boltzmann type equations. The so-called “Zitterbewegung”, i.e., random, chaotic motion of elementary particles, of Heisenberg, and Schroedinger, and the uncertainty relation, thus have a rational and natural explanation in terms of the Planck particles of Winterberg. The Planck particles are thought to be in thermal equilibrium with the blackbody radiation photons, which surround them. Incidentally, Winterberg did his doctoral thesis under Heisenberg.

If we consider the CMB photons, specifically, there is a relation between their peak frequency of oscillation, and temperature, T. It is,

$$\nu_{peak} = 2.8214\ (k_B T/h)$$

$$= 1.601\ E11 \quad Hz \qquad (2-3)$$

The, k_B, is Boltzmann’s constant, and the frequency calculation is for the current CMB temperature of, $T_0 = 2.726\ degrees\ Kelvin$. For this CMB temperature, $h\nu_{peak} = 1.061\ E-22\ Joules$. Substituting this into, $equation,(2-1)$, we obtain

$$\Delta E_{peak} = [.5 + .0633](1.061\ E-22)$$

$$= 5.976\ E-23\ \ Joules \qquad (2-4)$$

This is the peak energy being emitted and absorbed, in the present epoch, by the photons, when they interact with the surrounding planckions.

We also know that a Planck particle trapped in a three dimensional box has quantized energy levels, being in a bound state. Remember that they oscillate, or vibrate about their equilibrium positions, but they are essentially anchored in position due to the very strong restoring superfluid forces, acting upon them. According to a basic formula in quantum mechanics, the energy states (levels) for a particle trapped in a box are given by,

$$E_{n_x n_y n_z} = \pi^2 \hbar^2/(2mL^2) \ (n_x^2 + n_y^2 + n_z^2) \qquad (2-5)$$

The, n_x , n_y , n_z , are quantum numbers, which can take on the values, 1,2,3, …. The lowest energy level, or ground state, is specified by, $(n_x , n_y , n_z) = (1,1,1)$. The size of the box is, L^3, where, L, is the length on one side. The formula is still valid at zero temperature, and holds for both, the quantized positive, as well as negative mass, planckions. A transition between energy states or levels, positive or negative, would emit or absorb a finite amount of energy,

$$\Delta E = E_{n_x n_y n_z} - E_{n_x' \ n_y' \ n_z'} \qquad (2-6)$$

The unprimed quantum numbers refer to the situation before, and the primed quantum numbers correspond to the situation after the transition. This is completely analogous to the situation in the Hydrogen atom, where we have the Lyman series, the Balmer series, the Paschen series, etc.

By considering a few transitions with actual quantum numbers, such as, $211 \rightarrow 111$ (positive planckion emission), or, $-111 \rightarrow -112$ (negative planckion emission), it is easy to convince oneself that, E_{111}, is the most probable, i.e., most frequent amount of energy either emitted or absorbed. Thus, we are justified in setting,

$$(\Delta E)_{peak} = 2 \ E_{111} = 3\pi^2 \hbar^2/(\ m_{Pl} \ L^2) \qquad (2-7)$$

The factor of 2 is needed because the photon energy is, on average, equally divided between the two species of planckions, positive and negative. A negative mass particle will have its energy lowered, if it transitions upwards within the quantum mechanical box.

Now, we have a value for, $(\Delta E)_{peak}$. See, $equation, (2-4)$. We also know that the Planck mass have the values, $m_{Pl} = \pm \mid \ m_{Pl} \mid = \pm \ 2.176 \ E-8 \ \ kg$. Thus, $equation, (2-7)$, can be used to solve for L. We find that,

$$L = l_+(0) = l_-(0) = 5.032 \ E-19 \ meters \qquad (2-8)$$

This we consider to be the fundamental length scale for the vacuum (space), in the present epoch. It is also the nearest neighbor distance of separation between two positive, or two negative, Planck particles, within the two component superfluid.

We note that, once this distance is known, a typical number density for both the positive, and the negative, mass planckion, can be found. We calculate,

$$n_+(0) = l_+(0)^{-3} = 7.848\ E54\ m^{-3} \qquad (2-9a)$$

$$n_-(0) = l_-(0)^{-3} = 7.848\ E54\ m^{-3} \qquad (2-9b)$$

$$n_0 \equiv L^3 = l_\pm(0)^{-3} = 7.848\ E54\ m^{-3} \qquad (2-9c)$$

These results were derived in a previous work, reference, [11], by this author. The zero signifies a vacuum in the undisturbed, equilibrium state.

It is important to realize that as the CMB temperature increases, so does the peak frequency by, $equation$, $(2-3)$. Thus, $(\Delta E)_{peak}$, increases, as does, E_{111}. This shows us that at higher CMB temperatures, the "L" value actually decreases, which is what we would expect for the universe going back in cosmological time. We emphasize that the, ν_{peak}, is not the only frequency being emitted or absorbed. A whole distribution, (spectrum) of frequencies, for the oscillating planckions are present, either, emitted or, absorbed, through blackbody bombardment. This is because many different Planck particle energy levels are excited, positive and negative. Since the blackbody radiation follows a blackbody spectrum, we expect that, so too, will the transitions between individual Planck particle excited states, in their emission and absorption spectra.

Also very important is the realization that, because the Planck mass can now take on both positive and negative value, $m_{Pl} = \pm \mid m_{Pl} \mid$, in, $equation$, $(2-5)$, the average of positive with negative energy states, equals,

$$E_{Vacuum} = < E_{n_x n_y n_z} >_+ + < E_{n_x n_y n_z} >_- \qquad (2-10)$$

$$= \sum_{n_x n_y n_z} (E_{n_x n_y n_z})_+ /N + \sum_{n_x n_y n_z} (E_{n_x n_y n_z})_- /N = 0$$

This implies that under normal conditions (circumstances), the quantum mechanical vacuum has no energy density, nor net vacuum pressure, when the planckions are in a perfectly balanced state, in terms of numbers, and populated energy levels. The vacuum is also devoid of net mass or charge. The vacuum will appear empty, when, in fact, it is not.

A long standing problem in physics is the cosmological constant problem. If there were only one species of Planck particle, and if it had positive mass, then the mass density of the quantum mechanical vacuum would equal,

$$\rho_{QM} = m_{Pl}/l_{Pl}^3 = c^5/(\hbar G^2) = 5.155\ E96\ kg/m^3 \quad (Winterberg\ version) \qquad (2-11)$$

Here, l_{Pl} , is the Planck length, defined by, $l_{Pl} \equiv (\hbar G/c^3)^{1/2} = 1.616\ E-35\ meters$, and, G , is Newton's constant. In our version, we would substitute, $L = 5.032\ E-19\ meters$, for l_{Pl} , and obtain, correspondingly,

$$\rho_{QM} = m_{Pl}/L^3 = c^5/(\hbar G^2) = 1.708\ E47\ kg/m^3 \quad (Pilot\ version) \qquad (2-12)$$

The value of L is specified by, $equation, (2-8)$. Often, the cosmological constant, Λ, and the mass density associated with dark energy, $\rho_\Lambda \cong 5.96\ E-27\ kg/m^3$, in particular, have been compared to the value indicated by, $equation, (2-11)$. This has sometimes been referred to as the "worst fine-tuning problem" in physics. Even with the amended version, where we would use, $equation, (2-11)$, we are still very much at odds with the two values. We bypass this problem by, first, introducing two species of Planck particle, one with positive mass, and the other with negative mass. Second, we will deal with, "L" , versus l_{Pl} ,in all equations. The former is our intrinsic scale for the vacuum, and not, l_{Pl}, according to our reasoning. The length, "L" , is also the nearest neighbor distance between adjoining Planck particles for each species. And third, we will identify the dark energy mass density, ρ_Λ, with something else. It is not to be compared to either, m_{Pl}/L^3, nor, $\{|\ m_{Pl}\ |/L^3 + (-|\ m_{Pl|}\ |\}/L^3 = 0$. Rather, it is a *residual* part of, ρ_{vacuum}, left over after the Planck symmetry is broken, at much reduced CMB temperatures. More on this will be said later, in section IV.

III Review of the Higgs Potential Energy Equation, and Extension in Interpretation

In a previous work [1], we derived an equation which linked the Higgs potential energy with the potential energies of the positive, and negative, mass planckions. In short, we worked from the assumption that the energy stored by virtue of position for the Higgs field, φ, was equivalent to that, associated with a positive with negative mass planckion pair. In other words, $U(\varphi) = U(\psi_+) + U(\psi_-)$, where, $\psi_\pm$ are the positive, and negative, mass Planck particle wave functions. Very strong restoring superfluid forces cause the individual Planck particles to maintain a fixed distance of separation between them, not too near, or too far. This causes the individual species of Planck particle literally to rub shoulders with one another, because they are forced to occupy the same space. The species do not interact directly with one another, as shown by Winterberg. But, because the positive and the negative planckions occupy the same space, their respective wave functions are compelled to overlap.

This causes a coherence length for the Higgs boson, $\xi(\varphi)$, which is roughly, 3.13 times the inter-planckion distance of separation, in the present epoch. In the last section, we estimated that that the nearest neighbor distance of separation between individual planckions of the same species is of the order, $L = 5.032\ E-19\ meters$. The Higgs coherence length does not change with time, as its value is fixed, by its definition, $\xi(\varphi) \equiv \hbar/(m_\varphi c)$, where, m_φ, is the mass of the Higgs. The value of, L , however, must change with cosmological time, as its value is CMB temperature dependent. See, section II. As, L, decreases at higher CMB temperatures, the Higgs coherence length to L ratio, $\xi(\varphi)/L$, must therefore increase, as one goes back in cosmological time..

The Higgs potential energy equation derived in reference [1] reads,

$$U_{HPE} = (2\lambda\hbar^2/m_\varphi) \; |\varphi|^2 - m_\varphi c^2 = \hbar c \, L^2 \, (n_+ - n_-) \qquad (3-1)$$

In this equation, the Higgs self-coupling strength, $\lambda > 0$. Experimentally, its value has been determined to equal, $\lambda = .260$. The, m_φ , is the mass of the Higgs boson, $m_\varphi = 125.35 \; GeV/c^2 = 2.231 * 10^{-25} kg$. The, $L = l_+(0) = l_-(0)$, introduced in the previous section, is an inherent coupling constant having the dimension of length. This fundamental length scale for the vacuum is epoch dependent, with smaller values expected in previous epochs, and serves as a coupling constant as seen by the above equation. And the, $n_\pm = n_\pm(\vec{x})$, are the respective Planck particle *number densities* for the positive and negative Planck wave functions, $\psi_\pm$. Notice that if, $n_+ = n_-$, in a specified region of space, then we *still* have a rest mass for the Higgs field, $m_\varphi c^2 = 125.35 \; GeV$. The original vacuum symmetry is inherently broken at this scale.

We can raise or lower the effective mass of the Higgs by the term on the right hand side of, $equation, (3-1)$. The effective mass is obtained by bringing this term over to the left hand side. The effective mass now becomes,

$$m'_\varphi \, c^2 = \hbar c \, L^2 \, (n_+ - n_-) + 125.35 \; GeV \qquad (3-2)$$

The prime denotes an adjusted or effective mass. We can define a new coherence length for the Higgs in terms of this effective mass, m'_φ. Let, $\xi'(\varphi) \equiv \hbar/(m'_\varphi \, c)$. This is to be contrasted with the original rest mass coherence length, defined previously as, $\xi(\varphi) \equiv \hbar/(m_\varphi c)$. In the limit where, $(n_+ - n_-) \to 0$, the, $\xi'(\varphi) \to \xi(\varphi)$.

If, $n_+ > n_-$, then we have an increase in effective mass, and the $\psi_+ - \psi_-$ bond must become stronger. However, if, $(n_+ - n_-)$, increases to the point where,

$$\hbar c \, L^2 \, (n_+ - n_-) + 125.35 \; GeV > U_2 \qquad (3-3)$$

, where, U_2 , is some upper energy limit, then the Higgs ceases to exist. To see this, remember that the coherence length, $\xi'(\varphi) > L$, for a Higgs particle to exist, if it is to be made up of a $\psi_+ - \psi_-$ bond. Therefore, using the definition of, $\xi'(\varphi)$, we must have, $\hbar/(m'_\varphi \, c) > L$. Equivalently,

$$\hbar c/L > m'_\varphi\ c^2$$

$$392.9\ GeV > 125.35\ GeV + \hbar c\ L^2\ (n_+ - n_-) \qquad (3-4)$$

The term, on the left hand side in, $equation, (3-4)$, has been worked out numerically, and, $equation, (3-2)$, has been employed in the second line. $Equations, (3-3)$, and, $(3-4)$, are at odds with one another. A Higgs particle can only exist if, $equation, (3-4)$, holds.

If, on the other hand, $n_+ < n_-$, then we must have a weakening in the, $\psi_+ - \psi_-$ bond. But this also has its limits. What happens if the effective mass, given by, $equation, (3-2)$, reaches a point where it becomes negative? This also makes no sense. No sensible coherence length for the Higgs can be defined. If the mass of the Higgs, m'_φ , approaches zero, then the Higgs coherence length must approach infinity, i.e., $\xi'(\varphi) \to \infty$. The, $\psi_+ - \psi_-$ bond, cannot approach an infinite value, or go beyond that. Hence, no composite particle can exist. Also, a negative value for, $quation, (3-2)$, substituted into, $equation, (3-1)$, makes no sense. A zero value for, $equation, (3-2)$, substituted in, $equation, (3-3)$, would imply that, φ, is zero. This would also mean no Higgs.

Summarizing, for the Higgs field to exist, the planckion *number density imbalance*, must fall within the range,

$$-125.35\ GeV < \ \hbar c\ L^2\ (n_+ - n_-)\ < \ 392.9\ GeV - 125.35\ GeV \qquad (3-5)$$

Only in this way, can we guarantee that the effective Higgs coherence length lies within the range, $\infty > \ \ \xi'(\varphi)\ \ > \ L$. $Equation, (3-5)$, can be re-expressed, more elegantly, as,

$$-.319 < \ \ (n_+ - n_-)/n_0\ \ < \ .681 \qquad (3-6)$$

We have made use of, $equation, (2-9c)$.

Our basic equation, $equation, (3-1)$, can be re-written in an alternative form as,

$$U_{HPE} = (2\lambda\hbar^2/m_\varphi)\ \ |\varphi\ |^2 - m_\varphi c^2 = (\hbar c/L)\ L^3\ (n_+ - n_-)$$

$$= Mc^2\ (n_+ - n_-)/n_0 \qquad (3-7)$$

Here, we defined a new mass. $M = \hbar/(L\ c)$, which is positive definite, unlike the Planck mass. And, by, $equation, (2-9c)$, $n_0 = L^{-3}$. The mass density, n_0, is the current epoch, number density, for both positive and negative mass Planck particle. Using our expression for L, $equation, (2-8)$, we find that, $Mc^2 = 392.9\ GeV$, and the, $M = 6.994\ E-25\ kg$. We can think of M as a "coherence mass" for the Planck particle, one associated with its size in physical space. The actual physical mass for a Planck particle is, $m_{Pl} = \pm \mid\ m_{Pl} \mid = \pm 2.176\ E-8\ kg$.

According to Winterberg, the vacuum pressure equals the vacuum energy density, and this is given by the expression,

$$p_{Pl} = u_{Pl} = |\, m_{Pl} \,|\, c^2 \,(n_+ - n_-) \qquad (3-8)$$

The vacuum pressure, p_{Pl}, which is equivalent to the vacuum energy density, u_{Pl}, can be positive, negative, or zero, depending on whether, $(n_+ - n_-)$, is greater than zero, less than zero, or equal to zero. The increase or decrease in vacuum pressure, thus depends on the addition or removal of Planck particles, either positive or negative within a region of space. Only then can one create an imbalance in number density, between the positive versus the negative species. *In the Winterberg model, planckions do not have excited states or various energy levels, associated with them*.

Upon comparison of, $equation, (3-8)$, with $equation, (3-7)$, we see a similar structure. $Equation, (3-7)$, can be rewritten in an alternative form, upon bringing the number density, n_0 , from the right hand side of the equation, over to the left hand side. We then obtain,

$$u_{HPE} \equiv n_0\, U_{HPE} = Mc^2\,(n_+ - n_-) \qquad (3-9)$$

The subscript, HPE , stands for $Higgs\ Potential\ Energy$, because this is what both, the left hand side, and the right hand side of, $equation, (3-7)$, represent. An equivalent way to re-express, $equation, (3-9)$, is to retrace our steps. If we do this, we recognize that it can also be formulated as,

$$u_{HPE} \equiv n_0\, U_{HPE} = \hbar c\, L^2\,(n_+ - n_-) \qquad (3-10)$$

Upon comparison of, $equation, (3-9)$, with, $equation, (3-8)$, we see that there is a difference. The, $|\, m_{Pl} \,|\, c^2$, has been replaced by, Mc^2. Given the difference in the respective masses, this is a dramatic energy shift. We believe that, $equation, (3-9)$, is preferable to, $equation, (3-8)$. It is our extension of the original Winterberg equation, $equation, (3-8)$.

There is a second equally important modification, that we wish to make with regards to, $equation, (3-8)$. We will replace the, $(n_+ - n_-)$, by a weighted average over energy states, i.e., $(\overline{n_+} - \overline{n_-})$, where, by definition,

$$\overline{n_+} \equiv n_{111}E_{111} + n_{112}E_{112} + n_{121}E_{121} + n_{211}E_{211} + n_{222}E_{221} + \cdots / (E_{111} + E_{112} + E_{121} + \cdots)$$

$$= \sum_{n_x n_y n_z} (n_{n_x n_y n_z} E_{n_x n_y n_z}) / \sum_{n_x n_y n_z} (E_{n_x n_y n_z}) \qquad (3-11)$$

And,

$$\overline{n_-} \equiv n_{111}(-E_{111}) + n_{112}(-E_{112}) + n_{121}(-E_{121}) + n_{211}(-E_{211}) + n_{222}(-E_{221}) + \cdots / (-E_{111} - E_{112} - E_{121} + \cdots)$$

$$= \sum_{n_x\, n_y\, n_z} (n_{n_x\, n_y\, n_z} \left(-E_{n_x\, n_y\, n_z}\right)) / \sum_{n_x\, n_y\, n_z} (-E_{n_x\, n_y\, n_z}) \qquad (3-12)$$

We have used the notation of section II, where the, n_x , n_y , n_z , are quantum numbers, which can take on the values, 1,2,3, …. $Equations$, $(3-11)$ and, $(3-12)$, hold for the positive, and negative, mass planckions, respectively. The, $\overline{n_+}$, is an energy-weighted, *number density average* for positive mass Planck particles, where, n_{111}, n_{112}, n_{121}, etc., represent the individual number densities corresponding to Planck states energy levels, E_{111}, E_{112}, E_{121}, etc.. Similarly, $\overline{n_-}$, is an energy-weighted, *number density average* for negative mass Planck particles, where, n_{111}, n_{112}, n_{121}, etc., represent the individual number densities associated with Planck states having energy levels, $-E_{111}$, $-E_{112}$, $-E_{121}$, etc.. The energy levels in, $equation$ $(3-12)$, and negative definite, i.e., $-E_{n_x\, n_y\, n_z} = -|\ E_{n_x\, n_y\, n_z}\ |$. The two equations, $equations$, $(3-11)$, and, $(3-12)$, hold *within the same region of space*.

Obviously, if both the positive, and the negative, planckion energy levels, are equally populated, then,

$$(\overline{n_+} - \overline{n_-}) = 0 \qquad (balanced\ vacuum) \qquad (3-13)$$

This would represent a vacuum with no net pressure, nor net energy density. Space would also have no net mass. It would be analogous to a perfectly smooth ocean with no ripples or waves upon its surface. We replace the, $(n_+ - n_-)$, on the right hand side of, $equation$, $(3-9)$, by, $(\overline{n_+} - \overline{n_-})$, in order to obtain, our generalized vacuum pressure, or, equivalently, our generalized vacuum energy density,

$$p_{HPE} = u_{HPE} = Mc^2\ (\overline{n_+} - \overline{n_-}) \qquad (3-14)$$

Our extension (generalization) of, $equation$, $(3-8)$, is thus, $equation$, $(3-14)$. The difference between this equation, and Winterberg's original equation, $equation$, $(3-8)$, should be apparent.

Previously, we worked with, $equation$, $(3-8)$. See references, [9,11]. We will henceforth work with $equation$, $(3-14)$. Equation, (3-14), is preferable because, first, it makes an intimate connection with the Higgs field. Second, it introduces an inherent length scale for the vacuum, L, which is different from Winterberg's Planck length, l_{Pl}. In a follow up work, we will show that L scales appropriately with the expansion of the universe, whereas, l_{Pl} , does not. The scale, L, also leads to less fantastic number densities and volumes for the individual planckions. Third, $equation$, $(3-14)$, allows for Planck particle excited states transitions, whereas, $equation$, $(3-8)$, does not. If there were only *one permissible* energy state, per

Planck species, then the, $\overline{n_+}$, $in\ equation$, $(3-11)$, reduces to, n_+. And the, $\overline{n_-}$, simplifies to, n_- , by, $equation$, $(3-12)$. We would retrieve the Winterberg vacuum imbalance, $(n_+ - n_-)$, in this special limit.

We saw that the vacuum energy density is given by, $equation$, $(3-9)$, or equivalently, by, $equation$, $(3-10)$. Of course, within these equations, we now replace, $(n_+ - n_-)$, with the more general, $(\overline{n_+} - \overline{n_-})$. These equations imply that the potential energy of the vacuum,

$$U_{vacuum} = U_{HPE} = Mc^2(\overline{n_+} - \overline{n_-}) = \hbar c L^2\,(\overline{n_+} - \overline{n_-}). \qquad (3-15)$$

In the Winterberg model, the corresponding equations would read,

$$U_{vacuum} = U_{HPE} = \mid m_{Pl} \mid c^2\,(n_+ - n_-) = \hbar c\; l_{Pl}^2\,(n_+ - n_-) \qquad (3-16)$$

In, $equation$, $(3-15)$, the fundamental length scale is, L, and the number density is defined in terms of, $n_o = n_\pm = L^{-3}$. By contrast, in, $equation$, $(3-16)$, the fundamental length scale is, l_{pl}, and the number density is defined with respect to, $n_{Pl} = n_\pm = l_{Pl}^{-3}$. We emphasize that *no* physical addition or removal, of Planck particles, is required in our extension, in order to create an imbalance, and define a nontrivial net vacuum pressure or net energy density within space.

IV Focus on Ordinary Matter, Dark Matter, and Dark Energy

This section concerns itself with space, at large, when cosmic distance scales in excess of, $100\ Mpc$, are considered. We know that the Friedman equation connects the expansion rate of the universe with the energy density contained within it. In its simplest variant, we have,

$$\begin{aligned} H^2 &= 8\pi G/3\,(\rho_{RAD} + \rho_{OM} + \rho_{DM} + \rho_{DE}) \\ &= 8\pi G/3\,(\Omega_{RAD} + \Omega_{OM} + \Omega_{DM} + \Omega_{DE})\,\rho_{crit} \qquad (4-1) \end{aligned}$$

In this equation, H, is Hubble's constant, and, G, is the Newton's constant, The component mass (energy) densities, ρ_{RAD} , ρ_{OM}, ρ_{DM}, $\rho_{DM} = \rho_\Lambda$, are the radiation, ordinary matter, dark matter, and dark energy contributions, respectively, to the total critical mass density, ρ_{crit} . In terms of their relative weightings (proportions) to the total critical mass density, we have, in the current epoch, $(\Omega_{RAD},\ \Omega_{OM},\ \Omega_{DM},\ \Omega_\Lambda) = (9.12\,E-5,\ .0486,\ .2589,\ .6911)$. The sum of the density parameters, $\sum \Omega_i$, equals unity, since all indications are that the universe is flat.

For the Hubble constant, we obtain, in the present epoch, $H_0 = 67.74\ km/(s\ Mpc)$. This corresponds to a critical mass density of, $\rho_0 = 8.624\,E-27\ kg/m^3$, by the above equation. These estimates are consistent with data from the latest CMB Planck satellite collaboration [16,17]. The mass densities in, $equation$, $(4-1)$, are smeared values, valid only when immense distance scales are entertained, because only then can the individual galaxies be treated much like molecules within a gas.

If we accept a strict Winterberg interpretation, then we can easily find the associated imbalance in planckion density, for the values of dark matter, and dark energy mass densities, listed above. The dark matter component, has, as its present epoch mass density,

$$\rho_{DM} = .2589\, \rho_{crit} = 2.233\, E - 27\; kg/m^3 \qquad (4-2)$$

This can be set equal to, $equation$, $(3-8)$, divided by, c^2. If we do that we find that,

$$(n_+ - n_-)_{DM} = 1.026\, E - 19 \qquad (4-3)$$

Our model for dark matter, mentioned in the introduction, rested on the notion of a polarized positive, with negative, mass dipole. This forms bound matter which can add to the ordinary mass which induced it, provided the conditions are right. We right away have a problem with this interpretation within the Winterberg model. According to, $equation$, $(4-3\,)$, we must have unequal positive versus negative number densities. Dipole moments require equal numbers, positive with negative, within a given region (volume) of space.

It would be better to use, $equation$, $(3-14)$, divided by, c^2 . If we set this equal to the right hand side of, $equation$, $(4-2)$, we obtain a far different result,

$$(\overline{n_+} - \overline{n_-})_{DM} = 3.192\, E - 3 \qquad (4-4)$$

This is compatible with our model for dark matter. The energy levels can be differently populated in positive, versus, negative energy states. This can produce the required vacuum energy density, without the addition, or removal, of physical Planck particles. Notice that the new imbalance has a numerically greater value than before, because we are now dividing out by the much larger, $M, versus,$ | m_{Pl} |.

For dark energy, we can proceed analogously. In the present epoch, the dark energy mass density amounts to,

$$\rho_{DE} = .6911\, \rho_{crit} = 5.960\, E - 27\; kg/m^3 \qquad (4-5)$$

The Winterberg model would give, by, $equation$, $(3-8)$,

$$(n_+ - n_-)_{DE} = 2.74\, E - 19 \qquad (4-6)$$

The alternative, model, indicated by, $equation$, $(3-14)$, would indicate that,

$$(\overline{n_+} - \overline{n_-})_{DE} = 8.52\, E - 3 \qquad (4-7)$$

Again, we have quite a difference in value and interpretation, between our two equations, $equation, (4-6)$, and, $equation, (4-7)$. We believe that, $equation, (4-7)$, is the better alternative.

We also have to keep in mind that, in the Winterberg model, the planckion number density is defined differently, than in our alternative model. In the Winterberg model, the planckion number density equals, $n_{Pl} = n_{\pm} = l_{Pl}^{-3} = 2.37\,E104$. And so, the result indicated by, $equation, (4-6)$, for example, should use this as a basis, for comparing the disturbance. If we set up a ratio, we find that,

$$(n_+ - n_-)_{DE}/n_{Pl} = (2.74\,E-19)/(2.37\,E104) = 1.16\,E-123 \qquad (4-8)$$

This is a very extremely minute disturbance upon this sea (vast assembly) of positive and negative planckions, which are already occupying this space. It would be analogous to the very tiniest of waves rippling on the surface of a very large and deep ocean.

In the alternative model presented here, we must consider, $equation, (4-7)$, within the context of a different planckion number density, $n_o = n_{\pm} = L^{-3}$. The new Planck particle number density amounts to, $n_o = 7.85\,E54$. See, $equations, (2-8)\ and\ (2-9)$. Using this as our base, the value indicated by, $equation, (4-7)$, is also but a very small perturbation, or anomaly, in comparison to the vast number density of planckions already present. Setting up a ratio, as we did in, $equation, (4-8)$, we now obtain,

$$(\overline{n_+} - \overline{n_-})_{DE}/n_0 = (8.52\,E-3)/(7.85\,E54) = 1.09\,E-57 \qquad (4-9)$$

Clearly, this is again, a very small disturbance, but not nearly as small as that, indicated by, $equation, (4-8)$.

In summary, it is our view that ordinary matter, dark matter, and dark energy, can be treated as ripples upon a vast ocean of positive, and negative mass planckions. Their mass densities are minute disturbances when compared to a greater sea where positive cancels negative. The cosmological problem has thus been greatly reduced in scope. The new challenge is to discover what causes these disturbances in the first place, and how they are created.

V Summary and Conclusions

This paper offered a new interpretation for vacuum energy, the vacuum energy density, and the vacuum pressure, within the context of a Winterberg two-component planckion superfluid model. We assumed that "empty" space is made up of blackbody radiation, and also, material positive and negative mass Planck particles, referred to as planckions. Their masses are,

$m_{Pl} = \pm \mid m_{Pl} \mid = \pm\, 2.176\, E - 8\ \ kg$. These two species of Planck particle do not interact directly, but, indirectly, through very strong fluid forces, which act within their species. These are restoring forces when the planckions are displaced from their equilibrium positions, and they keep the particles within their respective species a fixed distance apart from one another, not to near and not too far. Due to CMB blackbody photon bombardment, the planckions vibrate or oscillate about their equilibrium positions. Because the two species of Planck particles occupy the same space, they are invariably forced to rub shoulders with one another, and we have the situation where their wave functions overlap. A positive Planck particle can pair up with a negative Planck particle to form a composite Higgs boson. See, $equation, (3-1)$, which is a consequence of setting the potential energy of the Higgs, equal to the combined potential energy of one individual positive mass Planck wave function added to the potential energy of one negative mass Planck wave function. The identification of a Higgs as a bound, $\psi_+ - \psi_-$ composite state was first made by this author in a previous paper.

The vacuum potential energy, the vacuum energy density, and the vacuum pressure, in a strict Winterberg interpretation, would be defined by, $equations, (3-8)$, and, $(3-16)$. In these equations, $l_{Pl} = 1.616\, E - 35\, meters$, is the fundamental length scale for space, and the corresponding number density for both positive and negative mass planckions is, $n_{Pl} = n_\pm = l_{Pl}^{-3} = 2.37\, E104\, particles/m^3$. Any imbalance in number density has to be compared to this value. See, for example, $equation, (4-8)$. In our alternative scheme, $equations, (3-14)$, and, $(3-15)$, would replace the former equations. Here the fundamental length scale for the vacuum is the much larger, $L = 5.032\, E - 19\, meters$. See, $uation, (2-8)$, and section II, where this quantity was derived. The corresponding $\pm\, mass$ planckion number density is specified by, $equations, (2-9)$, where we see that, $n_0 = n_\pm = L^{-3} = 7.85\, E54\, m^{-3}$. Any imbalance in number density should be compared to this number. We can refer to, $equation, (4-9)$, as but one example.

In both versions specified by, $equations, (4-8)\ and\ (4-9)$, respectively, the number density imbalance associated with dark energy, is very small. The same would hold true for dark matter. We can thus consider these energy densities to be very small perturbations, in this ocean (vast assembly) of positive and negative mass planckion particles. The cosmological constant problem has been reduced to finding out how these small perturbations arise in the first place, and what causes them. That would include ordinary matter. According to Winterberg, elementary particles are quasiparticle excitations, set up within the vacuum.

Our fundamental length, $L = 5.032\, E - 19\, meters$, ties in nicely with our Higgs composite model; it follows as a natural consequence, as seen by, $equations, (3-1)$, and, $(3-7)$. Refer also to, $equation, (3-15)$. All these equations involve, L. The other length scale, the one proposed by Winterberg, where, $l_{Pl} = 1.616\, E - 35\, meters$, has no connection to the Higgs field. Its' corresponding energy is the Planck energy, $1.22\,E19\,GeV$ which is many orders of magnitude removed (divorced) from the ordinary Higgs energy scale, $125.35\,GeV$.

Our alternative theory for vacuum energy density, and vacuum energy, $equations, (3-14)$, and, $(3-15)$, also contains the generalized number density imbalance, $(\overline{n_+} - \overline{n_-})$. This is to be contrasted with the, $(n_+ - n_-)$, which is found in, $equations, (3-8)$, and, $(3-15)$. The $\overline{n_\pm}$ are defined by, $equations, (3-11)$, and, $(3-12)$. These are the energy-weighted *number densities*, for both the positive, and the negative mass, Planck particles. If only one energy level is available for both species, then the, $\overline{n_\pm}$, reduce to, $n_\pm$. The, $\overline{n_\pm}$, is an important generalization or extension to, $n_\pm$. If it were not for this generalization, the physical addition or removal of planckions would be necessary to create an imbalance, and a non-trivial vacuum pressure, and energy density, within a region of space. The new formulation suffers under no such restriction. Our new generalized definition of "imbalance" in number density allows for the numbers of Planck particles to remain the same within a region of space. This may be important in specific models for dark matter, and dark energy. For example, in our polarization model for dark matter, the number density of positive Planck particles must match that of the negative Planck particles.

In summary, this paper is noteworthy because it,

a) Offers a *new* interpretation for the Higgs vacuum energy density and vacuum pressure
b) Shows (highlights) the fundamental difference between our model and a strict Winterberg interpretation, in the definition of vacuum energy and vacuum pressure
c) Introduces a new length scale for the vacuum, one based on transitions between excited planckion energy states, and box quantization. See section II.
d) Greatly reduces the cosmological constant problem to a residual perturbation about a mean. In this vast assembly (ocean) of positive and negative mass particles, small ripples or waves of net positive, and, net negative energy densities can manifest themselves.

The fundamental question remains as to what causes these waves or ripples. In other words, why do we have ordinary matter, dark matter, and dark energy? Even within a specific model, where, dark matter, and dark energy are related to ordinary matter, made possible if we assume $\pm mass$ planckions, what causes the ordinary matter, in the first place? Another fundamental challenge would be to determine how the excited energy states, specific to both the positive and the negative mass Planck particles, are populated. They must be populated in a certain fashion, in order to create the characteristics associated with dark matter, and dark energy, respectivelly. These questions, and others, must be left for future work.

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